

Solutions Manual

Corporate Finance

Ross, Westerfield, Jaffe, and Jordan
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CHAPTER 1

INTRODUCTION TO CORPORATE FINANCE

Answers to Concept Questions

1. In the corporate form of ownership, the shareholders are the owners of the firm. The shareholders elect the directors of the corporation, who in turn appoint the firm's management. This separation of ownership from control in the corporate form of organization is what causes agency problems to exist. Management may act in its own or someone else's best interests, rather than those of the shareholders. If such events occur, they may contradict the goal of maximizing the share price of the equity of the firm.
2. Such organizations frequently pursue social or political missions, so many different goals are conceivable. One goal that is often cited is revenue minimization; i.e., provide whatever goods and services are offered at the lowest possible cost to society. A better approach might be to observe that even a not-for-profit business has equity. Thus, one answer is that the appropriate goal is to maximize the value of the equity.
3. Presumably, the current stock value reflects the risk, timing, and magnitude of all future cash flows, both short-term *and* long-term. If this is correct, then the statement is false.
4. An argument can be made either way. At the one extreme, we could argue that in a market economy, all of these things are priced. There is thus an optimal level of, for example, ethical and/or illegal behavior, and the framework of stock valuation explicitly includes these. At the other extreme, we could argue that these are non-economic phenomena and are best handled through the political process. A classic (and highly relevant) thought question that illustrates this debate goes something like this: "A firm has estimated that the cost of improving the safety of one of its products is \$30 million. However, the firm believes that improving the safety of the product will only save \$20 million in product liability claims. What should the firm do?"
5. The goal will be the same, but the best course of action toward that goal may be different because of differing social, political, and economic institutions.
6. The goal of management should be to maximize the share price for the current shareholders. If management believes that it can improve the profitability of the firm so that the share price will exceed \$35, then they should fight the offer from the outside company. If management believes that this bidder, or other unidentified bidders, will actually pay more than \$35 per share to acquire the company, then they should still fight the offer. However, if the current management cannot increase the value of the firm beyond the bid price, and no other higher bids come in, then management is not acting in the interests of the shareholders by fighting the offer. Since current managers often lose their jobs when the corporation is acquired, poorly monitored managers have an incentive to fight corporate takeovers in situations such as this.

7. We would expect agency problems to be less severe in other countries, primarily due to the relatively small percentage of individual ownership. Fewer individual owners should reduce the number of diverse opinions concerning corporate goals. The high percentage of institutional ownership might lead to a higher degree of agreement between owners and managers on decisions concerning risky projects. In addition, institutions may be better able to implement effective monitoring mechanisms on managers than can individual owners, based on the institutions' deeper resources and experiences with their own management.
8. The increase in institutional ownership of stock in the United States and the growing activism of these large shareholder groups may lead to a reduction in agency problems for U.S. corporations and a more efficient market for corporate control. However, this may not always be the case. If the managers of the mutual fund or pension plan are not concerned with the interests of the investors, the agency problem could potentially remain the same, or even increase, since there is the possibility of agency problems between the fund and its investors.
9. How much is too much? Who is worth more, Larry Ellison or Tiger Woods? The simplest answer is that there is a market for executives just as there is for all types of labor. Executive compensation is the price that clears the market. The same is true for athletes and performers. Having said that, one aspect of executive compensation deserves comment. A primary reason executive compensation has grown so dramatically is that companies have increasingly moved to stock-based compensation. Such movement is obviously consistent with the attempt to better align stockholder and management interests. In recent years, stock prices have soared, so management has cleaned up. It is sometimes argued that much of this reward is due to rising stock prices in general, not managerial performance. Perhaps in the future, executive compensation will be designed to reward only differential performance, i.e., stock price increases in excess of general market increases.
10. Maximizing the current share price is the same as maximizing the future share price at any future period. The value of a share of stock depends on all of the future cash flows of company. Another way to look at this is that, barring large cash payments to shareholders, the expected price of the stock must be higher in the future than it is today. Who would buy a stock for \$100 today when the share price in one year is expected to be \$80?

CHAPTER 2

ACCOUNTING STATEMENTS, TAXES, AND CASH FLOW

Answers to Concepts Review and Critical Thinking Questions

1. True. Every asset can be converted to cash at some price. However, when we are referring to a liquid asset, the added assumption that the asset can be quickly converted to cash at or near market value is important.
2. The recognition and matching principles in financial accounting call for revenues, and the costs associated with producing those revenues, to be “booked” when the revenue process is essentially complete, not necessarily when the cash is collected or bills are paid. Note that this way is not necessarily correct; it’s the way accountants have chosen to do it.
3. The bottom line number shows the change in the cash balance on the balance sheet. As such, it is not a useful number for analyzing a company.
4. The major difference is the treatment of interest expense. The accounting statement of cash flows treats interest as an operating cash flow, while the financial cash flows treat interest as a financing cash flow. The logic of the accounting statement of cash flows is that since interest appears on the income statement, which shows the operations for the period, it is an operating cash flow. In reality, interest is a financing expense, which results from the company’s choice of debt and equity. We will have more to say about this in a later chapter. When comparing the two cash flow statements, the financial statement of cash flows is a more appropriate measure of the company’s performance because of its treatment of interest.
5. Market values can never be negative. Imagine a share of stock selling for –\$20. This would mean that if you placed an order for 100 shares, you would get the stock along with a check for \$2,000. How many shares do you want to buy? More generally, because of corporate and individual bankruptcy laws, net worth for a person or a corporation cannot be negative, implying that liabilities cannot exceed assets in market value.
6. For a successful company that is rapidly expanding, for example, capital outlays will be large, possibly leading to negative cash flow from assets. In general, what matters is whether the money is spent wisely, not whether cash flow from assets is positive or negative.
7. It’s probably not a good sign for an established company to have negative cash flow from operations, but it would be fairly ordinary for a start-up, so it depends.

8. For example, if a company were to become more efficient in inventory management, the amount of inventory needed would decline. The same might be true if the company becomes better at collecting its receivables. In general, anything that leads to a decline in ending NWC relative to beginning would have this effect. Negative net capital spending would mean more long-lived assets were liquidated than purchased.
9. If a company raises more money from selling stock than it pays in dividends in a particular period, its cash flow to stockholders will be negative. If a company borrows more than it pays in interest and principal, its cash flow to creditors will be negative.
10. The adjustments discussed were purely accounting changes; they had no cash flow or market value consequences unless the new accounting information caused stockholders to revalue the assets.

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. To find owners' equity, we must construct a balance sheet as follows:

<u>Balance Sheet</u>			
CA	\$ 4,300	CL	\$ 2,900
NFA	<u>24,000</u>	LTD	10,700
		OE	<u>??</u>
TA	<u>\$28,300</u>	TL & OE	<u>\$28,300</u>

We know that total liabilities and owners' equity (TL & OE) must equal total assets of \$28,300. We also know that TL & OE is equal to current liabilities plus long-term debt plus owners' equity, so owners' equity is:

$$\begin{aligned}\text{Owners' equity} &= \$28,300 - 10,700 - 2,900 \\ \text{Owners' equity} &= \$14,700\end{aligned}$$

And net working capital is current assets minus current liabilities, so:

$$\begin{aligned}\text{NWC} &= \text{Current assets} - \text{Current liabilities} \\ \text{NWC} &= \$4,300 - 2,900 \\ \text{NWC} &= \$1,400\end{aligned}$$

2. The income statement for the company is:

<u>Income Statement</u>	
Sales	\$473,000
Costs	275,000
Depreciation	<u>42,000</u>
EBIT	\$156,000
Interest	<u>23,000</u>
EBT	\$133,000
Taxes	<u>27,930</u>
Net income	<u>\$105,070</u>

One equation for net income is:

Net income = Dividends + Addition to retained earnings

Rearranging, we get:

Addition to retained earnings = Net income – Dividends

Addition to retained earnings = \$105,070 – 25,000

Addition to retained earnings = \$80,070

3. To find the book value of current assets, we use: $NWC = CA - CL$. Rearranging to solve for current assets, we get:

Current assets = Net working capital + Current liabilities

Current assets = \$850,000 + 2,200,000

Current assets = \$3,050,000

The market value of current assets and net fixed assets is given, so:

Book value CA = \$3,050,000

Book value NFA = \$4,900,000

Book value assets = \$7,950,000

Market value CA = \$2,700,000

Market value NFA = \$6,400,000

Market value assets = \$9,100,000

4. Taxes = $.10(\$9,525) + .12(\$38,700 - 9,525) + .22(\$82,500 - 38,700) + .24(\$157,500 - 82,500) + .32(\$189,000 - 157,500)$
 Taxes = \$42,169.50

The average tax rate is the total tax paid divided by taxable income, so:

Average tax rate = \$42,169.50/\$189,000

Average tax rate = .2231, or 22.31%

The marginal tax rate is the tax rate on the next \$1 of earnings, so the marginal tax rate is 32 percent.

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5. To calculate OCF, we first need the income statement:

<u>Income Statement</u>	
Sales	\$22,400
Costs	11,600
Depreciation	<u>2,200</u>
EBIT	\$8,600
Interest	<u>1,370</u>
Taxable income	\$7,230
Taxes	<u>1,591</u>
Net income	<u>\$5,639</u>

$$\text{OCF} = \text{EBIT} + \text{Depreciation} - \text{Taxes}$$

$$\text{OCF} = \$8,600 + 2,200 - 1,591$$

$$\text{OCF} = \$9,209$$

6. Net capital spending = $\text{NFA}_{\text{end}} - \text{NFA}_{\text{beg}} + \text{Depreciation}$
 Net capital spending = $\$1,430,000 - 1,280,000 + 146,000$
 Net capital spending = $\$296,000$
7. The long-term debt account will increase by \$30 million, the amount of the new long-term debt issue. Since the company sold 4.5 million new shares of stock with a \$1 par value, the common stock account will increase by \$4.5 million. The capital surplus account will increase by \$53.5 million, the value of the new stock sold above its par value. Since the company had a net income of \$7.5 million, and paid \$1.7 million in dividends, the addition to retained earnings was \$5.8 million, which will increase the accumulated retained earnings account. So, the new long-term debt and stockholders' equity portion of the balance sheet will be:

Long-term debt	<u>\$ 75,000,000</u>
Total long-term debt	\$ 75,000,000

Shareholders' equity	
Preferred stock	\$ 2,900,000
Common stock (\$1 par value)	15,500,000
Accumulated retained earnings	112,800,000
Capital surplus	<u>102,500,000</u>
Total equity	\$ 233,700,000

Total liabilities & equity	\$ 308,700,000
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8. Cash flow to creditors = Interest paid – Net new borrowing
 Cash flow to creditors = $\$170,000 - (\text{LTD}_{\text{end}} - \text{LTD}_{\text{beg}})$
 Cash flow to creditors = $\$170,000 - (\$1,645,000 - 1,565,000)$
 Cash flow to creditors = $\$170,000 - 80,000$
 Cash flow to creditors = $\$90,000$

9. Cash flow to stockholders = Dividends paid – Net new equity
 Cash flow to stockholders = $\$335,000 - [(\text{Common}_{\text{end}} + \text{APIS}_{\text{end}}) - (\text{Common}_{\text{beg}} + \text{APIS}_{\text{beg}})]$
 Cash flow to stockholders = $\$335,000 - [(\$525,000 + 3,750,000) - (\$490,000 + 3,400,000)]$
 Cash flow to stockholders = $\$335,000 - (\$4,275,000 - 3,890,000)$
 Cash flow to stockholders = $-\$50,000$

Note, APIS is the additional paid-in surplus.

