

# Macroeconometrics, Empirical project

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# 1 Idea

We want to look at the relationship between certain prices and the respective search interest on Google for these prices. Can we find granger causality for this relationship? What are possible issues? For example: modern trading algorithms scrape data from the internet and then buy or sell based on the sentiment. Large spikes in search interest may trigger such algorithms. As media spreads the news of price increases more people will look up prices of goods and commodities, again triggering the algorithms. This is basically a feedback loop.

# 2 Data

First some notes on the data. The data on the search index of certain prices is taken from Google trends which collects the search queries of people within a specific region (here: United States of America). This data is aggregated on a monthly basis and normalized with a range from zero to 100. Already filtered out are duplicate searches in the sense that the same user made the same search multiple times within a short time-frame. This way we exclude the users which have already invested and constantly checked the prices to look how their investment is doing. Data points are divided by total searches for the month and region to represent the relative popularity, i.e. no over-weighting of regions with more people than others which would, given the same search behavior, lead to differing popularities otherwise.

The data on gold prices comes from the London Bullion Market Association Gold Price and the Federal Reserve Bank of St. Louis. It is measured in USD per troy, daily at 3:00pm. Aggregation is done via prices at the end of each month and it is not seasonally adjusted.

In parts of this project we scale the price and search index to a range from zero to one in order to compare the relative movements more easily.

# 3 Project Code

```
# clear workspace
rm(list=ls())

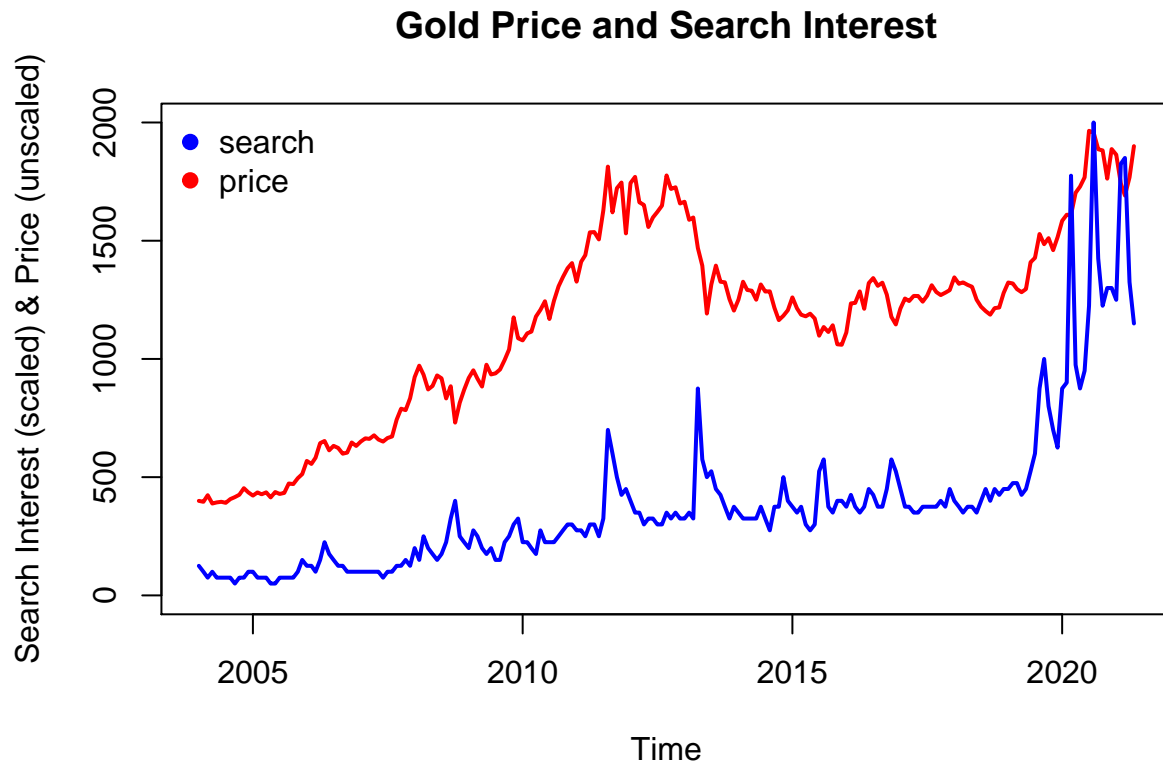
# load needed libraries
library(readr)
library(vars)
library(tseries)
library(rugarch)

# set working directory
#setwd("/Users/samue/Downloads/Studium/Economics (Master - Vienna)/2. Semester/Macroeconometrics/Project")

# import search trends
data <- read_csv("btc-vs-gold-2004.csv", col_types = cols(Month = col_date(format = "%Y-%m")))
# import prices data:
gold_pr <- read_csv("gold-2004.csv", col_types = cols(
  DATE = col_date(format = "%Y-%m-%d")))
# import high-frequency prices for gold:
gold_HF <- read_csv('gold-2001-HF.csv', col_types = cols(
  DATE = col_date(format = '%Y-%m-%d'), GOLDPMGBD228NLBM = col_double()))

# renaming variables
gold_price <- gold_pr$GOLDPMGBD228NLBM
gold_price_HF <- gold_HF$GOLDPMGBD228NLBM
gold_date <- gold_pr$DATE
gold_search <- data$GOLD
```

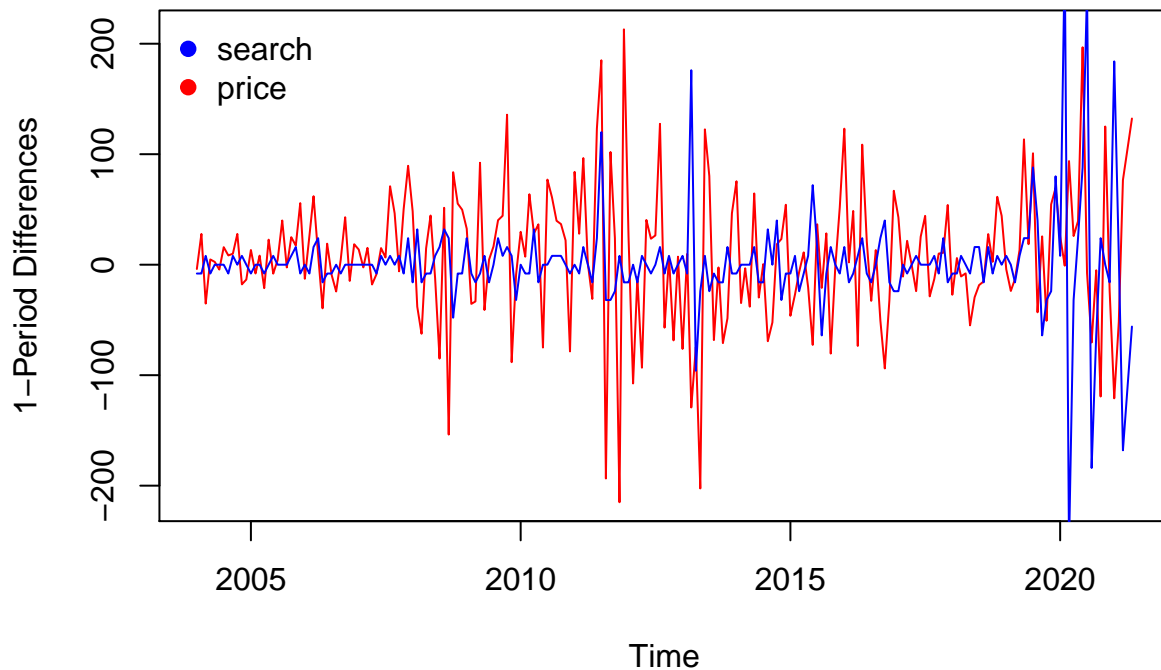
```
# plot gold price on monthly basis
plot(y=gold_price,x=gold_date,type = 'l', lwd = 2, col = 'red',
     ylim = c(0,2000), main = 'Gold Price and Search Interest',
     xlab = 'Time', ylab = 'Search Interest (scaled) & Price (unscaled)')
# add gold search interest scaled up
lines(y=25*gold_search,x=gold_date, lwd = 2, col = 'blue')
legend('topleft', legend = c('search','price'),
     col = c('blue','red'), bty = "n", pch = c(19,19))
```



```
# create first differenced prices and search interest
t <- length(gold_date)
gold_price_FD <- rep(0,t-1)
for(i in 2:209){gold_price_FD[i-1] <- gold_price[i]-gold_price[i-1]}
gold_search_FD <- rep(0,t-1)
for(i in 2:209){gold_search_FD[i-1] <- gold_search[i]-gold_search [i-1]}
t_1 <- length(gold_HF$DATE)
gold_daily_FD <- rep(0,t_1-1)
for(i in 2:5332){gold_daily_FD[i-1] <- gold_price_HF[i]-gold_price_HF[i-1]}

# plot first differenced variables
plot(y=gold_price_FD,x=gold_pr$DATE[1-209], type = 'l', lwd = 1, col = 'red',
     xlab = 'Time', ylab = '1-Period Differences',
     main = 'First Differences: Gold Price and Search Interest')
lines(y=gold_search_FD*8,x=gold_pr$DATE[1-209], lwd = 1, col = 'blue')
legend('topleft', legend = c('search','price'),
     col = c('blue','red'), bty = "n", pch = c(19,19))
```

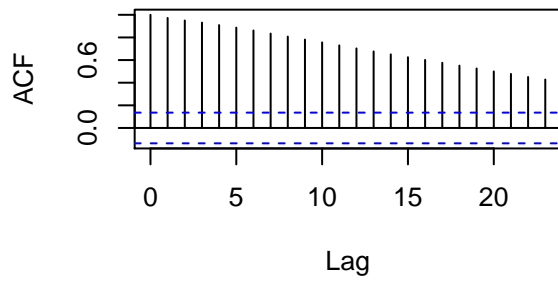
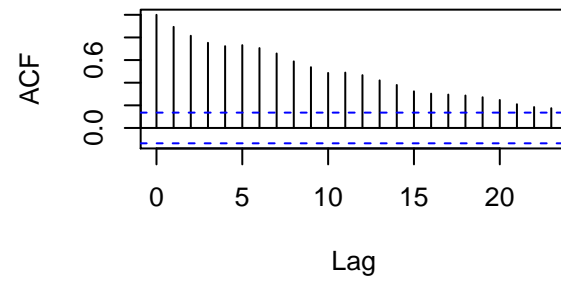
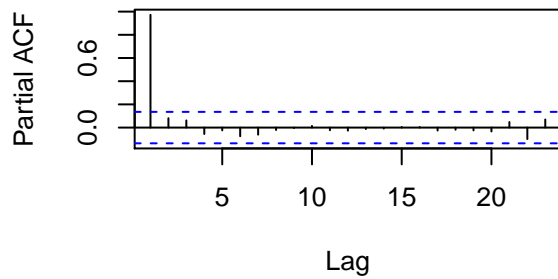
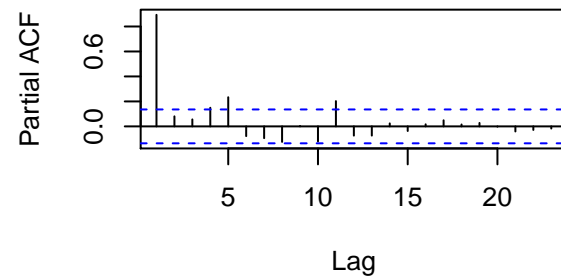
## First Differences: Gold Price and Search Interest



Visually, it appears that the more volatile periods match. An issue seems to be the scaling of the variables.

```
# plot ACF for unmodified variables:
par(mfrow=c(2,2)) # changes the plot layout to more easily compare them
acf(gold_pr$GOLDPMGBD228NLBM, main = 'ACF Gold Price')
acf(data$GOLD, main = 'ACF Gold Search Interest')

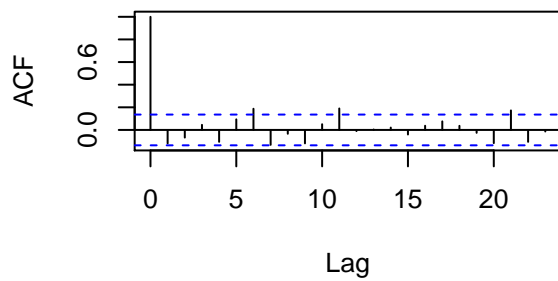
# plot PACF for unmodified variables:
pacf(gold_pr$GOLDPMGBD228NLBM, main = 'PACF Gold Price')
pacf(data$GOLD, main = 'PACF Gold Search Interest')
```

**ACF Gold Price****ACF Gold Search Interest****PACF Gold Price****PACF Gold Search Interest**

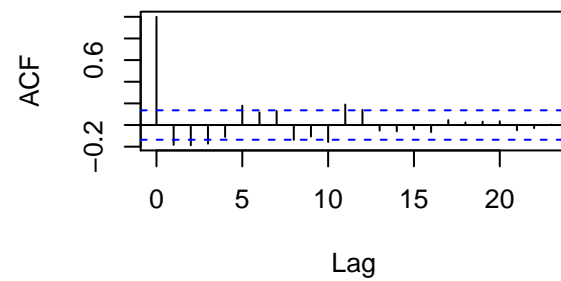
```
# plot ACF for differenced variables
acf(gold_price_FD, main = 'ACF Gold Price FD')
acf(gold_search_FD, main = 'ACF Gold Search Interest FD')

# plot PACF for differenced variables
pacf(gold_price_FD, main = 'PACF Gold Price FD')
pacf(gold_search_FD, main = 'PACF Gold Search Interest FD')
```

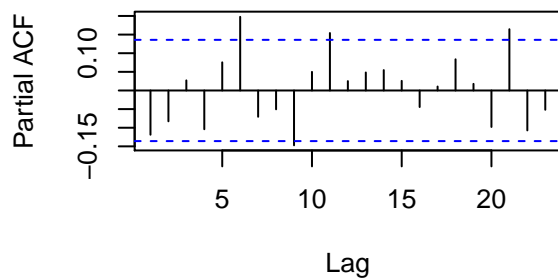
ACF Gold Price FD



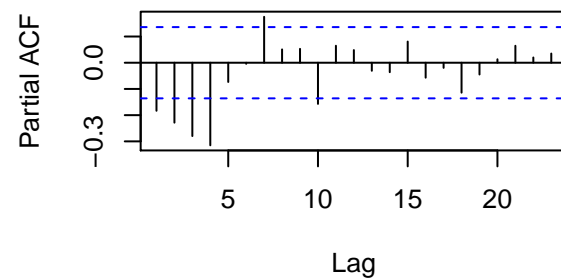
ACF Gold Search Interest FD



PACF Gold Price FD



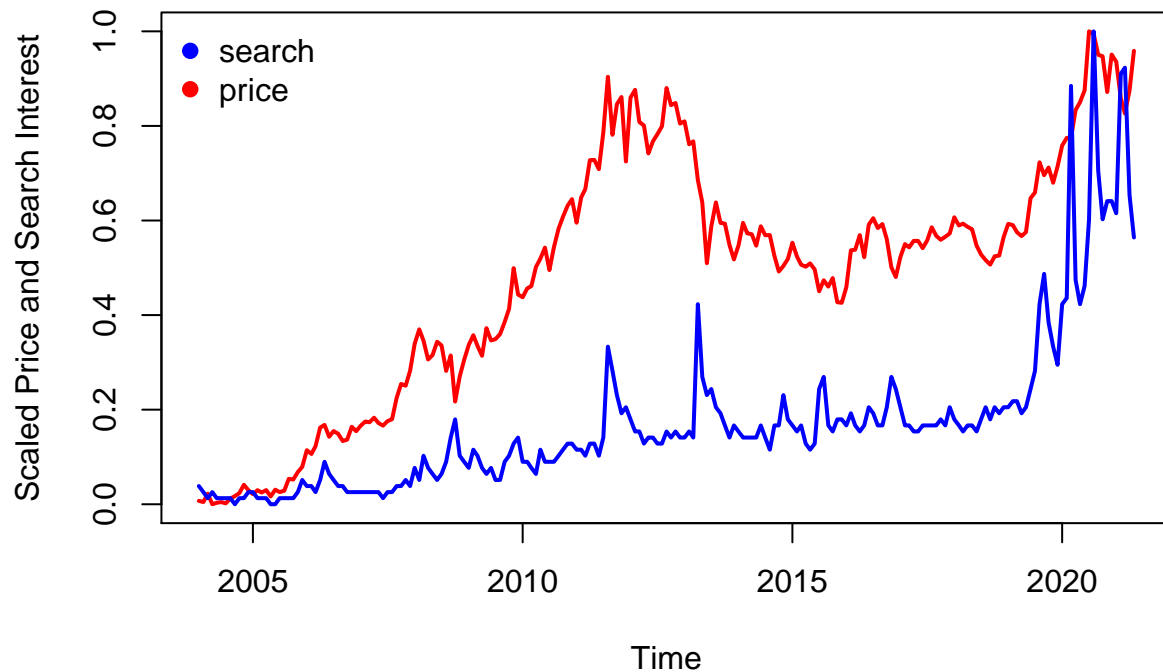
PACF Gold Search Interest FD



```
par(mfrow = c(1,1)) # revert layout changes
```

