The Artificial Intelligence Toolbox
Part II – CS26210

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Program

Week 1

7/02 Set Theory, Fuzzy Logic (319)
8/02 Fuzzy Logic (B20) - Hand-out Assignment 1

Week 2

14/02 Fuzzy Logic - Further Exercises (319)
15/02 Theory of Probability (B20)

Week 3

21/02 Conditional Probability (319)
22/02 Conditional Probability (B20) - Hand-in Assignment 1 (Blackboard)

Week 4

28/02 In Class Test (319) (Set Theory, Theory of Probability, Conditional Probability)
1/03 Bayesian Networks (B20)

Week 5

7/03 Bayesian networks (319) - Hand-out Assignment 2
8/03 Discussion, further exercises (B20)

22/03 Hand-in Assignment 2 (Blackboard)

Using Qwizdom QVR

- On any web-enabled device go to:

http://qvr.qwizdom.com

- Select **I have a Session Key**
- Enter the code **Q5VN94**

If you aren't already using AU Eduroam wireless have a look at
http://www.inf.aber.ac.uk/advisory/faq/253/

Thursday 7th February, 2013

- Set Theory
- Fuzzy Logic and Fuzzy Sets (graphical and vector) representation
- Crisp Sets
- Membership values and membership functions

Sets

A set is a collection of object (CAPITAL LETTERS)

Examples:”
A={a,b,c}; B={x:x=a,....,z}; C={x:x=1,....,100}; (: *read “such that”)*

An object that belongs to a particular set is called an element of that set. Elements are indicated by lower case letters.

$a \in A$; ($\in$ means belongs to)
$a \notin C$; ($\notin$ means does not belong to)

The Universal set is that set that has as elements all elements of every set entering into the discussion.

Subsets ($\subset$)

$A \subset B$ (A is a subset of B)

A is a subset of B if and only if every element that belongs to A also belongs to B

Example:

A={1};

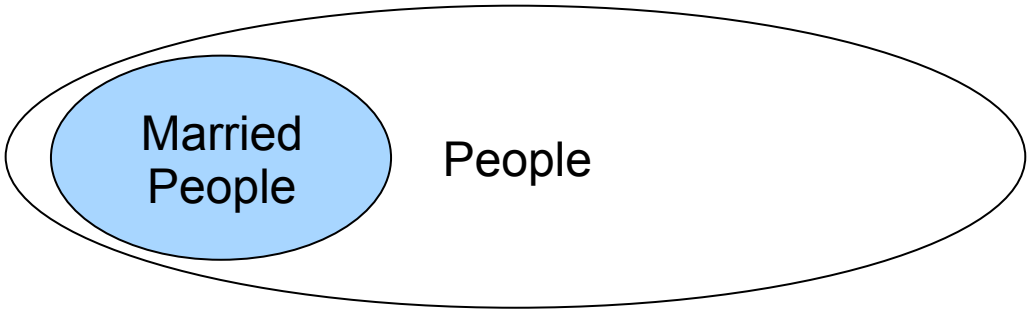
B={1,2};

C={1,2,z}

$A \subset B$; $B \subset C$; $A \subset C$;

The set of all married people is a subset of the set of all people.

Venn diagram



Union ($\cup$)

$A \cup B$ (The union of A and B)

The union of A and B is the set which consists of all the elements that belong to A, or to B, or to both.

$A \cup B = \{x: x \in A \text{ or } x \in B\}$;

$A=\{1,2\}$;

$B=\{1,3\}$;

$C=\{4\}$;

$A \cup B=\{1,2,3\}$; $A \cup C=\{1,2,4\}$; $B \cup C=\{1,3,4\}$

Venn diagram

Intersection (∩)

A ∩ B (The intersection of A and B)
The intersection of A and B is that set which consists of all elements that belong both to A and to B

.

$A \cap B = \{x: x \in A \text{ and } x \in B\}$

A={1,2};
B={1,3};
C={4};

$A \cap B = \{1\}$; $A \cap C = \varnothing$ $B \cap C = \varnothing$ $\varnothing = \{\ \}$ the empty set

Venn Diagram

The complement of A ($\bar{A}$)

$\bar{A}$ (The complement of A)
The complement of A, with respect to a given universal set, is the set of all elements not belonging to A.

$$\bar{A} = \{x: x \notin A\}$$

$U=\{x: 1 \leq x \leq 10\}$; The universal set

$A=\{x: 1 \leq x \leq 2\}$;
$E=\{1, 10\}$;

$\bar{A}=\{x: 3 \leq x \leq 10\}$;
$\bar{E}=\{1<x<10\}$;

True or False

$A=\{x: x =1, 0, 1\};$

$B=\{x: x =-1, 0, 1\};$

$A \subset B$?