10. Cash flow from assets = Cash flow to creditors + Cash flow to stockholders
 = $\$90,000 - 50,000$
 = $\$40,000$

Cash flow from assets = OCF – Change in NWC – Net capital spending
 $\$40,000 = \text{OCF} - (-\$96,000) - 735,000$
 Operating cash flow = $\$40,000 - 96,000 + 735,000$
 Operating cash flow = $\$679,000$

Intermediate

11. a. The accounting statement of cash flows explains the change in cash during the year. The accounting statement of cash flows will be:

Statement of cash flows

Operations

Net income	\$129
Depreciation	92
Changes in other current assets	(17)
Change in accounts payable	<u>17</u>
Total cash flow from operations	<u>\$221</u>

Investing activities

Acquisition of fixed assets	<u>\$(111)</u>
Total cash flow from investing activities	<u>\$(111)</u>

Financing activities

Proceeds of long-term debt	\$8
Dividends	<u>(97)</u>
Total cash flow from financing activities	<u>\$(89)</u>

Change in cash (on balance sheet)	<u>\$21</u>
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- b. $\text{Change in NWC} = \text{NWC}_{\text{end}} - \text{NWC}_{\text{beg}}$
 $= (\text{CA}_{\text{end}} - \text{CL}_{\text{end}}) - (\text{CA}_{\text{beg}} - \text{CL}_{\text{beg}})$
 $= [(\$84 + 192) - 146] - [(\$63 + 175) - 129]$
 $= \$130 - 109$
 $= \$21$
- c. To find the cash flow generated by the firm's assets, we need the operating cash flow and the capital spending. So, calculating each of these, we find:

Operating cash flow

Net income	\$129
Depreciation	<u>92</u>
Operating cash flow	\$221

Note that we can calculate OCF in this manner since there are no taxes.

Capital spending

Ending fixed assets	\$417
Beginning fixed assets	-398
Depreciation	<u>92</u>
Capital spending	\$111

Now we can calculate the cash flow generated by the firm's assets, which is:

Cash flow from assets

Operating cash flow	\$221
Capital spending	-111
Change in NWC	<u>-21</u>
Cash flow from assets	\$ 89

12. With the information provided, the cash flows from the firm are the capital spending and the change in net working capital, so:

Cash flows from the firm

Capital spending	-\$29,000
Additions to NWC	<u>-2,400</u>
Capital spending and NWC cash flow	-\$31,400

And the cash flows to the investors of the firm are:

Cash flows to investors of the firm

Sale of long-term debt	-\$16,400
Sale of common stock	-4,000
Dividends paid	<u>13,100</u>
Cash flows to investors of the firm	-\$7,300

13. a. The interest expense for the company is the amount of debt times the interest rate on the debt. So, the income statement for the company is:

<u>Income Statement</u>	
Sales	\$865,000
Cost of goods sold	455,000
Selling costs	210,000
Depreciation	<u>105,000</u>
EBIT	\$ 95,000
Interest	<u>27,200</u>
Taxable income	\$ 67,800
Taxes	<u>14,238</u>
Net income	<u>\$ 53,562</u>

- b. And the operating cash flow is:

$$\begin{aligned}\text{OCF} &= \text{EBIT} + \text{Depreciation} - \text{Taxes} \\ \text{OCF} &= \$95,000 + 105,000 - 14,238 \\ \text{OCF} &= \$185,762\end{aligned}$$

14. To find the OCF, we first calculate net income.

<u>Income Statement</u>	
Sales	\$246,000
Costs	135,000
Other expenses	7,100
Depreciation	<u>19,100</u>
EBIT	\$84,800
Interest	<u>10,000</u>
Taxable income	\$74,800
Taxes	<u>18,876</u>
Net income	<u>\$55,924</u>
Dividends	\$9,800
Additions to RE	\$46,124

- a. $\text{OCF} = \text{EBIT} + \text{Depreciation} - \text{Taxes}$
 $\text{OCF} = \$84,800 + 19,100 - 18,876$
 $\text{OCF} = \$85,024$
- b. $\text{CFC} = \text{Interest} - \text{Net new LTD}$
 $\text{CFC} = \$10,000 - (-\$6,800)$
 $\text{CFC} = \$16,800$

Note that the net new long-term debt is negative because the company repaid part of its long-term debt.

- c. $\text{CFS} = \text{Dividends} - \text{Net new equity}$
 $\text{CFS} = \$9,800 - 7,900$
 $\text{CFS} = \$1,900$

d. We know that $CFA = CFC + CFS$, so:

$$CFA = \$16,800 + 1,900$$

$$CFA = \$18,700$$

CFA is also equal to $OCF - \text{Net capital spending} - \text{Change in NWC}$. We already know OCF. Net capital spending is equal to:

$$\text{Net capital spending} = \text{Increase in NFA} + \text{Depreciation}$$

$$\text{Net capital spending} = \$41,900 + 19,100$$

$$\text{Net capital spending} = \$61,000$$

Now we can use:

$$CFA = OCF - \text{Net capital spending} - \text{Change in NWC}$$

$$\$18,700 = \$85,024 - 61,000 - \text{Change in NWC}$$

$$\text{Change in NWC} = \$5,324$$

This means the company increased its NWC by \$5,324.

15. The solution to this question works the income statement backwards. Starting at the bottom:

$$\text{Net income} = \text{Dividends} + \text{Addition to retained earnings}$$

$$\text{Net income} = \$1,720 + 5,300$$

$$\text{Net income} = \$7,020$$

Now, looking at the income statement:

$$EBT - (EBT \times \text{Tax rate}) = \text{Net income}$$

Recognize that $EBT \times \text{Tax rate}$ is the calculation for taxes. Solving this for EBT yields:

$$EBT = \text{Net income} / (1 - \text{Tax rate})$$

$$EBT = \$7,020 / (1 - .21)$$

$$EBT = \$8,886.08$$

Now we can calculate:

$$EBIT = EBT + \text{Interest}$$

$$EBIT = \$8,886.08 + 2,050$$

$$EBIT = \$10,936.08$$

The last step is to use:

$$EBIT = \text{Sales} - \text{Costs} - \text{Depreciation}$$

$$\$10,936.08 = \$54,000 - 29,500 - \text{Depreciation}$$

$$\text{Depreciation} = \$13,563.92$$

16. The market value of shareholders' equity cannot be negative. A negative market value in this case would imply that the company would pay you to own the stock. The market value of shareholders' equity can be stated as: Shareholders' equity = $\text{Max} [(TA - TL), 0]$. So, if TA is \$11,900, equity is equal to \$1,200, and if TA is \$9,400, equity is equal to \$0. We should note here that while the market value of equity cannot be negative, the book value of shareholders' equity can be negative.

17.

	<u>Income Statement</u>
Sales	\$630,000
COGS	465,000
A&S expenses	85,000
Depreciation	<u>135,000</u>
EBIT	-\$55,000
Interest	<u>70,000</u>
Taxable income	-\$125,000
Taxes (21%)	<u>0</u>
a. Net income	<u>-\$125,000</u>

b. $\text{OCF} = \text{EBIT} + \text{Depreciation} - \text{Taxes}$
 $\text{OCF} = -\$55,000 + 135,000 - 0$
 $\text{OCF} = \$80,000$

- c. Net income was negative because of the tax deductibility of depreciation and interest expense. However, the actual cash flow from operations was positive because depreciation is a non-cash expense and interest is a financing expense, not an operating expense.

18. A firm can still pay out dividends if net income is negative; it just has to be sure there is sufficient cash flow to make the dividend payments.

Change in NWC = Net capital spending = Net new equity = 0. (Given)

Cash flow from assets = $\text{OCF} - \text{Change in NWC} - \text{Net capital spending}$

Cash flow from assets = $\$80,000 - 0 - 0 = \$80,000$

Cash flow to stockholders = $\text{Dividends} - \text{Net new equity}$

Cash flow to stockholders = $\$34,000 - 0 = \$34,000$

Cash flow to creditors = $\text{Cash flow from assets} - \text{Cash flow to stockholders}$

Cash flow to creditors = $\$80,000 - 34,000$

Cash flow to creditors = \$46,000

Cash flow to creditors is also:

Cash flow to creditors = $\text{Interest} - \text{Net new LTD}$

So:

Net new LTD = $\text{Interest} - \text{Cash flow to creditors}$

Net new LTD = $\$70,000 - 46,000$

Net new LTD = \$24,000

19. a. The income statement is:

<u>Income Statement</u>	
Sales	\$24,360
Cost of goods sold	17,600
Depreciation	<u>3,400</u>
EBIT	\$ 3,360
Interest	<u>860</u>
Taxable income	\$ 2,500
Taxes	<u>525</u>
Net income	<u>\$1,975</u>

- b. $OCF = EBIT + Depreciation - Taxes$

$$OCF = \$3,360 + 3,400 - 525$$

$$OCF = \$6,235$$

- c. $Change\ in\ NWC = NWC_{end} - NWC_{beg}$
 $= (CA_{end} - CL_{end}) - (CA_{beg} - CL_{beg})$
 $= (\$6,410 - 3,445) - (\$5,560 - 3,040)$
 $= \$2,965 - 2,520$
 $= \$445$

$$\begin{aligned} \text{Net capital spending} &= NFA_{end} - NFA_{beg} + \text{Depreciation} \\ &= \$21,180 - 18,650 + 3,400 \\ &= \$5,930 \end{aligned}$$

$$\begin{aligned} CFA &= OCF - \text{Change in NWC} - \text{Net capital spending} \\ &= \$6,235 - 445 - 5,930 \\ &= -\$140 \end{aligned}$$

The cash flow from assets can be positive or negative, since it represents whether the firm raised funds or distributed funds on a net basis. In this problem, even though net income and OCF are positive, the firm invested heavily in both fixed assets and net working capital; it had to raise a net \$140 in funds from its stockholders and creditors to make these investments.

- d. $Cash\ flow\ to\ creditors = Interest - \text{Net new LTD}$
 $= \$860 - 0$
 $= \$860$

$$\begin{aligned} \text{Cash flow to stockholders} &= \text{Cash flow from assets} - \text{Cash flow to creditors} \\ &= -\$140 - 860 \\ &= -\$1,000 \end{aligned}$$

We can also calculate the cash flow to stockholders as:

$$\text{Cash flow to stockholders} = \text{Dividends} - \text{Net new equity}$$

Solving for net new equity, we get:

$$\begin{aligned} \text{Net new equity} &= \$1,000 - (-790) \\ &= \$1,790 \end{aligned}$$

The firm had positive earnings in an accounting sense ($NI > 0$) and had positive cash flow from operations. The firm invested \$445 in new net working capital and \$5,930 in new fixed assets. The firm had to raise \$140 from its stakeholders to support this new investment. It accomplished this by raising \$1,790 in the form of new equity. After paying out \$790 of this in the form of dividends to shareholders and \$860 in the form of interest to creditors, \$140 was left to meet the firm's cash flow needs for investment.

