Dynamic Macroeconomics with Numerics: Project II

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Part 1: Blanchard Kahn Approach

First note that we have

$$z_{t+1} = \exp(\epsilon_{t+1}) z_t^{\rho},$$

but we may also use

$$z_{t+1} = \exp(x_{t+1}) = \exp(\rho x_t + \epsilon_{t+1}),$$

Furthermore, we have from the Euler error defined in the last project two different capital utilization functions, differing only in the exponent

$$U_{t+1}^{\alpha} = \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} \exp\Big(\frac{(1-\alpha)\alpha}{\phi-\alpha} x_{t+1}\Big) k_{t+1}^{\frac{(\alpha-1)\alpha}{\phi-\alpha}}$$
$$U_{t+1}^{\phi} = \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} \exp\Big(\frac{(1-\alpha)\phi}{\phi-\alpha} x_{t+1}\Big) k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

We have a **three-equation system**, having already plugged the values for U_t, U_{t+1} given by

$$\mathbf{F}(\mathbf{s}_{t+1}, \mathbf{s}_t) = \begin{pmatrix} (1) \\ (2) \\ (3) \end{pmatrix},$$

with the elements being given by the equations below:

$$c_{t+1} - \beta c_t \Big[\alpha \exp\Big([1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}] x_{t+1}\Big) k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \exp\Big(x_{t+1}\frac{(1-\alpha)\phi}{\phi-\alpha}\Big)\Big] \tag{1}$$

$$\exp\Big(\Big[1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}\Big] x_t\Big) k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + k_t - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} \exp\Big(\frac{(1-\alpha)\phi}{\phi-\alpha} x_t\Big) k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} - c_t - k_{t+1} \tag{2}$$

$$x_{t+1} - \rho x_t \tag{3}$$

Now we take the partial derivatives w.r.t. $c_t, c_{t+1}, k_t, k_{t+1}, x_t, x_{t+1}$ for all three equations evaluated at the steady state. First the Euler error partial derivatives evaluated at the steady state:

$$\frac{\partial}{c_t}(1) = -\beta\Big[\alpha k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big]$$
$$\frac{\partial}{k_t}(1) = 0$$
$$\frac{\partial}{x_t}(1) = 0$$
$$\frac{\partial}{c_{t+1}}(1) = 1$$
$$\frac{\partial}{k_{t+1}}(1) = -\beta c_* \Big[\alpha\Big(\alpha - 1 + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) k_*^{\alpha-2+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}-1} \Big(\frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big]$$
$$\frac{\partial}{x_{t+1}}(1) = -\beta c_* \Big[\alpha\big[1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}\big] k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \frac{(1-\alpha)\phi}{\phi-\alpha}\Big]$$

Now the law of motion for capital's partial derivatives evaluated at the steady state:

$$\frac{\partial}{c_t}(2) = -1$$
$$\frac{\partial}{k_t}(2) = \Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} \Big(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\Big) k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

$$\frac{\partial}{x_t}(2) = \Big[1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}\Big]k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}}\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}\frac{(1-\alpha)\phi}{\phi-\alpha}k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}$$
$$\frac{\partial}{c_{t+1}}(2) = 0$$
$$\frac{\partial}{k_{t+1}}(2) = -1$$
$$\frac{\partial}{x_{t+1}}(2) = 0$$

And lastly the error term from the law of motion for x's partial derivatives evaluated at the steady state:

$$\frac{\partial}{c_t}(3) = 0$$
$$\frac{\partial}{k_t}(3) = 0$$
$$\frac{\partial}{x_t}(3) = -\rho$$
$$\frac{\partial}{c_{t+1}}(3) = 0$$
$$\frac{\partial}{k_{t+1}}(3) = 0$$
$$\frac{\partial}{x_{t+1}}(3) = 1$$