Is A a subset of B?

True or False

A={x: x =2, 3, 4, 5, 6, 7};

B={x: x =0, 1, 2, 3};

$\cup$ means “Union”

C =A $\cup$ B

C={x: x =0, 1, 2, 3, 4, 5, 6, 7};

True or False

$A=\{x: x =2, 3, 4, 5, 6, 7\};$

$B=\{x: x =0, 1, 2, 3\};$

$\cap$ means Intersection

$C = A \cap B$

$C=\{x: x =0, 1, 2, 3\};$

Fuzzy logic

Problem:
Let's assume you want to program an expert system to deal with elements that can be hot, pretty hot, warm, very warm, cold, very cold, not so hot, etc., or with persons that can be defined as short, medium, tall, very tall, etc., or with vehicle that can be fast, slow, etc.

... hot, pretty hot, cold, tall, fast, etc.
are ambiguous, vague, fuzzy, terms
because:

different values due to different ideas about meaning,
different values due to different context.

Fuzzy logic – a definition

A branch of logic that uses degrees of membership in sets rather than a strict true/ false membership

Fuzzy/Crisp set

- Fuzzy sets = collection of fuzzy values.
- Note that, the same element can belong to different fuzzy sets with a different membership value.

- Crisp sets = regular sets with membership value either 1 or 0

Fuzzy sets and membership function

Linguistic Variables	Universe of Discourse (X)	Linguistic Values
Temperature	[-30, 30]	Hot, Cold
Speed	[0, 250]	Fast, low
Height	[140, 210]	Short, Medium, Tall

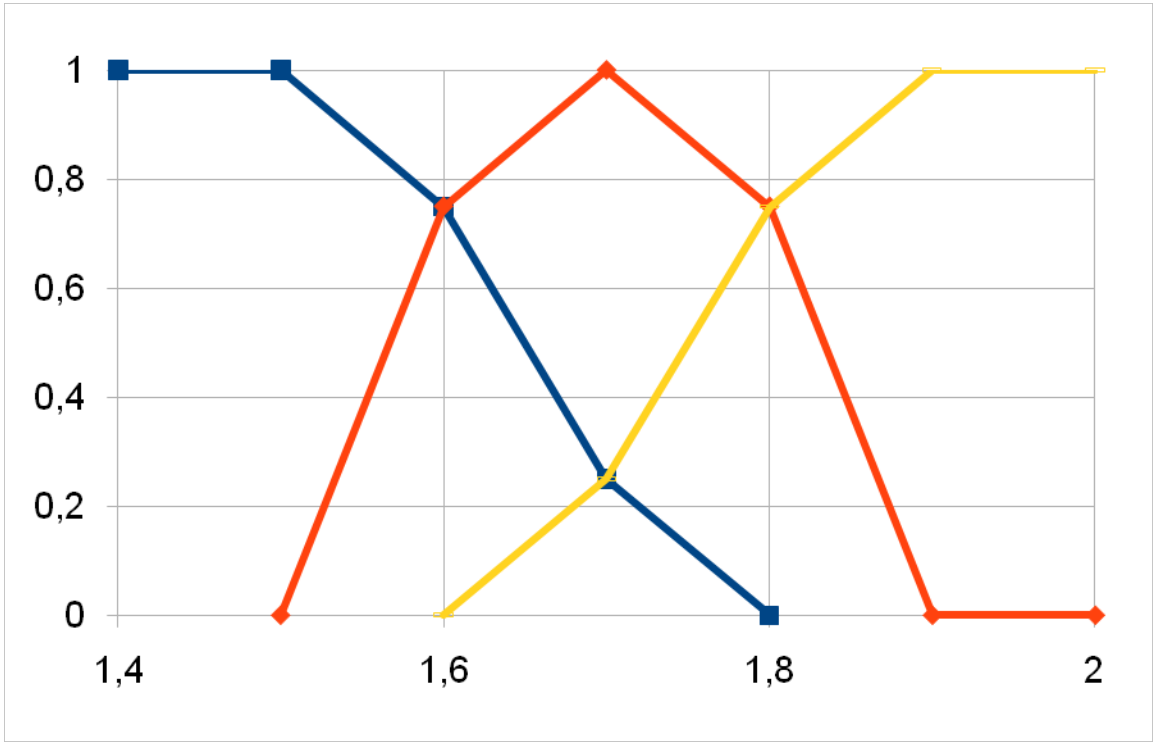
Let X be the universe of discourse, with elements of X denoted as x. A fuzzy set (A) of X is characterised by a membership function $f_A(x_i) = a_i$ that associate each element x with a degree of membership value in A.

