MASTER QUICK REFERENCE: EXAM CONCEPTS AND COUNTERFACTUALS

Based on Sample Exam Analysis (Part I & II)

December 7, 2025

1 Part I: Foundations of Econometrics

Sample Exam I (Example_Final_PtI_SampleExamI.pdf)

1. **Question 1(a)(i) Verbatim:** What would be the dimension of element $\beta, E(xx')$, and $E(xy)$?
   - **Potential Alternates:**
   - Provide the OLS estimator formula $\hat{\beta} = (X'X)^{-1}X'y$ [1, 2].
   - Given K parameters (including the constant), specify the dimensions of $X'X$ and $X'y$ [2, 3].
2. **Question 1(a)(ii) Verbatim:** Provide one reason why the authors could be interested in estimating vector β as just defined. Justify.
   - **Potential Alternates:**
   - Discuss the difference between the population parameter β and the sample estimator $\hat{\beta}$ [1, 2].
   - Explain why $\hat{\beta}$ is the Best Linear Approximation (BLA) to the Conditional Expectation Function (CEF) [4].
3. **Question 1(b) Verbatim:** Consider running a regression of the OLS residuals from estimating the model above with respect to a constant, $fbpen, netpen$ and $prights$. What would you expect the coefficient of determination of running this regression to be? Justify.
   - **Potential Alternates:**
   - Explain the meaning of R^2 as a measure of in-sample predictive power [5, 6].
   - List two things R^2 does NOT tell you (e.g., causality, correct specification) [6, 7].
4. **Question 1(c)(ii) Verbatim:** Perform test using a 1% significance level? Justify.
   - **Potential Alternates:**
   - Test a linear restriction like $H_0 : \beta_2 = 1$ [8, 9].
   - Perform a two-sided test and calculate the critical value for $\alpha = 5\%$ [10].
   - Draw the p-value for a one-sided test (e.g., $H_1 : \beta_2 > 0$) [9, 10].
5. **Question 1(d) Verbatim:** List the assumptions, regarding the dgp behind the data, that would be needed to justify the use of the exact $t-test$ statistic. Label each assumption and next to each label, provide the corresponding statistical expression that defines the assumption, considering regressors are stochastic. No need to justify.
   - **Potential Alternates:**
   - Which specific assumption is violated if there is measurement error in $fbpen$? [11-13].
   - Provide the statistical expression for Conditional Homoskedasticity (A_4) or No Conditional Autocorrelation (A_5) [11, 14].
6. **Question 1(e) Verbatim:** Consider testing whether $fbpen$ and $netpen$ are jointly significant at 1% significance level, using the exact F-test statistic.

- **Potential Alternates:**
- Test an equality restriction like $H_0 : \beta_2 = \beta_3$ [15, 16].
- Test a linear restriction like $H_0 : \beta_2 + \beta_3 = 1$ [15, 16].
- Write the restricted model used to calculate $RSSE$ for $H_0 : \beta_2 + \beta_3 = 0$ [17].
- Calculate the F-statistic using the R^2 formula: $F = (R^2_{UR} - R^2_R)/q \div (1 - R^2_{UR})/(n - K)$ [15, 16].

7. **Question 1(f) Verbatim:** If all so-called classical assumptions, except for normality, were holding: (i) What would the distribution of the test statistic used in 1c be? (ii) Would the failure of normality affect the size of the $p - value$ you drew in 1c(iii)? If so, how? Which one would be larger? Rigorously justify.

   - **Potential Alternates:**
   - Explain the asymptotic distribution of the F-statistic (converges to $\chi^2(q)/q$) [12, 18].
   - Justify why the Central Limit Theorem allows $t \overset{a}{\sim} N(0, 1)$ for large samples [18, 19].

8. **Question 1(g)(i) Verbatim:** Which of the 4 elements listed would require adjustment in calculating its value and which would not? Just state Yes ('adjustment required') or No ('adjustment not required').

   - **Potential Alternates:**
   - Justify why the OLS estimator ($\hat{\beta}$) remains unbiased even with heteroskedasticity [20, 21].
   - Justify why the standard error ($se(\hat{\beta})$) requires adjustment using robust estimation [21, 22].

9. **Question 1(h)(i) Verbatim:** Under what condition could we give the OLS estimates of β_2 and β_3 a causal interpretation? Rigorously explain, using the help of a causal path diagram.

