UNIVERSITY OF VIENNA
Department of Economics

Summer Term 2021

040068 UK Dynamic Macroeconomics with Numerics
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GROUP PROJECT NR. 1

Maximal Possible Points: 10

Due: Friday, May 28, 2021, noon

In this problem, you will explore how the nature of the standard one-sector stochastic growth model changes when variable rates of capital utilization are allowed.

1. Consider a representative agent with preferences given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \qquad 0 < \beta < 1.$$

It is assumed that u is strictly increasing, concave, and twice differentiable. Output depends on the actual number of machines used at time t, κ_t. Thus, the aggregate resource constrains is

$$c_t + i_t \leq z f(\kappa_t)$$

where c denotes consumption, i gross investment, and z the level of technology. The function f is strictly increasing, concave, and twice differentiable. In addition, f is such that the marginal product of capital converges to zero as the stock goes to infinity. Capital that is not used does not depreciate. Thus, capital accumulation satisfies

$$k_{t+1} \leq (1-\delta)\kappa_t + (k_t - \kappa_t) + i_t,$$

where it is required that the number of machines used, κ_t, does not exceed the number of machines available, k_t, or $k_t \geq \kappa_t$.

a. Formulate the social planner's dynamic optimization problem and analyze, as thoroughly as possible, the first-order necessary conditions. Briefly discuss your results.

b. Formally describe the steady state of this economy. If necessary, make additional assumptions to guarantee that a steady state exists and is unique. If you make additional assumptions, go as far as you can giving an economic interpretation of them.

c. Determine the optimal level of capacity utilization, κ^*, in this economy in the steady-state.

2. Consider the subsequent alternative formulation of a one-sector stochastic growth model with variable capital utilization.

Let the representative agent's preferences be given by

$$\sum_{t=0}^{\infty} \beta^t \ln(c_t), \qquad 0 < \beta < 1.$$

Output y in any period t is produced according to the following production function

$$y_t = z_t^{1-\alpha}(k_t U_t)^{\alpha}, \qquad 0 < \alpha < 1, \qquad 0 < U < 1,$$

where z denotes the level of technology, k the stock of physical capital, and U the rate of capital utilization. By assumption, using capital more intensely increases the rate at which it depreciates. Specifically, the time-varying rate of capital depreciation equals

$$\delta_t = \delta U_t^{\phi}, 0 < \delta < 1, \phi > 1.$$

The law of motion of physical capital equals $k_{t+1} = (1 - \delta_t)k_t + i_t$, where i denotes gross investment.

The level of technology is captured by the following stochastic process

$$z_t = exp(x_t), \qquad x_t = \rho x_{t-1} + \varepsilon_t,$$

where $0<\rho<1$ und ε_t is identically, independently and normally distributed with mean zero and standard deviation σ_ε. The error term ε_t is uncorrelated with all earlier and later error terms. In the stationary steady-state we have that $\sigma_\varepsilon = 0$.

Lastly, the aggregate resource constraint is given by

$$c_t + i_t \le y_t.$$

a. When markets are complete, the competitive equilibrium of this model economy corresponds to the solution of a social planning problem. Fully formulate this problem, specifying the planner's objective, constraints and the paths of all decision variables.

b. Denote by $\bar{\delta}$ the time-average of δ_t. Use Matlab to compute the stationary steady-state of this model for the given parameters

$$\beta = 1.03^{-0.25}, \alpha = 0.36, \delta = 0.0285, \bar{\delta} = 0.02, \phi = 1.5, \rho = 0.9.$$