It might help with the interpretation: scale all variables  $\mathbf{X}$  such that  $X_t \in [0, 1] \forall t \in T$ .

```
range01 <- function(x){(x-min(x))/(max(x)-min(x))}
plot(y=range01(gold_pr$GOLDPMGBD228NLBM),x=gold_pr$DATE, lwd = 2, type = 'l',
     ylab = 'Scaled Price and Search Interest',
     xlab = 'Time', col = 'red')
lines(y=range01(data$GOLD),x=gold_pr$DATE, lwd = 2, col = 'blue')
legend('topleft', legend = c('search','price'),
     col = c('blue','red'), bty = "n", pch = c(19,19))
```

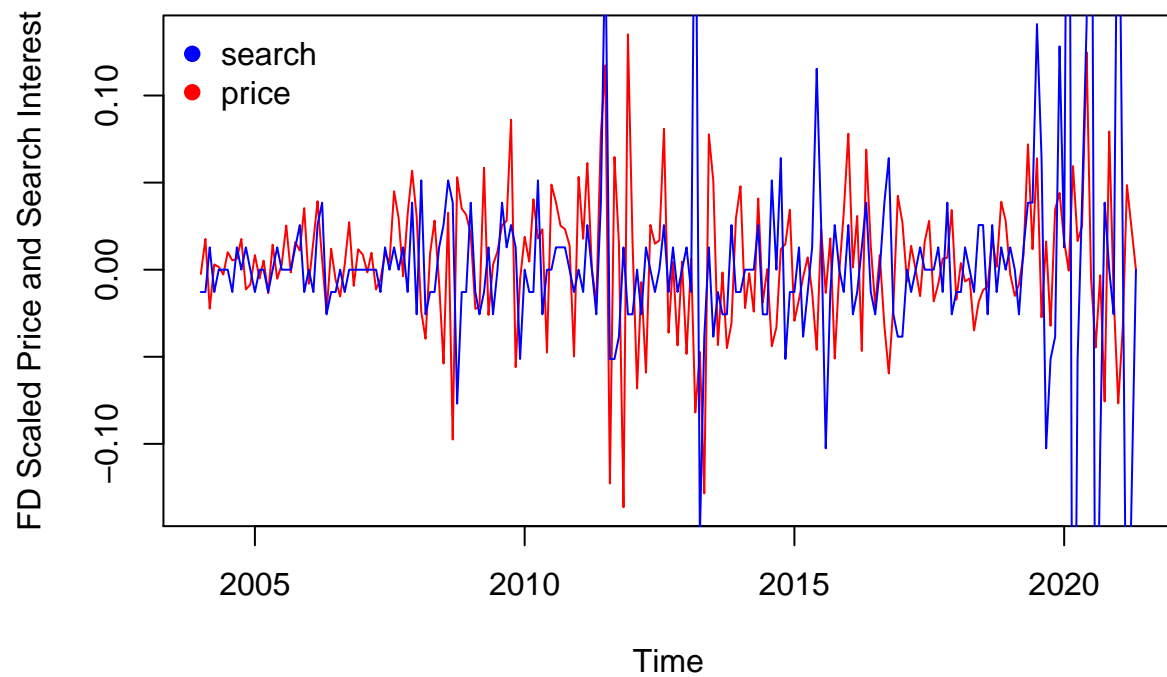


```
# save scaled variables
gold_price_scaled <- range01(gold_pr$GOLDPMGBD228NLBM)
gold_search_scaled <- range01(data$GOLD)

# create first difference on scaled variables:
gold_search_scaled_FD <- rep(0,t-1)
gold_price_scaled_FD <- rep(0,t-1)

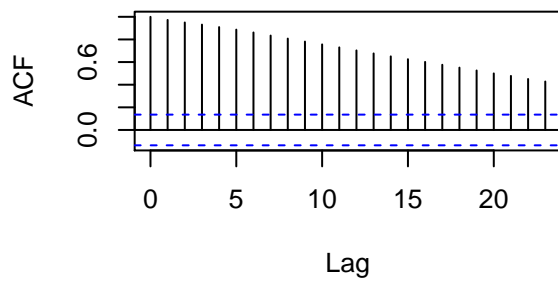
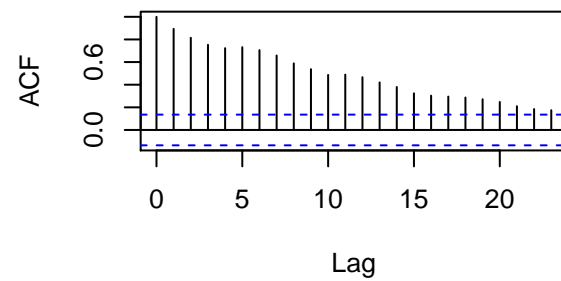
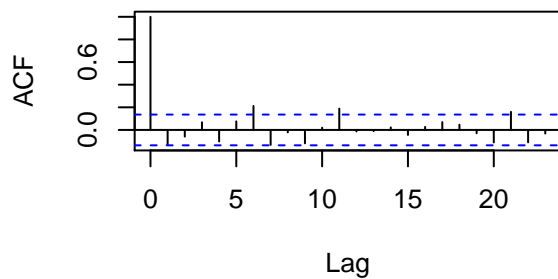
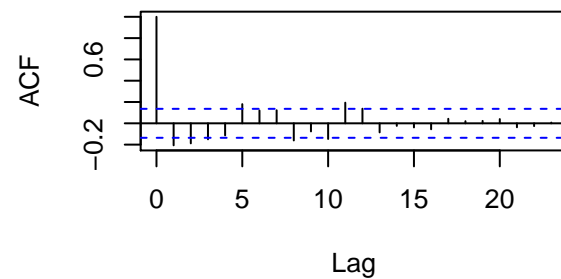
for(i in 2:t-1){
  gold_price_scaled_FD[i-1] <- gold_price_scaled[i]-gold_price_scaled[i-1]
}
for(i in 2:t-1){
  gold_search_scaled_FD[i-1] <- gold_search_scaled[i]-gold_search_scaled[i-1]
}

# plot first differenced:
plot(y=gold_price_scaled_FD, x=gold_pr$DATE[1-209], lwd = 1, type = 'l',
     ylab = 'FD Scaled Price and Search Interest',
     xlab = 'Time', col = 'red')
lines(y= gold_search_scaled_FD, x=gold_pr$DATE[1-209], lwd = 1, col = 'blue')
legend('topleft', legend = c('search','price'),
     col = c('blue','red'), bty = "n", pch = c(19,19))
```



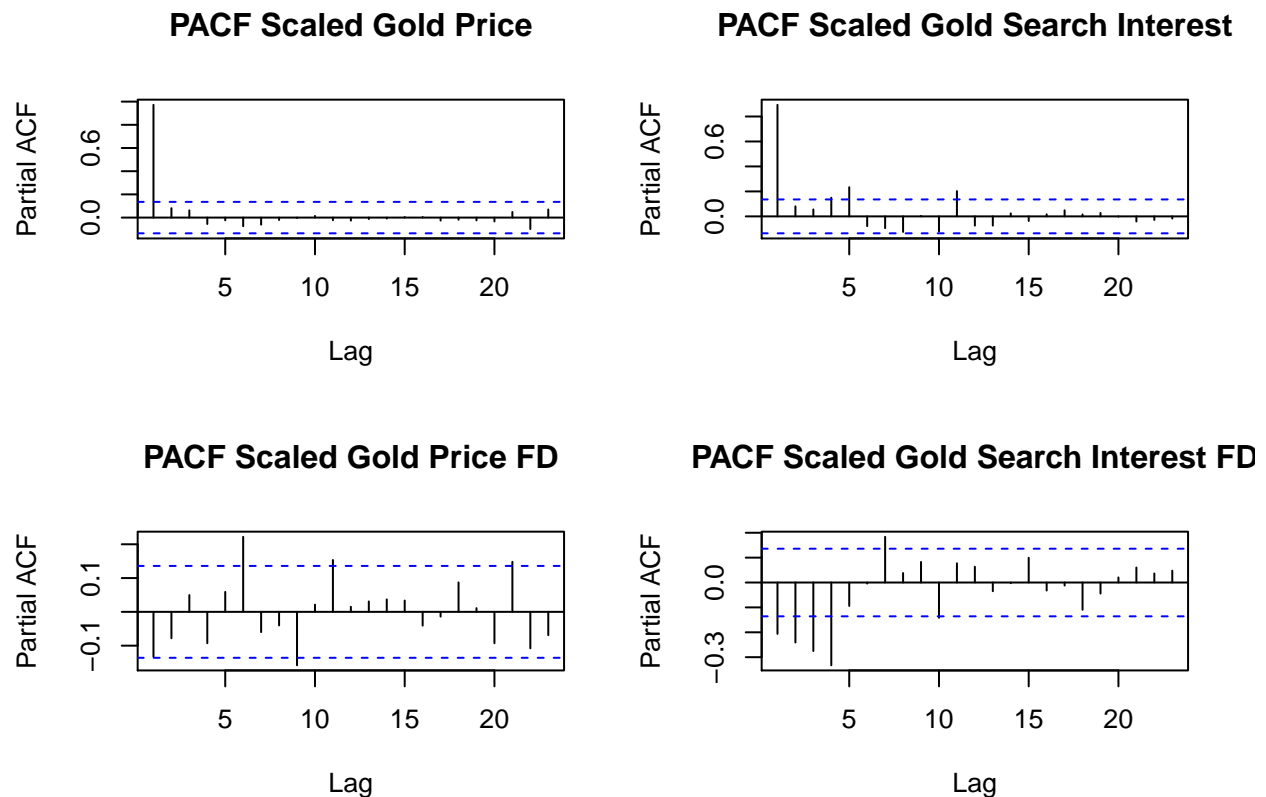
```
# plot ACFs
par(mfrow=c(2,2))      # changes the plot layout to more easily compare them
acf(gold_price_scaled, main = 'ACF Scaled Gold Price')
acf(gold_search_scaled, main = 'ACF Scaled Gold Search Interest')
acf(gold_price_scaled_FD, main = 'ACF Scaled Gold Price FD')
acf(gold_search_scaled_FD, main = 'ACF Scaled Gold Search Interest FD')
```



**ACF Scaled Gold Price****ACF Scaled Gold Search Interest****ACF Scaled Gold Price FD****ACF Scaled Gold Search Interest FD**

```
par(mfrow = c(1,1))  # revert layout changes

# plot PACFs
par(mfrow=c(2,2))    # changes the plot layout to more easily compare them
pacf(gold_price_scaled, main = 'PACF Scaled Gold Price')
pacf(gold_search_scaled, main = 'PACF Scaled Gold Search Interest')
pacf(gold_price_scaled_FD, main = 'PACF Scaled Gold Price FD')
pacf(gold_search_scaled_FD, main = 'PACF Scaled Gold Search Interest FD')
```



```
par(mfrow = c(1,1))  # revert layout changes
```

Unsurprisingly the rescaling does not matter for the autocorrelation as it is a scaled measure of linear relationships anyway.

ACF Scaled Gold Search Interest FD together with PACF Scaled Gold Search Interest FD gives evidence for an AR(4).

For the Gold Price it's ambiguous. Could be an MA, AR or an ARMA.

```
# Scaled non-differenced
```

```
# save variable vectors in time series format:
```

```
gold_price_scaled <- ts(gold_price_scaled, frequency = 12,
                        start = c(2004, 1), end = c(2021, 5))
```

```
gold_search_scaled <- ts(gold_search_scaled, frequency = 12,
                         start = c(2004,1), end = c(2021,5))
```

```
# set up data for estimation using `VAR()``
```

```
VAR_data_scaled <- window(ts.union(gold_price_scaled, gold_search_scaled),
                          start = c(2004, 1), end = c(2021, 5))
```

```
# estimate model coefficients using `VAR()``
```

```
VAR_est_scaled <- VAR(y = VAR_data_scaled, p = 6)  # lag order 6 is a guess
summary(VAR_est_scaled)
```

```
##
```

```
## VAR Estimation Results:
```

```

## =====
## Endogenous variables: gold_price_scaled, gold_search_scaled
## Deterministic variables: const
## Sample size: 203
## Log Likelihood: 654.89
## Roots of the characteristic polynomial:
## 1.022 0.9765 0.8811 0.8811 0.6951 0.6951 0.6883 0.6883 0.5855 0.5855 0.5225 0.5154
## Call:
## VAR(y = VAR_data_scaled, p = 6)
##
##
## Estimation results for equation gold_price_scaled:
## =====
## gold_price_scaled = gold_price_scaled.l1 + gold_search_scaled.l1 + gold_price_scaled.l2 + gold_search_scaled.l2 + gold_price_scaled.l3 + gold_search_scaled.l3 + gold_price_scaled.l4 + gold_search_scaled.l4 + gold_price_scaled.l5 + gold_search_scaled.l5 + gold_price_scaled.l6 + gold_search_scaled.l6 + const
##
##               Estimate Std. Error t value Pr(>|t|)
## gold_price_scaled.l1  0.867262   0.073103  11.864   <2e-16 ***
## gold_search_scaled.l1 -0.011567   0.045376  -0.255   0.7991
## gold_price_scaled.l2  0.056618   0.097182   0.583   0.5609
## gold_search_scaled.l2  0.002344   0.049475   0.047   0.9623
## gold_price_scaled.l3  0.098047   0.096568   1.015   0.3112
## gold_search_scaled.l3  0.003890   0.050533   0.077   0.9387
## gold_price_scaled.l4 -0.131698   0.096470  -1.365   0.1738
## gold_search_scaled.l4  0.098422   0.051738   1.902   0.0586 .
## gold_price_scaled.l5  0.133311   0.096784   1.377   0.1700
## gold_search_scaled.l5  0.001141   0.053101   0.021   0.9829
## gold_price_scaled.l6 -0.051117   0.075102  -0.681   0.4969
## gold_search_scaled.l6 -0.058714   0.050199  -1.170   0.2436
## const                0.012900   0.006248   2.065   0.0403 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.03952 on 190 degrees of freedom
## Multiple R-Squared: 0.9768, Adjusted R-squared: 0.9754
## F-statistic: 667.2 on 12 and 190 DF, p-value: < 2.2e-16
##
##
## Estimation results for equation gold_search_scaled:
## =====
## gold_search_scaled = gold_price_scaled.l1 + gold_search_scaled.l1 + gold_price_scaled.l2 + gold_search_scaled.l2 + gold_price_scaled.l3 + gold_search_scaled.l3 + gold_price_scaled.l4 + gold_search_scaled.l4 + gold_price_scaled.l5 + gold_search_scaled.l5 + gold_price_scaled.l6 + gold_search_scaled.l6 + const
##
##               Estimate Std. Error t value Pr(>|t|)
## gold_price_scaled.l1  0.245537   0.116443   2.109 0.036285 *
## gold_search_scaled.l1  0.505928   0.072278   7.000 4.29e-11 ***
## gold_price_scaled.l2 -0.040999   0.154798  -0.265 0.791406
## gold_search_scaled.l2 -0.032792   0.078806  -0.416 0.677798
## gold_price_scaled.l3  0.012483   0.153820   0.081 0.935403
## gold_search_scaled.l3  0.009076   0.080492   0.113 0.910341
## gold_price_scaled.l4 -0.110605   0.153663  -0.720 0.472540
## gold_search_scaled.l4  0.092291   0.082412   1.120 0.264180
## gold_price_scaled.l5  0.057624   0.154164   0.374 0.708981
## gold_search_scaled.l5  0.318943   0.084583   3.771 0.000217 ***
## gold_price_scaled.l6 -0.145838   0.119627  -1.219 0.224314

```

```
## gold_search_scaled.l6 0.152875 0.079960 1.912 0.057394 .
## const -0.009339 0.009953 -0.938 0.349260
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.06296 on 190 degrees of freedom
## Multiple R-Squared: 0.8743, Adjusted R-squared: 0.8664
## F-statistic: 110.1 on 12 and 190 DF, p-value: < 2.2e-16
##
##
## Covariance matrix of residuals:
##               gold_price_scaled gold_search_scaled
## gold_price_scaled      0.0015621      -0.0001397
## gold_search_scaled     -0.0001397      0.0039634
##
## Correlation matrix of residuals:
##               gold_price_scaled gold_search_scaled
## gold_price_scaled      1.00000      -0.05616
## gold_search_scaled     -0.05616      1.00000
# augmented DF test with a trend on gold price
df_test_gold_price <- urca::ur.df(gold_price_scaled, type = c('trend'),
                                selectlags = 'AIC')
summary(df_test_gold_price)

##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression trend
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.134978 -0.021219 -0.001006  0.022695  0.130576
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.833e-03  5.949e-03   1.485   0.139
## z.lag.1     -2.821e-02  1.742e-02  -1.620   0.107
## tt           9.382e-05  7.500e-05   1.251   0.212
## z.diff.lag  -1.077e-01  7.042e-02  -1.529   0.128
##
## Residual standard error: 0.0394 on 203 degrees of freedom
## Multiple R-squared: 0.02698, Adjusted R-squared: 0.0126
## F-statistic: 1.876 on 3 and 203 DF, p-value: 0.1348
##
##
## Value of test-statistic is: -1.6196 2.0259 1.3129
```

```
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau3 -3.99 -3.43 -3.13
## phi2  6.22  4.75  4.07
## phi3  8.43  6.49  5.47

# augmented DF test with a trend on gold search
df_test_gold_search <- urca::ur.df(gold_search_scaled, type = c('trend'),
                                  selectlags = 'AIC')
summary(df_test_gold_search)

##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression trend
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.28046 -0.02637 -0.00703  0.00958  0.45812
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.0101620  0.0101515  -1.001  0.318001
## z.lag.1      -0.1821778  0.0453131  -4.020  8.19e-05 ***
## tt           0.0004192  0.0001244   3.371  0.000897 ***
## z.diff.lag   -0.0896148  0.0708442  -1.265  0.207337
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.07057 on 203 degrees of freedom
## Multiple R-squared:  0.1063, Adjusted R-squared:  0.09308
## F-statistic: 8.047 on 3 and 203 DF,  p-value: 4.307e-05
##
##
## Value of test-statistic is: -4.0204 5.6202 8.2243
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau3 -3.99 -3.43 -3.13
## phi2  6.22  4.75  4.07
## phi3  8.43  6.49  5.47
```

For the gold price we cannot reject the null of a non-stationary process (the random walk with drift+trend is the null), seems to fit conventional wisdom on prices. Prices are often thought about as following a random walk and thus being non-stationary. For the gold search index, we reject the  $H_0$ , indicating a stationary process, given the data.

