The Artificial Intelligence Toolbox Part II – CS26210

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Program

Week 1

7/02 Set Theory, Fuzzy Logic (319)
8/02 Fuzzy Logic (B20) - Hand-out Assignment 1

Week 2

14/02 Fuzzy Logic - Further Exercises (319)
15/02 Theory of Probability (B20)

Week 3

21/02 Conditional Probability (319)
22/02 Probabilistic Methods (B20) - Hand-in Assignment 1 (Blackboard)

Week 4

28/02 Bayesian Networks (B20)
1/03 In Class Test (319) (Set Theory, Theory of Probability, Conditional Probability)

Week 5

7/03 Bayesian networks (319)
8/03 Discussion, further exercises (B20) - Hand-out Assignment 2
22/03 Hand-in Assignment 2 (Blackboard)

In-Class Test

1. Notion of Set Theory
2. Probability of Joint and Disjoint Events
3. Conditional Probability
4. Conditional Independence
5. Product Rule
6. Full Joint Probability Distribution Table
7. First-order Markov chain

Thursday 28th, March 2013

- First-order Markov chain
- Bayes' Theorem
- Bayes' Theorem - Law of total probability
- Bayes' Theorem - Multiple Causes
- Bayes' Theorem - Multiple Effects

First-order Markov Chain

system S = {s_1=sunny, s_2=cloudy, s_3=rainy, s_4=foggy}

The expected number of observations of, or duration d_i, within any state s_i, given that the first observation is in that state:

$$/\left(\qquad\qquad\right)$$

	S1	S2	S3	S4	T
S1	a_{11}	a_{12}	a_{13}	a_{14}	1
S2	a_{21}	a_{22}	a_{23}	a_{24}	1
S3	a_{31}	a_{32}	a_{33}	a_{34}	1
S4	a_{41}	a_{42}	a_{43}	a_{44}	1

First-order Markov Chain

Steady-State Matrix (S)
S = ST
With T = transition matrix
S is independent from initial state

$$\begin{bmatrix} X & Y \end{bmatrix} \begin{bmatrix} 0.1 & 0.9 \\ 0.6 & 0.4 \end{bmatrix} = \begin{bmatrix} X & Y \end{bmatrix}$$

$$0.1X + 0.6Y = X$$

$$0.9X + 0.4Y = Y$$

$$X = \frac{0.6}{0.9}Y = \frac{2}{3}Y$$

$$X + Y = 1$$

$$1 - Y = \frac{2}{3}Y$$

$$X = \frac{2}{5} \quad Y = \frac{3}{5}$$

Bayes' Rules

Bayes' Theorem relates cause and effect in such a way that by understanding the effect/s we can learn the probability of its cause

... from the effect/s to the cause in a probabilistic perspective ...

Bayes' Rules

... from the effect/s to the cause in a probabilistic perspective ...

This is particularly important in Medicine to determine :

- the causes of a disease
- the effects of some particular medication on that disease

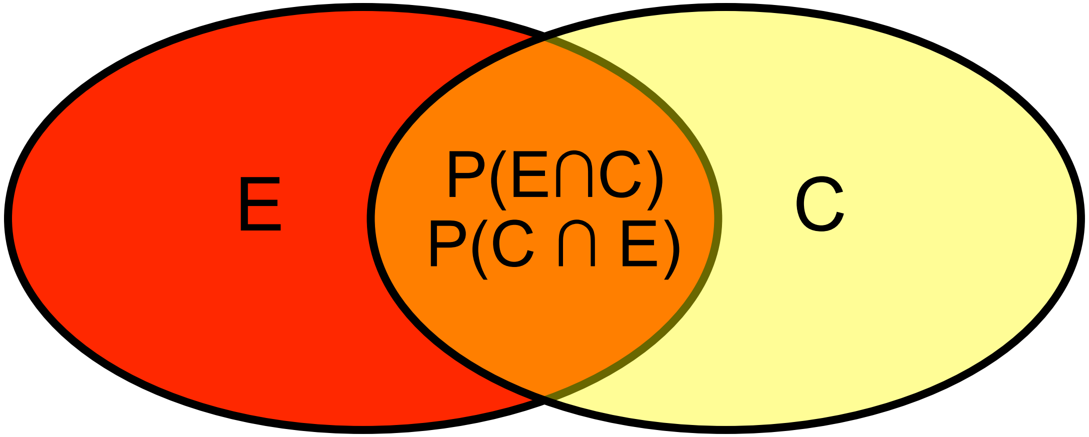
Bayes' Rules

Remember this:

$$P(\text{cause} \mid \text{effect}) = \frac{P(\text{cause})\, P(\text{effect} \mid \text{cause})}{P(\text{effect})}$$