$$0 \leq f_A(x_i) \leq 1$$

Fuzzy sets
graphic representation

Linguistic Variable: Height

Blue: short Red: medium Yellow: tall



Fuzzy sets
vector representation

Linguistic Variable: Height
Height is short
Height is medium
Height is tall

$A = (f_A(x_1)/x_1, f_A(x_2)/x_2, f_A(x_3)/x_3, f_A(x_4)/x_4, \text{etc.})$
(membership value / element i)

Short = (1/140, 1/150, 0.8/160, 0.6/170, 0.4/180, 0.2/190, 0/200, 0/210)

Medium = (0/140, 0/150, 0.4/160, 0.8/170, 1.0/180, 0.8/190, 0.4/200, 0.0/210)

Tall = (0/140, 0/150, 0/160, 0.2/170, 0.4/180, 0.6/190, 0.8/200, 1.0/210)

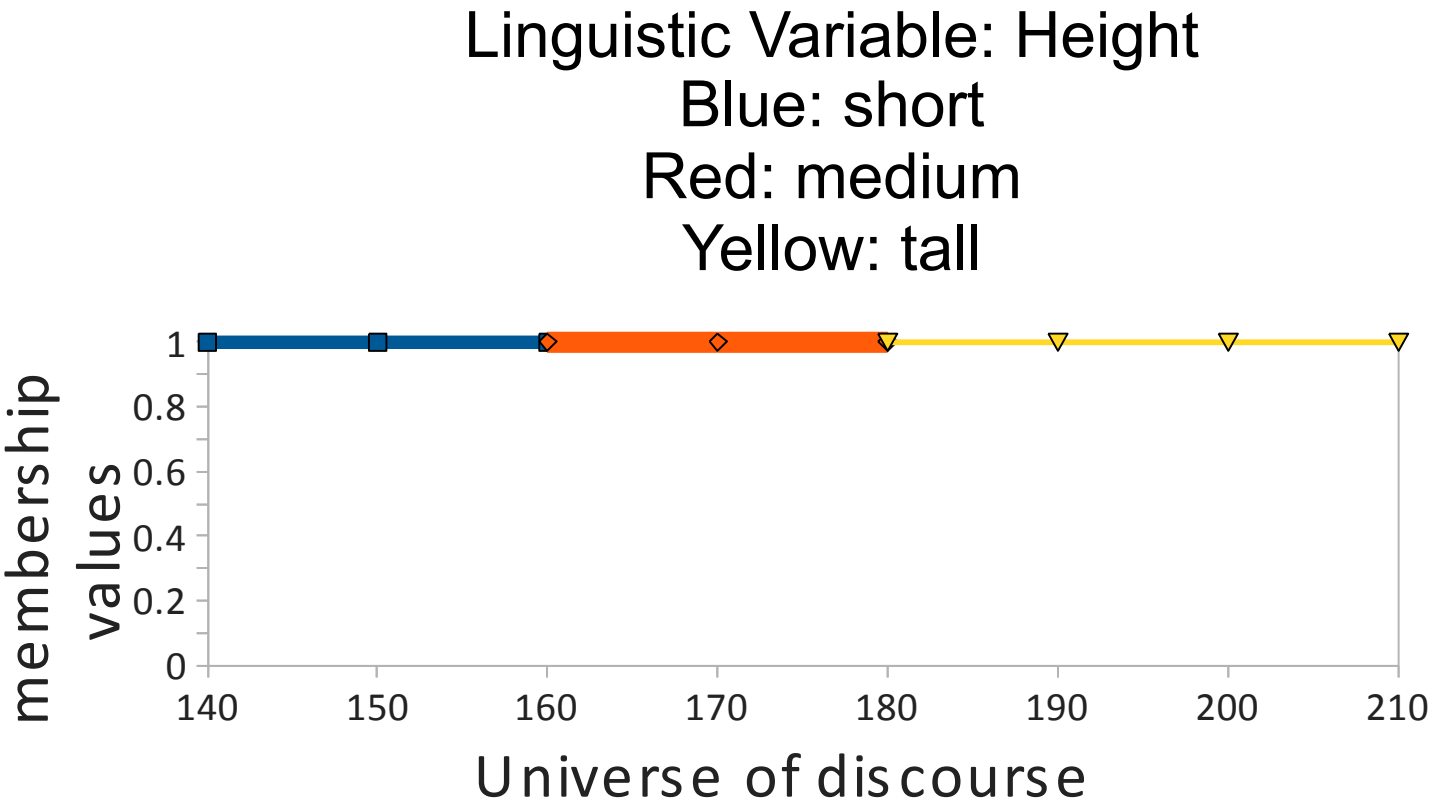
Crisp values

A crisp value is a clearly defined value with only one interpretation