For gold price, we look at difference-stationarity.

```

# Scaled First-differences

# save variable vectors as time series format:
gold_price_scaled_FD <- ts(gold_price_scaled_FD, frequency = 12,
                           start = c(2004, 2), end = c(2021, 5))      #excluding first observation.
gold_search_scaled_FD <- ts(gold_search_scaled, frequency = 12,
                            start = c(2004,2), end = c(2021,5))

# set up data for estimation using `VAR()`
VAR_data_scaled_FD <- window(ts.union(gold_price_scaled_FD, gold_search_scaled_FD),
                             start = c(2004, 2), end = c(2021, 5))

# estimate model coefficients using `VAR()`
VAR_est_scaled_FD <- VAR(y = VAR_data_scaled_FD, p = 5)               #lag order 6 is a guess
summary(VAR_est_scaled_FD)

##
## VAR Estimation Results:
## =====
## Endogenous variables: gold_price_scaled_FD, gold_search_scaled_FD
## Deterministic variables: const
## Sample size: 203
## Log Likelihood: 655.09
## Roots of the characteristic polynomial:
## 1.023 0.8636 0.8636 0.7413 0.7413 0.613 0.613 0.4371 0.4244 0.4244
## Call:
## VAR(y = VAR_data_scaled_FD, p = 5)
##
##
## Estimation results for equation gold_price_scaled_FD:
## =====
## gold_price_scaled_FD = gold_price_scaled_FD.l1 + gold_search_scaled_FD.l1 + gold_price_scaled_FD.l2 +
##
##
##               Estimate Std. Error t value Pr(>|t|)
## gold_price_scaled_FD.l1 -0.134490   0.071705  -1.876   0.0622 .
## gold_search_scaled_FD.l1 -0.018280   0.042583  -0.429   0.6682
## gold_price_scaled_FD.l2 -0.055488   0.072502  -0.765   0.4450
## gold_search_scaled_FD.l2 -0.020997   0.049795  -0.422   0.6737
## gold_price_scaled_FD.l3  0.057081   0.072767   0.784   0.4338
## gold_search_scaled_FD.l3  0.111142   0.050858   2.185   0.0301 *
## gold_price_scaled_FD.l4 -0.077790   0.074279  -1.047   0.2963
## gold_search_scaled_FD.l4 -0.006915   0.052089  -0.133   0.8945
## gold_price_scaled_FD.l5  0.028470   0.074084   0.384   0.7012
## gold_search_scaled_FD.l5 -0.075807   0.046115  -1.644   0.1018
## const                   0.006617   0.004237   1.562   0.1200
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.03903 on 192 degrees of freedom
## Multiple R-Squared: 0.07655, Adjusted R-squared: 0.02846
## F-statistic: 1.592 on 10 and 192 DF,  p-value: 0.1115
##
##

```

```
## Estimation results for equation gold_search_scaled_FD:
## =====
## gold_search_scaled_FD = gold_price_scaled_FD.l1 + gold_search_scaled_FD.l1 + gold_price_scaled_FD.l2
##
##               Estimate Std. Error t value Pr(>|t|)
## gold_price_scaled_FD.l1 -0.079646  0.115627 -0.689  0.4918
## gold_search_scaled_FD.l1  0.533597  0.068666  7.771 4.59e-13 ***
## gold_price_scaled_FD.l2  0.226373  0.116912  1.936  0.0543 .
## gold_search_scaled_FD.l2  0.009454  0.080296  0.118  0.9064
## gold_price_scaled_FD.l3  0.216376  0.117339  1.844  0.0667 .
## gold_search_scaled_FD.l3  0.044653  0.082010  0.544  0.5867
## gold_price_scaled_FD.l4  0.163597  0.119777  1.366  0.1736
## gold_search_scaled_FD.l4  0.093875  0.083995  1.118  0.2651
## gold_price_scaled_FD.l5  0.018623  0.119462  0.156  0.8763
## gold_search_scaled_FD.l5  0.386010  0.074361  5.191 5.30e-07 ***
## const                -0.003414  0.006833 -0.500  0.6179
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.06293 on 192 degrees of freedom
## Multiple R-Squared: 0.8703, Adjusted R-squared: 0.8636
## F-statistic: 128.9 on 10 and 192 DF, p-value: < 2.2e-16
##
## Covariance matrix of residuals:
##               gold_price_scaled_FD gold_search_scaled_FD
## gold_price_scaled_FD          1.523e-03          5.586e-06
## gold_search_scaled_FD          5.586e-06          3.960e-03
##
## Correlation matrix of residuals:
##               gold_price_scaled_FD gold_search_scaled_FD
## gold_price_scaled_FD          1.000000          0.002274
## gold_search_scaled_FD          0.002274          1.000000
##
## augmented df test on only the differenced gold price
df_test_gold_price_FD <- urca::ur.df(gold_price_scaled_FD, type = 'none',
                                     selectlags = 'AIC')
summary(df_test_gold_price_FD)
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression none
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.139705 -0.013946  0.004984  0.026891  0.129077
```

```
##
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## z.lag.1    -1.19119    0.10475 -11.372  <2e-16 ***
## z.diff.lag  0.06367    0.07010   0.908   0.365
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0393 on 204 degrees of freedom
## Multiple R-squared:  0.562, Adjusted R-squared:  0.5577
## F-statistic: 130.9 on 2 and 204 DF, p-value: < 2.2e-16
##
##
## Value of test-statistic is: -11.3722
##
## Critical values for test statistics:
##          1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

As the DF-test for the first-difference gold price rejects, we cannot say that the data is not stationary. Which gives evidence for the gold price being an I(1) process.

*# Unscaled Non-differenced*

```
# save variable vectors as time series format:
gold_price <- ts(gold_price, frequency = 12,
                 start = c(2004, 1), end = c(2021, 5))
gold_search <- ts(gold_search, frequency = 12,
                  start = c(2004,1), end = c(2021,5))

# set up data for estimation using `VAR()`
VAR_data <- window(ts.union(gold_price, gold_search),
                   start = c(2004, 1), end = c(2021, 5))

# estimate model coefficients using `VAR()`
VAR_est <- VAR(y = VAR_data, p = 6, type = 'both') #lag order 6 is a guess
summary(VAR_est)
```

```
##
## VAR Estimation Results:
## =====
## Endogenous variables: gold_price, gold_search
## Deterministic variables: both
## Sample size: 203
## Log Likelihood: -1723.474
## Roots of the characteristic polynomial:
## 1.009 0.9752 0.8789 0.8789 0.6993 0.6993 0.6893 0.6893 0.5958 0.5958 0.5399 0.5202
## Call:
## VAR(y = VAR_data, p = 6, type = "both")
##
##
## Estimation results for equation gold_price:
## =====
## gold_price = gold_price.l1 + gold_search.l1 + gold_price.l2 + gold_search.l2 + gold_price.l3 + gold_
```



```

##               Estimate Std. Error t value Pr(>|t|)
## gold_price.l1  0.865221  0.073328  11.799  <2e-16 ***
## gold_search.l1 -0.265052  0.920471  -0.288  0.7737
## gold_price.l2  0.056172  0.097363   0.577  0.5647
## gold_search.l2  0.028097  1.002323   0.028  0.9777
## gold_price.l3  0.098312  0.096746   1.016  0.3108
## gold_search.l3  0.045693  1.024869   0.045  0.9645
## gold_price.l4 -0.130782  0.096660  -1.353  0.1777
## gold_search.l4  1.946623  1.050342   1.853  0.0654 .
## gold_price.l5  0.133317  0.096961   1.375  0.1708
## gold_search.l5  0.005676  1.075607   0.005  0.9958
## gold_price.l6 -0.054977  0.075559  -0.728  0.4678
## gold_search.l6 -1.221659  1.018341  -1.200  0.2318
## const         29.843692  14.758319   2.022  0.0446 *
## trend         0.076827  0.138371   0.555  0.5794
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 62.42 on 189 degrees of freedom
## Multiple R-Squared:  0.9769, Adjusted R-squared:  0.9753
## F-statistic: 613.6 on 13 and 189 DF, p-value: < 2.2e-16
##
##
## Estimation results for equation gold_search:
## =====
## gold_search = gold_price.l1 + gold_search.l1 + gold_price.l2 + gold_search.l2 + gold_price.l3 + gold
##
##               Estimate Std. Error t value Pr(>|t|)
## gold_price.l1  0.0118659  0.0057694   2.057  0.041090 *
## gold_search.l1  0.5015848  0.0724220  6.926  6.59e-11 ***
## gold_price.l2 -0.0020905  0.0076604  -0.273  0.785228
## gold_search.l2 -0.0354688  0.0788620  -0.450  0.653401
## gold_price.l3  0.0006544  0.0076119   0.086  0.931578
## gold_search.l3  0.0045059  0.0806360   0.056  0.955497
## gold_price.l4 -0.0053455  0.0076052  -0.703  0.482998
## gold_search.l4  0.0863897  0.0826401   1.045  0.297186
## gold_price.l5  0.0028521  0.0076288   0.374  0.708930
## gold_search.l5  0.3165304  0.0846280  3.740  0.000244 ***
## gold_price.l6 -0.0077520  0.0059450  -1.304  0.193833
## gold_search.l6  0.1480114  0.0801223   1.847  0.066264 .
## const        -1.1391321  1.1611737  -0.981  0.327838
## trend         0.0106651  0.0108870   0.980  0.328524
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 4.911 on 189 degrees of freedom
## Multiple R-Squared:  0.8749, Adjusted R-squared:  0.8663
## F-statistic: 101.7 on 13 and 189 DF, p-value: < 2.2e-16
##
##
## Covariance matrix of residuals:

```

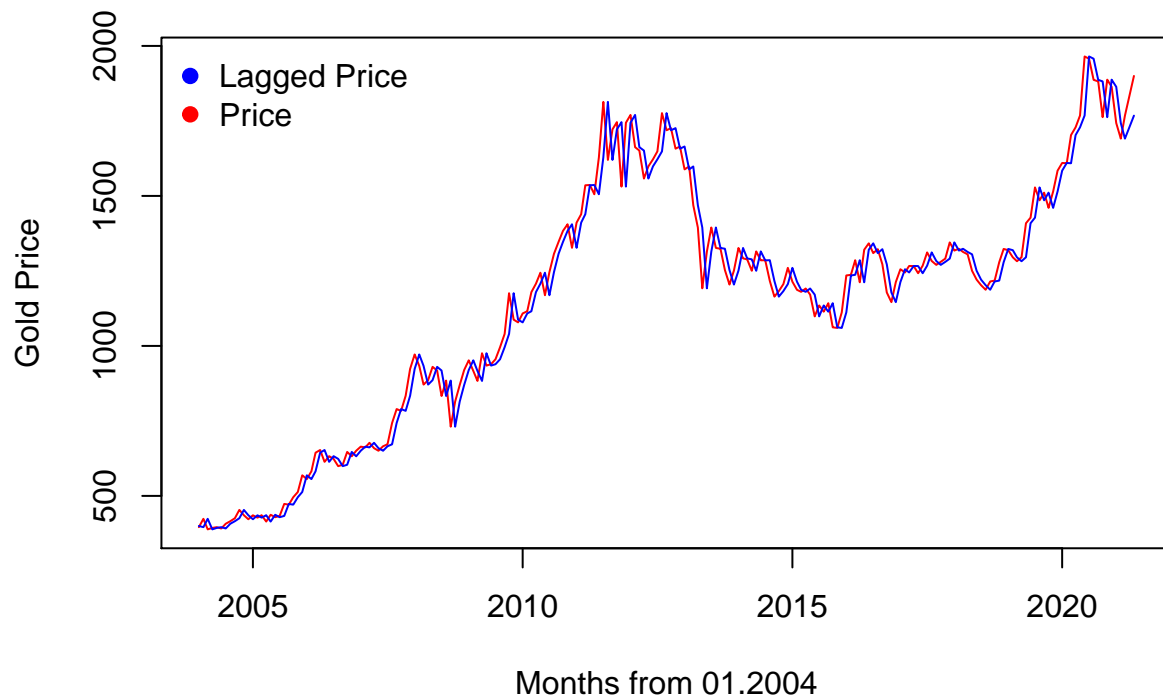
```
##          gold_price gold_search
## gold_price    3896.12    -18.16
## gold_search    -18.16     24.12
##
## Correlation matrix of residuals:
##          gold_price gold_search
## gold_price    1.00000    -0.05922
## gold_search   -0.05922     1.00000

#####
##### Sollten wir hier beim AR(1) nicht die First differences verwenden.
# Weil wir ja einen I(1) prozess haben. Und sollten wir nicht einfach mit
# dem besten ARMA modell arbeiten und nicht AR(1) ? #####
#####

# compare the VAR to the AR(1) model for the prices
T <-length(gold_price)
gold_price_2 <- as.numeric(gold_price[-1])
gold_price_lagged <- as.numeric(gold_price[-T])

plot(y=gold_price_2,x=gold_pr$DATE[1-209], type = 'l', lwd = 1, col = 'red',
     main = 'Gold Price and Lagged Gold Price',
     ylab = 'Gold Price', xlab = 'Months from 01.2004')
lines(y=gold_price_lagged,x=gold_pr$DATE[1-209], lwd = 1, col = 'blue')
legend('topleft', legend = c('Lagged Price','Price'),
     col = c('blue','red'), bty = "n", pch = c(19,19))
```

## Gold Price and Lagged Gold Price



```

# estimate AR(1) model
gold_price_AR1 <- lm(gold_price_2 ~ gold_price_lagged)
# estimate robust standard errors
coeftest(gold_price_AR1, vcov. = vcovHC, type = "HC1")

##
## t test of coefficients:
##
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    20.49216   10.59726   1.9337  0.05452 .
## gold_price_lagged  0.98841    0.01114  88.7232 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

gold_price_AR1_check <- arima(gold_price, order = c(1,0,0)) # check if we did correctly
summary(gold_price_AR1_check) #same

##           Length Class  Mode
## coef           2  -none- numeric
## sigma2          1  -none- numeric
## var.coef        4  -none- numeric
## mask            2  -none- logical
## loglik           1  -none- numeric
## aic              1  -none- numeric
## arma            7  -none- numeric
## residuals 209    ts      numeric
## call            3  -none- call
## series          1  -none- character
## code            1  -none- numeric
## n.cond          1  -none- numeric
## nobs            1  -none- numeric
## model           10 -none- list

# estimate MA(1) model, Auto ARIMA suggests an MA(0,1,1), see below
gold_price_MA1 <- arima(gold_price, order = c(0,0,1))
summary(gold_price_MA1)

##           Length Class  Mode
## coef           2  -none- numeric
## sigma2          1  -none- numeric
## var.coef        4  -none- numeric
## mask            2  -none- logical
## loglik           1  -none- numeric
## aic              1  -none- numeric
## arma            7  -none- numeric
## residuals 209    ts      numeric
## call            3  -none- call
## series          1  -none- character
## code            1  -none- numeric
## n.cond          1  -none- numeric
## nobs            1  -none- numeric
## model           10 -none- list

