Assignment 1

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September 29, 2025

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Question 1

(a) Describe, in one precise sentence, what term (1) captures.

Term 1, $E(y_i \mid d_i = 1)$, captures the expected value of of y_i given $d_i = 1$, specifically the expected monthly pay of a person given they have a college degree ($d_i = 1$)

(b) What sign would you expect for (1) – (2)? Briefly justify.

Term 1 is the expected monthly pay of individuals with a college degree, while Term 2 is the expected monthly pay of those without a college degree. Generally we would expect those with a college degree to, on average, have a higher monthly pay, so Term 1 would be larger than Term 2. As a result, we would expect (1) - (2) to be positive.

(c) Describe what does term (3) capture. Be specific.

(d) Explain how the sample can be used to provide information for term (1) – (2) but not (3) or (4).

(e) What would it mean for term (4) to be positive?

(f) What are the implications of term (4) being positive for measuring the effect of a degree on earnings?

Question 2

Discrete random variables X and Y can take 10 equally likely pairs:

$$(1,2),(1,4),(1,6),(2,1),(2,3),(2,5),(3,2),(3,4),(3,6),(3,8).$$

Verify the Law of Total Expectations: $E[E(Y|X)] = E(Y)$. **Answer:** ...

Question 3

Variable $X \sim N(0,1)$ and $Y = X^2 - 1$. Show how this example illustrates that uncorrelated $\not\Rightarrow$ independent. **Answer:** ...

Question 4

Joint pdf: $f(x,y) = \frac{3(x^2+y)}{11}$ for $0 \leq x \leq 2$, $0 \leq y \leq 1$.

(a) Find the best linear approximation to the CEF.

(b) Plot the CEF and linear fit (include code + figure).

Question 5

Simple regression model: $y = \beta_1 + \beta_2 x^2 + \epsilon$.

(a) Prove β_1, β_2 solve the least squares problem.

(b) Reverse regression $x^2 = \alpha_1 + \alpha_2 y + v$: write expressions for α_1, α_2.

(c) When does $\alpha_2 = 1/\beta_2$? Justify.