

Dynamic Macroeconomics with Numerics: Project II

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One-Sector Stochastic Growth Model

Notes on optimality conditions

We have

$$\mathbf{X}_{t+1} = \begin{pmatrix} c_{t+1} \\ k_{t+1} \\ U_{t+1} \\ z_{t+1} \end{pmatrix} = f(\mathbf{X}_t) = \begin{pmatrix} c_t \beta \left\{ \alpha (\rho x_t + \epsilon_{t+1})^{1-\alpha} U_{t+1}^\alpha k_{t+1}^{\alpha-1} + 1 - \delta U_{t+1}^\phi \right\} \\ k_t - \delta U_t^\phi k_t + i_t \\ \left\{ (\delta \phi)^{-1} \alpha \exp(\rho x_t + \epsilon_{t+1})^{1-\alpha} (k_t - \delta U_t^\phi k_t + i_t)^{\alpha-1} \right\}^{\frac{1}{\phi-\alpha}} \\ \rho x_t + \epsilon_{t+1} \end{pmatrix}$$

We now express all terms in terms of past values:

$$\begin{aligned} z_{t+1} &= \exp(\rho x_t + \epsilon_{t+1}) \\ k_{t+1} &= (1 - \delta U_t^\phi) k_t + z_t^{1-\alpha} k_t^\alpha U_t^\alpha - \beta c_t \left\{ \alpha U_t z_t^{1-\alpha} k_t^{\alpha-1} U_t^\alpha + 1 - \delta U_t^\phi \right\} \\ &= \left(1 - \left\{ \frac{\alpha}{\delta \phi} z_t^{1-\alpha} k_t^{\alpha-1} \right\}^{\frac{1}{\phi-\alpha}} \right) k_t + z_t^{1-\alpha} k_t^\alpha U_t^\alpha - \beta c_t \left\{ \alpha U_t z_t^{1-\alpha} k_t^{\alpha-1} U_t^\alpha + 1 - \delta U_t^\phi \right\} \\ &= k_t - k_t^{\frac{\phi-1}{\phi-\alpha}} \left(\frac{\alpha}{\delta \phi} \right)^{\frac{1}{\phi-\alpha}} z_t^{\frac{1-\alpha}{\phi-\alpha}} + z_t^{1-\alpha} k_t^\alpha U_t^\alpha - \beta c_t \left\{ \alpha U_t z_t^{1-\alpha} k_t^{\alpha-1} U_t^\alpha + 1 - \delta U_t^\phi \right\} \\ U_{t+1} &= \left(\frac{\alpha}{\delta \phi} \right)^{\frac{1}{\phi-\alpha}} \exp(\rho x_t + \epsilon_{t+1})^{\frac{1}{\phi-\alpha}} \left(k_t - k_t^{\frac{\phi-1}{\phi-\alpha}} \left(\frac{\alpha}{\delta \phi} \right)^{\frac{1}{\phi-\alpha}} z_t^{\frac{1-\alpha}{\phi-\alpha}} + z_t^{1-\alpha} k_t^\alpha U_t^\alpha - \beta c_t \left\{ \alpha U_t z_t^{1-\alpha} k_t^{\alpha-1} U_t^\alpha + 1 - \delta U_t^\phi \right\} \right)^{\frac{\alpha-1}{\phi-\alpha}} \\ c_{t+1} &= c_t \beta \left\{ \alpha (\rho x_t + \epsilon_{t+1})^{1-\alpha} U_{t+1}^\alpha k_{t+1}^{1-\alpha} + 1 - \delta U_{t+1}^\phi \right\} \\ i_{t+1} &= z_{t+1}^{1-\alpha} k_{t+1}^\alpha U_{t+1}^\alpha - \beta c_t \left\{ \alpha U_{t+1} z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} U_{t+1}^\alpha + 1 - \delta U_{t+1}^\phi \right\} \\ y_{t+1} &= z_{t+1}^{1-\alpha} (k_{t+1} U_{t+1})^\alpha \end{aligned}$$

