Dynamic Macroeconomics with Numerics: Project II

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June 17, 2021

One-Sector Stochastic Growth Model

Notes on optimality conditions

We have the Euler error given by

$$V_t = c_{t+1} - \beta c_t [f_{k_{t+1}}(k_{t+1}, U_{t+1}, z_{t+1}) + 1 - \delta_{t+1}(k_{t+1})],$$

We have

$$U_t = \Big(\frac{\alpha}{\delta\phi} z_t^{1-\alpha} k_t^{\alpha-1}\Big)^{\frac{1}{\phi-\alpha}}$$
$$U_{t+1} = \Big(\frac{\alpha}{\delta\phi} z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1}\Big)^{\frac{1}{\phi-\alpha}}$$

And

$$\begin{aligned}
c_t &= z_t^{1-\alpha} k_t^{\alpha} U_t^{\alpha} - k_{t+1} + k_t - \delta U_t^{\phi} k_t, \\
&= z_t^{1-\alpha} k_t^{\alpha} \Big(\frac{\alpha}{\delta\phi} z_t^{1-\alpha} k_t^{\alpha-1}\Big)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta \Big(\frac{\alpha}{\delta\phi} z_t^{1-\alpha} k_t^{\alpha-1}\Big)^{\frac{\phi}{\phi-\alpha}} k_t \\
&= z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \\
c_{t+1} &= z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - k_{t+2} + k_{t+1} - \delta \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}
\end{aligned}$$

Then we have

$$\begin{aligned}
f_{k_{t+1}}(\cdot) &= \alpha z_{t+1}^{1-\alpha} U_{t+1}^{\alpha} k_{t+1}^{\alpha-1} \\
&= \alpha z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} \Big(\frac{\alpha}{\delta\phi} z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1}\Big)^{\frac{\alpha}{\phi-\alpha}} \\
&= \alpha z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
&= \alpha \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}}
\end{aligned}$$

And lastly we have

$$\begin{aligned}
\delta_{t+1} = \delta U_{t+1}^{\phi} &= \delta \Big(\frac{\alpha}{\delta\phi} z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1}\Big)^{\frac{\phi}{\phi-\alpha}} \\
&= \delta \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}}
\end{aligned}$$

We can put together all terms and have

$$\begin{aligned}
V_t = {} & z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - k_{t+2} + k_{t+1} - \delta \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \\
& + \beta \underbrace{\Big\{ z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\}}_{=c_t} \alpha \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
& + \beta \underbrace{\Big\{ z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\}}_{=c_t} \\
& - \beta \underbrace{\Big\{ z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\}}_{=c_t} \delta \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}}
\end{aligned}$$

Now we can take the partial derivatives:

$$\frac{\partial}{\partial k_{t+2}} V_t = -1$$
$$\frac{\partial}{\partial k_{t+2}} V_t|_{k_t=k^*, z_t=z^*} = -1$$

Next we have

$$\begin{aligned}
\frac{\partial}{\partial k_{t+1}} V_t = &\Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1+\frac{(\alpha-1)\alpha}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
&+ \beta\Big\{ z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \\
&\cdot \alpha\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} \Big(\alpha - 1 + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-2+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
&+ (-1)\alpha\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
&+ \beta(-1) \\
&- \beta\Big\{ z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \\
&\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} \Big(\frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) z_{t+1}^{\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\alpha}{\phi-\alpha}-1} \\
&+ \beta\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}}
\end{aligned}$$

and evaluated at the steady state this is

$$\begin{aligned}
\frac{\partial}{\partial k_{t+1}} V_* = &\Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1+\frac{(\alpha-1)\alpha}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
&+ \beta\Big\{ k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \cdot \alpha\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} \Big(\alpha - 1 + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) k_*^{\alpha-2+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
&+ (-1)\alpha\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
&+ \beta(-1) \\
&- \beta\Big\{ k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} \Big(\frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) k_*^{\frac{(\alpha-1)\alpha}{\phi-\alpha}-1} \\
&+ \beta\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}}
\end{aligned}$$

