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| *Rhif y modiwl*<br>Module number | |
| *Dyddiad*<br>Date | |

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| *Nifer y tudalennau a gyflwynwyd*<br>Number of pages handed in | |
| **Rhif Cyfair y Myfyriwr/**<br>**Student Reference Number**<br><br>**.....** | |

Cyfenw/Surname …….

Enwau Cyntaf/ First Names …….

Llofnod/Signature ……

## Cyfarwyddiadau

1. Darllenwch lawlyfr y myfyrwyr ar y we i sicrhau eich bod yn ymwybodol beth yw llên-ladrad. Darllenwch y Datganiad Gwaith Gwreiddiol isod a'i arwyddo yng nghornel uchaf ochr dde y dudalen i ddweud mai eich gwaith chi yw hwn.
2. Cysylltwch y dudalen yma â'ch gwaith at ei gilydd trwy ddefnyddio'r styffylwr o'r dderbynfa.
3. Cyflwynwch eich aseiniad yn ôl cyfarwyddyd y darlithydd.

## Datganiad Gwaith Gwreiddiol

Trwy arwyddo'r uchod, yr wyf yn cadarnhau mai:

Fy ngwaith gwreiddiol i yw hwn, oni bai ei fod wedi nodi yn eglur fel arall.

Deallaf fod y gosb am lên-ladrad ac unrhyw ymddygiad annheg arall yn llym, ac y gallai arwain at golli marciau neu hyd yn oed beidio â dyfarnu gradd.

Yr wyf wedi darllen yr adrannau am ymddygiad annheg yn Llawlyfr Arholiadau'r Myfyrwyr a'r adrannau perthnasol yn rhifyn cyfredol Llawlyfr Myfyrwyr yr Adran Cyfrifiadureg.

Yr wyf yn deall rheoliadau'r Brifysgol ar y materion hyn a chytunaf i gadw atynt.

## Instructions

1. Please read the student handbook on the web to ensure that you are aware of the plagiarism issue. Read the Declaration of Originality below and sign in the space provided at the top right of this page to say that the work is your own.
2. Securely fasten this page to your work using a stapler.
3. Please submit your assignment as directed by your lecturer.

## Declaration of Originality

In signing above, I confirm that:

This submission is my own work, except where clearly indicated.

I understand that there are severe penalties for plagiarism and other unfair practice, which can lead to loss of marks or even the withholding of a degree.

I have read the sections on unfair practice in the Students' Examinations Handbook and the relevant sections of the current Student Handbook of the Department of Computer Science.

I understand and agree to abide by the University's regulations governing these issues.

Computer Science, CS26210: The Artificial Intelligence Toolbox
Part II: Programming in an Uncertain World
This exam counts for 15% of the total marks, and it will be marked out of 30
Calculators, laptops are not permitted. However, you don't need to do any calculation, leave fractions, do not multiply etc.

Time allowed: 50 min.

ANSWER ALL QUESTIONS

1) These questions are about Set Theory. Given the following sets: [*5 marks*]
U = {x: -20 ≤ x ≤ 20}; this is our universal set;
A = {x:x = -20, -19, -18, -17, -16, 16, 17, 18, 19, 20};
B = {x: -15 < x < 15};
C = {x:x = 16, 17, 18, 19, 20};

Compute:

a) $A \cup B$ ={x:x=-20 ≤ x ≤ 20, x≠15, x≠-15};
b) $A \cap C$ = {16, 17, 18, 19, 20};
c) $\bar{A}$ = { x:x=-15 ≤ x ≤ 15};
d) $\bar{A} \cap B$ = { x:x=-14 ≤ x ≤ 14};
e) $\bar{A} \cap \bar{B}$ = { x=-15, x=15};

2) This question is about Sample Space, Events, and Probability. Consider the following experiment: [*3 marks]*
**We roll a pair of fair dice once, with $x_1$ and $x_2$ indicating the number on each dice**

a) Define (using the synthax of set theory) the sample space of this experiment.
S ={$(x^1, x^2)$: $1 \le x^1 \le 6$; $1 \le x^2 \le 6$;}

b) Define (using the synthax of set theory) the set of those elementary events in which the sum of the two numbers ($x_1 + x_2$) is 6, and find the probability of the random variable $X_{(\omega)}^{(6)}$, with $\omega = x_1 + x_2$.
B={(3,3), (5,1), (1,5), (2,4), (4,2)}; $P(X_{(\omega)}^{(6)}) = 5/36$

c) Define (using the synthax of set theory) the set of those elementary events in which the product of the two numbers is 9, and find the probability of the random variable $X_{(\Phi)}^{(9)}$, with Φ = ($x_1$ multiply $x_2$).

C = {(3,3)}; $P(X_{(\Phi)}^{(9)}) = 1/36$

3) This question is about Probability. Consider the following experiment: [*3 marks*]
**We draw 1 card at random from a standard deck of 52 cards:**

a) What is the probability that the card is a queen (Q) or a king (K)? Explain how you computed the result.
$P(Q \cup K) = P(Q) + P(K) = 2/13$; Q and K are disjoint events

b) What is the probability that the card is a red ace? Explain how you computed the result.
P(RA) = 1/26;

c) What is the probability that the card is a queen (Q) or a heart (H)? Explain how you computed the result.