Important here are the partial derivatives of the the first three terms w.r.t. all past terms:

$$\begin{aligned} \frac{\partial}{\partial z_t} k_{t+1} &= -\frac{1-\alpha}{\phi-\alpha} k_t^{\frac{\phi-1}{\phi-\alpha}} \left(\frac{\alpha}{\delta \phi} \right)^{\frac{1}{\phi-\alpha}} z_t^{\frac{1-\phi}{\phi-\alpha}} + (1-\alpha) \left(\frac{k_t U_t}{z_t} \right)^\alpha - \beta c_t (1-\alpha) z_t^{-\alpha} U_t^{1+\alpha} k_t^{\alpha-1} \\ \frac{\partial}{\partial k_t} k_{t+1} &= 1 - \frac{\phi-1}{\phi-\alpha} k_t^{\frac{\alpha-1}{\phi-\alpha}} \left(\frac{\alpha}{\delta \phi} \right)^{\frac{1}{\phi-\alpha}} z_t^{\frac{1-\alpha}{\phi-\alpha}} + \alpha z_t^{1-\alpha} k_t^{\alpha-1} U_t^\alpha - \beta c_t \alpha U_t^{1+\alpha} z_t^{1-\alpha} k_t^{\alpha-2} \\ \frac{\partial}{\partial U_t} k_{t+1} &= \alpha z_t^{1-\alpha} k_t^\alpha U_t^{\alpha-1} - \beta c_t \left\{ \alpha (1+\alpha) z_t^{1-\alpha} k_t^{\alpha-1} U_t^\alpha - \delta \phi U_t^{\phi-1} \right\} \\ \frac{\partial}{\partial c_t} k_{t+1} &= \beta \left\{ \alpha z_t^{1-\alpha} k_t^{\alpha-1} U_t^{1+\alpha} + 1 - \delta U_t^\phi \right\} \\ \frac{\partial}{\partial i_t} k_{t+1} &= 1 \\ \frac{\partial}{\partial y_t} k_{t+1} &= 1 - \beta c_t (??) \end{aligned}$$

as we have $k_{t+1} = [\dots] + y_t - \beta c_t \frac{\partial}{\partial k_t} y_t + [\dots]$????

Next we have

$$\begin{aligned} \frac{\partial}{\partial z_t} U_{t+1} &= \left(\frac{\alpha}{\delta \phi} \right)^{\frac{1}{\phi-\alpha}} z_{t+1}^{\frac{1-\alpha}{\phi-\alpha}} \left\{ (1-\alpha) \left(\frac{k_t U_t}{z_t} \right)^\alpha - \frac{1-\alpha}{\phi-\alpha} k_t^{\frac{\phi-1}{\phi-\alpha}} \left(\frac{\alpha}{\delta \phi} \right)^{\frac{1}{\phi-\alpha}} z_t^{\frac{1-\phi}{\phi-\alpha}} - \beta c_t \alpha (1-\alpha) \left(\frac{U_t}{z_t} \right)^\alpha k_t^{\alpha-1} \right\} \\ &\quad \left\{ k_t - k_t^{\frac{\phi-1}{\phi-\alpha}} \left(\frac{\alpha}{\delta \phi} \right)^{\frac{1}{\phi-\alpha}} z_t^{\frac{1-\alpha}{\phi-\alpha}} + z_t^{1-\alpha} k_t^\alpha U_t^\alpha - \beta c_t \left\{ \alpha U_t z_t^{1-\alpha} k_t^{\alpha-1} U_t^\alpha + 1 - \delta U_t^\phi \right\} \right\}^{\frac{2\alpha-1-\phi}{\phi-\alpha}} \frac{\alpha-1}{\phi-\alpha} \\ \frac{\partial}{\partial k_t} U_{t+1} &= \left(\frac{\alpha}{\delta \phi} \right)^{\frac{1}{\phi-\alpha}} z_{t+1}^{\frac{1-\alpha}{\phi-\alpha}} \dots \text{fuck me} \end{aligned}$$

We have the Euler error given by

$$V_t = c_{t+1} - \beta c_t [f_{k_{t+1}}(k_{t+1}, U_{t+1}, z_{t+1}) + 1 - \delta_{t+1}(k_{t+1})],$$

We have

$$U_t = \left(\frac{\alpha}{\delta\phi} z_t^{1-\alpha} k_t^{\alpha-1} \right)^{\frac{1}{\phi-\alpha}}$$

$$U_{t+1} = \left(\frac{\alpha}{\delta\phi} z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} \right)^{\frac{1}{\phi-\alpha}}$$

And

$$\begin{aligned} c_t &= z_t^{1-\alpha} k_t^\alpha U_t^\alpha - k_{t+1} + k_t - \delta U_t^\phi k_t, \\ &= z_t^{1-\alpha} k_t^\alpha \left(\frac{\alpha}{\delta\phi} z_t^{1-\alpha} k_t^{\alpha-1} \right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta \left(\frac{\alpha}{\delta\phi} z_t^{1-\alpha} k_t^{\alpha-1} \right)^{\frac{\phi}{\phi-\alpha}} k_t \\ &= z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \\ c_{t+1} &= z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+2} + k_{t+1} - \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \end{aligned}$$