We have the two matrices

$$D\mathbf{F}_1(\cdot) = \begin{pmatrix} \frac{\partial}{\partial c_t}(1) & \frac{\partial}{\partial k_t}(1) & \frac{\partial}{\partial x_t}(1) \\ \frac{\partial}{\partial c_t}(2) & \frac{\partial}{\partial k_t}(2) & \frac{\partial}{\partial x_t}(2) \\ \frac{\partial}{\partial c_t}(3) & \frac{\partial}{\partial k_t}(3) & \frac{\partial}{\partial x_t}(3) \end{pmatrix}, \; D\mathbf{F}_2(\cdot) = \begin{pmatrix} \frac{\partial}{\partial c_{t+1}}(1) & \frac{\partial}{\partial k_{t+1}}(1) & \frac{\partial}{\partial x_{t+1}}(1) \\ \frac{\partial}{\partial c_{t+1}}(2) & \frac{\partial}{\partial k_{t+1}}(2) & \frac{\partial}{\partial x_{t+1}}(2) \\ \frac{\partial}{\partial c_{t+1}}(3) & \frac{\partial}{\partial k_{t+1}}(3) & \frac{\partial}{\partial x_{t+1}}(3) \end{pmatrix}$$

Due to the complexity of the following operation we do not provide any detailed analytical description, but it follows that we have the Jacobian of the state space given by

$$\mathbf{J} = -D\mathbf{F}_2(\mathbf{s}_*, \mathbf{s}_*)^{-1}D\mathbf{F}_1(\mathbf{s}_*, \mathbf{s}_*)$$

The result is a 3×3 matrix. We then proceed in the exact same way as we did in the fourth and fifth exercise session, so any details beyond the policy functions are omitted.

Note: our policy function is written to match the dynare output, so we take deviations as the input.