# Unscaled First-difference

# save variable vectors as time series format:

```

```

gold_price_FD <- ts(gold_price_FD, frequency = 12,
                    start = c(2004, 2), end = c(2021, 5))      # excluding first observation.
gold_search_FD <- ts(gold_search_FD, frequency = 12,
                     start = c(2004,2), end = c(2021,5))

# set up data for estimation using `VAR()`
VAR_data_FD <- window(ts.union(gold_price_FD, gold_search_FD),
                      start = c(2004, 2), end = c(2021, 5))

# estimate model coefficients using `VAR()`
VAR_est_FD <- VAR(y = VAR_data_FD, p = 6, type = 'both')      # lag order 6 is a guess
summary(VAR_est_FD)

##
## VAR Estimation Results:
## =====
## Endogenous variables: gold_price_FD, gold_search_FD
## Deterministic variables: both
## Sample size: 202
## Log Likelihood: -1712.892
## Roots of the characteristic polynomial:
## 0.8877 0.8877 0.7911 0.7911 0.7672 0.7672 0.76 0.5282 0.5282 0.492 0.3699 0.3699
## Call:
## VAR(y = VAR_data_FD, p = 6, type = "both")
##
##
## Estimation results for equation gold_price_FD:
## =====
## gold_price_FD = gold_price_FD.l1 + gold_search_FD.l1 + gold_price_FD.l2 + gold_search_FD.l2 + gold_p
##
##              Estimate Std. Error t value Pr(>|t|)
## gold_price_FD.l1 -0.135937  0.072487  -1.875  0.0623 .
## gold_search_FD.l1 -0.395049  0.921696  -0.429  0.6687
## gold_price_FD.l2 -0.053191  0.073979  -0.719  0.4730
## gold_search_FD.l2 -0.002274  1.038624  -0.002  0.9983
## gold_price_FD.l3  0.028746  0.074088   0.388  0.6985
## gold_search_FD.l3  0.016961  1.085642   0.016  0.9876
## gold_price_FD.l4 -0.082810  0.075108  -1.103  0.2716
## gold_search_FD.l4  1.776604  1.121423   1.584  0.1148
## gold_price_FD.l5  0.068094  0.075632   0.900  0.3691
## gold_search_FD.l5  1.361571  1.117061   1.219  0.2244
## gold_price_FD.l6  0.190986  0.075474   2.531  0.0122 *
## gold_search_FD.l6  1.005578  0.982060   1.024  0.3072
## const            9.063697  9.412017   0.963  0.3368
## trend            -0.024209  0.078188  -0.310  0.7572
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 61.73 on 188 degrees of freedom
## Multiple R-Squared: 0.1073, Adjusted R-squared: 0.04555
## F-statistic: 1.738 on 13 and 188 DF, p-value: 0.05617
##
##

```

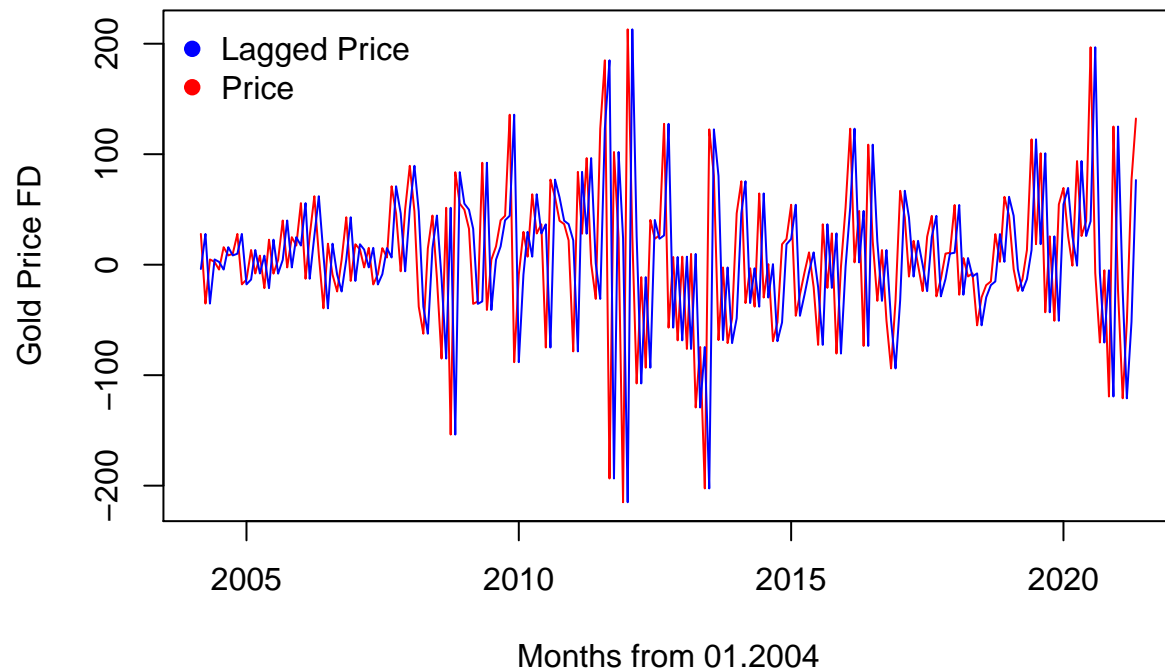
```
## Estimation results for equation gold_search_FD:
## =====
## gold_search_FD = gold_price_FD.l1 + gold_search_FD.l1 + gold_price_FD.l2 + gold_search_FD.l2 + gold_
##
##               Estimate Std. Error t value Pr(>|t|)
## gold_price_FD.l1  0.012344   0.005769   2.140  0.0337 *
## gold_search_FD.l1 -0.496526   0.073359  -6.768 1.61e-10 ***
## gold_price_FD.l2   0.009542   0.005888   1.621  0.1068
## gold_search_FD.l2 -0.548228   0.082665  -6.632 3.42e-10 ***
## gold_price_FD.l3   0.010279   0.005897   1.743  0.0829 .
## gold_search_FD.l3 -0.549871   0.086407  -6.364 1.47e-09 ***
## gold_price_FD.l4   0.004757   0.005978   0.796  0.4272
## gold_search_FD.l4 -0.466982   0.089255  -5.232 4.45e-07 ***
## gold_price_FD.l5   0.007543   0.006020   1.253  0.2117
## gold_search_FD.l5 -0.148191   0.088908  -1.667  0.0972 .
## gold_price_FD.l6  -0.004600   0.006007  -0.766  0.4448
## gold_search_FD.l6 -0.045917   0.078163  -0.587  0.5576
## const             -1.045454   0.749110  -1.396  0.1645
## trend              0.015290   0.006223   2.457  0.0149 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 4.913 on 188 degrees of freedom
## Multiple R-Squared:  0.3402, Adjusted R-squared:  0.2946
## F-statistic: 7.457 on 13 and 188 DF, p-value: 9.394e-12
##
##
## Covariance matrix of residuals:
##               gold_price_FD gold_search_FD
## gold_price_FD    3810.41      -13.72
## gold_search_FD   -13.72       24.14
##
## Correlation matrix of residuals:
##               gold_price_FD gold_search_FD
## gold_price_FD    1.00000      -0.04523
## gold_search_FD   -0.04523       1.00000
```

*# compare the VAR to the AR(1) model for prices first-differenced*

```
T <-length(gold_price_FD)
gold_price_FD_2 <- as.numeric(gold_price_FD[-1])
gold_price_FD_lagged <- as.numeric(gold_price_FD[-T])

plot(y=gold_price_FD_2,x=gold_pr$DATE[3:209], type = 'l', lwd = 1, col = 'red',
     main = 'Gold Price FD and Lagged Gold Price FD',
     ylab = 'Gold Price FD', xlab = 'Months from 01.2004')
lines(y=gold_price_FD_lagged,x=gold_pr$DATE[3:209], lwd = 1, col = 'blue')
legend('topleft', legend = c('Lagged Price','Price'),
     col = c('blue','red'), bty = "n", pch = c(19,19))
```

## Gold Price FD and Lagged Gold Price FD



```
# estimate AR(1) model
gold_price_FD_AR1 <- lm(gold_price_FD_2 ~ gold_price_FD_lagged)
# estimate robust standard errors
coeftest(gold_price_FD_AR1, vcov. = vcovHC, type = "HC1")

##
## t test of coefficients:
##
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      8.066698   4.377330   1.8428   0.0668 .
## gold_price_FD_lagged -0.121139   0.096469  -1.2557   0.2106
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

gold_price_AR1_FD_check <- arima(gold_price_FD, order = c(1,0,0)) # check if we did correctly
summary(gold_price_AR1_FD_check) # almost same

##           Length Class  Mode
## coef           2  -none- numeric
## sigma2          1  -none- numeric
## var.coef        4  -none- numeric
## mask            2  -none- logical
## loglik           1  -none- numeric
## aic              1  -none- numeric
## arma             7  -none- numeric
## residuals      208    ts    numeric
## call            3  -none- call
```

```
## series      1  -none- character
## code        1  -none- numeric
## n.cond      1  -none- numeric
## nobs        1  -none- numeric
## model       10 -none- list
```

```
# estimate MA(1)model, what Auto ARIMA suggests MA(0,1,1) for prices and MA(0,0,1) for FD prices, see b
gold_price_MA1 <- arima(gold_price, order = c(0,0,1))
summary(gold_price_MA1)
```

```
##           Length Class  Mode
## coef         2  -none- numeric
## sigma2        1  -none- numeric
## var.coef       4  -none- numeric
## mask          2  -none- logical
## loglik         1  -none- numeric
## aic            1  -none- numeric
## arma          7  -none- numeric
## residuals 209   ts      numeric
## call          3  -none- call
## series        1  -none- character
## code          1  -none- numeric
## n.cond        1  -none- numeric
## nobs          1  -none- numeric
## model         10 -none- list
```

The values on the intercept seem to differ, but the estimated coefficient on the lag seems to fit.

```
# verify the 'by-hand' results with built-in function for ARIMA
ar.ols(gold_price, order.max = 5, intercept = T)
```

```
##
## Call:
## ar.ols(x = gold_price, order.max = 5, intercept = T)
##
## Coefficients:
##      1
## 0.9884
##
## Intercept: 7.171 (4.301)
##
## Order selected 1  sigma^2 estimated as 3848
```

```
forecast::auto.arima(gold_price, ic = 'aic')
```

```
## Series: gold_price
## ARIMA(0,1,1) with drift
##
## Coefficients:
##           ma1    drift
##        -0.1411  7.1279
## s.e.    0.0740  3.6766
##
## sigma^2 estimated as 3842:  log likelihood=-1152.52
## AIC=2311.05  AICc=2311.16  BIC=2321.06
```

```
#for FD
ar.ols(gold_price_FD, order.max = 5, intercept = T) #just verifies the above ARIMA(0,1,1)

##
## Call:
## ar.ols(x = gold_price_FD, order.max = 5, intercept = T)
##
## Coefficients:
##      1
## -0.1211
##
## Intercept: -0.01952 (4.303)
##
## Order selected 1  sigma^2 estimated as  3832
forecast::auto.arima(gold_price_FD, ic = 'aic')
```

```
## Series: gold_price_FD
## ARIMA(0,0,1) with non-zero mean
##
## Coefficients:
##      ma1      mean
##    -0.1411  7.1279
## s.e.    0.0740  3.6766
##
## sigma^2 estimated as 3842:  log likelihood=-1152.52
## AIC=2311.05   AICc=2311.16   BIC=2321.06
```

The model is automated to difference such that the data is stationary, then the function finds the best forecasting model via the AIC. Here this would be an ARMA(0,1) model:

$$\widehat{\Delta \text{gold price}}_t = \underset{3.6766}{(7.1279)} + \epsilon_t + \underset{(0.0740)}{(-0.1411)}\epsilon_{t-1}$$

```
# estimate model coefficients for VAR using AIC
```

```
VAR_lag <- VAR(y = VAR_data, type = 'both', ic = 'AIC', lag.max = 15)
summary(VAR_lag)
```

```
##
## VAR Estimation Results:
## =====
## Endogenous variables: gold_price, gold_search
## Deterministic variables: both
## Sample size: 198
## Log Likelihood: -1654.705
## Roots of the characteristic polynomial:
## 0.9968 0.9968 0.9901 0.9901 0.9211 0.9211 0.9195 0.9195 0.9102 0.9102 0.8787 0.8416 0.8416 0.8333 0.8333
## Call:
## VAR(y = VAR_data, type = "both", lag.max = 15, ic = "AIC")
##
##
## Estimation results for equation gold_price:
## =====
## gold_price = gold_price.l1 + gold_search.l1 + gold_price.l2 + gold_search.l2 + gold_price.l3 + gold_search.l3 + ...
##
```



```

##               Estimate Std. Error t value Pr(>|t|)
## gold_price.l1    0.87441    0.07633  11.455 < 2e-16 ***
## gold_search.l1   -0.09896    0.97900  -0.101  0.91960
## gold_price.l2     0.08168    0.09980   0.818  0.41422
## gold_search.l2    0.43364    1.07400   0.404  0.68689
## gold_price.l3     0.09959    0.10068   0.989  0.32394
## gold_search.l3   -0.67663    1.08498  -0.624  0.53369
## gold_price.l4    -0.15542    0.10093  -1.540  0.12541
## gold_search.l4    1.87254    1.10489   1.695  0.09191 .
## gold_price.l5     0.10802    0.09939   1.087  0.27862
## gold_search.l5   -0.79868    1.17455  -0.680  0.49741
## gold_price.l6     0.16395    0.09995   1.640  0.10275
## gold_search.l6    0.19169    1.21396   0.158  0.87471
## gold_price.l7    -0.29844    0.10264  -2.908  0.00412 **
## gold_search.l7   -1.33615    1.13699  -1.175  0.24153
## gold_price.l8     0.05090    0.10696   0.476  0.63472
## gold_search.l8   -0.32111    1.15305  -0.278  0.78097
## gold_price.l9    -0.10628    0.10652  -0.998  0.31980
## gold_search.l9    1.60014    1.16680   1.371  0.17202
## gold_price.l10    0.23204    0.10700   2.169  0.03147 *
## gold_search.l10   0.65500    1.36858   0.479  0.63283
## gold_price.l11   -0.08451    0.08375  -1.009  0.31435
## gold_search.l11  -0.63300    1.25273  -0.505  0.61399
## const            29.87096   15.55263   1.921  0.05641 .
## trend            0.04859    0.15009   0.324  0.74651
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 61.6 on 174 degrees of freedom
## Multiple R-Squared:  0.9772, Adjusted R-squared:  0.9741
## F-statistic: 323.7 on 23 and 174 DF, p-value: < 2.2e-16
##
##
## Estimation results for equation gold_search:
## =====
## gold_search = gold_price.l1 + gold_search.l1 + gold_price.l2 + gold_search.l2 + gold_price.l3 + gold
##
##               Estimate Std. Error t value Pr(>|t|)
## gold_price.l1    0.008780    0.005712   1.537  0.126067
## gold_search.l1    0.494927    0.073254   6.756 2.05e-10 ***
## gold_price.l2   -0.005736    0.007468  -0.768  0.443507
## gold_search.l2   -0.067768    0.080363  -0.843  0.400230
## gold_price.l3     0.005739    0.007533   0.762  0.447230
## gold_search.l3    0.060180    0.081184   0.741  0.459524
## gold_price.l4   -0.004481    0.007552  -0.593  0.553680
## gold_search.l4    0.082621    0.082673   0.999  0.319003
## gold_price.l5     0.005676    0.007437   0.763  0.446357
## gold_search.l5    0.373837    0.087886   4.254 3.43e-05 ***
## gold_price.l6   -0.011852    0.007479  -1.585  0.114828
## gold_search.l6   -0.039448    0.090834  -0.434  0.664616
## gold_price.l7     0.022381    0.007680   2.914  0.004037 **
## gold_search.l7    0.137440    0.085075   1.616  0.108013
## gold_price.l8   -0.011973    0.008003  -1.496  0.136450

```

```
## gold_search.l8 -0.080319 0.086277 -0.931 0.353176
## gold_price.l9 0.001943 0.007970 0.244 0.807708
## gold_search.l19 -0.044945 0.087306 -0.515 0.607348
## gold_price.l10 -0.004495 0.008006 -0.561 0.575249
## gold_search.l10 -0.241279 0.102404 -2.356 0.019579 *
## gold_price.l11 -0.006814 0.006266 -1.087 0.278368
## gold_search.l11 0.342986 0.093736 3.659 0.000335 ***
## const -0.991835 1.163729 -0.852 0.395225
## trend 0.018826 0.011231 1.676 0.095485 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 4.609 on 174 degrees of freedom
## Multiple R-Squared: 0.8962, Adjusted R-squared: 0.8824
## F-statistic: 65.29 on 23 and 174 DF, p-value: < 2.2e-16
##
##
## Covariance matrix of residuals:
##      gold_price gold_search
## gold_price 3793.969 4.728
## gold_search 4.728 21.242
##
## Correlation matrix of residuals:
##      gold_price gold_search
## gold_price 1.00000 0.01665
## gold_search 0.01665 1.00000

