The Artificial Intelligence ToolBox Part II – CS26210

Elio Tuci
elt7@aber.ac.uk

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Program

Week 1

7/02 Set Theory, Fuzzy Logic (319)
8/02 Fuzzy Logic (B20) - Hand-out Assignment 1

Week 2

14/02 Fuzzy Logic - Further Exercises (319)
15/02 Theory of Probability (B20)

Week 3

21/02 Conditional Probability (319)
22/02 Conditional Probability (B20) - Hand-in Assignment 1 (Blackboard)

Week 4

28/02 Bayesian Networks (319)
1/03 In Class Test (B20) (Set Theory, Theory of Probability, Conditional Probability)

Week 5

7/03 Bayesian networks (319)
8/03 Discussion, further exercises (B20) - Hand-out Assignment 2
22/03 Hand-in Assignment 2 (Blackboard)

Friday 15th, February 2013

- Sample Space
- Events
- Probability
- Disjoint Events
- Joint Events
- How to calculate probabilities
- Conditional probability

Sample Space
the set of all possible outcomes

Probability theory
has to do with experiments that have a set of distinct outcomes

Experiments: any operation whose outcomes cannot be predicted with certainty, but we can predict all the possible outcomes

- Experiment: the role of a single die;
  Sample space = (the set of all possible outcomes)
  S = {1, 2, 3, 4, 5, 6}

- Experiment: drawing the top card from a deck of 52 cards
  Sample space S = {1, ..., 52}

Events

An Event is a subset of the sample space

Experiment: the role of a single die;
Sample space S = {1, 2, 3, 4, 5, 6};

E={2,4,6}; E (even numbers) is a subset of S or $E \subset S$

O={1,3,5}; O (odd numbers) is a subset of S or $O \subset S$

Elementary event G={1}, a set of one single element, $G \subset S$

$(E \cap O) = \emptyset$ the elements of E and O (Intersection)

$(E \cup O) = \{1,2,3,4,5,6\}$ the elements of E, or O or both (Union)

Events

An Event is a subset of the sample space

In an experiment,
an event occurs if any of its elements occurs.

Choose among the options

Experiment: the role of a single die;
The result is 3. What event occurred?

A) The event E={2,4,6};

B) The event O={1,3,5};

C) The event F={1,2,4,6};

D) The event G={1,2,4,5,6};

Probability

Probability can be called a measure (a number) applied to the events that can occur.

This number corresponds to the relative frequency of the event.

WE HAVE AN EXPERIMENT
WE HAVE A SAMPLE SPACE
WE HAVE AN EVENT
PROBABILITY IS A MEASURE ASSOCIATED TO THIS EVENT

Probability (Axioms)

1 - The probability of something we are sure will occur is 1

2 - Probability can not be negative

3 - $P(A_1 \cup A_2 \cup A_3 \cup \ldots A_n) = \sum P(A_n)$
non-overlapping events
$A_i \cap A_j = \emptyset$ for all $i \neq j$

Probability
first axiom

Experiment: the role of a single die

Sample space $S=\{e_1,e_2,e_3,e_4,e_5,e_6\}$;

$(\Sigma_i P(e_i) = 1)$ - FIRST AXIOM

Please make your selection

Experiment: the role of a single die
What is the probability to get 1,3, or 5?

A) 1/6

B) 3/5

C) 1/2

Probability
third axiom

Experiment: the role of a single die
What is the probability to get 1,3, or 5?

Sample space S = {1,2,3,4,5,6}
Event $O \subset S$ with O ={1,3,5}
OUR MODEL
$P(1) = P(2) = P(3) = P(4) = P(5) = P(6)$

$P(O) = m(O)/n(S)$ with $n(S)$ the number of elements in S
and $m(O)$ the number of elements in O

$P(O) = 3/6 = 1/2$
$P(O) = P(1) + P(3) + P(5)$

Probability

Experiment: drawing the top card from a deck of 52
What is the probability to get a queen?

Sample space S = {1,..., 52};

For $E \subset S$ with E ={queen}; $E=\{q_{hearts}, q_{diamonds}, q_{spades}, q_{clubs}\}$;

$P(E) = P(\{q_h\}) + P(\{q_d\}$ + etc.) = n(E)/n(S);

p(E) = 1/52 + 1/52 + 1/52 + 1/52 = 4/52 = 1/13

Please make your selection

Experiment = the role of a pair of dice

Sample space S = $\{(x_1,x_2); x_1=1,..,6; x_2=1,..,6\}$;

S = {(1,1),(1,2), ...(6,6)}; there are 36 elementary events

A is the event that the sum is 2 – What is P(A)?

