Dynamic Macroeconomics with Numerics: Project II

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One-Sector Stochastic Growth Model

Notes on optimality conditions

We have

$$\mathbf{X}_{t+1} = \begin{pmatrix} c_{t+1} \\ k_{t+1} \\ U_{t+1} \\ z_{t+1} \end{pmatrix} = f(\mathbf{X}_t) = \begin{pmatrix} c_t\beta\Big\{\alpha(\rho x_t + \epsilon_{t+1})^{1-\alpha}U_{t+1}^{\alpha}k_{t+1}^{\alpha-1} + 1 - \delta U_{t+1}^{\phi}\Big\} \\ k_t - \delta U_t^{\phi}k_t + i_t \\ \Big\{(\delta\phi)^{-1}\alpha\exp(\rho x_t + \epsilon_{t+1})^{1-\alpha}(k_t - \delta U_t^{\phi}k_t + i_t)^{\alpha-1}\Big\}^{\frac{1}{\phi-\alpha}} \\ \rho x_t + \epsilon_{t+1} \end{pmatrix}$$

We now express all terms in terms of past values:

$$\begin{aligned}
z_{t+1} &= \exp(\rho x_t + \epsilon_{t+1}) \\
k_{t+1} &= (1 - \delta U_t^{\phi})k_t + z_t^{1-\alpha}k_t^{\alpha}U_t^{\alpha} - \beta c_t\Big\{\alpha U_t z_t^{1-\alpha}k_t^{\alpha-1}U_t^{\alpha} + 1 - \delta U_t^{\phi}\Big\} \\
&= \left(1 - \left\{\frac{\alpha}{\delta\phi}z_t^{1-\alpha}k_t^{\alpha-1}\right\}^{\frac{1}{\phi-\alpha}}\right)k_t + z_t^{1-\alpha}k_t^{\alpha}U_t^{\alpha} - \beta c_t\Big\{\alpha U_t z_t^{1-\alpha}k_t^{\alpha-1}U_t^{\alpha} + 1 - \delta U_t^{\phi}\Big\} \\
&= k_t - k_t^{\frac{\phi-1}{\phi-\alpha}}\left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}z_t^{\frac{1-\alpha}{\phi-\alpha}} + z_t^{1-\alpha}k_t^{\alpha}U_t^{\alpha} - \beta c_t\Big\{\alpha U_t z_t^{1-\alpha}k_t^{\alpha-1}U_t^{\alpha} + 1 - \delta U_t^{\phi}\Big\} \\
U_{t+1} &= \left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}\exp(\rho x_t + \epsilon_{t+1})^{\frac{1-\alpha}{\phi-\alpha}}\left(k_t - k_t^{\frac{\phi-1}{\phi-\alpha}}\left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}z_t^{\frac{1-\alpha}{\phi-\alpha}} + z_t^{1-\alpha}k_t^{\alpha}U_t^{\alpha} - \beta c_t\Big\{\alpha U_t z_t^{1-\alpha}k_t^{\alpha-1}U_t^{\alpha} + 1 - \delta U_t^{\phi}\Big\}\right)^{\frac{\alpha-1}{\phi-\alpha}} \\
c_{t+1} &= c_t\beta\Big\{\alpha(\rho x_t + \epsilon_{t+1})^{1-\alpha}U_{t+1}^{\alpha}k_{t+1}^{1-\alpha} + 1 - \delta U_{t+1}^{\phi} \\
i_{t+1} &= z_{t+1}^{1-\alpha}k_{t+1}^{\alpha}U_{t+1}^{\alpha} - \beta c_t\Big\{\alpha U_{t+1}z_{t+1}^{1-\alpha}k_{t+1}^{\alpha-1}U_{t+1}^{\alpha} + 1 - \delta U_{t+1}^{\phi}\Big\} \\
y_{t+1} &= z_{t+1}^{1-\alpha}(k_{t+1}U_{t+1})^{\alpha}
\end{aligned}$$

Important here are the partial derivatives of the the first three terms w.r.t. all past terms:

$$\begin{aligned}
\frac{\partial}{\partial z_t}k_{t+1} &= -\frac{1-\alpha}{\phi-\alpha}k_t^{\frac{\phi-1}{\phi-\alpha}}\left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}z_t^{\frac{1-\phi}{\phi-\alpha}} + (1-\alpha)\left(\frac{k_tU_t}{z_t}\right)^{\alpha} - \beta c_t(1-\alpha)z_t^{-\alpha}U_t^{1+\alpha}k_t^{\alpha-1} \\
\frac{\partial}{\partial k_t}k_{t+1} &= 1 - \frac{\phi-1}{\phi-\alpha}k_t^{\frac{\alpha-1}{\phi-\alpha}}\left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}z_t^{\frac{1-\alpha}{\phi-\alpha}} + \alpha z_t^{1-\alpha}k_t^{\alpha-1}U_t^{\alpha} - \beta c_t\alpha U_t^{1+\alpha}z_t^{1-\alpha}k_t^{\alpha-2} \\
\frac{\partial}{\partial U_t}k_{t+1} &= \alpha z_t^{1-\alpha}k_t^{\alpha}U_t^{\alpha-1} - \beta c_t\Big\{\alpha(1+\alpha)z_t^{1-\alpha}k_t^{\alpha-1}U_t^{\alpha} - \delta\phi U_t^{\phi-1}\Big\} \\
\frac{\partial}{\partial c_t}k_{t+1} &= \beta\Big\{\alpha z_t^{1-\alpha}k_t^{\alpha-1}U_t^{1+\alpha} + 1 - \delta U_t^{\phi}\Big\} \\
\frac{\partial}{\partial i_t}k_{t+1} &= 1 \\
\frac{\partial}{\partial y_t}k_{t+1} &= 1 - \beta c_t(??)
\end{aligned}$$

as we have $k_{t+1} = [\dots] + y_t - \beta c_t\frac{\partial}{\partial k_t}y_t + [\dots]$????

Next we have

$$\begin{aligned}
\frac{\partial}{\partial z_t}U_{t+1} &= \left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}z_{t+1}^{\frac{1-\alpha}{\phi-\alpha}}\left\{(1-\alpha)\left(\frac{k_tU_t}{z_t}\right)^{\alpha} - \frac{1-\alpha}{\phi-\alpha}k_t^{\frac{\phi-1}{\phi-\alpha}}\left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}z_t^{\frac{1-\phi}{\phi-\alpha}} - \beta c_t\alpha(1-\alpha)\left(\frac{U_t}{z_t}\right)^{\alpha}k_t^{\alpha-1}\right\} \\
&\qquad \left\{k_t - k_t^{\frac{\phi-1}{\phi-\alpha}}\left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}z_t^{\frac{1-\alpha}{\phi-\alpha}} + z_t^{1-\alpha}k_t^{\alpha}U_t^{\alpha} - \beta c_t\Big\{\alpha U_t z_t^{1-\alpha}k_t^{\alpha-1}U_t^{\alpha} + 1 - \delta U_t^{\phi}\Big\}\right\}^{\frac{2\alpha-1-\phi}{\phi-\alpha}}\frac{\alpha-1}{\phi-\alpha} \\
\frac{\partial}{\partial k_t}U_{t+1} &= \left(\frac{\alpha}{\delta\phi}\right)^{\frac{1}{\phi-\alpha}}z_{t+1}^{\frac{1-\alpha}{\phi-\alpha}}\dots\text{fuck me}
\end{aligned}$$

We have the Euler error given by

$$V_t = c_{t+1} - \beta c_t[f_{k_{t+1}}(k_{t+1}, U_{t+1}, z_{t+1}) + 1 - \delta_{t+1}(k_{t+1})],$$

We have

$$U_t = \left(\frac{\alpha}{\delta\phi} z_t^{1-\alpha} k_t^{\alpha-1}\right)^{\frac{1}{\phi-\alpha}}$$
$$U_{t+1} = \left(\frac{\alpha}{\delta\phi} z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1}\right)^{\frac{1}{\phi-\alpha}}$$

And

$$\begin{aligned}
c_t &= z_t^{1-\alpha} k_t^{\alpha} U_t^{\alpha} - k_{t+1} + k_t - \delta U_t^{\phi} k_t,\\
&= z_t^{1-\alpha} k_t^{\alpha} \left(\frac{\alpha}{\delta\phi} z_t^{1-\alpha} k_t^{\alpha-1}\right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta\left(\frac{\alpha}{\delta\phi} z_t^{1-\alpha} k_t^{\alpha-1}\right)^{\frac{\phi}{\phi-\alpha}} k_t\\
&= z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\\
c_{t+1} &= z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+2} + k_{t+1} - \delta\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}
\end{aligned}$$

Then we have

$$\begin{aligned}
f_{k_{t+1}}(\cdot) &= \alpha z_{t+1}^{1-\alpha} U_{t+1}^{\alpha} k_{t+1}^{\alpha-1}\\
&= \alpha z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} \left(\frac{\alpha}{\delta\phi} z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1}\right)^{\frac{\alpha}{\phi-\alpha}}\\
&= \alpha z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\alpha}{\phi-\alpha}}\\
&= \alpha\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}}
\end{aligned}$$

