Dynamic Macroeconomics with Numerics: Project II

Samuel

June 17, 2021

One-Sector Stochastic Growth Model

Notes on optimality conditions

We have

$$
\mathbf{X}_{t+1} = \begin{pmatrix} c_{t+1} \\ k_{t+1} \\ U_{t+1} \\ z_{t+1} \end{pmatrix} = f(\mathbf{X}_t) = \begin{pmatrix} c_t\beta\left\{\alpha(\rho x_t + \epsilon_{t+1})^{1-\alpha}U_{t+1}^{\alpha}k_{t+1}^{\alpha-1} + 1 - \delta U_{t+1}^{\phi}\right\} \\ k_t - \delta U_t^{\phi}k_t + i_t \\ \left\{(\delta\phi)^{-1}\alpha\exp(\rho x_t + \epsilon_{t+1})^{1-\alpha}(k_t - \delta U_t^{\phi}k_t + i_t)^{\alpha-1}\right\}^{\frac{1}{\phi-\alpha}} \\ \rho x_t + \epsilon_{t+1} \end{pmatrix}
$$

We now express all terms in terms of past values:

$$z_{t+1} = \exp(\rho x_t + \epsilon_{t+1})$$

$$k_{t+1} = (1 - \delta U_t^{\phi})k_t + z_t^{1-\alpha}k_t^{\alpha}U_t^{\alpha} - \beta c_t\left\{\alpha U_t z_t^{1-\alpha}k_t^{\alpha-1}U_t^{\alpha} + 1 - \delta U_t^{\phi}\right\}$$

$$= \left(1 - \left\{\frac{\alpha}{\delta\phi}z_t^{1-\alpha}k_t^{\alpha-1}\right\}^{\frac{1}{\phi-\alpha}}\right)k_t + z_t^{1-\alpha}k_t^{\alpha}U_t^{\alpha} - \beta c_t\left\{\alpha U_t z_t^{1-\alpha}k_t^{\alpha-1}U_t^{\alpha} + 1 - \delta U_t^{\phi}\right\}$$

$$= k_t - k_t^{\frac{\phi-1}{\phi-\alpha}}\left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}z_t^{\frac{1-\alpha}{\phi-\alpha}} + z_t^{1-\alpha}k_t^{\alpha}U_t^{\alpha} - \beta c_t\left\{\alpha U_t z_t^{1-\alpha}k_t^{\alpha-1}U_t^{\alpha} + 1 - \delta U_t^{\phi}\right\}$$

$$U_{t+1} = \left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}\exp(\rho x_t + \epsilon_{t+1})^{\frac{1-\alpha}{\phi-\alpha}}\left(k_t - k_t^{\frac{\phi-1}{\phi-\alpha}}\left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}z_t^{\frac{1-\alpha}{\phi-\alpha}} + z_t^{1-\alpha}k_t^{\alpha}U_t^{\alpha} - \beta c_t\left\{\alpha U_t z_t^{1-\alpha}k_t^{\alpha-1}U_t^{\alpha} + 1 - \delta U_t^{\phi}\right\}\right)^{\frac{\alpha-1}{\phi-\alpha}}$$

$$c_{t+1} = c_t\beta\left\{\alpha(\rho x_t + \epsilon_{t+1})^{1-\alpha}U_{t+1}^{\alpha}k_{t+1}^{1-\alpha} + 1 - \delta U_{t+1}^{\phi}\right.$$

$$i_{t+1} = z_{t+1}^{1-\alpha}k_{t+1}^{\alpha}U_{t+1}^{\alpha} - \beta c_t\left\{\alpha U_{t+1}z_{t+1}^{1-\alpha}k_{t+1}^{\alpha-1}U_{t+1}^{\alpha} + 1 - \delta U_{t+1}^{\phi}\right\}$$

$$y_{t+1} = z_{t+1}^{1-\alpha}(k_{t+1}U_{t+1})^{\alpha}$$

Important here are the partial derivatives of the the first three terms w.r.t. all past terms:

$$\frac{\partial}{\partial z_t}k_{t+1} = -\frac{1-\alpha}{\phi-\alpha}k_t^{\frac{\phi-1}{\phi-\alpha}}\left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}z_t^{\frac{1-\phi}{\phi-\alpha}} + (1-\alpha)\left(\frac{k_tU_t}{z_t}\right)^{\alpha} - \beta c_t(1-\alpha)z_t^{-\alpha}U_t^{1+\alpha}k_t^{\alpha-1}$$

$$\frac{\partial}{\partial k_t}k_{t+1} = 1 - \frac{\phi-1}{\phi-\alpha}k_t^{\frac{\alpha-1}{\phi-\alpha}}\left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}z_t^{\frac{1-\alpha}{\phi-\alpha}} + \alpha z_t^{1-\alpha}k_t^{\alpha-1}U_t^{\alpha} - \beta c_t\alpha U_t^{1+\alpha}z_t^{1-\alpha}k_t^{\alpha-2}$$

$$\frac{\partial}{\partial U_t}k_{t+1} = \alpha z_t^{1-\alpha}k_t^{\alpha}U_t^{\alpha-1} - \beta c_t\left\{\alpha(1+\alpha)z_t^{1-\alpha}k_t^{\alpha-1}U_t^{\alpha} - \delta\phi U_t^{\phi-1}\right\}$$

$$\frac{\partial}{\partial c_t}k_{t+1} = \beta\left\{\alpha z_t^{1-\alpha}k_t^{\alpha-1}U_t^{1+\alpha} + 1 - \delta U_t^{\phi}\right\}$$

$$\frac{\partial}{\partial i_t}k_{t+1} = 1$$

$$\frac{\partial}{\partial y_t}k_{t+1} = 1 - \beta c_t(??)$$

as we have $k_{t+1} = [\dots] + y_t - \beta c_t\frac{\partial}{\partial k_t}y_t + [\dots]$????

Next we have

$$\frac{\partial}{\partial z_t}U_{t+1} = \left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}z_{t+1}^{\frac{1-\alpha}{\phi-\alpha}}\left\{(1-\alpha)\left(\frac{k_tU_t}{z_t}\right)^{\alpha} - \frac{1-\alpha}{\phi-\alpha}k_t^{\frac{\phi-1}{\phi-\alpha}}\left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}z_t^{\frac{1-\phi}{\phi-\alpha}} - \beta c_t\alpha(1-\alpha)\left(\frac{U_t}{z_t}\right)^{\alpha}k_t^{\alpha-1}\right\}$$

$$\left\{k_t - k_t^{\frac{\phi-1}{\phi-\alpha}}\left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}z_t^{\frac{1-\alpha}{\phi-\alpha}} + z_t^{1-\alpha}k_t^{\alpha}U_t^{\alpha} - \beta c_t\left\{\alpha U_t z_t^{1-\alpha}k_t^{\alpha-1}U_t^{\alpha} + 1 - \delta U_t^{\phi}\right\}\right\}^{\frac{2\alpha-1-\phi}{\phi-\alpha}}\frac{\alpha-1}{\phi-\alpha}$$

$$\frac{\partial}{\partial k_t}U_{t+1} = \left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}z_{t+1}^{\frac{1-\alpha}{\phi-\alpha}}\ \dots\text{fuck me}$$

We have the Euler error given by

$$V_t = c_{t+1} - \beta c_t [f_{k_{t+1}}(k_{t+1}, U_{t+1}, z_{t+1}) + 1 - \delta_{t+1}(k_{t+1})],$$

We have

$$U_t = \left(\frac{\alpha}{\delta\phi} z_t^{1-\alpha} k_t^{\alpha-1}\right)^{\frac{1}{\phi-\alpha}}$$

$$U_{t+1} = \left(\frac{\alpha}{\delta\phi} z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1}\right)^{\frac{1}{\phi-\alpha}}$$