A) 1/12

B) 2/15

C) 1/36

Probability

Experiment = the role of a pair of dice

Sample space S = $\{(x_1,x_2); x_1=1,..,6; x_2=1,..,6\}$;

S = {(1,1),(1,2), ...(6,6)}; there are 36 elementary events

A is the event that the sum is 2 – What is P(A)?

A = {(1,1)}

P(A) = n(A)/n(S) = 1/36

B is the event that the sum is 7 – What is P(B)?

B = {(1,6), (2,5),(3,4),(4,3),(5,2),(6,1)}

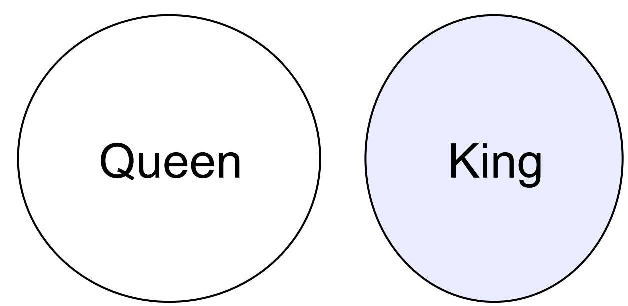
p(B) = 6/36

Disjoint Events

Experiment = drawing the top card from a deck of 52
Sample space S = {1,..., 52};
Which is the probability of picking up a queen or a king?

For $Q \subset S$ with Q={queen} and $K \subset S$ with K={king}
$Q = \{queen_{hearts}, queen_{diamonds}, queen_{spades}, queen_{clubs}\}$;
$K = \{king_{hearts}, king_{diamonds}, king_{spades}, king_{clubs}\}$;

$Q \cap K = \emptyset$ (Q and K are disjoint event)
$P(Q \cup K) = P(Q) + P(K)$
$P(queen \cup king) = P(queen) + P(king)$



Joint Events

Experiment = drawing the top card from a deck of 52
Sample space = {1,..., 52};
Which is the probability of picking up a queen or a heart?
$P(Q \cup H)$?
For $Q \subset S$ with Q = {queen}
$Q = \{queen_{hearts}, queen_{diamonds}, queen_{spades}, queen_{clubs}\}$;
$H \subset S$ with H = {heart}; H = {1,...,13};

$Q \cap H \neq \varnothing$ (Q and H are not disjoint)
$P(Q \cup H) = P(Q) + P(H) - P(Q \cap H)$
$P(queen \cup heart)=P(queen)+P(heart) - P(queen \cap heart) =$
$4/52 \quad + \quad 13/52 - \quad 1/52 \quad = \; 4/13$

Conditional Probability

Conditional probability is the probability of event A, given the occurrence of some other event B.

$$P(A|B)$$

This means, when we are asked the probability that the event A happens, the event B is already happened. Therefore, we have to take into account that B is already happened when calculating the probability of A.

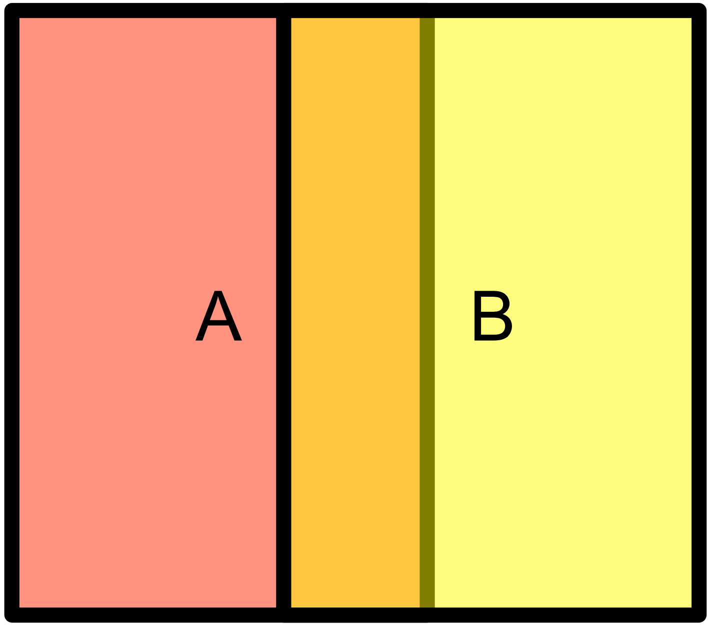
Conditional Probability

$P(A|B)$

The probability of event A given that we already have the evidence of event B

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Conditional Probability



A ∩ B
Intersection of A and B

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

The proportion of yellow area overlapping with the red area.

Conditional Probability

Experiment: select a card from a 52-card deck

Event B: the selected card is RED

Question: what is the probability that the selected card is the ace of hearts, given that the picked card is red?