20. a. $\text{Total assets 2018} = \$1,157 + 5,261 = \$6,418$
 $\text{Total liabilities 2018} = \$481 + 2,856 = \$3,337$
 $\text{Owners' equity 2018} = \$6,418 - 3,337 = \$3,081$
- $\text{Total assets 2019} = \$1,411 + 6,125 = \$7,536$
 $\text{Total liabilities 2019} = \$534 + 3,256 = \$3,790$
 $\text{Owners' equity 2019} = \$7,536 - 3,790 = \$3,746$
- b. $\text{NWC 2018} = \text{CA} - \text{CL} = \$1,157 - 481 = \$676$
 $\text{NWC 2019} = \text{CA} - \text{CL} = \$1,411 - 534 = \$877$
 $\text{Change in NWC} = \text{NWC}_{\text{end}} - \text{NWC}_{\text{beg}} = \$877 - 676 = \$201$

- c. We can calculate net capital spending as:

$\text{Net capital spending} = \text{Net fixed assets}_{\text{end}} - \text{Net fixed assets}_{\text{beg}} + \text{Depreciation}$

$\text{Net capital spending} = \$6,125 - 5,261 + 1,478$

$\text{Net capital spending} = \$2,342$

So, the company had a net capital spending cash flow of \$2,342. We also know that net capital spending is:

$\text{Net capital spending} = \text{Fixed assets bought} - \text{Fixed assets sold}$

$\$2,342 = \$2,820 - \text{Fixed assets sold}$

$\text{Fixed assets sold} = \$2,820 - 2,342$

$\text{Fixed assets sold} = \478

To calculate the cash flow from assets, we must first calculate the operating cash flow. The operating cash flow is calculated as follows (we could also prepare a traditional income statement):

$\text{EBIT} = \text{Sales} - \text{Costs} - \text{Depreciation}$

$\text{EBIT} = \$17,688 - 7,118 - 1,478$

$\text{EBIT} = \$9,092$

$\text{EBT} = \text{EBIT} - \text{Interest}$

$\text{EBT} = \$9,092 - 392$

$\text{EBT} = \$8,700$

$\text{Taxes} = \text{EBT} \times .22$

$\text{Taxes} = \$8,700 \times .22$

$\text{Taxes} = \$1,914$

$$\text{OCF} = \text{EBIT} + \text{Depreciation} - \text{Taxes}$$

$$\text{OCF} = \$9,092 + 1,478 - 1,914$$

$$\text{OCF} = \$8,656$$

$$\text{Cash flow from assets} = \text{OCF} - \text{Change in NWC} - \text{Net capital spending}$$

$$\text{Cash flow from assets} = \$8,656 - 201 - 2,342$$

$$\text{Cash flow from assets} = \$6,113$$

d. $\text{Net new borrowing} = \text{LTD}_{\text{end}} - \text{LTD}_{\text{beg}}$

$$\text{Net new borrowing} = \$3,256 - 2,856$$

$$\text{Net new borrowing} = \$400$$

$$\text{Net new borrowing} = \$400 = \text{Debt issued} - \text{Debt retired}$$

$$\text{Debt retired} = \$545 - 400$$

$$\text{Debt retired} = \$145$$

$$\text{Cash flow to creditors} = \text{Interest} - \text{Net new LTD}$$

$$\text{Cash flow to creditors} = \$392 - 400$$

$$\text{Cash flow to creditors} = -\$8$$

21.

Balance sheet as of Dec. 31, 2018

Cash	\$4,438	Accounts payable	\$4,661
Accounts receivable	4,874	Notes payable	<u>858</u>
Inventory	<u>10,444</u>	Current liabilities	\$5,519
Current assets	\$19,756		
		Long-term debt	\$14,537
Net fixed assets	<u>\$37,211</u>	Owners' equity	<u>36,911</u>
Total assets	<u><u>\$56,967</u></u>	Total liab. & equity	<u><u>\$56,967</u></u>

Balance sheet as of Dec. 31, 2019

Cash	\$5,620	Accounts payable	\$4,520
Accounts receivable	6,617	Notes payable	<u>806</u>
Inventory	<u>10,733</u>	Current liabilities	\$5,326
Current assets	\$22,970		
		Long-term debt	\$17,334
Net fixed assets	<u>\$39,049</u>	Owners' equity	<u>39,359</u>
Total assets	<u><u>\$62,019</u></u>	Total liab. & equity	<u><u>\$62,019</u></u>

<u>2018 Income Statement</u>		<u>2019 Income Statement</u>	
Sales	\$8,462.00	Sales	\$9,082.00
COGS	2,912.00	COGS	3,305.00
Other expenses	690.00	Other expenses	577.00
Depreciation	1,215.00	Depreciation	1,216.00
EBIT	\$3,645.00	EBIT	\$3,984.00
Interest	567.00	Interest	652.00
EBT	\$3,078.00	EBT	\$3,332.00
Taxes	646.38	Taxes	699.72
Net income	<u>\$2,431.62</u>	Net income	<u>\$2,632.28</u>
Dividends	\$1,032.00	Dividends	\$1,135.00
Additions to RE	1,399.62	Additions to RE	1,497.28

22. $OCF = EBIT + \text{Depreciation} - \text{Taxes}$
 $OCF = \$3,984 + 1,216 - 699.72$
 $OCF = \$4,500.28$

$\text{Change in NWC} = NWC_{\text{end}} - NWC_{\text{beg}} = (CA - CL)_{\text{end}} - (CA - CL)_{\text{beg}}$
 $\text{Change in NWC} = (\$22,970 - 5,326) - (\$19,756 - 5,519)$
 $\text{Change in NWC} = \$3,407$

$\text{Net capital spending} = NFA_{\text{end}} - NFA_{\text{beg}} + \text{Depreciation}$
 $\text{Net capital spending} = \$39,049 - 37,211 + 1,216$
 $\text{Net capital spending} = \$3,054$

$\text{Cash flow from assets} = OCF - \text{Change in NWC} - \text{Net capital spending}$
 $\text{Cash flow from assets} = \$4,500.28 - 3,407 - 3,054$
 $\text{Cash flow from assets} = -\$1,960.72$

$\text{Cash flow to creditors} = \text{Interest} - \text{Net new LTD}$
 $\text{Net new LTD} = LTD_{\text{end}} - LTD_{\text{beg}}$
 $\text{Cash flow to creditors} = \$652 - (\$17,334 - 14,537)$
 $\text{Cash flow to creditors} = -\$2,145$

$\text{Net new equity} = \text{Common stock}_{\text{end}} - \text{Common stock}_{\text{beg}}$
 $\text{Common stock} + \text{Retained earnings} = \text{Total owners' equity}$
 $\text{Net new equity} = (OE - RE)_{\text{end}} - (OE - RE)_{\text{beg}}$
 $\text{Net new equity} = OE_{\text{end}} - OE_{\text{beg}} + RE_{\text{beg}} - RE_{\text{end}}$
 $RE_{\text{end}} = RE_{\text{beg}} + \text{Additions to RE}$
 $\therefore \text{Net new equity} = OE_{\text{end}} - OE_{\text{beg}} + RE_{\text{beg}} - (RE_{\text{beg}} + \text{Additions to RE})$
 $\phantom{\therefore \text{Net new equity}} = OE_{\text{end}} - OE_{\text{beg}} - \text{Additions to RE}$
 $\text{Net new equity} = \$39,359 - 36,911 - 1,497.28 = \950.72

$\text{Cash flow to stockholders} = \text{Dividends} - \text{Net new equity}$
 $\text{Cash flow to stockholders} = \$1,135 - 950.72$
 $\text{Cash flow to stockholders} = \184.28

As a check, cash flow from assets is $-\$1,960.72$

Cash flow from assets = Cash flow from creditors + Cash flow to stockholders

Cash flow from assets = $-\$2,145 + 184.28$

Cash flow from assets = $-\$1,960.72$

Challenge

23. We will begin by calculating the operating cash flow. First, we need the EBIT, which can be calculated as:

EBIT = Net income + Current taxes + Deferred taxes + Interest

EBIT = $\$187 + 81 + 11 + 38$

EBIT = $\$317$

Now we can calculate the operating cash flow as:

Operating cash flow

Earnings before interest and taxes	\$317
Depreciation	74
Current taxes	<u>-81</u>
Operating cash flow	\$310

The cash flow from assets is found in the investing activities portion of the accounting statement of cash flows, so:

Cash flow from assets

Acquisition of fixed assets	\$194
Sale of fixed assets	<u>-19</u>
Capital spending	\$175

The net working capital cash flows are all found in the operations cash flow section of the accounting statement of cash flows. However, instead of calculating the net working capital cash flows as the change in net working capital, we must calculate each item individually. Doing so, we find:

Net working capital cash flow

Cash	\$12
Accounts receivable	12
Inventories	-15
Accounts payable	-11
Accrued expenses	6
Other	<u>-2</u>
NWC cash flow	\$2

Except for the interest expense, the cash flow to creditors is found in the financing activities of the accounting statement of cash flows. The interest expense from the income statement is given, so:

Cash flow to creditors

Interest	\$38
Retirement of debt	<u>145</u>
Debt service	\$183
Proceeds from sale of long-term debt	<u>-110</u>
Total	\$73

And we can find the cash flow to stockholders in the financing section of the accounting statement of cash flows. The cash flow to stockholders was:

Cash flow to stockholders

Dividends	\$67
Repurchase of stock	<u>32</u>
Cash to stockholders	\$99
Proceeds from new stock issue	<u>-39</u>
Total	\$60

$$\begin{aligned}
 24. \text{ Net capital spending} &= NFA_{\text{end}} - NFA_{\text{beg}} + \text{Depreciation} \\
 &= (NFA_{\text{end}} - NFA_{\text{beg}}) + (\text{Depreciation} + AD_{\text{beg}}) - AD_{\text{beg}} \\
 &= (NFA_{\text{end}} - NFA_{\text{beg}}) + AD_{\text{end}} - AD_{\text{beg}} \\
 &= (NFA_{\text{end}} + AD_{\text{end}}) - (NFA_{\text{beg}} + AD_{\text{beg}}) = FA_{\text{end}} - FA_{\text{beg}}
 \end{aligned}$$

CHAPTER 3

LONG-TERM FINANCIAL PLANNING AND GROWTH

Answers to Concepts Review and Critical Thinking Questions

1. Time trend analysis gives a picture of changes in the company's financial situation over time. Comparing a firm to itself over time allows the financial manager to evaluate whether some aspects of the firm's operations, finances, or investment activities have changed. Peer group analysis involves comparing the financial ratios and operating performance of a particular firm to a set of peer group firms in the same industry or line of business. Comparing a firm to its peers allows the financial manager to evaluate whether some aspects of the firm's operations, finances, or investment activities are out of line with the norm, thereby providing some guidance on appropriate actions to take to adjust these ratios if appropriate. Both allow an investigation into what is different about a company from a financial perspective, but neither method gives an indication of whether the difference is positive or negative. For example, suppose a company's current ratio is increasing over time. It could mean that the company had been facing liquidity problems in the past and is rectifying those problems, or it could mean the company has become less efficient in managing its current accounts. Similar arguments could be made for a peer group comparison. A company with a current ratio lower than its peers could be more efficient at managing its current accounts, or it could be facing liquidity problems. Neither analysis method tells us whether a ratio is good or bad; both show that something is different, and tell us where to look.
2. If a company is growing by opening new stores, then presumably total revenues would be rising. Comparing total sales at two different points in time might be misleading. Same-store sales control for this by only looking at revenues of stores open within a specific period.
3. The reason is that, ultimately, sales are the driving force behind a business. A firm's assets, employees, and, in fact, just about every aspect of its operations and financing exist to directly or indirectly support sales. Put differently, a firm's future need for things like capital assets, employees, inventory, and financing are determined by its future sales level.
4. Two assumptions of the sustainable growth formula are that the company does not want to sell new equity, and that financial policy is fixed. If the company raises outside equity, or increases its debt-equity ratio, it can grow at a higher rate than the sustainable growth rate. Of course, the company could also grow at a faster rate if its profit margin increases, if it changes its dividend policy by increasing the retention ratio, or its total asset turnover increases.

5. The sustainable growth rate is greater than 20 percent, because at a 20 percent growth rate the negative EFN indicates that there is excess financing still available. If the firm is 100 percent equity financed, then the sustainable and internal growth rates are equal and the internal growth rate would be greater than 20 percent. However, when the firm has some debt, the internal growth rate is always less than the sustainable growth rate, so it is ambiguous whether the internal growth rate would be greater than or less than 20 percent. If the retention ratio is increased, the firm will have more internal funding sources available, and it will have to take on more debt to keep the debt/equity ratio constant, so the EFN will decline. Conversely, if the retention ratio is decreased, the EFN will rise. If the retention rate is zero, both the internal and sustainable growth rates are zero, and the EFN will rise to the change in total assets.
6. Common-size financial statements provide the financial manager with a ratio analysis of the company. The common-size income statement can show, for example, that cost of goods sold as a percentage of sales is increasing. The common-size balance sheet can show a firm's increasing reliance on debt as a form of financing. Common-size statements of cash flows are not calculated for a simple reason: There is no possible denominator.
7. It would reduce the external funds needed. If the company is not operating at full capacity, it would be able to increase sales without a commensurate increase in fixed assets.
8. ROE is a better measure of the company's performance. ROE shows the percentage return earned on shareholder investment. Since the goal of a company is to maximize shareholder wealth, this ratio shows the company's performance in achieving this goal over the period.
9. The EBITD/Assets ratio shows the company's operating performance before interest, taxes, and depreciation. This ratio would show how a company has controlled costs. While taxes are a cost, and depreciation and amortization can be considered costs, they are not as easily controlled by company management. Conversely, depreciation and amortization can be altered by accounting choices. This ratio only uses costs directly related to operations in the numerator. As such, it gives a better metric to measure management performance over a period than does ROA.
10. Long-term liabilities and equity are investments made by investors in the company, either in the form of a loan or ownership. Return on investment is intended to measure the return the company earned from these investments. Return on investment will be higher than the return on assets for a company with current liabilities. To see this, realize that total assets must equal total debt and equity, and total debt and equity is equal to current liabilities plus long-term liabilities plus equity. So, return on investment could be calculated as net income divided by total assets minus current liabilities.
11. Presumably not, but, of course, if the product had been *much* less popular, then a similar fate would have awaited due to lack of sales.
12. Since customers did not pay until shipment, receivables rose. The firm's NWC, but not its cash, increased. At the same time, costs were rising faster than cash revenues, so operating cash flow declined. The firm's capital spending was also rising. Thus, all three components of cash flow from assets were negatively impacted.
13. Financing possibly could have been arranged if the company had taken quick enough action. Sometimes it becomes apparent that help is needed only when it is too late, again emphasizing the need for planning.