VAR_lag_select <- VARselect(y = VAR_data, type = 'both', lag.max = 15)
VAR_lag_select

## $selection
## AIC(n) HQ(n) SC(n) FPE(n)
## 11 5 1 11
##
## $criteria
##      1      2      3      4      5
## AIC(n) 11.82853 11.82522 11.82092 11.75218 11.67071
## HQ(n) 11.88309 11.90707 11.93005 11.88860 11.83441
## SC(n) 11.96328 12.02736 12.09043 12.08907 12.07498
## FPE(n) 137110.04209 136661.36323 136081.91073 127053.33408 117127.87221
##      6      7      8      9     10
## AIC(n) 11.67736 11.66562 11.61794 11.61773 11.61210
## HQ(n) 11.86835 11.88389 11.86349 11.89056 11.91222
## SC(n) 12.14901 12.20465 12.22435 12.29152 12.35327
## FPE(n) 117932.21775 116584.51726 111191.70935 111212.74329 110642.66475
##     11     12     13     14     15
## AIC(n) 11.56747 11.56862 11.58512 11.60526 11.59713
## HQ(n) 11.89487 11.92331 11.96709 12.01451 12.03366
## SC(n) 12.37601 12.44454 12.52842 12.61594 12.67518
## FPE(n) 105875.58850 106071.11783 107923.72935 110223.26619 109448.48022

#for FD
VAR_lag_FD <- VAR(y = VAR_data_FD, type = 'both', ic = 'AIC', lag.max = 15)
```

```
summary(VAR_lag_FD)
```

```
##
## VAR Estimation Results:
## =====
## Endogenous variables: gold_price_FD, gold_search_FD
## Deterministic variables: both
## Sample size: 198
## Log Likelihood: -1656.605
## Roots of the characteristic polynomial:
## 0.9889 0.9889 0.9207 0.9207 0.9202 0.9202 0.9084 0.9084 0.8755 0.8379 0.8379 0.8311 0.8311 0.8191 0.8191
## Call:
## VAR(y = VAR_data_FD, type = "both", lag.max = 15, ic = "AIC")
##
##
## Estimation results for equation gold_price_FD:
## =====
## gold_price_FD = gold_price_FD.l1 + gold_search_FD.l1 + gold_price_FD.l2 + gold_search_FD.l2 + gold_price_FD.l3 +
## gold_search_FD.l3 + gold_price_FD.l4 + gold_search_FD.l4 + gold_price_FD.l5 + gold_search_FD.l5 + gold_price_FD.l6 +
## gold_search_FD.l6 + gold_price_FD.l7 + gold_search_FD.l7 + gold_price_FD.l8 + gold_search_FD.l8 + gold_price_FD.l9 +
## gold_search_FD.l9 + gold_price_FD.l10 + gold_search_FD.l10 + const + trend
##
##               Estimate Std. Error t value Pr(>|t|)
## gold_price_FD.l1 -0.11056    0.07605  -1.454   0.1478
## gold_search_FD.l1 -0.08358    0.98071  -0.085   0.9322
## gold_price_FD.l2 -0.02939    0.07718  -0.381   0.7038
## gold_search_FD.l2  0.40212    1.10592   0.364   0.7166
## gold_price_FD.l3  0.06943    0.07658   0.907   0.3658
## gold_search_FD.l3 -0.28147    1.22923  -0.229   0.8191
## gold_price_FD.l4 -0.08251    0.07761  -1.063   0.2892
## gold_search_FD.l4  1.52539    1.30402   1.170   0.2437
## gold_price_FD.l5  0.02844    0.07693   0.370   0.7121
## gold_search_FD.l5  0.75620    1.38864   0.545   0.5867
## gold_price_FD.l6  0.19098    0.07760   2.461   0.0148 *
## gold_search_FD.l6  0.96453    1.34815   0.715   0.4753
## gold_price_FD.l7 -0.10576    0.07960  -1.329   0.1857
## gold_search_FD.l7 -0.52307    1.30032  -0.402   0.6880
## gold_price_FD.l8 -0.05299    0.08216  -0.645   0.5198
## gold_search_FD.l8 -0.95842    1.22935  -0.780   0.4367
## gold_price_FD.l9 -0.15845    0.08239  -1.923   0.0561 .
## gold_search_FD.l9  0.62239    1.21170   0.514   0.6081
## gold_price_FD.l10  0.07567    0.08355   0.906   0.3663
## gold_search_FD.l10 1.00005    1.16074   0.862   0.3901
## const            9.60691    10.13961   0.947   0.3447
## trend            -0.01649    0.08512  -0.194   0.8466
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 61.78 on 176 degrees of freedom
## Multiple R-Squared: 0.1622, Adjusted R-squared: 0.06226
## F-statistic: 1.623 on 21 and 176 DF, p-value: 0.04857
##
##
## Estimation results for equation gold_search_FD:
## =====
## gold_search_FD = gold_price_FD.l1 + gold_search_FD.l1 + gold_price_FD.l2 + gold_search_FD.l2 + gold_price_FD.l3 +
```

```
##
##               Estimate Std. Error t value Pr(>|t|)
## gold_price_FD.11    0.009149   0.005646   1.620 0.106924
## gold_search_FD.11  -0.504513   0.072807  -6.929 7.67e-11 ***
## gold_price_FD.12    0.003424   0.005730   0.598 0.550891
## gold_search_FD.12  -0.571249   0.082102  -6.958 6.55e-11 ***
## gold_price_FD.13    0.009154   0.005685   1.610 0.109122
## gold_search_FD.13  -0.511228   0.091257  -5.602 8.03e-08 ***
## gold_price_FD.14    0.004738   0.005762   0.822 0.412051
## gold_search_FD.14  -0.429967   0.096809  -4.441 1.58e-05 ***
## gold_price_FD.15    0.010462   0.005711   1.832 0.068667 .
## gold_search_FD.15  -0.055661   0.103092  -0.540 0.589939
## gold_price_FD.16   -0.001426   0.005761  -0.247 0.804819
## gold_search_FD.16  -0.095101   0.100085  -0.950 0.343312
## gold_price_FD.17    0.020980   0.005909   3.550 0.000493 ***
## gold_search_FD.17   0.039133   0.096535   0.405 0.685689
## gold_price_FD.18    0.009038   0.006100   1.482 0.140196
## gold_search_FD.18  -0.043579   0.091266  -0.477 0.633599
## gold_price_FD.19    0.011006   0.006117   1.799 0.073681 .
## gold_search_FD.19  -0.088944   0.089955  -0.989 0.324139
## gold_price_FD.110   0.006574   0.006202   1.060 0.290611
## gold_search_FD.110 -0.335833   0.086173  -3.897 0.000138 ***
## const              -1.470108   0.752756  -1.953 0.052409 .
## trend               0.016808   0.006319   2.660 0.008540 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 4.587 on 176 degrees of freedom
## Multiple R-Squared:  0.4615, Adjusted R-squared:  0.3972
## F-statistic: 7.181 on 21 and 176 DF, p-value: 7.4e-15
##
##
## Covariance matrix of residuals:
##               gold_price_FD gold_search_FD
## gold_price_FD    3817.324         6.271
## gold_search_FD     6.271        21.039
##
## Correlation matrix of residuals:
##               gold_price_FD gold_search_FD
## gold_price_FD     1.00000         0.02213
## gold_search_FD     0.02213         1.00000
VAR_lag_FD_select <- VARselect(y = VAR_data_FD, type = 'both', lag.max = 15)
VAR_lag_FD_select

## $selection
## AIC(n)  HQ(n)  SC(n) FPE(n)
##    10     4     4    10
##
## $criteria
##           1           2           3           4           5
## AIC(n)  11.89902  11.85386  11.75817  11.66242  11.66707
## HQ(n)   11.95379  11.93601  11.86771  11.79934  11.83138
```

```
## SC(n)      12.03426      12.05672      12.02865      12.00052      12.07280
## FPE(n) 147124.09896 140631.40080 127805.39947 116146.35986 116703.74827
##          6          7          8          9         10
## AIC(n)     11.66085     11.60882     11.61500     11.61547     11.56011
## HQ(n)      11.85254     11.82789     11.86146     11.88931     11.86133
## SC(n)      12.13419     12.14978     12.22359     12.29167     12.30393
## FPE(n) 116001.84875 110147.95111 110867.37695 110963.87937 105040.07209
##          11          12          13          14          15
## AIC(n)     11.56529     11.58768     11.60751     11.60284     11.62814
## HQ(n)      11.89390     11.94367     11.99088     12.01361     12.06629
## SC(n)      12.37674     12.46674     12.55419     12.61715     12.71007
## FPE(n) 105649.00712 108117.44187 110373.90910 109965.81854 112907.13064
```

*#Problem! All roots are inside the unit circle --> unstable. This is why we try it without a trend -->*

*# estimate model coefficients for VAR using AIC without a trend*

```
VAR_lag <- VAR(y = VAR_data, type = 'const', ic = 'AIC', lag.max = 15)
summary(VAR_lag)
```

```
##
## VAR Estimation Results:
## =====
## Endogenous variables: gold_price, gold_search
## Deterministic variables: const
## Sample size: 198
## Log Likelihood: -1656.34
## Roots of the characteristic polynomial:
## 1.03 0.9916 0.9916 0.9802 0.9221 0.9221 0.9201 0.9201 0.9151 0.9151 0.878 0.8434 0.8434 0.8344 0.8344
## Call:
## VAR(y = VAR_data, type = "const", lag.max = 15, ic = "AIC")
##
##
## Estimation results for equation gold_price:
## =====
## gold_price = gold_price.l1 + gold_search.l1 + gold_price.l2 + gold_search.l2 + gold_price.l3 + gold_search.l3 +
##
##          Estimate Std. Error t value Pr(>|t|)
## gold_price.l1    0.87513    0.07610  11.499 < 2e-16 ***
## gold_search.l1   -0.05621    0.96757  -0.058  0.95374
## gold_price.l2     0.08278    0.09949   0.832  0.40654
## gold_search.l2    0.43843    1.07115   0.409  0.68281
## gold_price.l3     0.10007    0.10041   0.997  0.32032
## gold_search.l3   -0.66179    1.08123  -0.612  0.54128
## gold_price.l4    -0.15626    0.10064  -1.553  0.12229
## gold_search.l4    1.88889    1.10091   1.716  0.08798 .
## gold_price.l5     0.10651    0.09902   1.076  0.28357
## gold_search.l5   -0.79764    1.17153  -0.681  0.49687
## gold_price.l6     0.16350    0.09969   1.640  0.10278
## gold_search.l6    0.16789    1.20862   0.139  0.88968
## gold_price.l7    -0.29871    0.10238  -2.918  0.00399 **
## gold_search.l7   -1.32694    1.13372  -1.170  0.24342
## gold_price.l8     0.04955    0.10660   0.465  0.64263
## gold_search.l8   -0.31966    1.15009  -0.278  0.78139
## gold_price.l9    -0.10611    0.10625  -0.999  0.31932
```

```

## gold_search.l9    1.59749    1.16378    1.373    0.17161
## gold_price.l10    0.23276    0.10670    2.181    0.03049 *
## gold_search.l10    0.68828    1.36122    0.506    0.61375
## gold_price.l11    -0.08065    0.08268    -0.975    0.33071
## gold_search.l11    -0.56506    1.23187    -0.459    0.64701
## const            30.00235    15.50751    1.935    0.05464 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 61.44 on 175 degrees of freedom
## Multiple R-Squared:  0.9771, Adjusted R-squared:  0.9743
## F-statistic: 340.1 on 22 and 175 DF, p-value: < 2.2e-16
##
##
## Estimation results for equation gold_search:
## =====
## gold_search = gold_price.l1 + gold_search.l1 + gold_price.l2 + gold_search.l2 + gold_price.l3 + gold
##
##              Estimate Std. Error t value Pr(>|t|)
## gold_price.l1    0.009061   0.005739   1.579 0.116152
## gold_search.l1    0.511486   0.072959   7.011 4.95e-11 ***
## gold_price.l2   -0.005312   0.007502   -0.708 0.479821
## gold_search.l2   -0.065911   0.080769   -0.816 0.415590
## gold_price.l3     0.005925   0.007571    0.783 0.434954
## gold_search.l3     0.065926   0.081529    0.809 0.419832
## gold_price.l4   -0.004810   0.007589   -0.634 0.527047
## gold_search.l4     0.088957   0.083013    1.072 0.285376
## gold_price.l5     0.005094   0.007467    0.682 0.496030
## gold_search.l5     0.374240   0.088339    4.236 3.67e-05 ***
## gold_price.l6   -0.012029   0.007517   -1.600 0.111336
## gold_search.l6   -0.048670   0.091135   -0.534 0.593991
## gold_price.l7     0.022277   0.007720    2.886 0.004397 **
## gold_search.l7     0.141009   0.085487    1.649 0.100845
## gold_price.l8   -0.012497   0.008038   -1.555 0.121829
## gold_search.l8   -0.079754   0.086721   -0.920 0.359020
## gold_price.l9     0.002008   0.008011    0.251 0.802349
## gold_search.l9   -0.045973   0.087754   -0.524 0.601024
## gold_price.l10   -0.004215   0.008046   -0.524 0.601041
## gold_search.l10  -0.228383   0.102642   -2.225 0.027355 *
## gold_price.l11   -0.005319   0.006235   -0.853 0.394709
## gold_search.l11    0.369307   0.092888    3.976 0.000102 ***
## const          -0.940933    1.169333   -0.805 0.422099
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 4.633 on 175 degrees of freedom
## Multiple R-Squared:  0.8945, Adjusted R-squared:  0.8812
## F-statistic: 67.43 on 22 and 175 DF, p-value: < 2.2e-16
##
##
## Covariance matrix of residuals:

```

```
##           gold_price gold_search
## gold_price    3774.561      5.581
## gold_search     5.581     21.461
##
```

```
## Correlation matrix of residuals:
##           gold_price gold_search
## gold_price    1.00000    0.01961
## gold_search    0.01961    1.00000
```

```
VAR_lag_select <- VARselect(y = VAR_data, type = 'const', lag.max = 15)
VAR_lag_select
```

```
## $selection
## AIC(n)  HQ(n)  SC(n) FPE(n)
##      11      5      1     11
##
```

```
## $criteria
##           1           2           3           4           5
## AIC(n)    11.83777    11.83070    11.81954    11.74277    11.65647
## HQ(n)     11.87869    11.89891    11.91504    11.86554    11.80653
## SC(n)     11.93884    11.99914    12.05537    12.04597    12.02705
## FPE(n) 138382.06056 137409.34210 135890.80282 125856.97520 115463.95383
##           6           7           8           9          10
## AIC(n)    11.66376    11.65285    11.61132    11.61462    11.61814
## HQ(n)     11.84110    11.85747    11.84323    11.87381    11.90461
## SC(n)     12.10172    12.15819    12.18404    12.25472    12.32561
## FPE(n) 116326.78161 115089.66397 110439.76870 110844.30016 111283.75901
##           11          12          13          14          15
## AIC(n)    11.56397    11.56907    11.58852    11.60400    11.60497
## HQ(n)     11.87773    11.91012    11.95685    11.99961    12.02786
## SC(n)     12.33882    12.41130    12.49813    12.58099    12.64933
## FPE(n) 105472.55908 106080.53909 108245.81312 110030.76954 110248.54472
```

```
#for FD
VAR_lag_FD <- VAR(y = VAR_data_FD, type = 'const', ic = 'AIC', lag.max = 15)
summary(VAR_lag_FD)
```

```
##
## VAR Estimation Results:
## =====
## Endogenous variables: gold_price_FD, gold_search_FD
## Deterministic variables: const
## Sample size: 198
## Log Likelihood: -1660.541
## Roots of the characteristic polynomial:
## 0.9857 0.9857 0.9186 0.9186 0.9167 0.9167 0.9003 0.9003 0.8751 0.8348 0.8348 0.8216 0.8216 0.8193 0.8193
## Call:
## VAR(y = VAR_data_FD, type = "const", lag.max = 15, ic = "AIC")
##
##
## Estimation results for equation gold_price_FD:
## =====
## gold_price_FD = gold_price_FD.l1 + gold_search_FD.l1 + gold_price_FD.l2 + gold_search_FD.l2 + gold_p
##
##           Estimate Std. Error t value Pr(>|t|)
```



