Scaled vs unscaled model

I will denote all unscaled variables with a hat over top of them,and all scaled variables without (i.e. $\hat{R}$ has units, R does not).

In its most general form, the unscaled model is

$$\begin{aligned}
\hat{E}(\hat{R},\hat{L};\hat{\psi}(\hat{r}),\delta\hat{\rho}(\hat{z})) &= \frac{2\pi}{\pi\hat{R}^2\hat{L}}\int_0^{\hat{L}} d\hat{z}\int_0^{\hat{R}}\hat{r}d\hat{r}\left[\frac{1}{2}\hat{K}_{22}\left(\hat{\psi}'+\frac{\sin 2\hat{\psi}}{2\hat{r}}-\hat{q}\right)^2+\frac{1}{2}\hat{K}_{33}\frac{\sin^4\hat{\psi}}{\hat{r}^2}\right] \\
&\quad+\frac{\hat{\Lambda}}{2}\frac{2\pi}{\pi\hat{R}^2\hat{L}}\int_0^{\hat{L}} d\hat{z}\int_0^{\hat{R}}\hat{r}d\hat{r}\delta\hat{\rho}\left(\frac{4\pi^2}{\hat{d}_0^2\cos^2\hat{\psi}}+\frac{\partial^2}{\partial\hat{z}^2}\right)^2\delta\hat{\rho} \\
&\quad+\hat{\omega}\frac{\pi\hat{R}^2}{\pi\hat{R}^2\hat{L}}\int_0^{\hat{L}} d\hat{z}\delta\hat{\rho}^2\left(\delta\hat{\rho}^2-\hat{\chi}^2\right)-\frac{(\hat{K}_{22}+\hat{k}_{24})}{\hat{R}^2}\sin\hat{\psi}(\hat{R})+\frac{2\hat{\gamma}}{\hat{R}} \\
&= \frac{2}{\hat{R}^2}\int_0^{\hat{R}}\hat{r}d\hat{r}\left[\frac{1}{2}\hat{K}_{22}\left(\hat{\psi}'+\frac{\sin 2\hat{\psi}}{2\hat{r}}-\hat{q}\right)^2+\frac{1}{2}\hat{K}_{33}\frac{\sin^4\hat{\psi}}{\hat{r}^2}\right] \\
&\quad+\frac{\hat{\Lambda}}{\hat{R}^2\hat{L}}\int_0^{\hat{L}} d\hat{z}\int_0^{\hat{R}}\hat{r}d\hat{r}\delta\hat{\rho}\left(\frac{4\pi^2}{\hat{d}_0^2\cos^2\hat{\psi}}+\frac{\partial^2}{\partial\hat{z}^2}\right)^2\delta\hat{\rho} \\
&\quad+\frac{\hat{\omega}}{\hat{L}}\int_0^{\hat{L}} d\hat{z}\delta\hat{\rho}^2\left(\delta\hat{\rho}^2-\hat{\chi}^2\right)-\frac{(\hat{K}_{22}+\hat{k}_{24})}{\hat{R}^2}\sin\hat{\psi}(\hat{R})+\frac{2\hat{\gamma}}{\hat{R}}
\end{aligned} \tag{1}$$

where I have ignored any surface contributions from the ends of the fibril. I have assumed that $\hat{\chi}^2 > 0$, as the density amplitude term (pre-factor $\hat{\omega}$) would be positive definite if not, meaning no density modulations would occur. The units of $\hat{\Lambda}$ are $\mathrm{pN}\cdot\mu\mathrm{m}^8$, the units of $\hat{\omega}$ are $\mathrm{pN}\cdot\mu\mathrm{m}^{10}$, and the units of $\hat{\chi}^2$ are $\mu\mathrm{m}^{-6}$. If I divide both side of 1 by $\hat{K}_{22}\hat{q}^2$, I can make the system dimensionless and reduce to the form

$$\begin{aligned}
E(R,L;\psi(r),\delta\rho(z)) &= \frac{2}{R^2}\int_0^{R} rdr\left[\frac{1}{2}\left(\psi'+\frac{\sin 2\psi}{2r}-1\right)^2+\frac{1}{2}K_{33}\frac{\sin^4\psi}{r^2}\right] \\
&\quad+\frac{\Lambda}{R^2L}\int_0^{L} dz\int_0^{R} rdr\delta\rho\left(\frac{4\pi^2}{d_0^2\cos^2\psi}+\frac{\partial^2}{\partial z^2}\right)^2\delta\rho \\
&\quad+\frac{\omega}{L}\int_0^{L} dz\delta\rho^2\left(\delta\rho^2-\chi^2\right)-\frac{(1+k_{24})}{R^2}\sin\psi(R)+\frac{2\gamma}{R}
\end{aligned}$$

$$. \tag{2}$$

The following is a list of the redefined, dimensionless quantites:

$$E = \frac{\hat{E}}{\hat{K}_{22}\hat{q}^2} \tag{3}$$

$$R = \hat{R}\hat{q} \tag{4}$$

$$r = \hat{r}\hat{q} \tag{5}$$

$$\psi(r) = \hat{\psi}(\hat{r}) \tag{6}$$

$$K_{33} = \frac{\hat{K}_{33}}{\hat{K}_{22}} \tag{7}$$

$$L = \hat{L}\hat{q} \tag{8}$$

$$\Lambda = \frac{\hat{\Lambda}\hat{q}^8}{\hat{K}_{22}} \tag{9}$$

$$\delta\rho = \frac{\delta\hat{\rho}}{\hat{q}^3} \tag{10}$$

$$d_0 = \hat{d}_0\hat{q} \tag{11}$$

$$\omega = \frac{\hat{\omega}\hat{q}^{10}}{\hat{K}_{22}} \tag{12}$$

$$\chi^2 = \frac{\hat{\chi}^2}{\hat{q}^6} \tag{13}$$

$$\gamma = \frac{\hat{\gamma}}{\hat{K}_{22}\hat{q}} \tag{14}$$

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References