The Artificial Intelligence Toolbox Part II – CS26210

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Program

Week 1

7/02 Set Theory, Fuzzy Logic (319)
8/02 Fuzzy Logic (B20) - Hand-out Assignment 1

Week 2

14/02 Fuzzy Logic - Further Exercises (319)
15/02 Theory of Probability (B20)

Week 3

21/02 Conditional Probability (319)
22/02 Probabilistic Methods (B20) - Hand-in Assignment 1 (Blackboard)

Week 4

28/02 Bayesian Networks (B20)
1/03 In Class Test (319) (Set Theory, Theory of Probability, Conditional Probability)

Week 5

7/03 Bayesian networks (319)
8/03 Discussion, further exercises (B20) - Hand-out Assignment 2
22/03 Hand-in Assignment 2 (Blackboard)

Friday 15st, February 2013

- Sample Space
- Events
- Probability
- Disjoint Events
- Joint Events
- How to calculate probabilities
- Conditional probability

Thursday 21st, February 2013

- $P(A|B)$, $P(A \cup B)$, and $P(A \cap B)$
- Cond. Independence
- Product Rule
- Discrete Random Variable

Friday 22nd, February 2013

- Joint Probability Distribution
- Full Joint Probability Distribution Table
- First-order Markov Chain

Please make your selection

If three fair coins are flipped, what is the probability that the three faces are alike?

A) 1/4
B) 1
C) 1/2
D) 1/3

True or False

We roll a pair of fair dice 1 time. The sample space is:

$S = \{(x_1, x_2): x_1=1,...,6; x_2=1,...,6;\}$

$S=\{(1,1),(1,2), ..., (6,6)\}$; 36 elements

$X(\omega) = (x_1 * x_2)/ x_1$; for $\omega = (x_1, x_2) \in S$

Is X a random variable?

Joint Probability Distribution
and
Full Joint Probability Distribution
Table

Probability in Artificial Intelligence

In AI, instead of talking about events,
we talk about propositions.
A = true or A = a
A = false or A = ¬a

Proposition can be combined using propositional logic

$P(\text{cavity} \mid \text{toothache}) = 0.6$
whenever toothache is true and we have no further information, cavity is true with probability P=0.6

$P(\text{cavity} \mid \text{toothache} \wedge \neg\text{cavity}) = 0$
$P(\text{cavity} \mid \text{toothache} \wedge \text{cavity}) = 1$

Domain and Probability Distribution

The Domain

is the set of possible values that a random variable can take on.

Role pair of fair dice {2,...,12}

Age {juvenile, teen, adult};

Weather {sunny, rain, cloudy, snow}

Probability distribution

Probability of all the possible values of a random variable

P(Weather) = <0.6, 0.1, 0.29, 0.01>

Joint Probability Distribution

In the study of probability, given two random variables X and Y, the joint probability distribution for X and Y defines the probability of events defined in terms of both X and Y.

$$P(X, Y)$$

Joint Probability Distribution

Two Random variables X and Y

We roll a pair of fair dice. The sample space is:

$S = \{(x_1, x_2): x_1=1,...,6; x_2=1,...,6;\}$, or $S=\{(1,1),..., (6,6)\}$;

Random Variable X -

Let X be the sum of the two numbers that occur

$X(\omega) = (x_1 + x_2)$; for $\omega = (x_1, x_2) \in S$; The domain of $X = \{2,..., 12\}$

Random Variable Y -

Let Y be assignment of "*odd*" to each pair of odd numbers and "*even*" to each pair in which there is at least one even number

The domain of Y ={*odd, even*}

Joint Probability Distribution

ω	$X^{(x)}$	$Y^{(y)}$
(1,1)	2	odd
(1,2)	3	even
...		
(2,1)	3	even
...		
(6,6)	12	even

The Joint Probability Distribution for X and Y P(X, Y), defines the probability of events defined in terms of both X and Y

$$P(X, Y) = P(X)^{(4)} \wedge P(Y)^{(odd)}$$

(3,1) (1,3) (2,2)

$$P(X, Y) = 1/36 + 1/36 = 1/18$$

Please make your selection

$$P(X, Y) = P(X)^{(5)} \wedge P(Y)^{(even)}$$

A) 2/3

B) 1/9

C) 1/2

Joint Probability Distribution with proposition

A person goes to the dentist with toothache or just for a check.