The next derivative is given by

$$\begin{aligned}
\frac{\partial}{\partial k_t} V_t = \beta\Big\{ &\Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1+\frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \\
&\alpha\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
&+ \beta\Big\{ \Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1+\frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \\
&- \beta\Big\{ \Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1+\frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \\
&\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\alpha}{\phi-\alpha}}
\end{aligned}$$

and evaluated at the steady state this is

$$
\begin{aligned}
\frac{\partial}{\partial k_t} V_* = \beta\Big\{ & \Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \alpha \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
& + \beta\Big\{ \Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \\
& - \beta\Big\{ \Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \delta \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\alpha}{\phi-\alpha}}
\end{aligned}
$$

Note: $\frac{\partial}{\partial k_t} c_t = \Big\{ \Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\}$
Next we have the derivatives w.r.t. the technology shocks

$$
\begin{aligned}
\frac{\partial}{\partial z_{t+1}} V_t = & \Big(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\alpha}{\phi-\alpha}-\alpha} k_{t+1}^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} - \delta\Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}-1} k_{t+1}^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \\
& + \beta\Big\{ z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \\
& \cdot \alpha \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} \Big(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi-\alpha}\Big) z_{t+1}^{\frac{(1-\alpha)\alpha}{\phi-\alpha}-\alpha} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
& + \beta\Big\{ z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \delta \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}-1} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}}
\end{aligned}
$$

which, at the steady state is

$$
\begin{aligned}
\frac{\partial}{\partial z_{t+1}} V_* = & \Big(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} - \delta\Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \\
& + \beta\Big\{ k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \alpha \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} \Big(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi-\alpha}\Big) k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
& + \beta\Big\{ k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \delta \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}}
\end{aligned}
$$

And as the last derivative, we have

$$
\begin{aligned}
\frac{\partial}{\partial z_t} V_t = \beta\Big\{ & \Big(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi-\alpha}\Big) z_t^{\frac{(1-\alpha)\alpha}{\phi-\alpha}-\alpha} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} \Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big) z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}-1} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \\
& \cdot \alpha \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
& + \beta\Big\{ \Big(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi-\alpha}\Big) z_t^{\frac{(1-\alpha)\alpha}{\phi-\alpha}-\alpha} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} \Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big) z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}-1} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \\
& - \beta\Big\{ \Big(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi-\alpha}\Big) z_t^{\frac{(1-\alpha)\alpha}{\phi-\alpha}-\alpha} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} \Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big) z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}-1} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \\
& \cdot \delta \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}},
\end{aligned}
$$

and evaluated at the steady state this is

$$
\begin{aligned}
\frac{\partial}{\partial z_t} V_* = \beta\Big\{ & \Big(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi-\alpha}\Big) k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} \Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big) k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \\
& \cdot \alpha \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
& + \beta\Big\{ \Big(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi-\alpha}\Big) k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} \Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big) k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\} \\
& - \beta\Big\{ \Big(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi-\alpha}\Big) k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} \Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big) k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}} \Big\}
\end{aligned}
$$

$$\cdot \, \delta \left(\frac{\alpha}{\delta \phi} \right)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

Note: $\frac{\partial}{\partial z_t} c_t = \left\{ \left(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi-\alpha} \right) z_t^{\frac{(1-\alpha)\alpha}{\phi-\alpha} - \alpha} k_t^{\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\alpha}{\phi-\alpha}} - \delta \left(\frac{\alpha}{\delta\phi} \right)^{\frac{\phi}{\phi-\alpha}} \left(\frac{(1-\alpha)\phi}{\phi-\alpha} \right) z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha} - 1} k_t^{1 + \frac{(\alpha-1)\phi}{\phi-\alpha}} \right\}$

Dynare: Policy Function using Parameter Set (i)

$$k_t = \underbrace{60.62}_{=G_0} + \underbrace{0.974}_{=G_1}(k_{t-1} - \bar{k}) + \underbrace{2.055}_{=G_2}(x_t - \bar{x})$$