Assignment 3

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Contents

Question 1: Assumptions for Random Samples

(a) Rewrite assumptions [A1]–[A4] from class for i.i.d. random samples $\{(x_i, y_i)\}$.

Hint: Express each assumption mathematically and briefly justify its meaning under i.i.d. sampling.

Example structure:

$$\begin{aligned} &\text{[A1]: } E[\varepsilon_i \mid X_i] = 0 \\ &\text{[A2]: } Var(\varepsilon_i \mid X_i) = \sigma^2 < \infty \\ &\qquad \vdots \end{aligned}$$

Insert explanations and intuition below.

Question 2: Conditional vs. Unconditional Properties of OLS

(a) (i) Prove that OLS is unconditionally unbiased given $E(\hat{\beta}_k \mid X) = \beta_k$. (ii) Explain in one sentence the interpretation of unconditional unbiasedness.

Hint: Use the law of iterated expectations. **Insert derivation:**

$$E(\hat{\beta}_k) = E[E(\hat{\beta}_k \mid X)] = \dots$$

(b) Starting from $\text{Var}(\hat{\beta}_k \mid X) \leq \text{Var}(\tilde{\beta}_k \mid X)$, show that $\text{Var}(\hat{\beta}_k) \leq \text{Var}(\tilde{\beta}_k)$.

Hint: Use the law of total variance: $\text{Var}(Z) = \text{Var}(E[Z \mid W]) + E[\text{Var}(Z \mid W)]$.

Insert derivation here.

Question 3: One-Sided Hypothesis Test

(a) Draw the acceptance and rejection regions for $H_0 : \beta_2 = 0$ vs. $H_1 : \beta_2 < 0$ at $\alpha = 5\%$.

Include a clear plot below (hand-drawn or generated in R/Python):

Q3a_test_region.png

(b) Explain the intuition behind the location of the critical region.

Write short reasoning below.

Question 4: Monte Carlo Simulation (Assig3Q4.R / Assig3Q4.py)

(a) Provide the expression of the DGP (data generating process) used to simulate samples.

Hint: Derive from line 8 in the code, e.g.

$$y_i = 10 + 5x_{2i} + \varepsilon_i, \quad \varepsilon_i \sim N(0, 6^2)$$

(b) (i) How many samples does the code generate? (ii) What do all samples have in common? (iii) Describe lines 1–16 (R) or 6–30 (Python) of the code.

Insert concise explanation below.

(c) Describe the function of lines 18–23 (R) or 32–39 (Python).

(d) Describe the function of lines 25–40 (R) or 41–56 (Python).

(e) Run the code and report the value of `ratiog` and include the plot below.

`Q4e_plot.png`

Comment on whether you are surprised by the value of `ratiog`.

(f) Explain what the plot illustrates and provide a rigorous interpretation.

(g) Discuss how increasing M from 100 to 10,000 would affect `ratiog`.

(h) Explain how changing confidence level (from 0.025/0.975 to 0.005/0.995) affects `Ratio` and the plot.

(i) What is the purpose of this simulation exercise? *Summarize the econometric concept it illustrates.*

Question 5: Fair's Model of U.S. Presidential Elections

(a) For each of Fair's four conditions, identify the relevant parameter(s) and expected sign.

Example Table:

Condition	Parameter	Expected Sign
Incumbent running again	β_3 (DPER)	$+$
Duration in power	β_4 (DUR)	$-$
Republican bias	β_2 (I)	$-$
State of economy	β_6, β_7, β_8	$+$

Table 1: Expected signs of parameters in Fair's model.

(b) Estimate the model using Stata with data `USelections.csv`. Include Stata output below. *Example:*

`Q5b_stata_output.png`

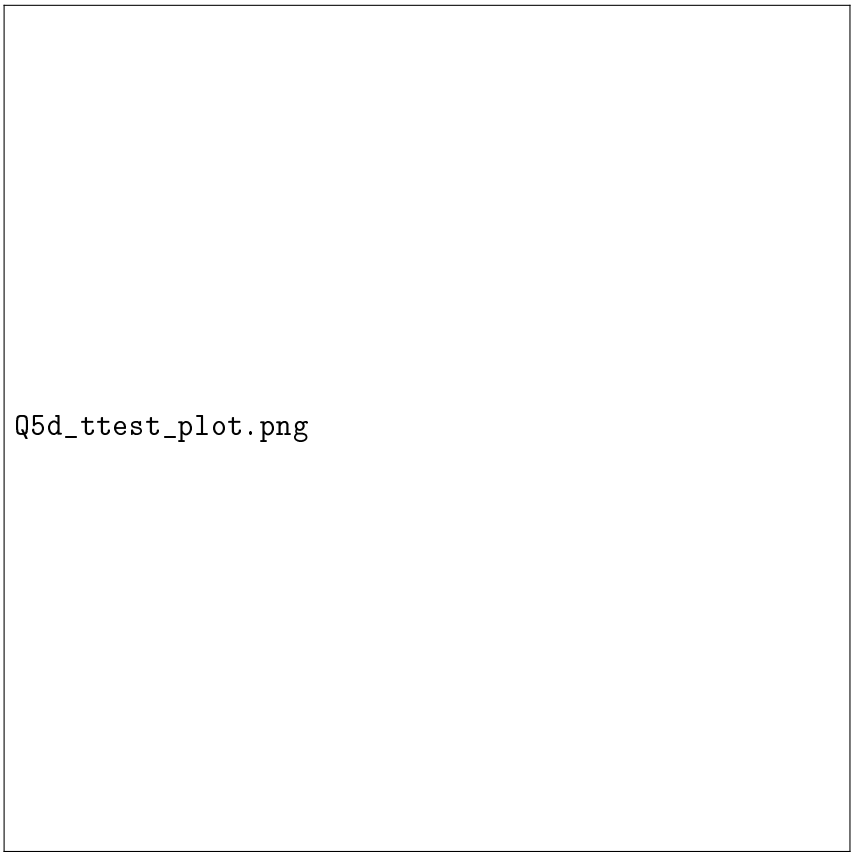
(c) Define and compute:

(i) $se(\hat{\beta}_6)$

(ii) t–value and p–value for regressor I

Include Python/R calculations and verify against Stata.

(d) Test $H_0 : \beta_7 = 0$ vs. $H_1 : \beta_7 \neq 0$ at 5% significance. Draw acceptance and rejection regions.



(e) Draw the p–value region and indicate the minimum significance level that makes β_7 significant.

(f) Test joint significance of $(G \cdot I)$, $(P \cdot I)$, and $(Z \cdot I)$ ("It's the economy, stupid!"). Include Stata output and steps.

(g) Compute prediction error for 2024 using given $VP = 49.25$, $G = 1.7$, $P = 4.54$, $Z = 4$. Interpret and comment on Fair's predictive method.

References

- Angrist, J.D. & Pischke, J.S. (2009). *Mostly Harmless Econometrics: An Empiricist's Companion.*
- Fair, R. (2024). *U.S. Presidential Vote Equation.* Yale University, `https://fairmodel.econ.yale.edu/`.
- Enikolopov, R., Makarin, A., Petrova, M. (2018). "Social Media and Corruption." *American Economic Journal: Applied Economics.*