Cold = 5˚
Warm = 20˚
Hot = 30˚

... or values that belong to only one set with degree of membership 1

Crisp sets
graphic representation



Crisp sets
vector representation

Linguistic Variable: Height
Height is short
Height is medium
Height is tall

$A = (f_A(x_1)/x_1, f_A(x_2)/x_2, f_A(x_3)/x_3, f_A(x_4)/x_4, \text{etc.})$
(membership value / element i)

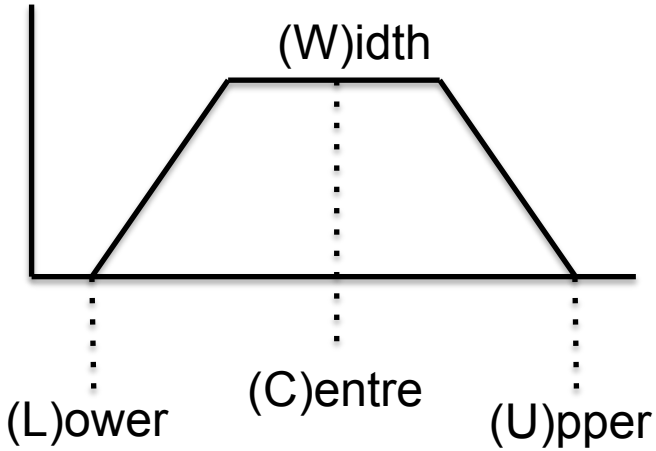
Short = (1/140, 1/150, 1/160, 0/170, 0/180, 0/190, 0/200, 0/210)

Medium = (0/140, 0/150, 1/161, 1/170, 1/180, 0/190, 0/200, 0/210)

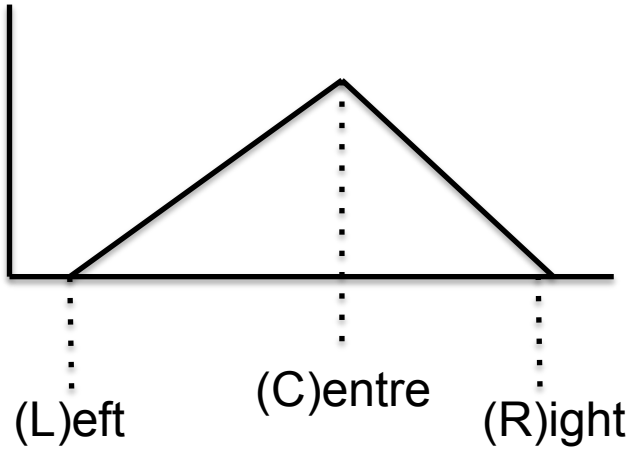
Tall = (0/140, 0/150, 0/160, 0/170, 1/181, 1/190, 1/200, 1/210)