The probability to find a cavity, without toothache, is P(Cavity)

The probability to find a cavity, with toothache, is
P(cavity | toothache)

The first thing the dentist does, is to check for cavities with a steel probe. There is a cavity if the probe catches in the person's teeth. P(Catches, Cavity)

Joint Probability Distribution with proposition

First Random Variable Toothache {toothache, ¬toothache};
Second Random Variable Cavity {cavity, ¬cavity}
Third Random Variable Catch{catch, ¬catch}

P(Toothache, Cavity, Catch)
denotes the joint probability distribution of all combinations of the values of the random variables Toothache, Cavity, Catch

The best way to represent all combinations of the values of *n* random variables is by drawing a

Full Joint Probability Distribution Table

Full Joint Probability Distribution Table

	toothache		*¬toothache*	
	catch	¬catch	*catch*	¬catch
cavity	P_1	p_2	P_3	p_4
¬cavity	p_5	p_6	p_7	p_8
Full Join Probability Distribution Table				

$$p_1+p_2+p_3+p_4+p_5+p_6+p_7+p_8 = 1$$

The probability of any proposition P(A) is given by the sum of the probabilities of those possible world in which the proposition A is true

$$P(\text{Cavity})^{(\text{cavity})} = p_1+p_2+p_3+p_4;\ P(\text{Cavity})^{(\neg \textit{cavity})} = p_5+p_6+p_7+p_8$$

Full Joint Probability Distribution Table

	toothache		*¬toothache*	
	catch	¬catch	*catch*	¬catch
cavity	*.108*	*.012*	*.072*	*.008*
¬cavity	*.016*	*.064*	*.144*	*.576*
Full Join Probability Distribution Table				

P(cavity ∧ toothache) = .108 + .012
P(cavity ∨ toothache) = .108 + .012 + .072 + .008 + .016 + .064

P(cavity | toothache) = P(cavity ∧ toothache)/P(toothache)
P(¬cavity | toothache) = P(¬cavity ∧ toothache)/P(toothache)

Joint Probability Distribution

First Random Variable Toothache {toothache, ¬toothache};
Second Random Variable Cavity {cavity, ¬ cavity}
Third Random Variable Catch{catch, ¬ catch}
Forth Random Variable Weather{sunny, rain, cloudy, snow}

P(Toothache, Cavity, Catch, Weather)
2 x 2x 2 x 4 = 32 entries for the JPDT
or
4 times the 2x2x2 original table

Full Joint Prob. Distribution Table

sunny

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬*
cavity	*.108*	*.012*	*.072*	
¬cavity	*.016*	*.064*	*.144*	
Full Join Probability Distribution Ta				

snow

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	*.108*	*.012*	*.072*	*.008*
¬cavity	*.016*	*.064*	*.144*	*.576*
Full Join Probability Distribution Table				

cloudy

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬c*
cavity	*.108*	*.012*	*.072*	*.0*
¬cavity	*.016*	*.064*	*.144*	*.5*
Full Join Probability Distribution Tabl				

rain

	Toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	*.108*	*.012*	*.072*	*.008*
¬cavity	*.016*	*.064*	*.144*	*.576*
Full Join Probability Distribution Table				

What relationship do these table have to each other?

Joint Probability Distribution one more variable

Let's assume we are interested in this relationship:
P(sunny, toothache, cavity, ¬catch)

The product rule
$P(A \wedge B) = P(A \mid B) * P(B)$

A B
P(sunny, toothache, cavity, ¬catch) =
P(sunny | toothache, cavity, ¬catch) * P(toothache, cavity, ¬catch)

Joint Probability Distribution one more variable

We can assume that sunny is conditionally independent from all the rest, because the weather has noting to do with toothache

Conditional Independence

$P(A \mid B) = P(A) \qquad P(B \mid A) = P(B)$

$P(A \wedge B) = P(A) * P(B)$

$P(sunny \mid toothache, cavity, \neg catch) = P(sunny)$

$P(sunny, toothache, cavity, \neg catch) = P(sunny) * P(toothache, cavity, \neg catch)$

P(Weather, Toothache, Cavity, Catch,) = P(Weather) * P(Toothache, Cavity, Catch)

Joint Probability Distribution
one more variable

P(Toothache, Cavity, Catch, Weather) = P(Weather) * P(Toothache, Cavity, Catch)

... therefore, we need a 4 entries table and a 2x2x2 entries table to make any inference (to calculate joint and conditional probabilities), instead of a 32 entries table.