   - **Potential Alternates:**
   - Explain how reverse causality ($CI \rightarrow netpen$) violates the strict exogeneity assumption (A_2) [12, 23].
   - Use a causal path diagram to illustrate positive or negative Omitted Variable Bias (OVB) [12, 23].

10. **Question 2(d)(i) Verbatim:** Change one of the elements of the dgp above to create perfect collinearity. Clearly indicate the element that you changed and how. Using the expression of the variance decomposition provided, rigorously argue why we would not be able to uniquely estimate β_2. Additionally, describe how the histogram provided above would look like.

    - **Potential Alternates:**
    - Explain the role of the Variance Inflation Factor (VIF) in this process, noting that $VIF \rightarrow \infty$ as $R^2_j \rightarrow 1$ [24, 25].
    - Explain the geometry of perfect collinearity (singular $X'X$ matrix) [12, 24, 26].

11. **Question 3(b) Verbatim:** Derive, step by step, a test statistic to perform the test above using this estimator tilde, for the case where matrix Ω is known.

    - **Potential Alternates:**
    - Derive the Lagrange Multiplier (LM) or Likelihood Ratio (LR) test statistic [27, 28].
    - State the necessary dimensions for R and r when testing three restrictions, e.g., $H_0 : \beta_2 = \beta_3 = \beta_4$ [29].

Sample Exam II (Eample_Final_PtI_SampleExamII.pdf)

1. **Question 1(a)(i) Verbatim:** Provide the general expression that has been used to calculate the standard error, value 0.2183563 underlined in the output above, in as much detail as possible. Then, substitute the available values provided in the output. Any element missing? If so, properly identify it.

   - **Potential Alternates:**
   - Given the full variance decomposition formula $\text{Var}(\hat{\beta}_j|X) = \sigma^2/SST_j \times 1/(1-R_j^2)$, explain how increasing the variation in the regressor (SST_j) would affect the standard error [25, 30].

2. **Question 1(a)(ii) Verbatim:** What information is this value meant to provide? Be as specific as you can.

   - **Potential Alternates:**
   - Explain that the standard error measures the **uncertainty or precision** of the estimate, quantifying the variability across potential samples [30, 31].
   - Define what the t-statistic measures (the number of standard errors the estimate is away from the hypothesized value) [32].

3. **Question 1(d)(i) Verbatim:** Write down the null and alternative hypotheses associated with this test.

   - **Potential Alternates:**
   - Test an exclusion restriction where $q = 1$ (e.g., $H_0 : \beta_3 = 0$) and show that $F = t^2$ [33].
   - Write the restricted model for an equality restriction ($H_0 : \beta_3 = \beta_4$) [17, 34].

4. **Question 1(d)(iii) Verbatim:** Provide the intuition behind where the acceptance region is located.

   - **Potential Alternates:**
   - Provide the intuition behind the **rejection region** (large F means restricted model fits poorly) [35, 36].
   - Describe the decision rule: Reject H_0 if $F > F_\alpha(q, n-K)$ [33].

5. **Question 1(d)(iv) Verbatim:** Provide the exact expression of the model you would have to estimate to calculate $RSSE$.

   - **Potential Alternates:**
   - Write the restricted model for $H_0 : \beta_2 = \beta_3$ (by estimating $y_i = \beta_1 + \beta_2(x_{2i} + x_{3i}) + \ldots$) [17].

6. **Question 2(b) Verbatim:** Now, we repeat the experiment, but setting $\sigma_v^2 = 1$. How would you expect the histogram you drew in question 2a to change (location, spread, shape)? Rigorously argue using the expression you feel is most appropriate to support your argument.

   - **Potential Alternates:**
   - Explain how the histogram spread would change if the sample size n were increased from $n = 30$ to $n = 300$ [37, 38].
   - Explain how the histogram spread would change if the error variance σ^2 (currently 16) were decreased to 1 [37, 38].

7. **Question 2(c) Verbatim:** Consider that using the same samples generated in 2a we estimate instead regression: (2) $y = \beta_1 + \beta_2 \cdot x_2 + \epsilon$. Would you expect the location of the histogram to change? If you do not, explain why. If you do, explain why and how. Rigorously justify in either case.