And

$$c_t = z_t^{1-\alpha} k_t^{\alpha} U_t^{\alpha} - k_{t+1} + k_t - \delta U_t^{\phi} k_t,$$

$$= z_t^{1-\alpha} k_t^{\alpha} \left(\frac{\alpha}{\delta\phi} z_t^{1-\alpha} k_t^{\alpha-1}\right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta \left(\frac{\alpha}{\delta\phi} z_t^{1-\alpha} k_t^{\alpha-1}\right)^{\frac{\phi}{\phi-\alpha}} k_t$$

$$= z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

$$c_{t+1} = z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+2} + k_{t+1} - \delta \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

Then we have

$$f_{k_{t+1}}(\cdot) = \alpha z_{t+1}^{1-\alpha} U_{t+1}^{\alpha} k_{t+1}^{\alpha-1}$$

$$= \alpha z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} \left(\frac{\alpha}{\delta\phi} z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1}\right)^{\frac{\alpha}{\phi-\alpha}}$$

$$= \alpha z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

$$= \alpha \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

And lastly we have

$$\delta_{t+1} = \delta U_{t+1}^{\phi} = \delta \left(\frac{\alpha}{\delta\phi} z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1}\right)^{\frac{\phi}{\phi-\alpha}}$$

$$= \delta \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

We can put together all terms and have

$$V_t = z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+2} + k_{t+1} - \delta \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

$$+ \beta \underbrace{\left\{ z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \right\}}_{=c_t} \alpha \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

$$+ \beta \underbrace{\left\{ z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \right\}}_{=c_t}$$

$$- \beta \underbrace{\left\{ z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \right\}}_{=c_t} \delta \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

Now we can take the partial derivatives:

$$\frac{\partial}{\partial k_{t+2}} V_t = -1$$

$$\frac{\partial}{\partial k_{t+2}} V_t \big|_{k_t = k^*, z_t = z^*} = -1$$

Next we have

$$\frac{\partial}{\partial k_{t+1}} V_t = \left(\alpha + \frac{(\alpha - 1)\alpha}{\phi - \alpha}\right) z_{t+1}^{1-\alpha + \frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha - 1 + \frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\left(1 + \frac{(\alpha-1)\alpha}{\phi-\alpha}\right)\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

$$+ \beta\left\{ z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \right\}$$

$$\cdot \alpha \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} \left(\alpha - 1 + \frac{(\alpha-1)\alpha}{\phi-\alpha}\right) z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-2+\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

$$+ (-1)\alpha \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

$$+ \beta(-1)$$

$$- \beta\left\{ z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \right\}$$

$$\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} \left(\frac{(\alpha-1)\alpha}{\phi-\alpha}\right) z_{t+1}^{\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\alpha}{\phi-\alpha}-1}$$

$$+ \beta\delta\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

and evaluated at the steady state this is

$$\frac{\partial}{\partial k_{t+1}} V_* = \left(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\right) k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\left(1 + \frac{(\alpha-1)\alpha}{\phi-\alpha}\right)\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

$$+ \beta\left\{ k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} - \delta\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \right\} \cdot \alpha \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} \left(\alpha - 1 + \frac{(\alpha-1)\alpha}{\phi-\alpha}\right) k_*^{\alpha-2+\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

$$+ (-1)\alpha \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

$$+ \beta(-1)$$

$$- \beta\left\{ k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} - \delta\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \right\} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} \left(\frac{(\alpha-1)\alpha}{\phi-\alpha}\right) k_*^{\frac{(\alpha-1)\alpha}{\phi-\alpha}-1}$$

$$+ \beta\delta\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

The next derivative is given by

$$\frac{\partial}{\partial k_t} V_t = \beta\left\{ \left(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\right) z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\left(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\right)\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \right\}$$