Bayes' Rules

$P(E \cap C) = P(C \cap E)$



Product Rule

$P(E \cap C) = P(E|C)P(C)$ $\quad$ $P(C \cap E) = P(C|E)P(E)$

Bayes' Rules

$P(E \cap C) = P(E|C)P(C)$ $\qquad$ $P(C \cap E) = P(C|E)P(E)$

$$P(E \cap C) = P(C \cap E)$$

$$P(E|C)P(C) = P(C|E)P(E)$$

$$\frac{P(E|C)P(C)}{P(E)} = P(C|E)$$

Bayes' Rules

$$\frac{P(E|C)P(C)}{P(E)} = P(C|E)$$

$$P(Cause|Effect)=\frac{P(Effect|Cause)P(Cause)}{P(Effect)}$$

Bayes ‘ Rules
Law of total probability

$E = (E \cap C) \cup (E \cap \neg C)$

$P(E) = (E \cap C) + P(E \cap \neg C)$

$P(E) = P(E \mid C)P(C) + P(E \mid \neg C)P(\neg C)$

Bayes ‘ Rules
Law of total probability

$$P(E) = P(E \mid C)P(C) + P(E \mid \neg C)P(\neg C)$$

$$P(C \mid E) = \frac{P(E \mid C)P(C)}{P(E)}$$

$$P(C \mid E) = \frac{P(E \mid C)P(C)}{P(E \mid C)P(C) + P(E \mid \neg C)P(\neg C)}$$

Bayes ‘ Rules
Exercise: screening test for breast cancer

$P(\text{cancer}) = 0.01$ (one in hundred)

$P(\text{pos} \mid \text{cancer}) = 0.9$

False Positive $P(\text{pos} \mid \neg\text{cancer}) = 0.2$

False Negative $P(\text{neg} \mid \text{cancer}) = 0.1$

What is the probability that a woman tested positive has a cancer?

$P(\text{cancer} \mid \text{pos})$

Bayes ‘ Rules
Exercise: screening test for breast cancer

What is the probability that a woman tested positive has a cancer?

$$P(\text{cancer}|\text{pos}) = \frac{P(\text{pos}|\text{cancer})P(\text{cancer})}{P(\text{pos})}$$

$$P(\text{pos}) = P(\text{pos}|\text{cancer})P(\text{cancer}) + P(\text{pos}|\neg\text{cancer})P(\neg\text{cancer})$$

$$P(\text{cancer}|\text{pos}) = \frac{P(\text{positive}|\text{cancer})P(\text{cancer})}{P(\text{pos}|\text{cancer})P(\text{cancer}) + P(\text{pos}|\neg\text{cancer})P(\neg\text{cancer})}$$

$$P(\text{cancer}|\text{pos}) = 0.043$$

Bayes' Rules

We have multiple mutually exclusive cause c_i
for the effect E.

$E \wedge c_1$	$E \wedge c_2$	$E \wedge c_3$	$E \wedge c_4$
$E \wedge c_5$	$E \wedge c_6$	$E \wedge c_7$	$E \wedge c_8$
$E \wedge c_9$	$E \wedge c_{10}$	$E \wedge c_{11}$	$E \wedge c_{12}$

$$E = (E \wedge c_1) \vee (E \wedge c_2) \vee \ldots \vee (E \wedge c_{12}$$

$$P(E) = P(E \wedge c_1)+P(E \wedge c_2) +.+ P(E \wedge c_{12}$$

$$P(E) = \Sigma\, P(E \wedge c_i) = \Sigma P(E|c_i)P(c_i)$$

$$P(c_k\,|E)=\frac{P(E|\,c_k)P(c_k)}{\Sigma P(E|c_i)P(c_i)}$$

Bayes' Rules with multiple mutually exclusive causes

$$P(\text{Cause}_k|\text{Effect}) = \frac{P(\text{Effect}|\text{Cause}_k)P(\text{Cause}_k)}{\Sigma P(\text{Effect}|\text{cause}_i)P(\text{cause}_i)}$$

Bayes' Rules
Exercise

Suppose you go out to buy a car.