   - **Potential Alternates:**
   - Explain what condition must hold for the estimate of β_2 to remain unbiased even if x_3 is omitted (i.e., if x_2 and x_3 are uncorrelated) [39].
   - Describe the expected direction of bias (e.g., positive shift) if the omitted variable (x_3) is positively correlated with both x_2 and y [40].

Sample Exam III (Eample_Final_PtI_SampleExamIII.pdf)

1. **Question 1(a) Verbatim:** From the output above, excluding number of observations and degrees of freedom, select 2 different statistics whose calculation does not depend on any assumption regarding the data generating process (dgp). Clearly label the statistics you selected. Be rigorous with the notation. No need to justify.

   - **Potential Alternates:**
   - Select 2 statistics whose calculation *does* depend on classical assumptions (e.g., $se(\hat{\beta}_j)$ or Confidence Intervals) [9, 41, 42].
   - Justify why R^2 is unaffected by heteroskedasticity [21, 43, 44].

2. **Question 1(c) Verbatim:** Consider we changed the units of measurement of regressor *sxp* from the current ratio over 1 to a percentage. From the output above, which statistics would change value? List them by name. Be rigorous with the notation.

   - **Potential Alternates:**
   - Explain how changing the units of the **dependent variable** (Y) would affect the coefficient estimates, standard errors, and R^2 [45].

3. **Question 1(d) Verbatim:** Looking at the output above, we can tell that not all countries in the sample had exactly the same quality of institutions, as measured by regressor *rl*. Why? Rigorously argue using the expression you feel is most appropriate to support your argument.

   - **Potential Alternates:**
   - Define the Variance Inflation Factor (VIF) and explain its implication as $VIF \to \infty$ (perfect collinearity) [25, 46].

4. **Question 1(k)(ii) Verbatim:** Explain the intuition behind the location of the acceptance region (for the Jarque-Bera test).

   - **Potential Alternates:**
   - If the calculated JB statistic were 10.0 and $\alpha = 0.05$, what would the conclusion be? (Reject H_0, errors are non-normal, since $10.0 > 5.99$) [47, 48].
   - Explain why the JB test is a one-sided (right-tailed) test [48].

5. **Question 2(a) Verbatim:** Could the range of values of the vertical axis have been greater than 1? Justify your answer.

   - **Potential Alternates:**
   - Explain why the total **area** under the density histogram must equal 1 [49].
   - Explain what property of the OLS estimator cannot be illustrated by a Monte Carlo simulation with a fixed $n = 50$ [37, 38, 50, 51].

6. **Question 3(c)(ii) Verbatim:** Provide a reason why authors chose to use robust standard errors.

   - **Potential Alternates:**
   - Explain why the OLS residuals in the partial regression plot (Figure 2) must be centered around zero [7, 52].
   - Explain why controlling for GDP per capita helps isolate the effect of social media (removing confounding effects) [53].

2 Part II: Causal Inference Methods

Final Exam 2021 (Econometrics_Final_Pt_2__Exam_2021b.pdf)

1. **Question 1(a) Verbatim:** Describe briefly how you would implement the experiment. Discuss on which level you think the randomization should be (country, city, accommodation)? Are you worried about take-up and why?

- **Potential Alternates:**
- Discuss the risks of **spillover effects** (e.g., if control hosts learn from treated hosts) [54, 55].
- Explain the trade-off between individual randomization and cluster randomization (e.g., randomizing at the city level) [56, 57].

2. **Question 1(b) Verbatim:** Give the formulas for the ATE and ATT, how would you estimate them?

   - **Potential Alternates:**
   - Provide the formulas for the Intent-to-Treat (ITT) and Treatment-on-Treated (TOT) estimators, explaining when they are used [54, 58].
   - Explain why ATE = ATT only under perfect randomization [57, 59].

3. **Question 3(a) Verbatim:** Which conditions must Z satisfy?

   - **Potential Alternates:**
   - If treatment effects are heterogeneous, list the two additional assumptions required (Exclusion Restriction and Monotonicity) [60, 61].
   - Define **Monotonicity** and specify the compliance type it rules out (defiers) [60, 61].

4. **Question 3(b) Verbatim:** Assume that treatment effects are heterogeneous, i.e. not all individuals respond to the treatment in the same way. What type of treatment effect can you compute with your instrument? Are there individuals for whom you cannot compute an effect? State the formula of the treatment effect.