$$\alpha \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

$$+ \beta\left\{ \left(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\right) z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\left(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\right)\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \right\}$$

$$- \beta\left\{ \left(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\right) z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\left(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\right)\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \right\}$$

$$\delta\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

and evaluated at the steady state this is

$$\frac{\partial}{\partial k_t} V_* = \beta\left\{ \left(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\right) k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\left(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\right)\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \right\} \alpha \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

$$+ \beta\left\{ \left(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\right) k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\left(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\right)\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \right\}$$

$$-\beta\Big\{\Big(\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}\Big)k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}}\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}+1-\delta\Big(1+\frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}k_*^{\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

Note: $\frac{\partial}{\partial k_t}c_t=\Big\{\Big(\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}\Big)z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}}k_t^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}}\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}+1-\delta\Big(1+\frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}}k_t^{\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}$

Next we have the derivatives w.r.t. the technology shocks

$$\frac{\partial}{\partial z_{t+1}}V_t=\Big(1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}z_{t+1}^{\frac{(1-\alpha)\alpha}{\phi-\alpha}-\alpha}k_{t+1}^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}}-\delta\Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}-1}k_{t+1}^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

$$+\beta\Big\{z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}}k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}}\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}-k_{t+1}+k_t-\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}}k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}$$

$$\cdot\alpha\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}\Big(1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}\Big)z_{t+1}^{\frac{(1-\alpha)\alpha}{\phi-\alpha}-\alpha}k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

$$+\beta\Big\{z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}}k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}}\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}-k_{t+1}+k_t-\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}}k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}-1}k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

which, at the steady state is

$$\frac{\partial}{\partial z_{t+1}}V_*=\Big(1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}}-\delta\Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

$$+\beta\Big\{k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}}\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}-\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}\alpha\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}\Big(1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}\Big)k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

$$+\beta\Big\{k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}}\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}-\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

And as the last derivative, we have

$$\frac{\partial}{\partial z_t}V_t=\beta\Big\{\Big(1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}\Big)z_t^{\frac{(1-\alpha)\alpha}{\phi-\alpha}-\alpha}k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}}\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}-\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}\Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big)z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}-1}k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}$$

$$\cdot\alpha\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}}k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

$$+\beta\Big\{\Big(1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}\Big)z_t^{\frac{(1-\alpha)\alpha}{\phi-\alpha}-\alpha}k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}}\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}-\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}\Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big)z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}-1}k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}$$

$$-\beta\Big\{\Big(1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}\Big)z_t^{\frac{(1-\alpha)\alpha}{\phi-\alpha}-\alpha}k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}}\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}-\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}\Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big)z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}-1}k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}$$

$$\cdot\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}}k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}},$$

and evaluated at the steady state this is

$$\frac{\partial}{\partial z_t}V_*=\beta\Big\{\Big(1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}\Big)k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}}\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}-\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}\Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big)k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}$$

$$\cdot\alpha\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$

$$+\beta\Big\{\Big(1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}\Big)k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}}\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}-\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}\Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big)k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}$$

$$-\beta\Big\{\Big(1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}\Big)k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}}\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}-\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}\Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big)k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}$$

$$\cdot\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

Note: $\frac{\partial}{\partial z_t}c_t=\Big\{\Big(1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}\Big)z_t^{\frac{(1-\alpha)\alpha}{\phi-\alpha}-\alpha}k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}}\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}-\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}\Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big)z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}-1}k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}$

Dynare: Policy Function using Parameter Set (i)

$$k_t = \underbrace{60.62}_{=G_0} + \underbrace{0.974}_{=G_1}(k_{t-1} - \bar{k}) + \underbrace{2.055}_{=G_2}(x_t - \bar{x})$$

$$k_t = \underbrace{60.62}_{=G_0} + \underbrace{0.974}_{=G_1}(k_{t-1} - \bar{k}) + \underbrace{2.055}_{=G_2}(x_t - \bar{x})$$