14. All three were important, but the lack of cash or, more generally, financial resources, ultimately spelled doom. An inadequate cash resource is usually cited as the most common cause of small business failure.
15. Demanding cash up front, increasing prices, subcontracting production, and improving financial resources via new owners or new sources of credit are some of the options. When orders exceed capacity, price increases may be especially beneficial.

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. Using the DuPont identity, the ROE is:

$$\begin{aligned}\text{ROE} &= (\text{Profit margin})(\text{Total asset turnover})(\text{Equity multiplier}) \\ \text{ROE} &= (.061)(1.87)(1.35) \\ \text{ROE} &= .1540, \text{ or } 15.40\%\end{aligned}$$

2. The equity multiplier is:

$$\begin{aligned}\text{Equity multiplier} &= 1 + D/E \\ \text{Equity multiplier} &= 1 + .85 \\ \text{Equity multiplier} &= 1.85\end{aligned}$$

One formula to calculate return on equity is:

$$\begin{aligned}\text{ROE} &= (\text{ROA})(\text{Equity multiplier}) \\ \text{ROE} &= .073(1.85) \\ \text{ROE} &= .1351, \text{ or } 13.51\%\end{aligned}$$

ROE can also be calculated as:

$$\text{ROE} = \text{Net income} / \text{Total equity}$$

So, net income is:

$$\begin{aligned}\text{Net income} &= \text{ROE}(\text{Total equity}) \\ \text{Net income} &= .1351(\$910,000) \\ \text{Net income} &= \$122,895.50\end{aligned}$$

3. This is a multi-step problem involving several ratios. The ratios given are all part of the DuPont Identity. The only DuPont Identity ratio not given is the profit margin. If we know the profit margin, we can find the net income since sales are given. So, we begin with the DuPont Identity:

$$\begin{aligned}\text{ROE} &= .14 = (\text{Profit margin})(\text{Total asset turnover})(\text{Equity multiplier}) \\ \text{ROE} &= (\text{Profit margin})(\text{Sales} / \text{Total assets})(1 + D/E)\end{aligned}$$

Solving the DuPont Identity for profit margin, we get:

$$\text{Profit margin} = [(\text{ROE})(\text{Total assets})]/[(1 + D/E)(\text{Sales})]$$

$$\text{Profit margin} = [(.14)(\$1,520)]/[(1 + 1.35)(\$3,300)]$$

$$\text{Profit margin} = .0274$$

Now that we have the profit margin, we can use this number and the given sales figure to solve for net income:

$$\text{Profit margin} = .0274 = \text{Net income}/\text{Sales}$$

$$\text{Net income} = .0274(\$3,300)$$

$$\text{Net income} = \$90.55$$

4. An increase of sales to \$42,112 is an increase of:

$$\text{Sales increase} = (\$42,112 - \$37,600)/\$37,600$$

$$\text{Sales increase} = .1200, \text{ or } 12.00\%$$

Assuming costs and assets increase proportionally, the pro forma financial statements will look like this:

Pro forma income statement

Sales	\$42,112.00
Costs	<u>29,232.00</u>
EBIT	12,880.00
Taxes (21%)	<u>2,704.80</u>
Net income	<u>\$10,175.20</u>

Pro forma balance sheet

Assets	\$ 151,200.00	Debt	\$ 37,000.00
		Equity	<u>105,151.20</u>
Total	<u>\$ 151,200.00</u>	Total	<u>\$142,151.20</u>

The payout ratio is constant, so the dividends paid this year is the payout ratio from last year times net income, or:

$$\text{Dividends} = (\$2,700/\$9,085)(\$10,175.20)$$

$$\text{Dividends} = \$3,024$$

The addition to retained earnings is:

$$\text{Addition to retained earnings} = \$10,175.20 - 3,024$$

$$\text{Addition to retained earnings} = \$7,151.20$$

And the new equity balance is:

$$\text{Equity} = \$98,000 + 7,151.20$$

$$\text{Equity} = \$105,151.20$$

So the EFN is:

$$\text{EFN} = \text{Total assets} - \text{Total liabilities and equity}$$

$$\text{EFN} = \$151,200 - 142,151.20$$

$$\text{EFN} = \$9,048.80$$

5. The maximum percentage sales increase without issuing new equity is the sustainable growth rate. To calculate the sustainable growth rate, we first need to calculate the ROE, which is:

$$\text{ROE} = \text{NI}/\text{TE}$$

$$\text{ROE} = \$20,066/\$88,000$$

$$\text{ROE} = .2280, \text{ or } 22.80\%$$

The plowback ratio, b , is one minus the payout ratio, so:

$$b = 1 - .30$$

$$b = .70$$

Now we can use the sustainable growth rate equation to get:

$$\text{Sustainable growth rate} = (\text{ROE} \times b)/[1 - (\text{ROE} \times b)]$$

$$\text{Sustainable growth rate} = [.2280(.70)]/[1 - .2280(.70)]$$

$$\text{Sustainable growth rate} = .1899, \text{ or } 18.99\%$$

So, the maximum dollar increase in sales is:

$$\text{Maximum increase in sales} = \$49,000(.1899)$$

$$\text{Maximum increase in sales} = \$9,306.67$$

6. We need to calculate the retention ratio to calculate the sustainable growth rate. The retention ratio is:

$$b = 1 - .20$$

$$b = .80$$

Now we can use the sustainable growth rate equation to get:

$$\text{Sustainable growth rate} = (\text{ROE} \times b)/[1 - (\text{ROE} \times b)]$$

$$\text{Sustainable growth rate} = [.11(.80)]/[1 - .11(.80)]$$

$$\text{Sustainable growth rate} = .0965, \text{ or } 9.65\%$$

7. We must first calculate the ROE using the DuPont ratio to calculate the sustainable growth rate. The ROE is:

$$\text{ROE} = (\text{PM})(\text{TAT})(\text{EM})$$

$$\text{ROE} = (.057)(2.65)(1.60)$$

$$\text{ROE} = .2417, \text{ or } 24.17\%$$

The plowback ratio is one minus the dividend payout ratio, so:

$$b = 1 - .70$$

$$b = .30$$

Now, we can use the sustainable growth rate equation to get:

$$\begin{aligned}\text{Sustainable growth rate} &= (\text{ROE} \times b) / [1 - (\text{ROE} \times b)] \\ \text{Sustainable growth rate} &= [.2417(.30)] / [1 - .2417(.30)] \\ \text{Sustainable growth rate} &= .0782, \text{ or } 7.82\%\end{aligned}$$

8. An increase of sales to \$9,462 is an increase of:

$$\begin{aligned}\text{Sales increase} &= (\$9,462 - \$8,300) / \$8,300 \\ \text{Sales increase} &= .14, \text{ or } 14\%\end{aligned}$$

Assuming costs and assets increase proportionally, the pro forma financial statements will look like this:

Pro forma income statement

Sales	\$ 9,462
Costs	<u>7,399</u>
Net income	<u>\$ 2,063</u>

Pro forma balance sheet

Assets	\$ 21,774	Debt	\$ 8,400
		Equity	<u>12,763</u>
Total	<u>\$ 21,774</u>	Total	<u>\$ 21,163</u>

If no dividends are paid, the equity account will increase by the net income, so:

$$\begin{aligned}\text{Equity} &= \$10,700 + 2,063 \\ \text{Equity} &= \$12,763\end{aligned}$$

So the EFN is:

$$\begin{aligned}\text{EFN} &= \text{Total assets} - \text{Total liabilities and equity} \\ \text{EFN} &= \$21,774 - 21,163 \\ \text{EFN} &= \$611\end{aligned}$$

9. a. First, we need to calculate the current sales and change in sales. The current sales are next year's sales divided by one plus the growth rate, so:

$$\begin{aligned}\text{Current sales} &= \text{Next year's sales} / (1 + g) \\ \text{Current sales} &= \$320,000,000 / (1 + .12) \\ \text{Current sales} &= \$285,714,286\end{aligned}$$

And the change in sales is:

$$\begin{aligned}\text{Change in sales} &= \$320,000,000 - 285,714,286 \\ \text{Change in sales} &= \$34,285,714\end{aligned}$$

We can now complete the current balance sheet. The current assets, fixed assets, and short-term debt are calculated as a percentage of current sales. The long-term debt and par value of stock are given. The plug variable is the addition to retained earnings. So:

<u>Assets</u>		<u>Liabilities and equity</u>	
Current assets	\$57,142,857	Short-term debt	\$42,857,143
		Long-term debt	\$110,000,000
Fixed assets	<u>200,000,000</u>	Common stock	\$45,000,000
		Accumulated retained earnings	<u>59,285,714</u>
		Total equity	<u>\$104,285,714</u>
Total assets	<u>\$257,142,857</u>	Total liabilities and equity	<u>\$257,142,857</u>

- b. We can use the equation from the text to answer this question. The assets/sales and debt/sales are the percentages given in the problem, so:

$$\begin{aligned}
 \text{EFN} &= \left(\frac{\text{Assets}}{\text{Sales}} \right) \times \Delta \text{Sales} - \left(\frac{\text{Debt}}{\text{Sales}} \right) \times \Delta \text{Sales} - (\text{PM} \times \text{Projected sales}) \times (1 - d) \\
 \text{EFN} &= (.20 + .70) \times \$34,285,714 - (.15 \times \$34,285,714) - [(.09 \times \$320,000,000) \times (1 - .30)] \\
 \text{EFN} &= \$5,554,286
 \end{aligned}$$

- c. The current assets, fixed assets, and short-term debt will all increase at the same percentage as sales. The long-term debt and common stock will remain constant. The accumulated retained earnings will increase by the addition to retained earnings for the year. We can calculate the addition to retained earnings for the year as:

$$\begin{aligned}
 \text{Net income} &= \text{Profit margin} \times \text{Sales} \\
 \text{Net income} &= .09(\$320,000,000) \\
 \text{Net income} &= \$28,800,000
 \end{aligned}$$

The addition to retained earnings for the year will be the net income times one minus the dividend payout ratio, which is:

$$\begin{aligned}
 \text{Addition to retained earnings} &= \text{Net income}(1 - d) \\
 \text{Addition to retained earnings} &= \$28,800,000(1 - .30) \\
 \text{Addition to retained earnings} &= \$20,160,000
 \end{aligned}$$

So, the new accumulated retained earnings will be:

$$\begin{aligned}
 \text{Accumulated retained earnings} &= \$59,285,714 + 20,160,000 \\
 \text{Accumulated retained earnings} &= \$79,445,714
 \end{aligned}$$

The pro forma balance sheet will be:

<u>Assets</u>		<u>Liabilities and equity</u>	
Current assets	\$64,000,000	Short-term debt	\$48,000,000
		Long-term debt	\$110,000,000
Fixed assets	<u>\$224,000,000</u>	Common stock	\$45,000,000
		Accumulated retained earnings	<u>79,445,714</u>
		Total equity	<u>\$124,445,714</u>
Total assets	<u>\$288,000,000</u>	Total liabilities and equity	<u>\$282,445,714</u>

The EFN is:

EFN = Total assets – Total liabilities and equity

EFN = \$288,000,000 – 282,445,714

EFN = \$5,554,286

10. a. The plowback ratio is one minus the dividend payout ratio, so:

$$b = 1 - .25$$

$$b = .75$$

Now, we can use the sustainable growth rate equation to get:

$$\text{Sustainable growth rate} = (\text{ROE} \times b) / [1 - (\text{ROE} \times b)]$$

$$\text{Sustainable growth rate} = [.121(.75)] / [1 - .121(.75)]$$

$$\text{Sustainable growth rate} = .0998, \text{ or } 9.98\%$$

- b. It is possible for the sustainable growth rate and the actual growth rate to differ. If any of the actual parameters in the sustainable growth rate equation differ from those used to compute the sustainable growth rate, the actual growth rate will differ from the sustainable growth rate. Since the sustainable growth rate includes ROE in the calculation, this also implies that changes in the profit margin, total asset turnover, or equity multiplier will affect the sustainable growth rate.
- c. The company can increase its growth rate by doing any of the following:
- Increase the debt-to-equity ratio by selling more debt or repurchasing stock.
 - Increase the profit margin, most likely by better controlling costs.
 - Decrease its total assets/sales ratio; in other words, utilize its assets more efficiently.
 - Reduce the dividend payout ratio.