```

## gold_price_FD.l1    -0.11001    0.07579   -1.451    0.1484
## gold_search_FD.l1   -0.11483    0.96472   -0.119    0.9054
## gold_price_FD.l2    -0.02844    0.07681   -0.370    0.7116
## gold_search_FD.l2    0.35291    1.07343    0.329    0.7427
## gold_price_FD.l3     0.07057    0.07614    0.927    0.3552
## gold_search_FD.l3   -0.34611    1.17986   -0.293    0.7696
## gold_price_FD.l4    -0.08162    0.07726   -1.056    0.2922
## gold_search_FD.l4    1.45542    1.24960    1.165    0.2457
## gold_price_FD.l5     0.02949    0.07653    0.385    0.7005
## gold_search_FD.l5    0.66982    1.31153    0.511    0.6102
## gold_price_FD.l6     0.19237    0.07705    2.497    0.0135 *
## gold_search_FD.l6    0.87667    1.26613    0.692    0.4896
## gold_price_FD.l7    -0.10507    0.07930   -1.325    0.1869
## gold_search_FD.l7   -0.59765    1.23864   -0.483    0.6300
## gold_price_FD.l8    -0.05174    0.08169   -0.633    0.5273
## gold_search_FD.l8   -1.01551    1.19026   -0.853    0.3947
## gold_price_FD.l9    -0.15695    0.08181   -1.919    0.0566 .
## gold_search_FD.l9    0.57031    1.17828    0.484    0.6290
## gold_price_FD.l10    0.07740    0.08284    0.934    0.3514
## gold_search_FD.l10   0.97309    1.14923    0.847    0.3983
## const               7.88628    4.87849    1.617    0.1078
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 61.62 on 177 degrees of freedom
## Multiple R-Squared:  0.162,    Adjusted R-squared:  0.06736
## F-statistic: 1.711 on 20 and 177 DF,  p-value: 0.03509
##
##
## Estimation results for equation gold_search_FD:
## =====
## gold_search_FD = gold_price_FD.l1 + gold_search_FD.l1 + gold_price_FD.l2 + gold_search_FD.l2 + gold_
##
##
##               Estimate Std. Error t value Pr(>|t|)
## gold_price_FD.l1    0.008587   0.005738   1.496 0.136312
## gold_search_FD.l1  -0.472657   0.073038  -6.471 9.21e-10 ***
## gold_price_FD.l2     0.002451   0.005815   0.421 0.673902
## gold_search_FD.l2  -0.521095   0.081268  -6.412 1.26e-09 ***
## gold_price_FD.l3     0.007988   0.005764   1.386 0.167578
## gold_search_FD.l3  -0.445345   0.089325  -4.986 1.46e-06 ***
## gold_price_FD.l4     0.003826   0.005850   0.654 0.513886
## gold_search_FD.l4  -0.358655   0.094605  -3.791 0.000206 ***
## gold_price_FD.l5     0.009390   0.005794   1.621 0.106844
## gold_search_FD.l5    0.032388   0.099294   0.326 0.744673
## gold_price_FD.l6    -0.002851   0.005834  -0.489 0.625674
## gold_search_FD.l6  -0.005552   0.095857  -0.058 0.953877
## gold_price_FD.l7     0.020280   0.006004   3.378 0.000898 ***
## gold_search_FD.l7    0.115154   0.093775   1.228 0.221086
## gold_price_FD.l8     0.007763   0.006184   1.255 0.211047
## gold_search_FD.l8    0.014607   0.090113   0.162 0.871413
## gold_price_FD.l9     0.009474   0.006193   1.530 0.127860
## gold_search_FD.l9   -0.035861   0.089206  -0.402 0.688171
## gold_price_FD.l10    0.004807   0.006272   0.767 0.444377

```



```
## gold_search_FD.110 -0.308345 0.087006 -3.544 0.000504 ***
## const 0.283681 0.369343 0.768 0.443470
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 4.665 on 177 degrees of freedom
## Multiple R-Squared: 0.4398, Adjusted R-squared: 0.3765
## F-statistic: 6.948 on 20 and 177 DF, p-value: 5.647e-14
##
##
## Covariance matrix of residuals:
##      gold_price_FD gold_search_FD
## gold_price_FD 3796.57 5.41
## gold_search_FD 5.41 21.76
##
## Correlation matrix of residuals:
##      gold_price_FD gold_search_FD
## gold_price_FD 1.00000 0.01882
## gold_search_FD 0.01882 1.00000
VAR_lag_FD_select <- VARselect(y = VAR_data_FD, type = 'const', lag.max = 15)
VAR_lag_FD_select
```

```
## $selection
## AIC(n) HQ(n) SC(n) FPE(n)
## 10 4 4 10
##
## $criteria
##      1 2 3 4 5
## AIC(n) 11.87983 11.83746 11.74978 11.66585 11.67760
## HQ(n) 11.92090 11.90592 11.84563 11.78908 11.82821
## SC(n) 11.98126 12.00651 11.98645 11.97014 12.04951
## FPE(n) 144326.02180 138341.25377 126733.82294 116539.10567 117930.03872
##      6 7 8 9 10
## AIC(n) 11.67244 11.61517 11.61999 11.62050 11.58150
## HQ(n) 11.85044 11.82055 11.85276 11.88065 11.86903
## SC(n) 12.11197 12.12233 12.19477 12.26290 12.29152
## FPE(n) 117341.99966 110835.17776 111403.09662 111500.44501 107283.44156
##      11 12 13 14 15
## AIC(n) 11.58501 11.59947 11.61361 11.60789 11.63345
## HQ(n) 11.89993 11.94177 11.98330 12.00496 12.05791
## SC(n) 12.36265 12.44473 12.52649 12.58839 12.68157
## FPE(n) 107719.62262 109359.83845 111002.49194 110467.49347 113444.11328
```

```
#Force it to 1 lag
```

```
# estimate model coefficients using AIC
```

```
VAR_lag <- VAR(y = VAR_data, type = 'both', p=1)
summary(VAR_lag)
```

```
##
## VAR Estimation Results:
```

```

## =====
## Endogenous variables: gold_price, gold_search
## Deterministic variables: both
## Sample size: 208
## Log Likelihood: -1799.017
## Roots of the characteristic polynomial:
## 0.9694 0.7759
## Call:
## VAR(y = VAR_data, p = 1, type = "both")
##
##
## Estimation results for equation gold_price:
## =====
## gold_price = gold_price.l1 + gold_search.l1 + const + trend
##
##               Estimate Std. Error t value Pr(>|t|)
## gold_price.l1  0.96788    0.01798  53.837  <2e-16 ***
## gold_search.l1 0.10532    0.50011   0.211  0.8334
## const         25.95499   13.78646   1.883  0.0612 .
## trend         0.15617    0.13017   1.200  0.2317
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 62.32 on 204 degrees of freedom
## Multiple R-Squared: 0.9772, Adjusted R-squared: 0.9769
## F-statistic: 2913 on 3 and 204 DF, p-value: < 2.2e-16
##
##
## Estimation results for equation gold_search:
## =====
## gold_search = gold_price.l1 + gold_search.l1 + const + trend
##
##               Estimate Std. Error t value Pr(>|t|)
## gold_price.l1  0.002848   0.001578   1.805  0.0725 .
## gold_search.l1 0.777456   0.043888  17.715  <2e-16 ***
## const         -2.158855   1.209834  -1.784  0.0758 .
## trend         0.023485   0.011423   2.056  0.0411 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 5.469 on 204 degrees of freedom
## Multiple R-Squared: 0.8359, Adjusted R-squared: 0.8335
## F-statistic: 346.5 on 3 and 204 DF, p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##           gold_price gold_search
## gold_price   3883.7    -12.50
## gold_search  -12.5     29.91
##
## Correlation matrix of residuals:

```

```
##           gold_price gold_search
## gold_price      1.00000   -0.03669
## gold_search    -0.03669    1.00000

#for FD
VAR_lag_FD <- VAR(y = VAR_data_FD, type = 'both', p=1)
summary(VAR_lag_FD)

##
## VAR Estimation Results:
## =====
## Endogenous variables: gold_price_FD, gold_search_FD
## Deterministic variables: both
## Sample size: 207
## Log Likelihood: -1796.533
## Roots of the characteristic polynomial:
## 0.1819 0.1819
## Call:
## VAR(y = VAR_data_FD, p = 1, type = "both")
##
##
## Estimation results for equation gold_price_FD:
## =====
## gold_price_FD = gold_price_FD.l1 + gold_search_FD.l1 + const + trend
##
##              Estimate Std. Error t value Pr(>|t|)
## gold_price_FD.l1 -0.123089   0.070274  -1.752   0.0814 .
## gold_search_FD.l1 -0.662835   0.755885  -0.877   0.3816
## const             8.352675   8.788579   0.950   0.3430
## trend             -0.001137   0.072641  -0.016   0.9875
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 62.39 on 203 degrees of freedom
## Multiple R-Squared: 0.01812, Adjusted R-squared: 0.003612
## F-statistic: 1.249 on 3 and 203 DF,  p-value: 0.2931
##
##
## Estimation results for equation gold_search_FD:
## =====
## gold_search_FD = gold_price_FD.l1 + gold_search_FD.l1 + const + trend
##
##              Estimate Std. Error t value Pr(>|t|)
## gold_price_FD.l1  0.016363   0.006338   2.582  0.01054 *
## gold_search_FD.l1 -0.180752   0.068176  -2.651  0.00865 **
## const            -0.250411   0.792679  -0.316  0.75240
## trend             0.003687   0.006552   0.563  0.57427
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 5.627 on 203 degrees of freedom
## Multiple R-Squared: 0.06579, Adjusted R-squared: 0.05199
## F-statistic: 4.766 on 3 and 203 DF,  p-value: 0.003117
```

```
##
##
##
## Covariance matrix of residuals:
##           gold_price_FD gold_search_FD
## gold_price_FD      3892.9      -11.90
## gold_search_FD     -11.9       31.67
##
## Correlation matrix of residuals:
##           gold_price_FD gold_search_FD
## gold_price_FD      1.0000      -0.0339
## gold_search_FD     -0.0339      1.0000
VAR_lag <- VAR(y = VAR_data, type = 'const', p=1)
summary(VAR_lag)

##
## VAR Estimation Results:
## =====
## Endogenous variables: gold_price, gold_search
## Deterministic variables: const
## Sample size: 208
## Log Likelihood: -1801.96
## Roots of the characteristic polynomial:
## 0.9901 0.8059
## Call:
## VAR(y = VAR_data, p = 1, type = "const")
##
##
## Estimation results for equation gold_price:
## =====
## gold_price = gold_price.l1 + gold_search.l1 + const
##
##           Estimate Std. Error t value Pr(>|t|)
## gold_price.l1  0.98036   0.01467  66.810 <2e-16 ***
## gold_search.l1 0.35938   0.45355   0.792  0.4291
## const         24.25247  13.72791   1.767  0.0788 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 62.39 on 205 degrees of freedom
## Multiple R-Squared: 0.977, Adjusted R-squared: 0.9768
## F-statistic: 4359 on 2 and 205 DF, p-value: < 2.2e-16
##
##
## Estimation results for equation gold_search:
## =====
## gold_search = gold_price.l1 + gold_search.l1 + const
##
##           Estimate Std. Error t value Pr(>|t|)
## gold_price.l1  0.004726   0.001296   3.646 0.000338 ***
## gold_search.l1 0.815662   0.040070  20.356 < 2e-16 ***
## const         -2.414886   1.212840  -1.991 0.047798 *
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 5.512 on 205 degrees of freedom
## Multiple R-Squared:  0.8325,    Adjusted R-squared:  0.8309
## F-statistic: 509.6 on 2 and 205 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##           gold_price gold_search
## gold_price    3892.000    -8.343
## gold_search    -8.343     30.379
##
## Correlation matrix of residuals:
##           gold_price gold_search
## gold_price     1.00000    -0.02426
## gold_search    -0.02426     1.00000

#for FD
VAR_lag_FD <- VAR(y = VAR_data_FD, type = 'const', p=1)
summary(VAR_lag_FD)

##
## VAR Estimation Results:
## =====
## Endogenous variables: gold_price_FD, gold_search_FD
## Deterministic variables: const
## Sample size: 207
## Log Likelihood: -1796.695
## Roots of the characteristic polynomial:
## 0.1813 0.1813
## Call:
## VAR(y = VAR_data_FD, p = 1, type = "const")
##
##
## Estimation results for equation gold_price_FD:
## =====
## gold_price_FD = gold_price_FD.l1 + gold_search_FD.l1 + const
##
##           Estimate Std. Error t value Pr(>|t|)
## gold_price_FD.l1 -0.12307    0.07009  -1.756   0.0806 .
## gold_search_FD.l1 -0.66331    0.75343  -0.880   0.3797
## const            8.23328    4.35479   1.891   0.0601 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 62.24 on 204 degrees of freedom
## Multiple R-Squared:  0.01812, Adjusted R-squared:  0.008495
## F-statistic: 1.882 on 2 and 204 DF,  p-value: 0.1549
##
##
## Estimation results for equation gold_search_FD:
## =====
```

```
## gold_search_FD = gold_price_FD.l1 + gold_search_FD.l1 + const
##
##               Estimate Std. Error t value Pr(>|t|)
## gold_price_FD.l1  0.016307   0.006327   2.577  0.01066 *
## gold_search_FD.l1 -0.179223   0.068008  -2.635  0.00905 **
## const            0.136697   0.393083   0.348  0.72838
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 5.618 on 204 degrees of freedom
## Multiple R-Squared:  0.06434, Adjusted R-squared:  0.05516
## F-statistic: 7.014 on 2 and 204 DF,  p-value: 0.001133
##
##
## Covariance matrix of residuals:
##               gold_price_FD gold_search_FD
## gold_price_FD      3873.81      -11.86
## gold_search_FD     -11.86       31.56
##
## Correlation matrix of residuals:
##               gold_price_FD gold_search_FD
## gold_price_FD      1.00000      -0.03391
## gold_search_FD     -0.03391       1.00000
```

*# doesn't change much. Still the root problem (inside unit circle)*  
*# I think the main problem is, that we regress an I(1) on a stationary variable.*

## 4 Phillips-Ouliaris Cointegration Test

```
po.test(VAR_data, demean = TRUE, lshort = TRUE)
```

```
##
## Phillips-Ouliaris Cointegration Test
##
## data:  VAR_data
## Phillips-Ouliaris demeaned = -19.431, Truncation lag parameter = 2,
## p-value = 0.06318
```

```
# for FD
po.test(VAR_data_FD, demean = TRUE, lshort = TRUE)
```

```
## Warning in po.test(VAR_data_FD, demean = TRUE, lshort = TRUE): p-value smaller
## than printed p-value
```

```
##
## Phillips-Ouliaris Cointegration Test
##
## data:  VAR_data_FD
## Phillips-Ouliaris demeaned = -224.2, Truncation lag parameter = 2,
## p-value = 0.01
```

We cannot reject the null of the residuals being I(1). Thus we cannot rule out the case of a spurious or unbalanced regression. Note: as we only have one I(1) process and one non I(1) process this test makes no

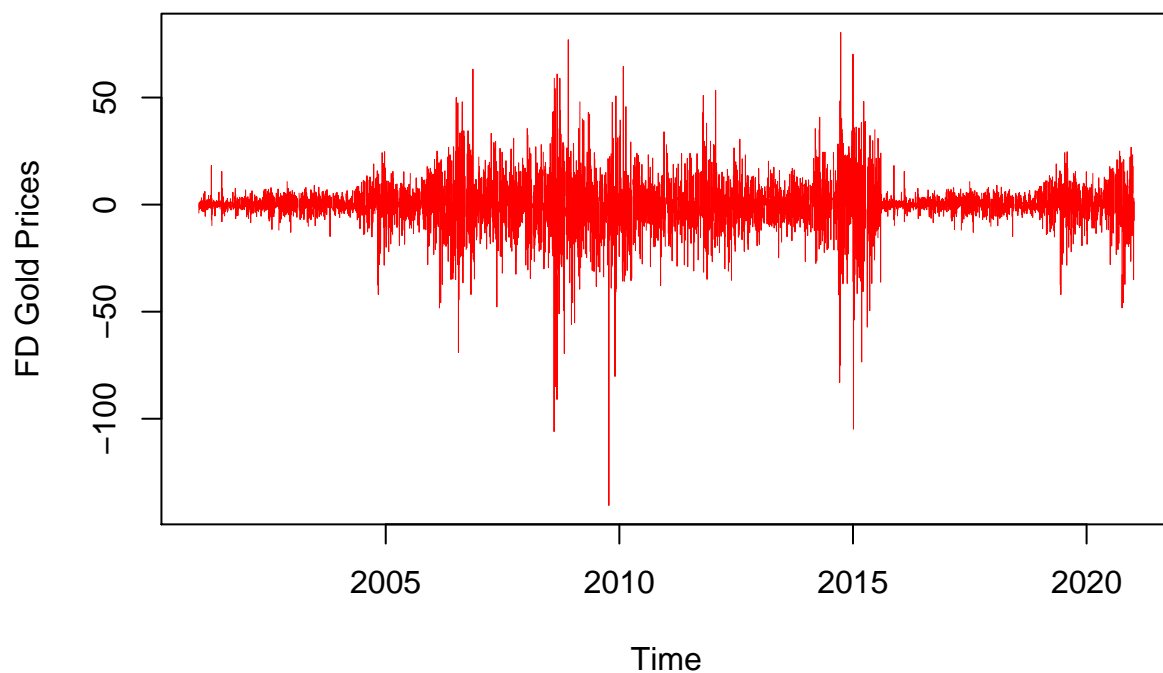
real sense.

## 5 Ideas for the first part: Univariate Time Series

## 6 Differenced MA(1) and ARCH Model for Gold Prices