B={x: x=1,...,13; hearts; x=1,...13; diamonds}

A={x: x = ace of hearts}

P(A|B)

Please make your selection

P(ace of hearts | red)

A) 1/26
B) 1/52
C) 1/2

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Exercise:

We roll a pair of faire dice 1 time and are given that the 2 numbers that happen are not the same. Compute the probability that :

- the sum is 7
- the sum is 4
- the sum is 12

Conditional Probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

- **Event B:** the numbers are not the same
- **Event A:** the sum is 7
- **Event C:** the sum is 4
- **Event D:** the sum is 12.

We proceed in the following way:

Step 1- Define the sample space, and count the elementary events.

Step 2 - Calculate how many elementary events of type A,B,C,D, there are in the sample space

Step 3- Calculate the conditional probability

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Step 1

Define the sample space, and count the elementary events.

$$U = \{(x_1, x_2): \text{with } x_1 = 1,\ldots,6;\ x_2=1,\ldots,6\}$$

36 elementary events

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Step 2 – count elementary events of type B with B – The numbers are not the same

$B=\{(x_1, x_2): (x_1 \neq x_2)$ with $x_1 = 1,\ldots,6; x_2=1,\ldots,6)$

We exclude (1,1) (2,2) (3,3) (4,4) (5,5) (6,6)

36 -6 = 30 elementary events

$P(B) = 30/36 = 5/6$

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Step 2 and 3 - count elementary event of type A and $P(A|B)$ with A - The sum is 7 (remember $P(B) = 5/6$)

$$A=\{(x_1, x_2): (x_1+x_2=7) \text{ with } x_1 = 1,...,6; x_2=1,...,6)\}$$

(4+3) (3+4) (1+6) (6+1) (5+2) (2+5)

$$P(A) = 6/36=1/6$$

$$P(A \cap B) = 6/36 = 1/6$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = (1/6)/(5/6) = 1/5$$

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Step 2 and 3 - count elementary event of type C and $P(C|B)$ with C - The sum is 4 (remember $P(B) = 5/6$)

$$C=\{(x_1, x_2): (x_1+x_2=4) \text{ with } x_1 = 1,\ldots,6;\ x_2=1,\ldots,6)\}$$

(3+1) (1+3) (2+2)

$P(C) = 3/36 = 1/12;$

$P(C \cap B) = 2/36 = 1/18$

$$P(C|B) = \frac{P(C \cap B)}{P(B)} = (1/18)/(5/6) = 1/15$$

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{(B)}$$

Step 2 and 3 - count elementary event of type C and P(C | B) with D - The sum is 12 (remember P(B) = 5/6)

$$D=\{(x_1, x_2): (x_1+x_2=12) \text{ with } x_1 = 1,...,6; x_2=1,...,6)\}$$

$$(6+6)$$

$$P(D) = 1/36 = 1/6$$

$$P(D \cap B) = 0/36 = 0$$

$$P(D | B) = \frac{P(D \cap B)}{P(B)} = (0)/(5/6)=0$$

True or False

An event is not a subset of the sample space

True or False

Experiment = drawing the top card from a deck of 52

Sample space S = {1,..., 52};

The probability of picking up a queen or a king is:

$$P(Q \cap K)$$

Choose among the options

If two fair coins are flipped, what is the probability that the two faces are alike?

A) $\frac{1}{2}$
B) $\frac{1}{4}$
C) $\frac{3}{4}$
D) 1

Disjoint Events

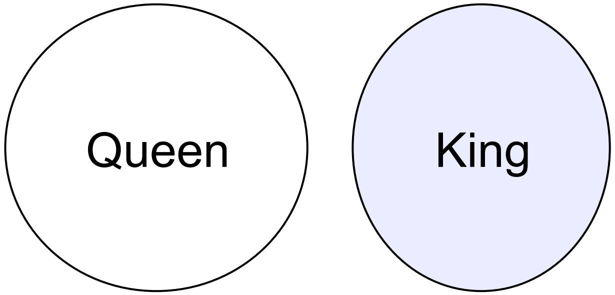
Disjoint events are events that have no intersection.

$Q \cap K = \varnothing$ (Q and K are disjoint event)

If I ask you the probability of $P(Q \cup K)$

then

$$P(Q \cup K) = P(Q) + P(K)$$



Joint Events

Joint events are events that intersect
$Q \cap R \neq \emptyset$ (Q and R are joint event)

If I ask you the probability of $P(Q \cup R)$

$$P(\text{queen} \cup \text{red}) = P(\text{queen}) + P(\text{red}) - P(\text{queen} \cap \text{red}) =$$
$$4/52 + 26/52 - 2/52 = 7/13$$