Intermediate

11. The solution requires substituting two ratios into a third ratio. Rearranging D/TA:

Firm A

$$D/TA = .35$$

$$(TA - E)/TA = .35$$

$$(TA/TA) - (E/TA) = .35$$

$$1 - (E/TA) = .35$$

$$E/TA = .65$$

$$E = .65(TA)$$

Firm B

$$D/TA = .45$$

$$(TA - E)/TA = .45$$

$$(TA/TA) - (E/TA) = .45$$

$$1 - (E/TA) = .45$$

$$E/TA = .55$$

$$E = .55(TA)$$

Rearranging ROA, we find:

$$NI/TA = .08$$

$$NI = .08(TA)$$

$$NI/TA = .07$$

$$NI = .07(TA)$$

Since $ROE = NI/E$, we can substitute the above equations into the ROE formula, which yields:

$$ROE = .08(TA)/.65(TA)$$

$$ROE = .08/.65$$

$$ROE = .1231, \text{ or } 12.31\%$$

$$ROE = .07(TA)/.55(TA)$$

$$ROE = .07/.55$$

$$ROE = .1273, \text{ or } 12.73\%$$

12. Profit margin = Net income/Sales
 Profit margin = $-\text{£}18,137/\text{£}279,386$
 Profit margin = $-.0649$, or 6.49%

As long as both net income and sales are measured in the same currency, there is no problem; in fact, except for some market value ratios like EPS and BVPS, none of the financial ratios discussed in the text are measured in terms of currency. This is one reason why financial ratio analysis is widely used in international finance to compare the business operations of firms and/or divisions across national economic borders. The net income in dollars is:

$$\text{Net income} = \text{Profit margin} \times \text{Sales}$$

$$\text{Net income} = -.0649(\$359,815)$$

$$\text{Net income} = -\$23,358.24$$

13. a. The equation for external funds needed is:

$$EFN = \left(\frac{\text{Assets}}{\text{Sales}} \right) \times \Delta \text{Sales} - \left(\frac{\text{Debt}}{\text{Sales}} \right) \times \Delta \text{Sales} - (\text{PM} \times \text{Projected sales}) \times (1 - d)$$

where:

$$\text{Assets/Sales} = \$24,200,000/\$21,860,000 = 1.11$$

$$\Delta \text{Sales} = \text{Current sales} \times \text{Sales growth rate} = \$21,860,000(.15) = \$3,279,000$$

$$\text{Short-term debt/Sales} = \$5,100,000/\$21,860,000 = .2333$$

$$\text{Profit margin} = \text{Net income/Sales} = \$1,807,500/\$21,860,000 = .0827$$

$$\text{Projected sales} = \text{Current sales} \times (1 + \text{Sales growth rate}) = \$21,860,000(1 + .15) = \$25,139,000$$

$$d = \text{Dividends/Net income} = \$361,500/\$1,807,500 = .20$$

so:

$$\text{EFN} = (1.11 \times \$3,279,000) - (.2333 \times \$3,279,000) - (.0827 \times \$25,139,000) \times (1 - .20)$$

$$\text{EFN} = \$1,202,100$$

- b. The current assets, fixed assets, and short-term debt will all increase at the same percentage as sales. The long-term debt and common stock will remain constant. The accumulated retained earnings will increase by the addition to retained earnings for the year. We can calculate the addition to retained earnings for the year as:

$$\text{Net income} = \text{Profit margin} \times \text{Sales}$$

$$\text{Net income} = .0827(\$25,139,000)$$

$$\text{Net income} = \$2,078,625$$

The addition to retained earnings for the year will be the net income times one minus the dividend payout ratio, which is:

$$\text{Addition to retained earnings} = \text{Net income}(1 - d)$$

$$\text{Addition to retained earnings} = \$2,078,625(1 - .20)$$

$$\text{Addition to retained earnings} = \$1,662,900$$

So, the new accumulated retained earnings will be:

$$\text{Accumulated retained earnings} = \$10,200,000 + 1,622,900$$

$$\text{Accumulated retained earnings} = \$11,862,900$$

The pro forma balance sheet will be:

<u>Assets</u>		<u>Liabilities and equity</u>	
Current assets	\$7,935,000	Short-term debt	\$5,865,000
		Long-term debt	\$5,800,000
Fixed assets	<u>19,895,000</u>	Common stock	\$3,100,000
		Accumulated retained earnings	<u>11,862,900</u>
		Total equity	<u>\$14,962,900</u>
Total assets	<u>\$27,830,000</u>	Total liabilities and equity	<u>\$26,627,900</u>

The EFN is:

$$\text{EFN} = \text{Total assets} - \text{Total liabilities and equity}$$

$$\text{EFN} = \$27,830,000 - 26,627,900$$

$$\text{EFN} = \$1,202,100$$

- c. The ROE is:

$$\text{ROE} = \text{Net income} / \text{Total equity}$$

$$\text{ROE} = \$1,807,500 / \$13,300,000$$

$$\text{ROE} = .1359, \text{ or } 13.59\%$$

And the retention ratio is:

$$b = \text{Retention ratio} = \text{Retained earnings}/\text{Net income}$$

$$b = \$1,446,000/\$1,807,500$$

$$b = .80$$

Now, we can use the sustainable growth rate equation to get:

$$\text{Sustainable growth rate} = (\text{ROE} \times b)/[1 - (\text{ROE} \times b)]$$

$$\text{Sustainable growth rate} = [.1359(.80)]/[1 - .1359(.80)]$$

$$\text{Sustainable growth rate} = .1220, \text{ or } 12.20\%$$

- d. The company cannot just cut its dividends to achieve the forecast growth rate. As shown below, even with a zero dividend policy, the EFN will still be \$786,375.

<u>Assets</u>		<u>Liabilities and equity</u>	
Current assets	\$7,935,000	Short-term debt	\$5,865,000
		Long-term debt	\$5,800,000
Fixed assets	<u>19,895,000</u>	Common stock	\$3,100,000
		Accumulated retained earnings	<u>12,278,625</u>
		Total equity	<u>\$15,378,625</u>
Total assets	<u>\$27,830,000</u>	Total liabilities and equity	<u>\$27,043,625</u>

The EFN is:

$$\text{EFN} = \text{Total assets} - \text{Total liabilities and equity}$$

$$\text{EFN} = \$27,830,000 - 27,043,625$$

$$\text{EFN} = \$786,375$$

The company does have several alternatives. It can increase its asset utilization and/or its profit margin. The company could also increase the debt in its capital structure. This will decrease the equity account, thereby increasing ROE.

14. This is a multi-step problem involving several ratios. It is often easier to look backward to determine where to start. We need receivables turnover to find days' sales in receivables. To calculate receivables turnover, we need credit sales, and to find credit sales, we need total sales. Since we are given the profit margin and net income, we can use these to calculate total sales as:

$$\text{Profit margin} = \text{Net income}/\text{Sales}$$

$$.0860 = \$386,000/\text{Sales}$$

$$\text{Sales} = \$4,488,372$$

Credit sales are 80 percent of total sales, so:

$$\text{Credit sales} = \$4,488,372(.80)$$

$$\text{Credit sales} = \$3,590,698$$

Now we can find receivables turnover by:

$$\text{Receivables turnover} = \text{Credit sales}/\text{Accounts receivable}$$

$$\text{Receivables turnover} = \$3,590,698/\$191,300$$

$$\text{Receivables turnover} = 18.77 \text{ times}$$

$$\text{Days' sales in receivables} = 365 \text{ days}/\text{Receivables turnover}$$

$$\text{Days' sales in receivables} = 365/18.77$$

$$\text{Days' sales in receivables} = 19.45 \text{ days}$$

15. The solution to this problem requires a number of steps. First, remember that:

$$\text{Current assets} + \text{Net fixed assets} = \text{Total assets}$$

So, if we find the current assets and the total assets, we can solve for net fixed assets. Using the numbers given for the current ratio and the current liabilities, we solve for current assets:

$$\text{Current ratio} = \text{Current assets}/\text{Current liabilities}$$

$$\text{Current assets} = \text{Current ratio}(\text{Current liabilities})$$

$$\text{Current assets} = 1.29(\$1,450)$$

$$\text{Current assets} = \$1,870.50$$

To find the total assets, we must first find the total debt and equity from the information given. So, we find the net income using the profit margin:

$$\text{Profit margin} = \text{Net income}/\text{Sales}$$

$$\text{Net income} = \text{Profit margin} \times \text{Sales}$$

$$\text{Net income} = .081(\$7,380)$$

$$\text{Net income} = \$597.78$$

We now use the net income figure as an input into ROE to find the total equity:

$$\text{ROE} = \text{Net income}/\text{Total equity}$$

$$\text{Total equity} = \text{Net income}/\text{ROE}$$

$$\text{Total equity} = \$597.78/.143$$

$$\text{Total equity} = \$4,180.28$$

Next, we need to find the long-term debt. The long-term debt ratio is:

$$\text{Long-term debt ratio} = .34 = \text{Long-term debt}/(\text{Long-term debt} + \text{Total equity})$$

Inverting both sides gives:

$$1/.34 = (\text{Long-term debt} + \text{Total equity})/\text{Long-term debt} = 1 + (\text{Total equity}/\text{Long-term debt})$$

Substituting the total equity into the equation and solving for long-term debt gives the following:

$$1 + (\$4,180.28/\text{Long-term debt}) = 2.94$$

$$\text{Long-term debt} = \$4,180.28/1.94$$

$$\text{Long-term debt} = \$2,153.48$$

Now, we can find the total debt of the company:

$$\text{Total debt} = \text{Current liabilities} + \text{Long-term debt}$$

$$\text{Total debt} = \$1,450 + 2,153.48$$

$$\text{Total debt} = \$3,603.48$$

And, with the total debt, we can find the total debt & equity, which is equal to total assets:

$$\text{Total assets} = \text{Total debt} + \text{Total equity}$$

$$\text{Total assets} = \$3,603.48 + 4,180.28$$

$$\text{Total assets} = \$7,783.76$$

And finally, we are ready to solve the balance sheet identity as:

$$\text{Net fixed assets} = \text{Total assets} - \text{Current assets}$$

$$\text{Net fixed assets} = \$7,783.76 - 1,870.50$$

$$\text{Net fixed assets} = \$5,913.26$$

16. This problem requires you to work backward through the income statement. First, recognize that $\text{Net income} = (1 - T_c)\text{EBT}$. Plugging in the numbers given and solving for EBT, we get:

$$\text{EBT} = \$13,150/(1 - .24)$$

$$\text{EBT} = \$17,302.63$$

Now, we can add interest to EBT to get EBIT as follows:

$$\text{EBIT} = \text{EBT} + \text{Interest paid}$$

$$\text{EBIT} = \$17,302.63 + 3,460$$

$$\text{EBIT} = \$20,762.63$$

To get EBITD (earnings before interest, taxes, and depreciation), the numerator in the cash coverage ratio, add depreciation to EBIT. Note, since there is no amortization in this problem, EBITDA equals EBITD.