Joint Probability Distribution one more variable

	Toothache		*¬toothache*	
	Catch	¬catch	*Catch*	¬catch
Cavity	*.108*	*.012*	*.072*	*.008*
¬cavity	*.016*	*.064*	*.144*	*.576*
Full Join Probability Distribution Table				

Weather
sunny
cloudy
rain
snow

P(Weather, Toothache, Cavity, Catch,) =

P(Weather) * P(Toothache, Cavity, Catch)

Temporal Reasoning

…. is reasoning about events that depend on time. That is the value of random variables changes during diagnosis.

The system's progression through a sequence of status is called stochastic process if it is probabilistic.

First-order Markov Chain

A stochastic process with the following characteristics:

1. There is a finite number of possible states.
2. The process can be in one and only one state at time.
3. The process moves or steps successively from one state to another over time.
4. The probability of a move depends only on the immediate proceeding state.

…. is a First-order Markov decision process or Markov chain.

First-order Markov Chain

Let's consider the system $S = \{s_1, s_2, s_3, s_4\}$

The system changes state at regular discrete time intervals $T = \{t_1, t_2, t_3, t_4\}$

The system changes state according to the distribution of probabilities associated with each state.

σ_t = current state
σ_{t-1} = previous state

$$P(\sigma_t) = P(\sigma_t | \sigma_{t-1})$$

This is a conditional probability

First-order Markov Chain

$S = \{s_1, s_2, s_3, s_4\}$

The probabilistic relationships between states do not change over time

Transition probability $a_{ij} = P(\sigma_t = s_i \mid \sigma_{t-1} = s_j)$, for each i:

	S1	S2	S3	S4	T
S1	a_{11}	a_{12}	a_{13}	a_{14}	1
S2	a_{21}	a_{22}	a_{23}	a_{24}	1
S3	a_{31}	a_{32}	a_{33}	a_{34}	1
S4	a_{41}	a_{42}	a_{43}	a_{44}	1

First-order Markov Chain

system S = {s_1=sunny, s_2=cloudy, s_3=rainy, s_4=foggy}

Today is sunny. What is the probability of the next four days being sunny, cloudy, foggy, rainy?

$E = \{s_1, s_1, s_2, s_4, s_3\}$;

$P(E) = P(s_1) \times P(s_1|s_1) \times P(s_2|s_1) \times P(s_4|s_2) \times P(s_3|s_4)$

	S1	S2	S3	S4	T
S1	a_{11}	a_{12}	a_{13}	a_{14}	1
S2	a_{21}	a_{22}	a_{23}	a_{24}	1
S3	a_{31}	a_{32}	a_{33}	a_{34}	1
S4	a_{41}	a_{42}	a_{43}	a_{44}	1

First-order Markov Chain

system S = {s_1=sunny, s_2=cloudy, s_3=rainy, s_4=foggy}

The expected number of observations of, or duration d_i, within any state s_i, given that the first observation is in that state:

$$d_i = 1 / (1 - a_{ii})$$

	S1	S2	S3	S4	T
S1	a_{11}	a_{12}	a_{13}	a_{14}	1
S2	a_{21}	a_{22}	a_{23}	a_{24}	1
S3	a_{31}	a_{32}	a_{33}	a_{34}	1
S4	a_{41}	a_{42}	a_{43}	a_{44}	1

First-order Markov Chain

10% of al people who now have a Renault will buy another Renault

60% of the people that have not a Renault now will buy a Renault.

Let’s assume that 80% of the people have a Renault now.
The question is: over a long period of time (i.e., 3 unit of time), how many people will have a Renault?

Start from: Transitional matrix
Tree Diagram of States

First-order Markov Chain

Steady-State Matrix (S)
S = ST
With T = transition matrix
S is independent from initial state

$$\begin{bmatrix} X & Y \end{bmatrix}\begin{bmatrix} 0.1 & 0.9 \\ 0.6 & 0.4 \end{bmatrix} = \begin{bmatrix} X & Y \end{bmatrix}$$

$$0.1X + 0.6Y = X$$

$$0.9X + 0.4Y = Y$$

$$X = \frac{0.6}{0.9}Y = \frac{2}{3}Y$$

$$X + Y = 1$$

$$1 - Y = \frac{2}{3}Y$$

$$X = \frac{2}{5} \quad Y = \frac{3}{5}$$

True or False

Given that Weather is independent from Dental problems, I can assume that

P(Weather, Toothache, Cavity, Catch,) = P(Weather) * P(Toothache, Cavity, Catch)