UNIVERSITY OF VIENNA
Department of Economics

Summer Term 2021

040068 UK Dynamic Macroeconomics with Numerics
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GROUP PROJECT NR. 2

Maximal Possible Points: 15

Due: Thursday, June 24th, 2021, noon

Consider the by now familiar formulation of a one-sector stochastic growth model with variable capital utilization.

Let the representative agent's preferences be given by

$$\sum_{t=0}^{\infty} \beta^t \, ln(c_t), \qquad 0 < \beta < 1.$$

Output y in any period t is produced according to the following production function

$$y_t = z_t^{1-\alpha}(k_t U_t)^{\alpha}, \qquad 0 < \alpha < 1, \qquad 0 < U < 1,$$

where z denotes the level of technology, k the stock of physical capital, and U the rate of capital utilization. By assumption, using capital more intensely increases the rate at which it depreciates. Specifically, the time-varying rate of capital depreciation equals

$$\delta_t = \delta U_t^{\phi}, 0 < \delta < 1, \phi > 1.$$

The law of motion of physical capital equals $k_{t+1} = (1 - \delta_t)k_t + i_t$, where i denotes gross investment.

The level of technology is captured by the following stochastic process

$$z_t = exp(x_t), \qquad x_t = \rho x_{t-1} + \varepsilon_t,$$

where $0<\rho<1$ und ε_t is identically, independently and normally distributed with mean zero and standard deviation σ_ε. The error term ε_t is uncorrelated with all earlier and later error terms. In the stationary steady-state we have that $\sigma_\varepsilon = 0$.

Lastly, the aggregate resource constraint is given by $c_t + i_t \leq y_t$.

You have already computed the stationary steady-state of this model for the following parameters

$$\beta = 1.03^{-0.25}, \alpha = 0.36, \delta = 0.0285, \bar{\delta} = 0.02, \phi = 1.5, \rho = 0.9.$$

a. Use the Blanchard-Kahn (1980) state-space approach to linearize the model around the stationary steady-state and determine the coefficients G_0, G_1, and G_2 in the policy rule

$$k_t = G_0 + G_1 k_{t-1} + G_2 z_t.$$

Show all necessary details of the different steps of this numerical solution procedure. Compute the values (G_0, G_1, G_2) of the policy rule which solves the linearized Euler equation corresponding to each of the following set of parameter values:

i. ϕ=1.5, δ=.0285, ρ=.9
ii. ϕ=1.5, δ=.9, ρ=.9
iii. ϕ=1.5, δ=.0285, ρ=.99
iv. ϕ=1.1, δ=.0285, ρ=.9

b. Use Dynare to determine the coefficients G_0, G_1, and G_2 in the linear policy rule:

$$k_t = G_0 + G_1 k_{t-1} + G_2 x_t.$$

Note the following peculiarity in Dynare. Normally, we write

$$k_t = \bar{k} + a_{k,k}\left(k_{t-1} - \bar{k}\right) + a_{k,x}(x_t - \bar{x}), \text{ and } x_t = \rho x_{t-1} + \epsilon_t.$$

However, Dynare reports

$$k_t = \bar{k} + a_{k,k}\left(k_{t-1} - \bar{k}\right) + a_{k,x_{-1}}(x_{t-1} - \bar{x}) + a_{k,x}\epsilon_t.$$

You can transform the output from Dynare into a more common expression by using

$$a_{k,x_{-1}} = \rho a_{k,x}.$$

c. Write a function in Matlab that calculates the time path of deviations from steady state as $\tau_t - \tau^*$ as a function of the predetermined variables.

d. Plot the impulse responses separately for consumption, investment, capital, output, and the rate of capital utilization to a one-standard deviation shock to x_t. Plot each variable's response as deviation from its steady-state value.

e. Compare the degree of variability between the stock of physical capital and the rate of capital utilization, especially across the specifications (i) and (iv) in part (a). Briefly explain the underlying economic causes of these adjustment patterns.