$$\text{EBITD} = \text{EBIT} + \text{Depreciation}$$

$$\text{EBITD} = \$20,762.63 + 4,380$$

$$\text{EBITD} = \$25,142.63$$

Now, plug the numbers into the cash coverage ratio and calculate:

$$\text{Cash coverage ratio} = \text{EBITD}/\text{Interest}$$

$$\text{Cash coverage ratio} = \$25,142.63/\$3,460$$

$$\text{Cash coverage ratio} = 7.27 \text{ times}$$

17. We can start by multiplying ROE by Total assets/Total assets

$$\text{ROE} = \frac{\text{Net income}}{\text{Equity}} = \frac{\text{Net income}}{\text{Equity}} \times \frac{\text{Total assets}}{\text{Total assets}}$$

Rearranging, we get:

$$\text{ROE} = \frac{\text{Net income}}{\text{Total assets}} \times \frac{\text{Total assets}}{\text{Equity}}$$

Next, we can multiply by Sales/Sales, which yields:

$$\text{ROE} = \frac{\text{Net income}}{\text{Total assets}} \times \frac{\text{Equity}}{\text{Total assets}} \times \frac{\text{Sales}}{\text{Sales}}$$

Rearranging, we get:

$$\text{ROE} = \frac{\text{Net income}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Total assets}} \times \frac{\text{Total assets}}{\text{Equity}}$$

Next, we can multiply the preceding three factor DuPont equation by EBT/EBT, which yields:

$$\text{ROE} = \frac{\text{Net income}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Total assets}} \times \frac{\text{Total assets}}{\text{Equity}} \times \frac{\text{EBT}}{\text{EBT}}$$

We can rearrange as:

$$\text{ROE} = \frac{\text{Net income}}{\text{EBT}} \times \frac{\text{EBT}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Total assets}} \times \frac{\text{Total assets}}{\text{Equity}}$$

Finally, multiplying this equation EBIT/EBIT and rearranging yields:

$$\text{ROE} = \frac{\text{Net income}}{\text{EBT}} \times \frac{\text{EBT}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Total assets}} \times \frac{\text{Total assets}}{\text{Equity}} \times \frac{\text{EBIT}}{\text{EBIT}}$$

$$\text{ROE} = \frac{\text{Net income}}{\text{EBT}} \times \frac{\text{EBT}}{\text{EBIT}} \times \frac{\text{EBIT}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Total assets}} \times \frac{\text{Total assets}}{\text{Equity}}$$

(1) (2) (3) (4) (5)

The interpretation of each term is as follows:

- (1) This is the company's tax burden. This is the proportion of the company's profits retained after paying income taxes.
- (2) This is the company's interest burden. It will be 1.00 for a company with no debt or financial leverage.
- (3) This is the company's operating profit margin. It is the operating profit before interest and taxes per dollar of sales.
- (4) This is the company's operating efficiency as measured by dollar of sales per dollar of total assets.

- (5) This is the company's financial leverage as measured by the equity multiplier.

18.		2018	Common size	2019	Common size	Common base year
	Assets					
	Current assets					
	Cash	\$11,459	2.90%	\$14,453	3.13%	1.2613
	Accounts receivable	29,247	7.40%	33,304	7.21%	1.1387
	Inventory	52,655	13.32%	60,689	13.14%	1.1526
	Total	\$93,361	23.62%	\$108,446	23.48%	1.1616
	Fixed assets					
	Net plant and equipment	301,978	76.38%	353,330	76.52%	1.1701
	Total assets	\$395,339	100%	\$461,776	100%	1.1681
	Liabilities and Owners' Equity					
	Current liabilities					
	Accounts payable	\$58,483	14.79%	\$66,623	14.43%	1.1392
	Notes payable	24,973	6.32%	24,735	5.36%	.9905
	Total	\$83,456	21.11%	\$91,358	19.78%	1.0947
	Long-term debt	34,500	8.73%	44,700	9.68%	1.2957
	Owners' equity					
	Common stock and paid-in surplus	\$54,000	13.66%	\$56,500	12.24%	1.0463
	Accumulated retained earnings	223,383	56.50%	269,218	58.30%	1.2052
	Total	\$277,383	70.16%	\$325,718	70.54%	1.1743
	Total liabilities and owners' equity	\$395,339	100%	\$461,776	100%	1.1681

The common-size balance sheet answers are found by dividing each category by total assets. For example, the cash percentage for 2018 is:

$$\$11,459/\$395,339 = .0290, \text{ or } 2.90\%$$

This means that cash is 2.90 percent of total assets.

The common-base year answers are found by dividing each category value for 2019 by the same category value for 2018. For example, the cash common-base year number is found by:

$$\$14,453/\$11,459 = 1.2613$$

This means the cash balance in 2019 is 1.2613 times as large as the cash balance in 2018.

19. To determine full capacity sales, we divide the current sales by the capacity the company is currently using, so:

$$\text{Full capacity sales} = \$530,000/.90$$

$$\text{Full capacity sales} = \$588,889$$

So, the dollar growth rate in sales is:

$$\text{Sales growth} = \$588,889 - 530,000$$

$$\text{Sales growth} = \$58,889$$

20. To find the new level of fixed assets, we need to find the current percentage of fixed assets to full capacity sales. Doing so, we find:

$$\text{Fixed assets/Full capacity sales} = \$620,000/\$588,889$$

$$\text{Fixed assets/Full capacity sales} = 1.0528$$

Next, we calculate the total dollar amount of fixed assets needed at the new sales figure.

$$\text{Total fixed assets} = 1.0528(\$605,000)$$

$$\text{Total fixed assets} = \$636,962$$

The new fixed assets necessary is the total fixed assets at the new sales figure minus the current level of fixed assets.

$$\text{New fixed assets} = \$636,962 - 620,000$$

$$\text{New fixed assets} = \$16,962$$

21. Assuming costs vary with sales and a 20 percent increase in sales, the pro forma income statement will look like this:

Pro Forma Income Statement	
Sales	\$ 1,069,920
Costs	873,480
Other expenses	<u>21,888</u>
EBIT	\$ 174,552
Interest	<u>13,400</u>
Taxable income	\$ 161,152
Taxes (22%)	<u>35,453</u>
Net income	<u>\$ 125,699</u>

The payout ratio is constant, so the dividends paid this year is the payout ratio from last year times net income, or:

$$\text{Dividends} = (\$36,224/\$103,007)(\$125,699)$$

$$\text{Dividends} = \$44,204$$

And the addition to retained earnings will be:

$$\text{Addition to retained earnings} = \$125,699 - 44,204$$

$$\text{Addition to retained earnings} = \$81,495$$

The new retained earnings on the pro forma balance sheet will be:

$$\text{New retained earnings} = \$174,730 + 81,495$$

$$\text{New retained earnings} = \$256,225$$

The pro forma balance sheet will look like this:

Pro Forma Balance Sheet			
Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 29,136	Accounts payable	\$ 78,240
Accounts receivable	44,484	Notes payable	<u>16,320</u>
Inventory	<u>100,080</u>	Total	\$ 94,560
Total	\$ 173,700	Long-term debt	<u>155,000</u>
Fixed assets		Owners' equity	
Net plant and equipment	<u>475,800</u>	Common stock and paid-in surplus	\$ 130,000
		Retained earnings	<u>256,225</u>
		Total	<u>\$ 386,225</u>
Total assets	<u>\$ 649,500</u>	Total liabilities and owners' equity	<u>\$ 635,785</u>

So the EFN is:

EFN = Total assets – Total liabilities and equity

EFN = \$649,500 – 635,785

EFN = \$13,715

22. First, we need to calculate full capacity sales, which is:

Full capacity sales = \$891,600/.80

Full capacity sales = \$1,114,500

The full capacity ratio at full capacity sales is:

Full capacity ratio = Fixed assets/Full capacity sales

Full capacity ratio = \$396,500/\$1,114,500

Full capacity ratio = .35576

The fixed assets required at the projected sales figure is the full capacity ratio times the projected sales level:

Total fixed assets = .35576(\$1,069,920) = \$380,640

So, EFN is:

EFN = (\$173,700 + 380,640) – \$635,785 = –\$81,445

Note that this solution assumes that fixed assets are decreased (sold) so the company has a 100 percent fixed asset utilization. If we assume fixed assets are not sold, the answer becomes:

EFN = (\$173,700 + 396,500) – \$635,785 = –\$65,585

23. The D/E ratio of the company is:

$$D/E = (\$81,520 + 155,000)/\$304,730$$

$$D/E = .7762$$

So the new total debt amount will be:

$$\text{New total debt} = .7762(\$386,225)$$

$$\text{New total debt} = \$299,773$$

This is the new total debt for the company. Given that our calculation for EFN is the amount that must be raised externally and does not increase spontaneously with sales, we need to subtract the spontaneous increase in accounts payable. The new level of accounts payable, which is the current accounts payable times the sales growth, will be:

$$\text{Spontaneous increase in accounts payable} = \$65,200(.20)$$

$$\text{Spontaneous increase in accounts payable} = \$13,040$$

This means that \$13,040 of the new total debt is not raised externally. So, the debt raised externally, which will be the EFN, is:

$$\text{EFN} = \text{New total debt} - (\text{Beginning LTD} + \text{Beginning CL} + \text{Spontaneous increase in AP})$$

$$\text{EFN} = \$299,773 - (\$155,000 + 81,520 + 13,040) = \$50,213$$

The pro forma balance sheet with the new long-term debt will be:

Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 29,136	Accounts payable	\$ 78,240
Accounts receivable	44,484	Notes payable	<u>16,320</u>
Inventory	<u>100,080</u>	Total	\$ 94,560
Total	\$ 173,700	Long-term debt	<u>205,213</u>
Fixed assets		Owners' equity	
Net plant and equipment	<u>475,800</u>	Common stock and paid-in surplus	\$ 130,000
		Retained earnings	<u>256,225</u>
		Total	\$ 386,225
Total assets	<u>\$ 649,500</u>	Total liabilities and owners' equity	<u>\$ 685,998</u>

The funds raised by the debt issue can be put into an excess cash account to make the balance sheet balance. The excess debt will be:

$$\text{Excess debt} = \$685,998 - 649,500 = \$36,498$$

To make the balance sheet balance, the company will have to increase its assets. We will put this amount in an account called excess cash, which will give us the following balance sheet:

Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 29,136	Accounts payable	\$ 78,240
Excess cash	36,498	Notes payable	<u>16,320</u>
Accounts receivable	44,484	Total	\$ 94,560
Inventory	<u>100,080</u>		
Total	\$ 210,198	Long-term debt	<u>205,213</u>
Fixed assets		Owners' equity	
Net plant and equipment	<u>475,800</u>	Common stock and paid-in surplus	\$ 130,000
		Retained earnings	<u>256,225</u>
		Total	<u>\$ 386,225</u>
Total assets	<u>\$ 685,998</u>	Total liabilities and owners' equity	<u>\$ 685,998</u>

The excess cash has an opportunity cost that we discussed earlier. Increasing fixed assets would also not be a good idea since the company already has enough fixed assets. A likely scenario would be the repurchase of debt and equity in its current capital structure weights. The company's debt-assets and equity-assets are:

$$\text{Debt-assets} = .7762 / (1 + .7762) = .44$$

$$\text{Equity-assets} = 1 / (1 + .7762) = .56$$

So, the amount of debt and equity needed will be:

$$\text{Total debt needed} = .44(\$649,500) = \$283,824$$

$$\text{Equity needed} = .56(\$649,500) = \$365,676$$

So, the repurchases of debt and equity will be:

$$\text{Debt repurchase} = (\$94,560 + 205,213) - 283,824 = \$15,949$$

$$\text{Equity repurchase} = \$386,225 - 365,676 = \$20,549$$

Assuming all of the debt repurchase is from long-term debt, and the equity repurchase is entirely from the retained earnings, the final pro forma balance sheet will be:

Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 29,136	Accounts payable	\$ 78,240
Accounts receivable	44,484	Notes payable	<u>16,320</u>
Inventory	<u>100,080</u>	Total	\$ 94,560
Total	\$ 173,700	Long-term debt	<u>189,264</u>
Fixed assets		Owners' equity	
Net plant and equipment	<u>475,800</u>	Common stock and paid-in surplus	\$ 130,000
		Retained earnings	<u>235,676</u>
		Total	<u>\$ 365,676</u>
Total assets	<u>\$ 649,500</u>	Total liabilities and owners' equity	<u>\$ 649,500</u>

Challenge

24. The pro forma income statements for all three growth rates will be:

Pro Forma Income Statement

	<i>15 % Sales Growth</i>	<i>20% Sales Growth</i>	<i>25% Sales Growth</i>
Sales	\$1,025,340	\$1,069,920	\$1,114,500
Costs	837,085	873,480	909,875
Other expenses	<u>20,976</u>	<u>21,888</u>	<u>22,800</u>
EBIT	\$167,279	\$174,552	\$181,825
Interest	<u>13,400</u>	<u>13,400</u>	<u>13,400</u>
Taxable income	\$153,879	\$161,152	\$168,425
Taxes (22%)	<u>33,853</u>	<u>35,453</u>	<u>37,054</u>
Net income	<u>\$120,026</u>	<u>\$125,699</u>	<u>\$131,372</u>
Dividends	\$42,209	\$44,204	\$46,199
Add to RE	77,817	81,495	85,173

We will calculate the EFN for the 15 percent growth rate first. Assuming the payout ratio is constant, the dividends paid will be:

$$\text{Dividends} = (\$36,224/\$103,007)(\$120,026)$$

$$\text{Dividends} = \$42,209$$

And the addition to retained earnings will be:

$$\text{Addition to retained earnings} = \$120,026 - 42,209$$

$$\text{Addition to retained earnings} = \$77,817$$

The new retained earnings on the pro forma balance sheet will be:

$$\text{New retained earnings} = \$174,730 + 77,817$$

$$\text{New retained earnings} = \$252,547$$

The pro forma balance sheet will look like this:

15% Sales Growth:

Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 27,922	Accounts payable	\$ 74,980
Accounts receivable	42,631	Notes payable	<u>16,320</u>
Inventory	<u>95,910</u>	Total	\$ 91,300
Total	\$ 166,463	Long-term debt	\$ 155,000
Fixed assets		Owners' equity	
Net plant and equipment	<u>455,975</u>	Common stock and paid-in surplus	\$ 130,000
		Retained earnings	<u>252,547</u>
		Total	<u>\$ 382,547</u>
Total assets	<u>\$ 622,438</u>	Total liabilities and owners' equity	<u>\$ 628,847</u>

So the EFN is:

$$\text{EFN} = \text{Total assets} - \text{Total liabilities and equity}$$

$$\text{EFN} = \$622,438 - 628,847$$

$$\text{EFN} = -\$6,409$$

At a 20 percent growth rate, and assuming the payout ratio is constant, the dividends paid will be:

$$\text{Dividends} = (\$36,224 / \$103,007)(\$125,699)$$

$$\text{Dividends} = \$44,204$$

And the addition to retained earnings will be:

$$\text{Addition to retained earnings} = \$125,699 - 44,204$$

$$\text{Addition to retained earnings} = \$81,495$$

The new retained earnings on the pro forma balance sheet will be:

$$\text{New retained earnings} = \$174,730 + 81,495$$

$$\text{New retained earnings} = \$256,225$$

The pro forma balance sheet will look like this:

20% Sales Growth:

Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 29,136	Accounts payable	\$ 78,240
Accounts receivable	44,484	Notes payable	<u>16,320</u>
Inventory	<u>100,080</u>	Total	\$ 94,560
Total	\$ 173,700	Long-term debt	<u>155,000</u>
Fixed assets		Owners' equity	
Net plant and equipment	<u>475,800</u>	Common stock and paid-in surplus	\$ 130,000
		Retained earnings	<u>256,225</u>
		Total	\$ <u>386,225</u>
Total assets	\$ <u>649,500</u>	Total liabilities and owners' equity	\$ <u>635,785</u>

So the EFN is:

EFN = Total assets – Total liabilities and equity

EFN = \$649,500 – 635,785

EFN = \$13,715

At a 25 percent growth rate, and assuming the payout ratio is constant, the dividends paid will be:

Dividends = $(\$36,224 / \$103,007)(\$131,372)$

Dividends = \$46,199

And the addition to retained earnings will be:

Addition to retained earnings = \$131,372 – 46,199

Addition to retained earnings = \$85,173

The new retained earnings on the pro forma balance sheet will be:

New retained earnings = \$174,730 + 85,173

New retained earnings = \$259,903

The pro forma balance sheet will look like this:

25% Sales Growth:

Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 30,350	Accounts payable	\$ 81,500
Accounts receivable	46,338	Notes payable	<u>16,320</u>
Inventory	<u>104,250</u>	Total	\$ 97,820
Total	\$ 180,938	Long-term debt	\$ 155,000
Fixed assets		Owners' equity	
Net plant and equipment	<u>495,625</u>	Common stock and paid-in surplus	\$ 130,000
		Retained earnings	<u>259,903</u>
		Total	<u>\$ 389,903</u>
Total assets	<u>\$ 676,563</u>	Total liabilities and owners' equity	<u>\$ 642,723</u>

So the EFN is:

EFN = Total assets – Total liabilities and equity

EFN = \$676,563 – 642,723

EFN = \$33,840

25. The pro forma income statements for all three growth rates will be:

Pro Forma Income Statement			
	20% Sales Growth	30% Sales Growth	35% Sales Growth
Sales	\$1,069,920	\$1,159,080	\$1,203,660
Costs	873,480	946,270	982,665
Other expenses	<u>21,888</u>	<u>23,712</u>	<u>24,624</u>
EBIT	\$174,552	\$189,098	\$196,371
Interest	<u>13,400</u>	<u>13,400</u>	<u>13,400</u>
Taxable income	\$161,152	\$175,698	\$182,971
Taxes (22%)	<u>35,453</u>	<u>38,654</u>	<u>40,254</u>
Net income	<u>\$125,699</u>	<u>\$137,044</u>	<u>\$142,717</u>
Dividends	\$44,204	\$48,194	\$50,189
Add to RE	81,495	88,851	92,529

At a 30 percent growth rate, and assuming the payout ratio is constant, the dividends paid will be:

Dividends = (\$36,224/\$103,007)(\$137,044)

Dividends = \$48,194

And the addition to retained earnings will be:

Addition to retained earnings = \$137,044 – 48,194

Addition to retained earnings = \$88,851

The new retained earnings on the pro forma balance sheet will be:

New addition to retained earnings = \$174,730 + 88,851

New addition to retained earnings = \$263,581

The new total debt will be:

New total debt = .7762(\$393,581)

New total debt = \$305,482

So, the new long-term debt will be the new total debt minus the new short-term debt, or:

New long-term debt = \$305,482 – 101,080

New long-term debt = \$204,402

The pro forma balance sheet will look like this:

Sales growth rate = 30% and debt/equity ratio = .7762:

Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 31,564	Accounts payable	\$ 84,760
Accounts receivable	48,191	Notes payable	<u>16,320</u>
Inventory	<u>108,420</u>	Total	\$ 101,080
Total	\$ 188,175	Long-term debt	<u>204,402</u>
Fixed assets		Owners' equity	
Net plant and equipment	<u>515,450</u>	Common stock and paid-in surplus	\$ 130,000
		Retained earnings	<u>263,581</u>
		Total	\$ 393,581
Total assets	<u>\$ 703,625</u>	Total liabilities and owners' equity	<u>\$ 699,063</u>

So the excess debt raised is:

Excess debt = \$699,063 – 703,625

Excess debt = -\$4,562

At a 30 percent growth rate, the firm will need funds in the amount of \$4,562 in addition to the external debt already raised. So, the EFN will be:

EFN = \$49,402 + 4,562

EFN = \$53,964

At a 35 percent growth rate, and assuming the payout ratio is constant, the dividends paid will be:

$$\text{Dividends} = (\$36,224/\$103,007)(\$142,717)$$

$$\text{Dividends} = \$50,189$$

And the addition to retained earnings will be:

$$\text{Addition to retained earnings} = \$142,717 - 50,189$$

$$\text{Addition to retained earnings} = \$92,529$$

The new retained earnings on the pro forma balance sheet will be:

$$\text{New retained earnings} = \$174,730 + 92,529$$

$$\text{New retained earnings} = \$267,259$$

The new total debt will be:

$$\text{New total debt} = .7762(\$397,259)$$

$$\text{New total debt} = \$308,337$$

So, the new long-term debt will be the new total debt minus the new short-term debt, or:

$$\text{New long-term debt} = \$308,337 - 104,340$$

$$\text{New long-term debt} = \$203,997$$

Sales growth rate = 35% and debt/equity ratio = .7762:

Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 32,778	Accounts payable	\$ 88,020
Accounts receivable	50,045	Notes payable	<u>16,320</u>
Inventory	<u>112,590</u>	Total	\$ 104,340
Total	\$ 195,413	Long-term debt	\$ 203,997
Fixed assets		Owners' equity	
Net plant and equipment	<u>535,275</u>	Common stock and paid-in surplus	\$ 130,000
		Retained earnings	<u>267,259</u>
		Total	<u>\$ 397,259</u>
Total assets	<u>\$ 730,688</u>	Total liabilities and owners' equity	<u>\$ 705,596</u>

So the excess debt raised is:

$$\text{Excess debt} = \$705,596 - 730,688$$

$$\text{Excess debt} = -\$25,092$$

At a 35 percent growth rate, the firm will need funds in the amount of \$25,092 in addition to the external debt already raised. So, the EFN will be:

$$\text{EFN} = \$48,997 + 25,092$$

$$\text{EFN} = \$74,089$$

26. We need the ROE to calculate the sustainable growth rate. The ROE is:

$$\text{ROE} = (\text{Profit margin})(\text{Total asset turnover})(\text{Equity multiplier})$$

$$\text{ROE} = (.042)(1/.80)(1 + .37)$$

$$\text{ROE} = .0719, \text{ or } 7.19\%$$

Now, we can use the sustainable growth rate equation to find the retention ratio as:

$$\text{Sustainable growth rate} = (\text{ROE} \times b) / [1 - (\text{ROE} \times b)]$$

$$\text{Sustainable growth rate} = .11 = [.0719b] / [1 - .0719b]$$

$$b = 1.38$$

This implies the payout ratio is:

$$\text{Payout ratio} = 1 - b$$

$$\text{Payout ratio} = 1 - 1.38$$

$$\text{Payout ratio} = -.38, \text{ or } -38\%$$

This answer indicates a dividend payout ratio of negative 38 percent, which is impossible. So, the growth rate is inconsistent with the other constraints. The lowest possible payout rate is 0 (without

issuing stock), which corresponds to a retention ratio of 1, or total earnings retention. This problem illustrates a key point we made in the chapter: Sustainable growth analysis forces the user to make internally consistent assumptions.

As an aside, we should note that it is possible to have a retention ratio greater than 1 if the company issues new stock. However, since the growth rate we are evaluating is perpetual, the company would have to issue stock every year, forever. But, doing so violates our underlying assumption that the sustainable growth rate requires no new equity.