Types of membership functions

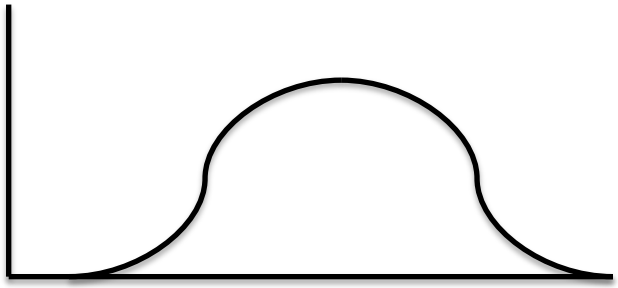
Trapezoid



Triangular

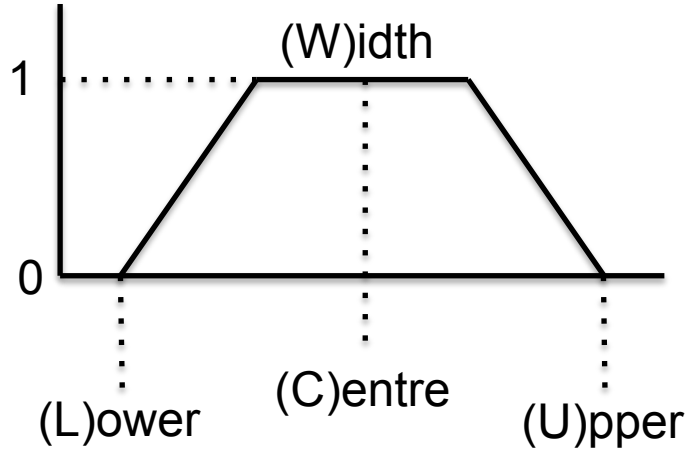


Gaussian



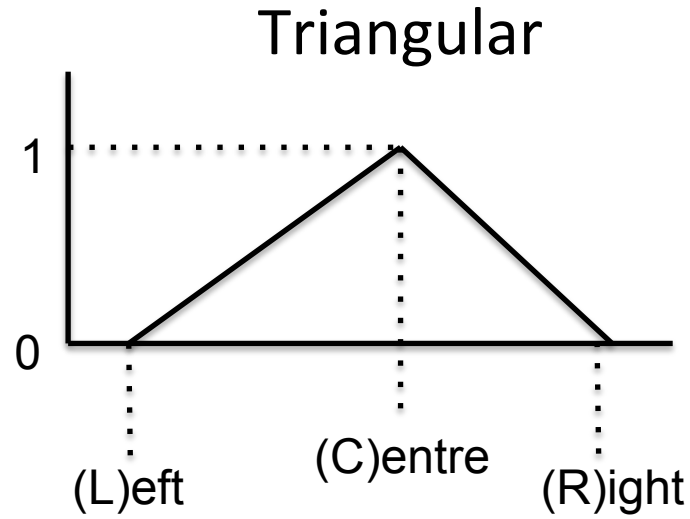
Types of membership functions

Trapezoid



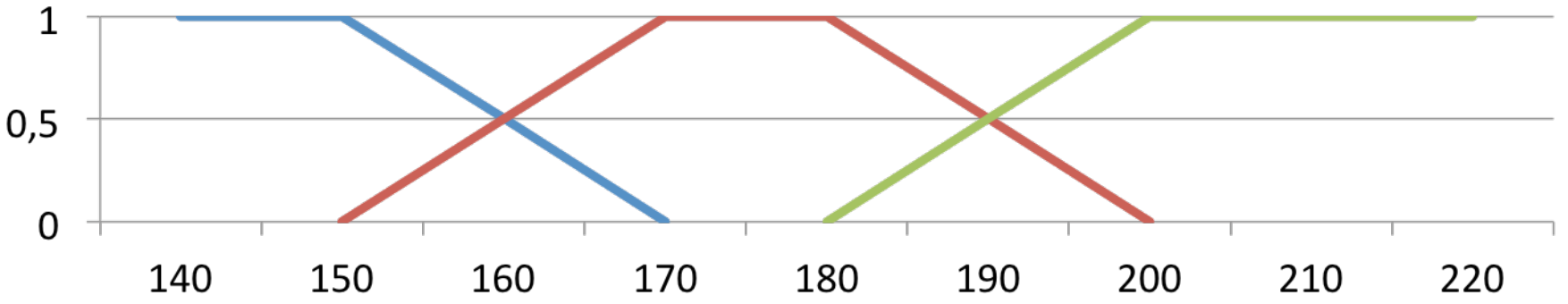
$$f_{trapez}(x) = \begin{cases} 0 & x < L \text{ or } x > U \\ (x-L)/(C-w/2-L) & L < x < (C-w/2) \\ 1 & (C-w/2) < x < (C+w/2) \\ (U-x)/(U-(C+w/2)) & (C+w/2) < x < U \end{cases}$$