Then we have

$$\begin{aligned} f_{k_{t+1}}(\cdot) &= \alpha z_{t+1}^{1-\alpha} U_{t+1}^\alpha k_{t+1}^{\alpha-1} \\ &= \alpha z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} \left(\frac{\alpha}{\delta\phi} z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} \right)^{\frac{\alpha}{\phi-\alpha}} \\ &= \alpha z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\ &= \alpha \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \end{aligned}$$

And lastly we have

$$\begin{aligned} \delta_{t+1} &= \delta U_{t+1}^\phi = \delta \left(\frac{\alpha}{\delta\phi} z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} \right)^{\frac{\phi}{\phi-\alpha}} \\ &= \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \end{aligned}$$

We can put together all terms and have

$$\begin{aligned} V_t &= z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+2} + k_{t+1} - \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \\ &\quad + \beta \underbrace{\left\{ z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \right\}}_{=c_t} \alpha \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\ &\quad + \beta \underbrace{\left\{ z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \right\}}_{=c_t} \\ &\quad - \beta \underbrace{\left\{ z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \right\}}_{=c_t} \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \end{aligned}$$

Now we can take the partial derivatives:

$$\frac{\partial}{\partial k_{t+2}} V_t = -1$$

$$\begin{aligned} \frac{\partial}{\partial z_{t+1}} V_t &= \left(1 - \alpha + \frac{(1 - \alpha)\alpha}{\phi - \alpha}\right) \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi - \alpha}} z_{t+1}^{\frac{(1 - \alpha)\alpha}{\phi - \alpha} - \alpha} k_{t+1}^{\alpha + \frac{(\alpha - 1)\alpha}{\phi - \alpha}} - \delta \left(\frac{(1 - \alpha)\phi}{\phi - \alpha}\right) \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi - \alpha}} z_{t+1}^{\frac{(1 - \alpha)\phi}{\phi - \alpha} - 1} k_{t+1}^{1 + \frac{(\alpha - 1)\phi}{\phi - \alpha}} \\ &\quad + \beta \left\{ z_t^{1 - \alpha + \frac{(1 - \alpha)\alpha}{\phi - \alpha}} k_t^{\alpha + \frac{(\alpha - 1)\alpha}{\phi - \alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi - \alpha}} - k_{t+1} + k_t - \delta \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi - \alpha}} z_t^{\frac{(1 - \alpha)\phi}{\phi - \alpha}} k_t^{1 + \frac{(\alpha - 1)\phi}{\phi - \alpha}} \right\} \\ &\quad \cdot \alpha \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi - \alpha}} \left(1 - \alpha + \frac{(1 - \alpha)\alpha}{\phi - \alpha}\right) z_{t+1}^{\frac{(1 - \alpha)\alpha}{\phi - \alpha} - \alpha} k_{t+1}^{\alpha - 1 + \frac{(\alpha - 1)\alpha}{\phi - \alpha}} \end{aligned}$$

$$+ \beta \left\{ z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \right\} \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}-1} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

which, at the steady state is

$$\begin{aligned} \frac{\partial}{\partial z_{t+1}} V_* &= \left(1 - \alpha + \frac{(1 - \alpha)\alpha}{\phi - \alpha}\right) \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi - \alpha}} k_*^{\alpha + \frac{(\alpha - 1)\alpha}{\phi - \alpha}} - \delta \left(\frac{(1 - \alpha)\phi}{\phi - \alpha}\right) \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi - \alpha}} k_*^{1 + \frac{(\alpha - 1)\phi}{\phi - \alpha}} \\ &+ \beta \left\{ k_*^{\alpha + \frac{(\alpha - 1)\alpha}{\phi - \alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi - \alpha}} - \delta \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi - \alpha}} k_*^{1 + \frac{(\alpha - 1)\phi}{\phi - \alpha}} \right\} \alpha \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi - \alpha}} \left(1 - \alpha + \frac{(1 - \alpha)\alpha}{\phi - \alpha}\right) k_*^{\alpha - 1 + \frac{(\alpha - 1)\alpha}{\phi - \alpha}} \\ &+ \beta \left\{ k_*^{\alpha + \frac{(\alpha - 1)\alpha}{\phi - \alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi - \alpha}} - \delta \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi - \alpha}} k_*^{1 + \frac{(\alpha - 1)\phi}{\phi - \alpha}} \right\} \delta \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi - \alpha}} k_*^{\frac{(\alpha - 1)\phi}{\phi - \alpha}} \end{aligned}$$