```
# Differenced MA(1) and ARCH Model for Gold Prices
gold_price_FD <- ts(gold_daily_FD, frequency = 365,
                    start = c(2001, 1, 1), end = c(2021, 6, 10)) # with higher frequency

plot(gold_price_FD, ylab = 'FD Gold Prices', col = 'red', lwd = 0.5)
```



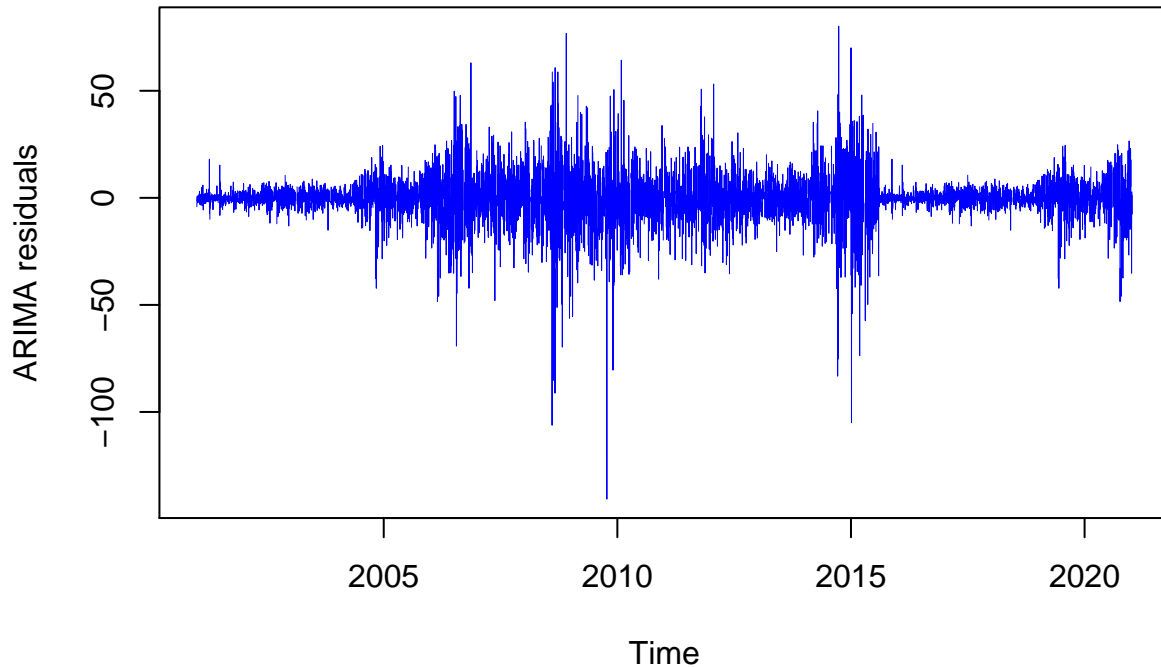
```
arima_model_HF_FD <- forecast::auto.arima(gold_price_FD, ic = 'aic')
arima_model_HF_FD
```

```
## Series: gold_price_FD
## ARIMA(4,0,2) with non-zero mean
##
## Coefficients:
##      ar1      ar2      ar3      ar4      ma1      ma2      mean
##      -0.0270  0.6155 -0.0033 -0.0384  0.0278 -0.6164  0.2022
## s.e.    0.2394  0.2277  0.0162  0.0162  0.2398  0.2285  0.1213
##
## sigma^2 estimated as 120.9: log likelihood=-26138.47
## AIC=52292.95  AICc=52292.97  BIC=52348.12
```

```
# Auto-ARIMA suggests an ARMA(4,2) for daily prices
```

```
plot(forecast::arima.errors(arima_model_HF_FD), type = 'l', lwd = 0.5, col = 'blue',  
     ylab = 'ARIMA residuals')
```

```
## Deprecated, use residuals.Arima(object, type='regression') instead
```



Going by the plot, it does not appear that the variance of the residuals is constant over time but rather has times of higher and lower volatility.

```
#####  
##      CHECK      ##  
#####
```

```
#ARCH by hand
```

```
resi_arima_FD_2 <- (forecast::arima.errors(arima_model_HF_FD))^2
```

```
## Deprecated, use residuals.Arima(object, type='regression') instead
```

```
arch1_FD_model <- arima(resi_arima_FD_2, order = c(1,0,0))  
arch1_FD_model
```

```
##  
## Call:  
## arima(x = resi_arima_FD_2, order = c(1, 0, 0))  
##  
## Coefficients:  
##      ar1  intercept
```



```
##      0.112   121.0271
## s.e.  0.012    6.4950
##
## sigma^2 estimated as 229290:  log likelihood = -51994.71,  aic = 103995.4
AIC_ARCH_1<-AIC(arch1_FD_model)
AIC_ARCH_1

## [1] 103995.4

#ARCH with garch()
gold_price_FD_clean<- na.remove(gold_price_FD) #Remove NAs for garch()
gold_price_FD_arch1 <- garch(gold_price_FD_clean,c(0,1))      #ARCH

##
## ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
##
##
##      I      INITIAL X(I)      D(I)
##
##      1      1.149935e+02      1.000e+00
##      2      5.000000e-02      1.000e+00
##
##      IT  NF      F      RELDF      PRELDF      RELDX      STPPAR      D*STEP      NPRELDF
##      0   1  1.970e+04
##      1   3  1.965e+04  2.73e-03  3.94e-02  2.0e-03  3.6e+03  4.7e-01  7.01e+01
##      2   7  1.959e+04  2.59e-03  1.38e-03  2.7e-02  0.0e+00  6.0e+00  1.38e-03
##      3   8  1.951e+04  4.31e-03  6.63e-03  1.2e-01  5.5e-01  2.4e+01  8.01e-03
##      4   9  1.950e+04  4.39e-04  6.30e-04  2.5e-02  0.0e+00  4.4e+00  6.30e-04
##      5  10  1.950e+04  2.22e-04  1.40e-04  1.8e-02  0.0e+00  3.2e+00  1.40e-04
##      6  11  1.949e+04  1.53e-04  1.49e-04  2.9e-02  0.0e+00  4.8e+00  1.49e-04
##      7  12  1.949e+04  1.83e-05  1.27e-05  4.8e-04  0.0e+00  7.9e-02  1.27e-05
##      8  13  1.949e+04  4.95e-06  4.69e-06  2.8e-04  0.0e+00  4.7e-02  4.69e-06
##      9  14  1.949e+04  2.25e-08  2.17e-08  6.0e-05  0.0e+00  9.8e-03  2.17e-08
##     10  15  1.949e+04  2.58e-11  2.57e-11  1.7e-06  0.0e+00  2.8e-04  2.57e-11
##
## ***** RELATIVE FUNCTION CONVERGENCE *****
##
##      FUNCTION      1.949344e+04      RELDX      1.721e-06
##      FUNC. EVALS      15      GRAD. EVALS      11
##      PRELDF      2.569e-11      NPRELDF      2.569e-11
##
##      I      FINAL X(I)      D(I)      G(I)
##
##      1      8.135467e+01      1.000e+00      4.064e-07
##      2      4.434166e-01      1.000e+00      -7.434e-05

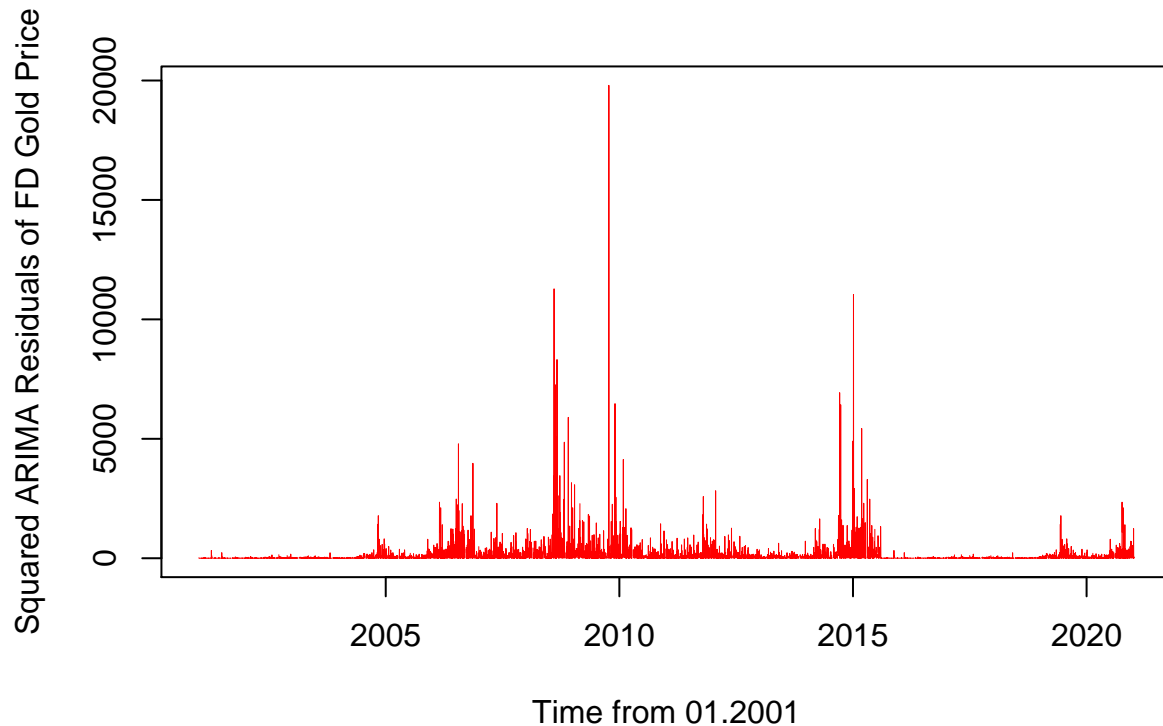
gold_price_FD_arch1

##
## Call:
## garch(x = gold_price_FD_clean, order = c(0, 1))
##
## Coefficient(s):
##      a0      a1
## 81.3547  0.4434
```

```
AIC_arch_1<-AIC(gold_price_FD_arch1)
AIC_arch_1

## [1] 51578.5

# plot the squared residuals:
plot(resi_arma_FD_2, ylab = 'Squared ARIMA Residuals of FD Gold Price',
     xlab = 'Time from 01.2001', col = 'red', lwd = 0.5)
```



## 7 GARCH

```
#Try GARCH(1,1)
garch_gold_price_FD <- garch(x=gold_price_FD_clean,order=c(1,1))
```

```
##
## ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
##
##      I      INITIAL X(I)      D(I)
##
##      1      1.089412e+02      1.000e+00
##      2      5.000000e-02      1.000e+00
##      3      5.000000e-02      1.000e+00
##
##      IT   NF      F      RELDF      PRELDF      RELDX      STPPAR      D*STEP      NPRELDF
##      0    1  1.968e+04
```

```

##      1      3  1.959e+04  4.67e-03  4.66e-02  2.2e-03  4.0e+03  4.8e-01  9.42e+01
##      2      5  1.958e+04  5.08e-04  9.24e-04  1.8e-04  4.7e+00  4.8e-02  2.39e+00
##      3      6  1.957e+04  3.06e-04  3.87e-04  1.6e-04  2.0e+00  4.8e-02  1.01e+00
##      4      7  1.956e+04  8.15e-04  1.04e-03  4.1e-04  2.0e+00  9.5e-02  7.67e-01
##      5      9  1.955e+04  1.50e-04  5.87e-04  2.7e-04  2.0e+00  7.4e-02  1.79e-01
##      6     10  1.955e+04  1.72e-04  4.98e-04  2.6e-04  2.0e+00  7.4e-02  2.80e-02
##      7     11  1.955e+04  9.75e-05  1.01e-04  3.2e-04  2.0e+00  7.4e-02  1.95e-03
##      8     14  1.955e+04  3.77e-06  1.19e-05  3.6e-05  4.1e+00  9.7e-03  2.19e-03
##      9     15  1.955e+04  4.93e-06  9.15e-06  3.4e-05  2.0e+00  9.7e-03  1.38e-03
##     10     16  1.955e+04  5.23e-06  5.24e-06  4.3e-05  2.0e+00  9.7e-03  1.26e-03
##     11     17  1.955e+04  5.04e-06  5.92e-06  8.7e-05  2.0e+00  1.9e-02  1.26e-03
##     12     22  1.950e+04  2.58e-03  1.26e-03  3.4e-02  0.0e+00  7.1e+00  1.26e-03
##     13     24  1.883e+04  3.42e-02  1.80e-02  8.4e-01  0.0e+00  9.3e+01  1.80e-02
##     14     25  1.875e+04  4.28e-03  6.49e-02  5.8e-01  0.0e+00  2.4e+01  6.49e-02
##     15     27  1.843e+04  1.69e-02  1.28e-02  2.2e-01  8.1e-01  1.2e+01  5.94e-02
##     16     29  1.836e+04  4.14e-03  4.01e-03  6.1e-02  2.0e+00  2.4e+00  1.57e+01
##     17     31  1.820e+04  8.37e-03  8.41e-03  1.5e-01  2.0e+00  4.7e+00  9.41e+02
##     18     32  1.805e+04  8.30e-03  1.61e-02  5.5e-01  2.0e+00  9.4e+00  2.70e+01
##     19     34  1.753e+04  2.89e-02  2.80e-02  1.4e-01  2.0e+00  9.4e-01  3.99e-01
##     20     36  1.744e+04  5.34e-03  5.28e-03  1.9e-01  2.0e+00  9.4e-01  2.04e+00
##     21     37  1.733e+04  6.40e-03  1.07e-02  8.6e-01  2.0e+00  1.9e+00  5.43e+00
##     22     40  1.731e+04  7.08e-04  1.72e-03  1.2e-02  8.6e+00  1.9e-02  5.40e-02
##     23     41  1.730e+04  6.70e-04  6.58e-04  1.0e-02  2.0e+00  1.9e-02  1.39e-02
##     24     42  1.729e+04  8.99e-04  8.83e-04  1.7e-02  1.9e+00  3.8e-02  1.38e-02
##     25     44  1.722e+04  3.80e-03  3.55e-03  4.1e-02  5.7e-01  7.5e-02  1.33e-02
##     26     45  1.718e+04  2.46e-03  2.84e-03  8.3e-02  1.9e+00  1.5e-01  6.74e-02
##     27     48  1.718e+04  9.66e-05  7.50e-04  4.8e-03  2.9e+00  1.0e-02  3.28e-02
##     28     49  1.717e+04  4.73e-04  4.72e-04  4.7e-03  2.0e+00  1.0e-02  3.15e-02
##     29     50  1.716e+04  3.83e-04  4.63e-04  9.9e-03  2.0e+00  2.0e-02  3.27e-02
##     30     51  1.715e+04  8.00e-04  1.02e-03  1.9e-02  1.9e+00  4.1e-02  2.14e-02
##     31     52  1.714e+04  4.32e-04  6.32e-04  1.9e-02  1.8e+00  4.1e-02  7.81e-03
##     32     53  1.714e+04  1.39e-04  6.04e-04  1.6e-02  8.9e-01  4.1e-02  1.04e-03
##     33     54  1.714e+04  1.60e-05  1.27e-04  2.2e-02  1.2e+00  4.1e-02  3.14e-04
##     34     55  1.714e+04  9.36e-05  1.51e-04  2.0e-02  6.9e-01  4.1e-02  1.89e-04
##     35     56  1.714e+04  2.15e-06  1.97e-05  4.8e-03  0.0e+00  8.9e-03  1.97e-05
##     36     57  1.714e+04  9.16e-06  9.29e-06  2.8e-03  0.0e+00  5.1e-03  9.29e-06
##     37     58  1.714e+04  2.65e-08  2.66e-08  1.0e-04  0.0e+00  2.7e-04  2.66e-08
##     38     59  1.714e+04  2.22e-10  8.62e-11  1.0e-05  0.0e+00  1.9e-05  8.62e-11
##     39     60  1.714e+04  2.92e-11  7.86e-13  2.2e-06  0.0e+00  4.1e-06  7.86e-13
##     40     61  1.714e+04  2.14e-12  4.95e-15  1.7e-07  0.0e+00  3.1e-07  4.95e-15
##     41     62  1.714e+04  7.43e-15  1.22e-18  4.7e-10  0.0e+00  1.1e-09  1.22e-18
##     42     63  1.714e+04 -1.70e-15  3.34e-22  1.9e-11  0.0e+00  3.6e-11  3.34e-22

```