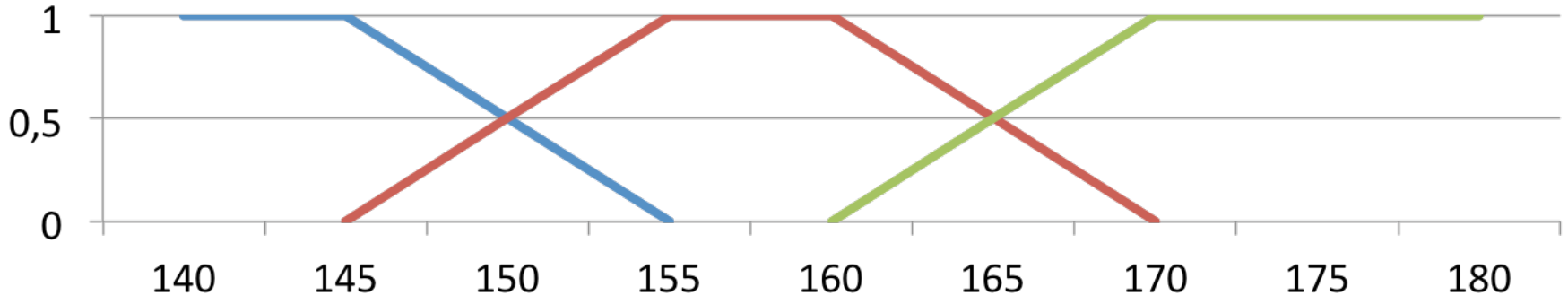
Types of membership functions



$$f_{triang}(x) = \begin{cases} 0 & x < L \\ 1 - \left| C - x \right| \Big/ (R - L) * 0.5 & L < x < R \\ 0 & x > R \end{cases}$$

Context


1
0,5
0
140
150
160
170
180
190
200
210
220


1
0,5
0
140
145
150
155
160
165
170
175
180

True or False

A is a crisp set:

A = (0/10, 0/15, 1/20, 1/25, 1/30, 0.5/35)

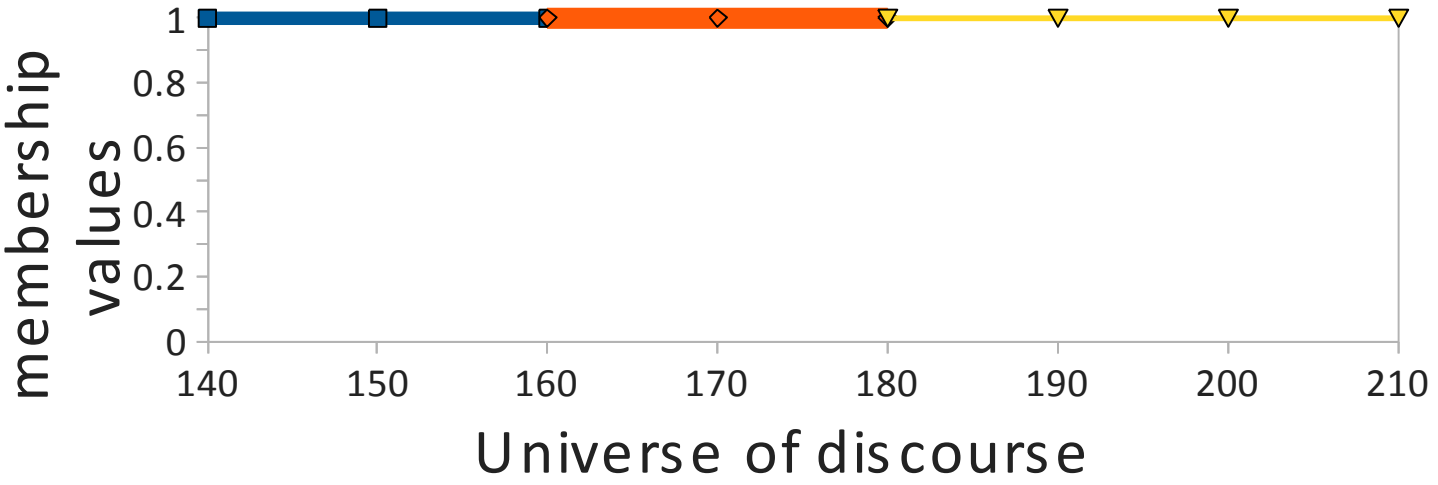
True or False

The following is a vector representation of the fuzzy set A:

A = (0/10, 0/15, 1/20, 1/25, 1/30, 0.5/35);

True or False

The following is the graphical representation of a crisp set:



Tomorrow
Fuzzy Inference

Rule 1
IF Speed is **slow**
THEN Make the acceleration **high**

Rule 2
IF Temperature is **low**
AND Pressure is **medium**
THEN Make the speed **very slow**