And lastly we have

$$\begin{aligned}
\delta_{t+1} = \delta U_{t+1}^{\phi} &= \delta\left(\frac{\alpha}{\delta\phi} z_{t+1}^{1-\alpha} k_{t+1}^{\alpha-1}\right)^{\frac{\phi}{\phi-\alpha}}\\
&= \delta\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}}
\end{aligned}$$

We can put together all terms and have

$$\begin{aligned}
V_t = {} & z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+2} + k_{t+1} - \delta\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\\
& + \beta\underbrace{\left\{z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\right\}}_{=c_t} \alpha\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}}\\
& + \beta\underbrace{\left\{z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\right\}}_{=c_t}\\
& - \beta\underbrace{\left\{z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \left(\frac{\alpha}{\delta\phi}\right)^{\frac{\alpha}{\phi-\alpha}} - k_{t+1} + k_t - \delta\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\right\}}_{=c_t} \delta\left(\frac{\alpha}{\delta\phi}\right)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}}
\end{aligned}$$

Now we can take the partial derivatives:

$$\frac{\partial}{\partial k_{t+2}} V_t = -1$$

$$\frac{\partial}{\partial k_{t+2}} V_t|_{k_t=k^*,z_t=z^*} = -1$$

$$\begin{aligned}
\frac{\partial}{\partial k_{t+1}} V_t = &\Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1 + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
&+ \beta c_t \alpha \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} \Big(\alpha - 1 + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-2+\frac{(\alpha-1)\alpha}{\phi-\alpha}} + (-1)\alpha\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
&+ \beta(-1) \\
&- \beta c_t \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} \Big(\frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) z_{t+1}^{\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\alpha}{\phi-\alpha}-1} + \beta\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}}
\end{aligned}$$

and evaluated at the steady state this is

$$\begin{aligned}
\frac{\partial}{\partial k_{t+1}} V_t^* = &\Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1 + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
&+ \beta\Big\{k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}\alpha\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}}\Big(\alpha - 1 + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) k_*^{\alpha-2+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
&+ (-1)\alpha\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
&+ \beta(-1) \\
&- \beta\Big\{k_*^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} - \delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{1+\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}}\Big(\frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) k_*^{\frac{(\alpha-1)\alpha}{\phi-\alpha}-1} + \beta\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}}
\end{aligned}$$

The next derivative is given by

$$\begin{aligned}
\frac{\partial}{\partial k_t} V_t = \beta\Big\{&\Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\} \\
&\alpha\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
&+ \beta\Big\{\Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\} \\
&- \beta\Big\{\Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\} \\
&\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_{t+1}^{\frac{(\alpha-1)\alpha}{\phi-\alpha}}
\end{aligned}$$

and evaluated at the steady state this is

$$\begin{aligned}
\frac{\partial}{\partial k_t} V_* = \beta\Big\{&\Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}\alpha\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \\
&+ \beta\Big\{\Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\} \\
&- \beta\Big\{\Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) k_*^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}\delta\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} k_*^{\frac{(\alpha-1)\alpha}{\phi-\alpha}}
\end{aligned}$$

Note: $\frac{\partial}{\partial k_t} c_t = \Big\{\Big(\alpha + \frac{(\alpha-1)\alpha}{\phi-\alpha}\Big) z_t^{1-\alpha+\frac{(1-\alpha)\alpha}{\phi-\alpha}} k_t^{\alpha-1+\frac{(\alpha-1)\alpha}{\phi-\alpha}} \Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} + 1 - \delta\Big(1 + \frac{(\alpha-1)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_t^{\frac{(1-\alpha)\phi}{\phi-\alpha}} k_t^{\frac{(\alpha-1)\phi}{\phi-\alpha}}\Big\}$
Next we have the derivatives w.r.t. the technology shocks

$$\frac{\partial}{\partial z_{t+1}} V_t = \Big(1 - \alpha + \frac{(1-\alpha)\alpha}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\alpha}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\alpha}{\phi-\alpha}-\alpha} k_{t+1}^{\alpha+\frac{(\alpha-1)\alpha}{\phi-\alpha}} - \delta\Big(\frac{(1-\alpha)\phi}{\phi-\alpha}\Big)\Big(\frac{\alpha}{\delta\phi}\Big)^{\frac{\phi}{\phi-\alpha}} z_{t+1}^{\frac{(1-\alpha)\phi}{\phi-\alpha}-1} k_{t+1}^{\frac{(\alpha-1)\phi}{\phi-\alpha}}$$

Dynare: Policy Function using Parameter Set (i)

$$k_t = \underbrace{60.62}_{=G_0} + \underbrace{0.974}_{=G_1}(k_{t-1} - \bar{k}) + \underbrace{2.055}_{=G_2}(x_t - \bar{x})$$