And as the last derivative, we have

$$\begin{aligned} \frac{\partial}{\partial z_t} V_t &= \beta \left\{ \left(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi - \alpha} \right) z_t^{\frac{(1-\alpha)\alpha}{\phi - \alpha} - \alpha} k_t^{\alpha + \frac{(\alpha-1)\alpha}{\phi - \alpha}} \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi - \alpha}} - \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi - \alpha}} \left(\frac{(1-\alpha)\phi}{\phi - \alpha} \right) z_t^{\frac{(1-\alpha)\phi}{\phi - \alpha} - 1} k_t^{1 + \frac{(\alpha-1)\phi}{\phi - \alpha}} \right\} \\ &\quad \cdot \alpha \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi - \alpha}} z_{t+1}^{1 - \alpha + \frac{(1-\alpha)\alpha}{\phi - \alpha}} k_{t+1}^{\alpha - 1 + \frac{(\alpha-1)\alpha}{\phi - \alpha}} \\ &\quad + \beta \left\{ \left(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi - \alpha} \right) z_t^{\frac{(1-\alpha)\alpha}{\phi - \alpha} - \alpha} k_t^{\alpha + \frac{(\alpha-1)\alpha}{\phi - \alpha}} \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi - \alpha}} - \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi - \alpha}} \left(\frac{(1-\alpha)\phi}{\phi - \alpha} \right) z_t^{\frac{(1-\alpha)\phi}{\phi - \alpha} - 1} k_t^{1 + \frac{(\alpha-1)\phi}{\phi - \alpha}} \right\} \\ &\quad - \beta \left\{ \left(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi - \alpha} \right) z_t^{\frac{(1-\alpha)\alpha}{\phi - \alpha} - \alpha} k_t^{\alpha + \frac{(\alpha-1)\alpha}{\phi - \alpha}} \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi - \alpha}} - \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi - \alpha}} \left(\frac{(1-\alpha)\phi}{\phi - \alpha} \right) z_t^{\frac{(1-\alpha)\phi}{\phi - \alpha} - 1} k_t^{1 + \frac{(\alpha-1)\phi}{\phi - \alpha}} \right\} \\ &\quad \cdot \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi - \alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi - \alpha}} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi - \alpha}}, \end{aligned}$$

and evaluated at the steady state this is

$$\begin{aligned} \frac{\partial}{\partial z_t} V_* &= \beta \left\{ \left(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi - \alpha} \right) k_*^{\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi-\alpha}} - \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi-\alpha}} \left(\frac{(1-\alpha)\phi}{\phi - \alpha} \right) k_*^{1 + \frac{(\alpha-1)\phi}{\phi-\alpha}} \right\} \\ &\quad \cdot \alpha \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi-\alpha}} k_*^{\alpha-1 + \frac{(\alpha-1)\alpha}{\phi-\alpha}} \\ &\quad + \beta \left\{ \left(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi - \alpha} \right) k_*^{\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi-\alpha}} - \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi-\alpha}} \left(\frac{(1-\alpha)\phi}{\phi - \alpha} \right) k_*^{1 + \frac{(\alpha-1)\phi}{\phi-\alpha}} \right\} \\ &\quad - \beta \left\{ \left(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi - \alpha} \right) k_*^{\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi-\alpha}} - \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi-\alpha}} \left(\frac{(1-\alpha)\phi}{\phi - \alpha} \right) k_*^{1 + \frac{(\alpha-1)\phi}{\phi-\alpha}} \right\} \\ &\quad \cdot \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \end{aligned}$$

Note: $\frac{\partial}{\partial z_t} c_t = \left\{ \left(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi-\alpha} \right) z_t^{\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi-\alpha}} - \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi-\alpha}} \left(\frac{(1-\alpha)\phi}{\phi-\alpha} \right) z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}-1} k_t^{1 + \frac{(\alpha-1)\phi}{\phi-\alpha}} \right\}$

Dynare: Policy Function using Parameter Set (i)

$$k_t = \underbrace{60.62}_{=G_0} + \underbrace{0.974}_{=G_1}(k_{t-1} - \bar{k}) + \underbrace{2.055}_{=G_2}(x_t - \bar{x})$$