(d) IRFs

Figure 1: (i): $\phi = 1.5,\ \delta = 0.0285,\ \rho = 0.9$

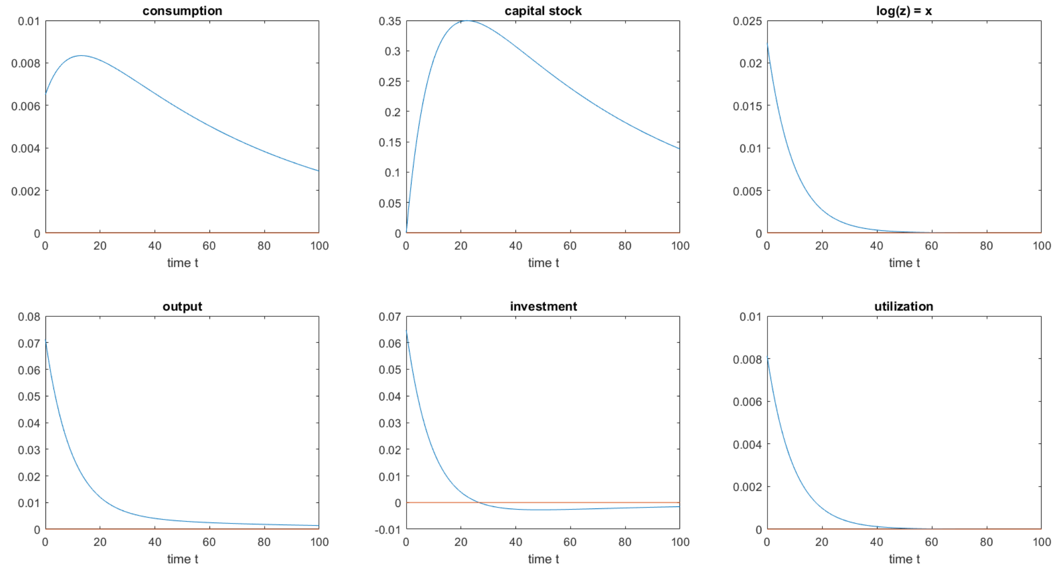

Figure 2: (ii): $\phi = 1.5,\ \delta = 0.9,\ \rho = 0.9$

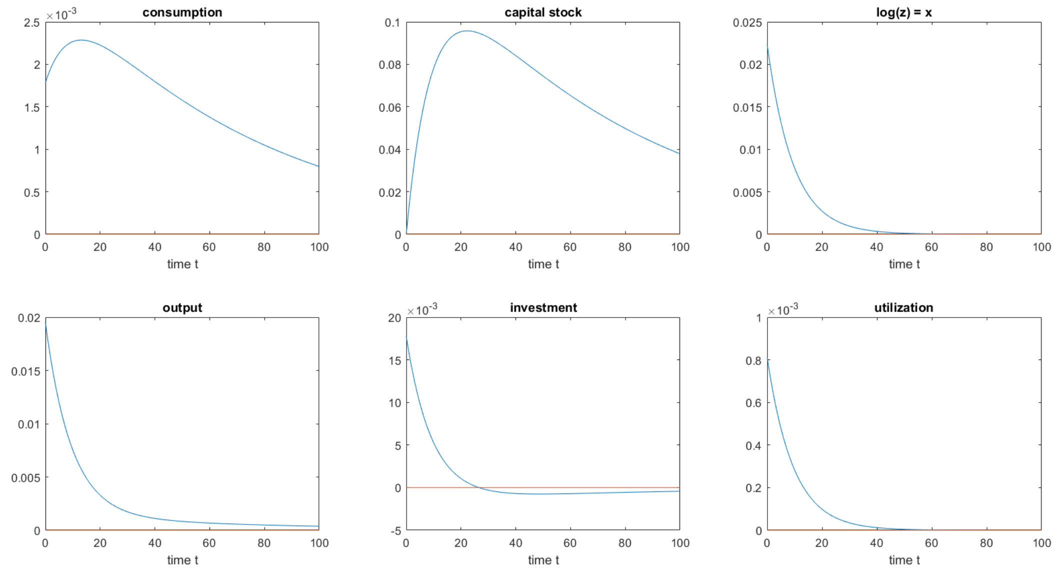

Figure 3: (iii): $\phi = 1.5,\ \delta = 0.0285,\ \rho = 0.99$

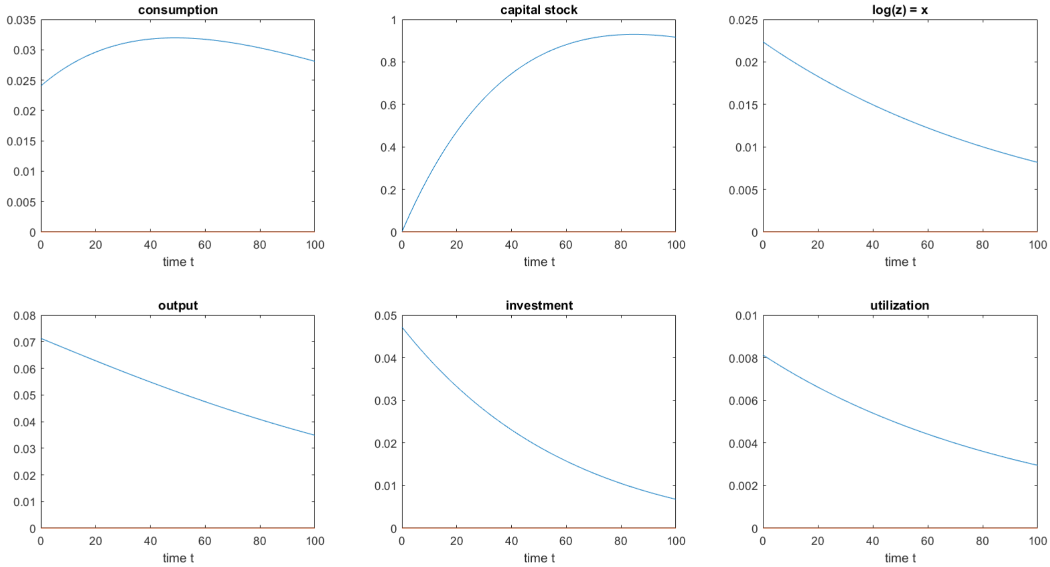

Figure 4: (iv): $\phi = 1.1,\ \delta = 0.0285,\ \rho = 0.9$