```

##
## ***** X- AND RELATIVE FUNCTION CONVERGENCE *****
##
## FUNCTION      1.713619e+04  RELDX      1.946e-11
## FUNC. EVALS    63          GRAD. EVALS    42
## PRELDF         3.337e-22    NPRELDF     3.337e-22
##
##      I      FINAL X(I)      D(I)      G(I)
##
##      1      1.679893e-01      1.000e+00      3.931e-09
##      2      8.431307e-02      1.000e+00      -1.874e-06
##      3      9.200609e-01      1.000e+00      -2.105e-06

```

```
## Warning in sqrt(pred$e): NaNs produced
```

```
summary(garch_gold_price_FD)
```

```
##
## Call:
## garch(x = gold_price_FD_clean, order = c(1, 1))
##
## Model:
## GARCH(1,1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -10.19636  -0.49038   0.04738   0.61628   7.36207
##
## Coefficient(s):
##      Estimate Std. Error t value Pr(>|t|)
## a0  0.167989    0.011448   14.67  <2e-16 ***
## a1  0.084313    0.002692   31.32  <2e-16 ***
## b1  0.920061    0.002636  349.06  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Diagnostic Tests:
##  Jarque Bera Test
##
## data:  Residuals
## X-squared = 5586.5, df = 2, p-value < 2.2e-16
##
##
##  Box-Ljung test
##
## data:  Squared.Residuals
## X-squared = 0.00050691, df = 1, p-value = 0.982
```

```
AIC_GARCH_1<-AIC(garch_gold_price_FD)
```

```
AIC_GARCH_1
```

```
## [1] 46816.38
```

```
# Check, if the above GARCH(1,1) works with rugarch
```

```
#fit the rugarch sGarch model
```

```
spec = ugarchspec(variance.model=list(model="sGARCH", garchOrder=c(1,1)), mean.model=list(armaOrder=c(0
test_garch_gold_price_FD<- ugarchfit(spec=spec, data=gold_price_FD_clean)
test_garch_gold_price_FD
```

```
##
## *-----*
## *          GARCH Model Fit          *
## *-----*
##
## Conditional Variance Dynamics
## -----
## GARCH Model   : sGARCH(1,1)
```

```

## Mean Model      : ARFIMA(0,0,0)
## Distribution : norm
##
## Optimal Parameters
## -----
##           Estimate Std. Error  t value Pr(>|t|)
## mu         0.192548   0.057573   3.3444 0.000825
## omega       0.191148   0.029141   6.5594 0.000000
## alpha1      0.075676   0.006018  12.5746 0.000000
## beta1       0.923324   0.005853 157.7537 0.000000
##
## Robust Standard Errors:
##           Estimate Std. Error  t value Pr(>|t|)
## mu         0.192548   0.059760   3.2220 0.001273
## omega       0.191148   0.072008   2.6545 0.007942
## alpha1      0.075676   0.017852   4.2390 0.000022
## beta1       0.923324   0.016907  54.6125 0.000000
##
## LogLikelihood : -23432.46
##
## Information Criteria
## -----
##
## Akaike          6.8428
## Bayes           6.8468
## Shibata         6.8428
## Hannan-Quinn    6.8441
##
## Weighted Ljung-Box Test on Standardized Residuals
## -----
##                               statistic p-value
## Lag[1]                      4.426 0.03539
## Lag[2*(p+q)+(p+q)-1] [2]    5.153 0.03701
## Lag[4*(p+q)+(p+q)-1] [5]    6.695 0.06146
## d.o.f=0
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## -----
##                               statistic p-value
## Lag[1]                      0.1029 0.7484
## Lag[2*(p+q)+(p+q)-1] [5]    2.6455 0.4756
## Lag[4*(p+q)+(p+q)-1] [9]    5.6242 0.3437
## d.o.f=2
##
## Weighted ARCH LM Tests
## -----
##           Statistic Shape Scale P-Value
## ARCH Lag[3]  0.00802 0.500 2.000 0.92864
## ARCH Lag[5]  5.06256 1.440 1.667 0.09940
## ARCH Lag[7]  6.72811 2.315 1.543 0.09931
##
## Nyblom stability test
## -----

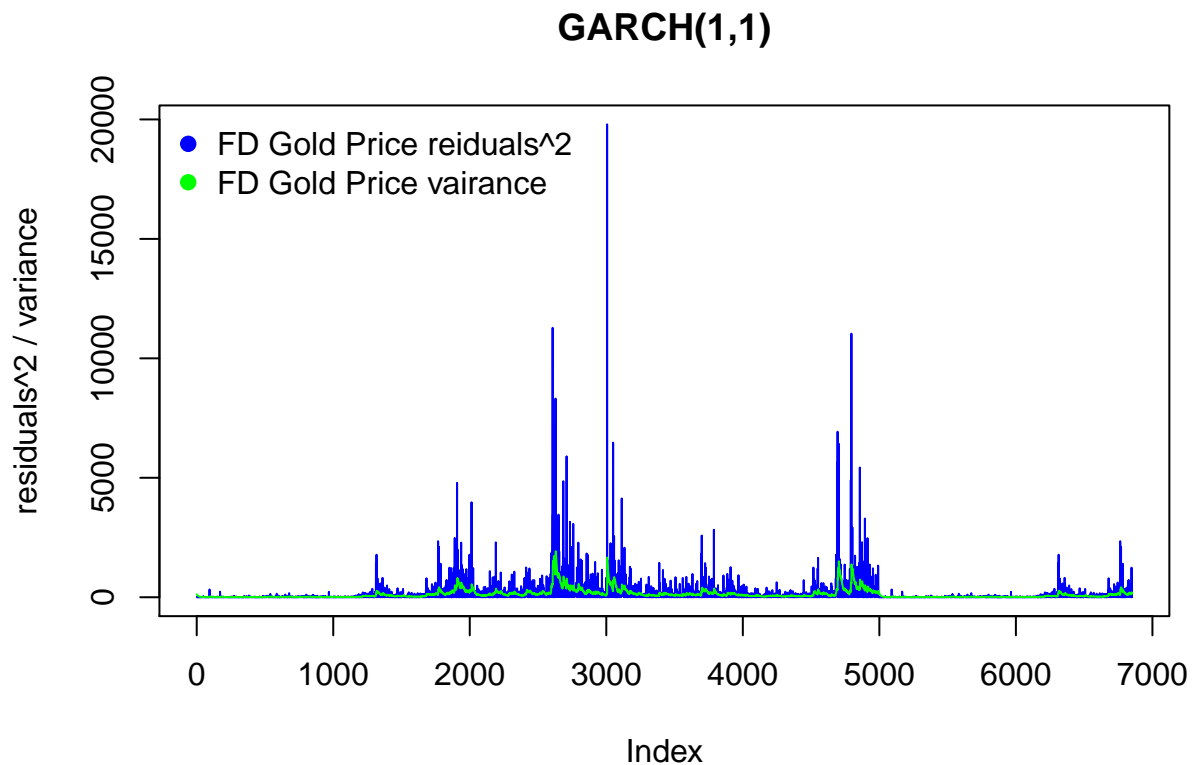
```

```
## Joint Statistic: 2.0948
## Individual Statistics:
## mu      0.01941
## omega   0.02079
## alpha1  1.15660
## beta1   1.07772
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:      1.07 1.24 1.6
## Individual Statistic: 0.35 0.47 0.75
##
## Sign Bias Test
## -----
##              t-value   prob sig
## Sign Bias      1.6935 0.09041  *
## Negative Sign Bias 0.2509 0.80192
## Positive Sign Bias 1.0492 0.29410
## Joint Effect      8.0823 0.04434  **
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## -----
##   group statistic p-value(g-1)
## 1    20      288.1    5.465e-50
## 2    30      317.4    2.829e-50
## 3    40      352.1    4.965e-52
## 4    50      369.0    1.204e-50
##
##
## Elapsed time : 0.4428611
#Summarizes coeff.
coef(test_garch_gold_price_FD)

##          mu      omega      alpha1      beta1
## 0.19254758 0.19114774 0.07567601 0.92332398
garch_gold_price_FD$coef # for comparison with the package tseries and garch from above, Checked online

##          a0          a1          b1
## 0.16798931 0.08431307 0.92006094
#calculating AIC:
AIC_GARCH_2 <- 6.8428*length(gold_price_FD_clean)
AIC_GARCH_2

## [1] 46873.18
#Plot of squared residuals and est. cond. variance
gold_price_FD_res<-test_garch_gold_price_FD@fit$residuals
gold_price_FD_var<-test_garch_gold_price_FD@fit$var
plot((gold_price_FD_res)^2, type = "l", col="blue",ylab = 'residuals^2 / variance', main="GARCH(1,1)")
lines(gold_price_FD_var, col="green")
legend('topleft', legend = c('FD Gold Price reiduals^2','FD Gold Price vairance'),
      col = c('blue','green'), bty = "n", pch = c(19,19))
```



Since rugarch uses a different version of AIC one needs to mutiply it with the length:  $6.8428 \times 6850 = 46873.18$

*#Try eGARCH*

```
spec = ugarchspec(variance.model=list(model="eGARCH", garchOrder=c(1,1)), mean.model=list(armaOrder=c(0
egarch_gold_price_FD<- ugarchfit(spec=spec, data=gold_price_FD_clean, solver = 'hybrid')
egarch_gold_price_FD
```

```
##
## *-----*
## *          GARCH Model Fit          *
## *-----*
##
## Conditional Variance Dynamics
## -----
## GARCH Model   : eGARCH(1,1)
## Mean Model    : ARFIMA(0,0,0)
## Distribution   : norm
##
## Optimal Parameters
## -----
##      Estimate  Std. Error   t value Pr(>|t|)
## mu      0.299998   0.040487  7.4097e+00    0
## omega    0.023218   0.001112  2.0882e+01    0
## alpha1   0.045680   0.005196  8.7922e+00    0
## beta1    0.996762   0.000003  3.2067e+05    0
## gamma1   0.145699   0.001177  1.2381e+02    0
##
```

```

## Robust Standard Errors:
##      Estimate   Std. Error   t value Pr(>|t|)
## mu      0.299998    0.028973   10.3544 0.000000
## omega   0.023218    0.002226   10.4308 0.000000
## alpha1  0.045680    0.011903    3.8377 0.000124
## beta1   0.996762    0.000011 90587.0311 0.000000
## gamma1  0.145699    0.002738   53.2104 0.000000
##
## LogLikelihood : -23391.54
##
## Information Criteria
## -----
##
## Akaike          6.8311
## Bayes           6.8361
## Shibata         6.8311
## Hannan-Quinn    6.8328
##
## Weighted Ljung-Box Test on Standardized Residuals
## -----
##
##              statistic p-value
## Lag[1]                2.695 0.10067
## Lag[2*(p+q)+(p+q)-1] [2] 3.777 0.08657
## Lag[4*(p+q)+(p+q)-1] [5] 5.570 0.11338
## d.o.f=0
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## -----
##
##              statistic p-value
## Lag[1]                3.362 0.06673
## Lag[2*(p+q)+(p+q)-1] [5] 4.544 0.19352
## Lag[4*(p+q)+(p+q)-1] [9] 6.393 0.25554
## d.o.f=2
##
## Weighted ARCH LM Tests
## -----
##
##      Statistic Shape Scale P-Value
## ARCH Lag[3]    0.03511 0.500 2.000 0.8514
## ARCH Lag[5]    2.29017 1.440 1.667 0.4106
## ARCH Lag[7]    3.74396 2.315 1.543 0.3850
##
## Nyblom stability test
## -----
## Joint Statistic: 0.9622
## Individual Statistics:
## mu      0.07732
## omega   0.15325
## alpha1  0.32760
## beta1   0.22526
## gamma1  0.04049
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:      1.28 1.47 1.88

```



```
## Individual Statistic:      0.35 0.47 0.75
##
## Sign Bias Test
## -----
##              t-value      prob sig
## Sign Bias      1.6731 0.094350  *
## Negative Sign Bias 0.9765 0.328865
## Positive Sign Bias 1.2452 0.213100
## Joint Effect      13.8384 0.003134 ***
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## -----
##   group statistic p-value(g-1)
## 1    20      282.0   9.803e-49
## 2    30      318.2   1.988e-50
## 3    40      331.8   4.283e-48
## 4    50      342.7   1.104e-45
##
##
## Elapsed time : 2.977272
```

```
AIC_eGARCH <- 6.8311*length(gold_price_FD_clean)
AIC_eGARCH
```

```
## [1] 46793.04
```

```
#Try iGARCH
```

```
spec = ugarchspec(variance.model=list(model="iGARCH", garchOrder=c(1,1)), mean.model=list(armaOrder=c(0,0)),
  igarch_gold_price_FD<- ugarchfit(spec=spec, data=gold_price_FD_clean)
igarch_gold_price_FD
```

```
##
## *-----*
## *          GARCH Model Fit          *
## *-----*
##
## Conditional Variance Dynamics
## -----
## GARCH Model   : iGARCH(1,1)
## Mean Model    : ARFIMA(0,0,0)
## Distribution   : norm
##
## Optimal Parameters
## -----
##      Estimate Std. Error t value Pr(>|t|)
## mu      0.19294   0.057572  3.3513 0.000804
## omega    0.18470   0.025701  7.1863 0.000000
## alpha1   0.07675   0.005293 14.5015 0.000000
## beta1    0.92325        NA      NA      NA
##
## Robust Standard Errors:
##      Estimate Std. Error t value Pr(>|t|)
## mu      0.19294   0.061023  3.1618 0.001568
## omega    0.18470   0.064045  2.8838 0.003929
```

```

## alpha1    0.07675    0.011744    6.5355 0.000000
## beta1     0.92325           NA         NA         NA
##
## LogLikelihood : -23430.77
##
## Information Criteria
## -----
##
## Akaike          6.842
## Bayes           6.845
## Shibata         6.842
## Hannan-Quinn    6.843
##
## Weighted Ljung-Box Test on Standardized Residuals
## -----
##
##                statistic p-value
## Lag[1]                4.397 0.03601
## Lag[2*(p+q)+(p+q)-1][2]  5.137 0.03738
## Lag[4*(p+q)+(p+q)-1][5]  6.700 0.06127
## d.o.f=0
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## -----
##
##                statistic p-value
## Lag[1]                0.09725 0.7552
## Lag[2*(p+q)+(p+q)-1][5]  2.60608 0.4837
## Lag[4*(p+q)+(p+q)-1][9]  5.53124 0.3557
## d.o.f=2
##
## Weighted ARCH LM Tests
## -----
##
##      Statistic Shape Scale P-Value
## ARCH Lag[3]  0.007351 0.500 2.000 0.9317
## ARCH Lag[5]  4.974918 1.440 1.667 0.1041
## ARCH Lag[7]  6.585417 2.315 1.543 0.1065
##
## Nyblom stability test
## -----
## Joint Statistic: 0.4255
## Individual Statistics:
## mu      0.01943
## omega   0.01849
## alpha1  0.24788
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:      0.846 1.01 1.35
## Individual Statistic: 0.35 0.47 0.75
##
## Sign Bias Test
## -----
##
##                t-value    prob sig
## Sign Bias          1.6990 0.08937  *
## Negative Sign Bias 0.3058 0.75973

```

```
## Positive Sign Bias  1.0883 0.27650
## Joint Effect      8.2011 0.04203  **
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## -----
##   group statistic p-value(g-1)
## 1    20      285.4   1.972e-49
## 2    30      326.5   4.527e-52
## 3    40      348.7   2.213e-51
## 4    50      378.9   1.584e-52
##
##
## Elapsed time : 0.2004662
AIC_iGARCH <- 6.842*length(gold_price_FD_clean)
AIC_iGARCH

## [1] 46867.7
# Summarizing all coeff:

garch_gold_price_FD$coef #GARCH t-series

##          a0          a1          b1
## 0.16798931 0.08431307 0.92006094
coef(test_garch_gold_price_FD) #sGARCH

##          mu          omega          alpha1          beta1
## 0.19254758 0.19114774 0.07567601 0.92332398
coef(egarch_gold_price_FD) #eGARCH

##          mu          omega          alpha1          beta1          gamma1
## 0.29999825 0.02321839 0.04568049 0.99676155 0.14569858
coef(igarch_gold_price_FD) #iGARCH

##          mu          omega          alpha1          beta1
## 0.19294107 0.18469719 0.07674977 0.92325023
# Summarizing AICs:
AIC_GARCH_1 #GARCH t-series

## [1] 46816.38
AIC_GARCH_2 #sGARCH

## [1] 46873.18
AIC_eGARCH #eGARCH

## [1] 46793.04
AIC_iGARCH #iGARCH

## [1] 46867.7
# According to AIC it seems like eGARCH performs the best, but isn't better than MA(1)
```