$P(Q \cup H) = P(Q) + P(H) - P(Q \cap H) = 4/52 + 13/52 - 1/52 = 4/13$; Q and H are joint events

4) This question is about Conditional Probability and Conditional Independence. Consider the following experiment: [*1 mark*]

**We draw the top card from a standard deck of 52 cards and we are told that the card is a club:**

Demonstrate that the clubs and aces are conditionally independent. Explain how you computed the result.

$P(A|B) = P(A)$ with A = aces, B = clubs

$P(A) = 4/52 = 1/13$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/52}{13/52} = 1/13$

5) This question is about Conditional Probability and Conditional Independence. Consider the following experiment: [*2 marks*]

**A fair coin is flipped 4 times.**

a) What is the probability that the fourth flip is a head, given that 3 heads occurred in the four flips? Explain how you computed the result.

$P(3 \text{ heads}) = ¼$

$P(4 \text{ flip head} \cap 3 \text{ heads}) = 3/16$

$P(4 \text{ flip head} \mid 3 \text{ heads}) = \frac{P(4 \text{ flip head} \cap 3 \text{ heads})}{P(3 \text{ heads})} = 3/16 * 4 = ¾$

H H T H
H T H H
H H H T
T H H H

b) What is the probability that the fourth flip is a head, given that 2 heads occurred in the four flips? Explain how you computed the result.

$P(2 \text{ heads}) = 1/4$

$P(4 \text{ flip head} \cap 2 \text{ heads}) = 1/8$

$P(4 \text{ flip head} \mid 2 \text{ heads}) = \frac{P(4 \text{ flip head} \cap 2 \text{ heads})}{P(2 \text{ heads})} = 1/8 * 4 = 1/2$

6) This experiment is about the Product Rule. Consider the following experiment: [*1 marks*]

**Two cards are drawn at random without replacement from a deck of 52 cards. B is the event of drawing a black card on the first trial, and A is the event of drawing a black card on the second trial:**

What is the probability that both cards are black (make use of the Product Rule)? Explain how you compute the result.

$P(B) = 26/52$

$P(A|B) = 25/51$

$P(A \cap B) = P(A|B)\ P(B) = 26/52 \text{ x } 25/51$

7) This experiment is about the Product Rule. Consider the following experiment: [*6 marks*]

**Box 1 contains 4 defective and 16 non-defective light bulbs. Box 2 contains 1 defective and 1 non-defective light bulb. We roll a fair dice 1 time. If we get a 1 or a 2, then we select a bulb at random from box 1. Otherwise we select a bulb at random from box 2.**

What is the probability that the selected bulb will be defective? Explain how you compute the result.

P(A) = 1/3; we select a bulb from box 1
P(Ā) = 2/3; we select a bulb from box 2
P(B) = (A ∩ B) ∪ (Ā ∩ B) = P(A|B) P(B) + P(Ā|B) P(B); The selected bulb is defective
P(B|A) = 1/5
P(B|Ā) = 1/2
P(B ∩ A) = P(B|A) P(A) = 1/5 * 1/3
P(B ∩ Ā) = P(B|Ā) P(Ā) = 1/2 * 2/3

8) This experiment is about using the Full Joint Probability Distribution Table to draw inferences.
**Consider the following four random variables:** [*3 marks*]
**- variable H, Headheache, with H= {h, ¬h}**
**- variable A, just attented a CS262 lecture, with A= {a, ¬a}**
**- variable S, the slides were clear, with S= {s, ¬s}**
**- variable W, weather, with W ={sunny, rainy, cloudy, snow}, P(W) = <0.6, 0.1, 0.29. 0.01>;**
Given the table below, compute P(cloudy, s, a, ¬h), and P(sunny | s, ¬a, ¬h) considering that W is conditionally independent from H, A, and S.

| | H | | ¬h | |
|---|---|---|---|---|
| | s | ¬s | s | ¬s |
| a | .108 | .012 | .072 | .008 |
| ¬a | .016 | .064 | .144 | .576 |

P(cloudy, s, a, ¬h) = P(cloudy) x P(s, a, ¬h) = 0.29 x .072
P(sunny | s, ¬a, ¬h) = P(sunny) = 0.6

9) This experiment is about First-order Markov Decision Process [*6 marks*]

**Consider the following: 30% of all people who now have a Vauxhall will buy another Vauxhall. 60% of the people that have not a Vauxhall now will buy a Vauxhall. Let's assume that 40% of the people have a Vauxhall now.**

a) Over a period of 2 units of time, how many people will have a Vauxhall? Explain how you computed the result.

$$\begin{bmatrix} .4 & .6 \end{bmatrix}\begin{bmatrix} .3 & .7 \\ .6 & .4 \end{bmatrix}=\begin{bmatrix} .4*.3+.6*.6 & .4*.7+.6*.4 \end{bmatrix}$$

$$\begin{bmatrix} .4*.3+.6*.6 & .4*.7+.6*.4 \end{bmatrix}\begin{bmatrix} .3 & .7 \\ .6 & .4 \end{bmatrix}=$$

b) Show how to compute the steady-state matrix and briefly say what this matrix is.

$$\begin{bmatrix} X & Y \end{bmatrix}\begin{bmatrix} 0.3 & 0.7 \\ 0.6 & 0.4 \end{bmatrix}=\begin{bmatrix} X & Y \end{bmatrix}$$

$$0.3X + 0.6Y = X$$

$$0.7X + 0.4Y = Y$$

$$X = \frac{0.6}{0.7}Y$$

$$X + Y = 1$$