   - **Potential Alternates:**
   - Define the four compliance types (compliers, always-takers, never-takers, defiers) [61].
   - Explain the primary limitation of LATE (limited external validity) [62, 63].

5. **Question 4 Verbatim:** State the two conditions for Y and D around cut-off z_0 that are necessary for RD. State the regression equation that you would estimate for a sharp design, assuming equal slopes to the right and left of the cut-off.

   - **Potential Alternates:**
   - Write the RD regression equation for **different slopes** around the cutoff z_0 [64].
   - Explain the importance of checking the continuity of observed covariates near the cutoff [65, 66].

Final Exam 2022 (Econometrics_Final_Pt_2__Exam_2021.pdf)

1. **Question 1(a) Verbatim:** Explain briefly how you would calculate the average treatment effect. Suppose you had additional information on individual customers, e.g. age, gender, race, would you use such information?

   - **Potential Alternates:**
   - Define ATE mathematically as the difference in expected values: ATE $= E[Y|D = 1] - E[Y|D = 0]$ [57, 67].
   - Discuss the concept of **attrition** and how non-random dropouts threaten the internal validity of an RCT [54].

2. **Question 2(a) Verbatim:** State the formula of the propensity score estimator for the ATE. State one reason why it is suitable to use propensity score some cases rather than direct matching.

   - **Potential Alternates:**
   - State the two key matching assumptions: Conditional Independence and Common Support [68, 69].

- Explain the consequence of violating Common Support (e.g., loss of matching partners, limited generalizability) [69, 70].

3. **Question 3(a) Verbatim:** Describe in a few sentences why an instrument might be needed in this case.

   - **Potential Alternates:**
   - Explain how an unobserved factor (like socio-economic status) leads to endogeneity ($\text{Cov}(D, \epsilon) \neq 0$) [12, 71].
   - Discuss the potential issue of a **weak instrument** and how the First Stage F-statistic addresses this [55, 72].

4. **Question 4 Verbatim:** State the two conditions for Y and D around cut-off z_0 that are necessary for RD. Graphically illustrate the two conditions for a sharp design.

   - **Potential Alternates:**
   - Explain the concept of checking the **continuity of covariates** [65].
   - Define the RD estimand (τ) as a local effect for units near the cutoff [73].

Final Exam 2024 (Econometrics_Final_Pt_2__Exam_2024.pdf)

1. **Question 1 Verbatim:** Given this information, do you think that comparing treated and control average heights provides a good estimate for the average treatment effect? Do you need to control by any regressors? Why?

   - **Potential Alternates:**
   - If the p-value for Mother's education was 0.01 instead of 0.062, explain the necessary action (controlling for the covariate) [74, 75].
   - Define the purpose of balance checks (verifying random assignment) [75].

2. **Question 2 Verbatim:** Are you convinced by this approach? Why?

   - **Potential Alternates:**
   - Explain the **Direction of Bias**: why unobserved factors like ability or motivation would lead to a positive bias in the estimated treatment effect [63, 76].
   - Justify why Propensity Score Matching (PSM) does *not* solve problems related to unobserved confounders [76, 77].

3. **Question 3 Verbatim:** Do you expect the comparison of treated and control weeks to provide a valid causal effect? Why? Explain briefly, connecting the empirical context described and the assumptions discussed in the course.

   - **Potential Alternates:**
   - Identify which classical assumption is violated by seasonality/holidays (Strict Exogeneity or $E[\epsilon|X] = 0$) [11, 78, 79].
   - Suggest an alternative research design, such as Difference-in-Differences, if suitable time/control data were available [80].

4. **Question 5 Verbatim:** Would you implement a sharp or a fuzzy design? Why? What treatment effect will you identify?

   - **Potential Alternates:**
   - Explain the key difference: Sharp requires $P(D = 1|Z)$ to jump strictly from 0 to 1 at the cutoff [65].
   - Explain why Fuzzy RD requires using an Instrumental Variable approach [73, 81].

5. **Question 6 Verbatim:** Does a difference-in-differences approach (exposed to TWEA vs not) provide a consistent estimate in this case? Why? If the UK was exposed to the same tariff increase but did not implement TWEA, can you suggest a method to obtain a valid causal effect?

- **Potential Alternates:**
- Mathematically state the **Parallel Trends Assumption** and how the common shock (tariffs) violates it [82, 83].
- Define the Triple-Difference (DDD) estimator and when it is necessary (common shock) [84].