The probability you go to dealer 1 $P(d_1)=0.2$
The probability you go to dealer 2 $P(d_2)=0.4$
The probability you go to dealer 3 $P(d_3)=0.4$
The probability of buying car (E) from $d_1 = P(E|d_1) = 0.2$
The probability of buying car (E) from $d_2 = P(E|d_2) = 0.4$
The probability of buying car (E) from $d_3 = P(E|d_3) = 0.3$

You buy E

What is the probability that you bought it from d_2? $P(d_2|E)$

Dealers = The causes

Buy a car (E) = The effect

Bayes' Rules
Exercise

$$P(d_2|E) = 0.5$$

Bayes' Rules
multiple conditionally independent effects

Let's assume we are in a medical context with multiple symptoms e_1, e_2, e_3, ..., e_n from set E one disease c

If you can assume that, for example, the symptoms are conditionally independent, Bayes' probability is easy to compute

Bayes' Rules
multiple conditionally independent effects

Symptoms (Effects) are conditionally independent if they are directly caused by the cause but they do not have a direct effect on each other.

(e.g., toothache and catch are conditionally independent given Cavity.
Toothache and catch are caused by cavity but neither has a direct effect on the other).

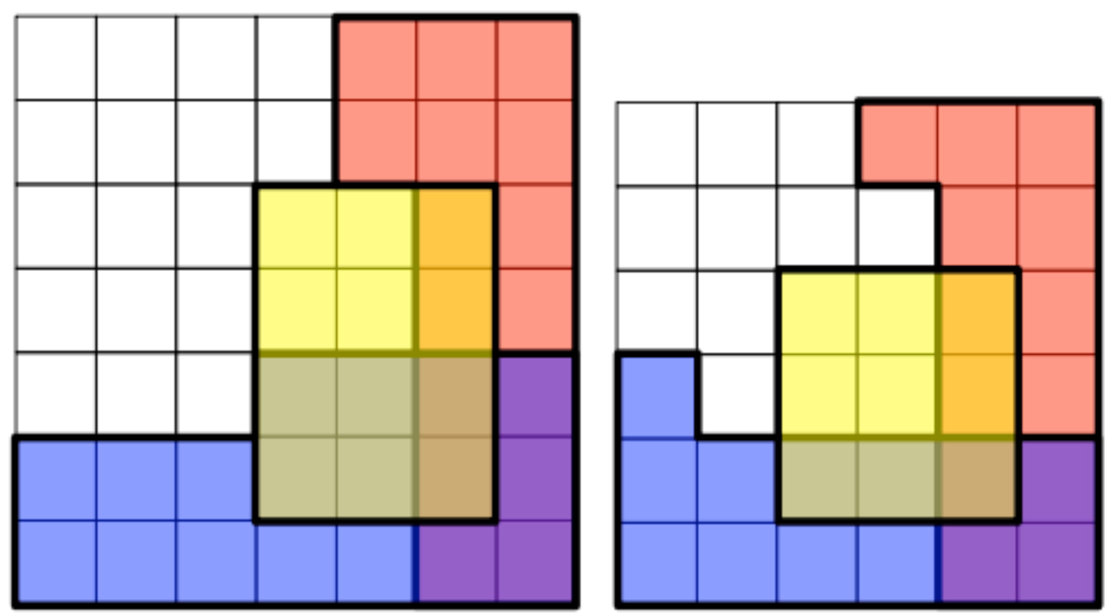
Conditional Independence

$P(A \cap B \mid C) = P(A \mid C)\, P(B \mid C)$

A and B are independent given C

R(red), Y(yellow), B (blue) examples

$P(R \cap B \mid Y) = P(R \mid Y)\, P(B \mid Y)$



R (A), and B (B) are independent given Y (C)

Bayes' Rules

multiple conditionally independent effects

$$P(A \cap B \,|\, C) = P(A \,|\, C)\, P(B \,|\, C)$$

One cause, multiple symptoms

$$P(e_1 \wedge e_2|c) = P(e_1|c)\, P(e_2|c)$$

$$P(c|\, e_1,e_2,\text{etc.}) = \frac{P(c) \prod P(e_i|c)}{P(e_1,e_2, \ldots, e_j)} = P(c) \prod P(e_i|c)$$

See Naive Bayes Classifier on Wikipedia
for a formal demonstration

A or B

The Bayes ‘ Rule is for computing:

A) The Probability of Causes given Effects

B) The Probability of Effects given Causes

A or B

$E \wedge c_1$	$E \wedge c_2$	$E \wedge c_3$	$E \wedge c_4$
$E \wedge c_5$	$E \wedge c_6$	$E \wedge c_7$	$E \wedge c_8$
$E \wedge c_9$	$E \wedge c_{10}$	$E \wedge c_{11}$	$E \wedge c_{12}$

The Table on this slide, represent a scenario in which:

A) The causes are disjoint events

B) The causes are joint events

Choose among the options

If three fair coins are flipped, what is the probability that the two faces are alike?

A) 1/8

B) 1/3

C) 1/4

D) 1/2