In this case, the maximum sustainable growth rate for this company is:

$$\text{Maximum sustainable growth rate} = (\text{ROE} \times b) / [1 - (\text{ROE} \times b)]$$

$$\text{Maximum sustainable growth rate} = [.0719(1)] / [1 - .0719(1)]$$

$$\text{Maximum sustainable growth rate} = .0775, \text{ or } 7.75\%$$

27. We know that EFN is:

$$\text{EFN} = \text{Increase in assets} - \text{Addition to retained earnings}$$

The increase in assets is the beginning assets times the growth rate, so:

$$\text{Increase in assets} = A \times g$$

The addition to retained earnings next year is the current net income times the retention ratio, times one plus the growth rate, so:

$$\text{Addition to retained earnings} = (\text{NI} \times b)(1 + g)$$

And rearranging the profit margin to solve for net income, we get:

$$\text{NI} = \text{PM}(S)$$

Substituting the last three equations into the EFN equation we started with and rearranging, we get:

$$\text{EFN} = A(g) - \text{PM}(S)b(1 + g)$$

$$\text{EFN} = A(g) - \text{PM}(S)b - [\text{PM}(S)b]g$$

$$\text{EFN} = -\text{PM}(S)b + [A - \text{PM}(S)b]g$$

28. We start with the EFN equation we derived in Problem 27 and set it equal to zero:

$$\text{EFN} = 0 = -\text{PM}(S)b + [A - \text{PM}(S)b]g$$

Substituting the rearranged profit margin equation into the internal growth rate equation, we have:

$$\text{Internal growth rate} = [\text{PM}(S)b] / [A - \text{PM}(S)b]$$

Since:

$$\text{ROA} = \text{NI}/A$$

$$\text{ROA} = \text{PM}(S)/A$$

We can substitute this into the internal growth rate equation and divide both the numerator and denominator by A. This gives:

$$\text{Internal growth rate} = \{[PM(S)b]/A\} / \{[A - PM(S)b]/A\}$$

$$\text{Internal growth rate} = b(ROA) / [1 - b(ROA)]$$

To derive the sustainable growth rate, we must realize that to maintain a constant D/E ratio with no external equity financing, EFN must equal the addition to retained earnings times the D/E ratio:

$$EFN = (D/E)[PM(S)b(1 + g)]$$

$$EFN = A(g) - PM(S)b(1 + g)$$

Solving for g and then dividing both the numerator and denominator by A:

$$\text{Sustainable growth rate} = PM(S)b(1 + D/E) / [A - PM(S)b(1 + D/E)]$$

$$\text{Sustainable growth rate} = [ROA(1 + D/E)b] / [1 - ROA(1 + D/E)b]$$

$$\text{Sustainable growth rate} = b(ROE) / [1 - b(ROE)]$$

29. In the following derivations, the subscript “E” refers to end of period numbers, and the subscript “B” refers to beginning of period numbers. TE is total equity and TA is total assets.

For the sustainable growth rate:

$$\text{Sustainable growth rate} = (ROE_E \times b) / (1 - ROE_E \times b)$$

$$\text{Sustainable growth rate} = (NI/TE_E \times b) / (1 - NI/TE_E \times b)$$

We multiply this equation by:

$$(TE_E/TE_E)$$

$$\text{Sustainable growth rate} = (NI/TE_E \times b) / (1 - NI/TE_E \times b) \times (TE_E/TE_E)$$

$$\text{Sustainable growth rate} = (NI \times b) / (TE_E - NI \times b)$$

Recognize that the denominator is equal to beginning of period equity, that is:

$$(TE_E - NI \times b) = TE_B$$

Substituting this into the previous equation, we get:

$$\text{Sustainable rate} = (NI \times b) / TE_B$$

Which is equivalent to:

$$\text{Sustainable rate} = (NI/TE_B) \times b$$

$$\text{Since } ROE_B = NI/TE_B$$

The sustainable growth rate equation is:

$$\text{Sustainable growth rate} = ROE_B \times b$$

For the internal growth rate:

$$\text{Internal growth rate} = (\text{ROA}_E \times b) / (1 - \text{ROA}_E \times b)$$

$$\text{Internal growth rate} = (\text{NI}/\text{TA}_E \times b) / (1 - \text{NI}/\text{TA}_E \times b)$$

We multiply this equation by:

$$(\text{TA}_E/\text{TA}_E)$$

$$\text{Internal growth rate} = (\text{NI}/\text{TA}_E \times b) / [(1 - \text{NI}/\text{TA}_E \times b) \times (\text{TA}_E/\text{TA}_E)]$$

$$\text{Internal growth rate} = (\text{NI} \times b) / (\text{TA}_E - \text{NI} \times b)$$

Recognize that the denominator is equal to beginning of period assets, that is:

$$(\text{TA}_E - \text{NI} \times b) = \text{TA}_B$$

Substituting this into the previous equation, we get:

$$\text{Internal growth rate} = (\text{NI} \times b) / \text{TA}_B$$

Which is equivalent to:

$$\text{Internal growth rate} = (\text{NI}/\text{TA}_B) \times b$$

$$\text{Since } \text{ROA}_B = \text{NI}/\text{TA}_B$$

The internal growth rate equation is:

$$\text{Internal growth rate} = \text{ROA}_B \times b$$

- 30.** Since the company issued no new equity, shareholders' equity increased by retained earnings. Retained earnings for the year were:

$$\text{Retained earnings} = \text{NI} - \text{Dividends}$$

$$\text{Retained earnings} = \$80,000 - 44,000$$

$$\text{Retained earnings} = \$36,000$$

So, the equity at the end of the year was:

$$\text{Ending equity} = \$260,000 + 36,000$$

$$\text{Ending equity} = \$296,000$$

The ROE based on the end of period equity is:

$$\text{ROE} = \$80,000 / \$296,000$$

$$\text{ROE} = .2703, \text{ or } 27.03\%$$

The plowback ratio is:

$$\text{Plowback ratio} = \text{Addition to retained earnings} / \text{NI}$$

$$\text{Plowback ratio} = \$36,000 / \$80,000$$

Plowback ratio = .45, or 45%

Using the equation presented in the text for the sustainable growth rate, we get:

$$\begin{aligned}\text{Sustainable growth rate} &= (\text{ROE} \times b) / [1 - (\text{ROE} \times b)] \\ \text{Sustainable growth rate} &= [.2703(.45)] / [1 - .2703(.45)] \\ \text{Sustainable growth rate} &= .1385, \text{ or } 13.85\%\end{aligned}$$

The ROE based on the beginning of period equity is

$$\begin{aligned}\text{ROE} &= \$80,000 / \$260,000 \\ \text{ROE} &= .3077, \text{ or } 30.77\%\end{aligned}$$

Using the shortened equation for the sustainable growth rate and the beginning of period ROE, we get:

$$\begin{aligned}\text{Sustainable growth rate} &= \text{ROE} \times b \\ \text{Sustainable growth rate} &= .3077 \times .45 \\ \text{Sustainable growth rate} &= .1385, \text{ or } 13.85\%\end{aligned}$$

Using the shortened equation for the sustainable growth rate and the end of period ROE, we get:

$$\begin{aligned}\text{Sustainable growth rate} &= \text{ROE} \times b \\ \text{Sustainable growth rate} &= .2703 \times .45 \\ \text{Sustainable growth rate} &= .1216, \text{ or } 12.16\%\end{aligned}$$

Using the end of period ROE in the shortened sustainable growth rate results in a growth rate that is too low. This will always occur whenever the equity increases. If equity increases, the ROE based on end of period equity is lower than the ROE based on the beginning of period equity. The ROE (and sustainable growth rate) in the abbreviated equation is based on equity that did not exist when the net income was earned.

CHAPTER 4

DISCOUNTED CASH FLOW VALUATION

Answers to Concepts Review and Critical Thinking Questions

1. Assuming positive cash flows and interest rates, the future value increases and the present value decreases.
2. Assuming positive cash flows and interest rates, the present value will fall and the future value will rise.
3. The better deal is the one with equal installments.
4. Yes, they should. APRs generally don't provide the relevant rate. The only advantage is that they are easier to compute, but, with modern computing equipment, that advantage is not very important.
5. A freshman does. The reason is that the freshman gets to use the money for much longer before interest starts to accrue.
6. It's a reflection of the time value of money. TMCC gets to use the \$24,099 immediately. If TMCC uses it wisely, it will be worth more than \$100,000 in thirty years.
7. This will probably make the security less desirable. TMCC will only repurchase the security prior to maturity if it is to its advantage, i.e. interest rates decline. Given the drop in interest rates needed to make this viable for TMCC, it is unlikely the company will repurchase the security. This is an example of a "call" feature. Such features are discussed at length in a later chapter.
8. The key considerations would be: (1) Is the rate of return implicit in the offer attractive relative to other, similar risk investments? and (2) How risky is the investment; i.e., how certain are we that we will actually get the \$100,000? Thus, our answer does depend on who is making the promise to repay.
9. The Treasury security would have a somewhat higher price because the Treasury is the strongest of all borrowers.
10. The price would be higher because, as time passes, the price of the security will tend to rise toward \$100,000. This rise is a reflection of the time value of money. As time passes, the time until receipt of the \$100,000 grows shorter, and the present value rises. In 2019, the price will probably be higher for the same reason. We cannot be sure, however, because interest rates could be much higher, or TMCC's financial position could deteriorate. Either event would tend to depress the security's price.

Solutions to Questions and Problems

NOTE: All-end-of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The time line for the cash flows is:



The simple interest per year is:

$$\$7,800 \times .077 = \$600.60$$

So, after 10 years, you will have:

$$\$600.60 \times 10 = \$6,006 \text{ in interest.}$$

The total balance will be $\$7,800 + 6,006 = \$13,806$

With compound interest, we use the future value formula:

$$FV = PV(1 + r)^t$$

$$FV = \$7,800(1.077)^{10} = \$16,377.65$$

The difference is:

$$\$16,377.65 - 13,806 = \$2,571.65$$

2. To find the FV of a lump sum, we use:

$$FV = PV(1 + r)^t$$



$$FV = \$1,250(1.05)^{10} = \$2,036.12$$



$$FV = \$1,250(1.10)^{10} = \$3,242.18$$

c.



$$FV = \$1,250(1.05)^{20} = \$3,316.62$$

- d. Because interest compounds on the interest already earned, the interest earned in part c is more than twice the interest earned in part a. With compound interest, future values grow exponentially.

3. To find the PV of a lump sum, we use:

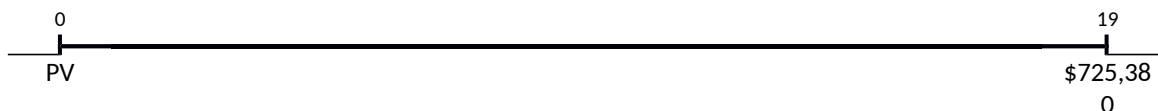
$$PV = FV/(1 + r)^t$$



$$PV = \$13,827/(1.07)^6 = \$9,213.51$$



$$PV = \$43,852/(1.15)^{11} = \$9,425.69$$



$$PV = \$725,380/(1.11)^{19} = \$99,868.60$$



$$PV = \$590,710/(1.18)^{29} = \$4,861.79$$

4. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$$FV = PV(1 + r)^t$$

Solving for r , we get:

$$r = (FV/PV)^{1/t} - 1$$



$$FV = \$287 = \$189(1 + r)^4; \quad r = (\$287/\$189)^{1/4} - 1 = .1101, \text{ or } 11.01\%$$



$$FV = \$887 = \$410(1 + r)^8; \quad r = (\$887/\$410)^{1/8} - 1 = .1013, \text{ or } 10.13\%$$



$$FV = \$152,184 = \$51,700(1 + r)^{14}; \quad r = (\$152,184/\$51,700)^{1/14} - 1 = .0802, \text{ or } 8.02\%$$



$$FV = \$538,600 = \$21,400(1 + r)^{27}; \quad r = (\$538,600/\$21,400)^{1/27} - 1 = .1269, \text{ or } 12.69\%$$

5. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$$FV = PV(1 + r)^t$$

Solving for t , we get:

$$t = \ln(FV/PV)/\ln(1 + r)$$



$$FV = \$1,104 = \$625(1.07)^t; \quad t = \ln(\$1,104/\$625)/\ln 1.07 = 8.41 \text{ years}$$



$$FV = \$5,275 = \$810(1.112)^t; \quad t = \ln(\$5,275/\$810)/\ln 1.12 = 16.53 \text{ years}$$



$$FV = \$245,830 = \$16,500(1.17)^t; \quad t = \ln(\$245,830/\$16,500)/\ln 1.17 = 17.21 \text{ years}$$



$$FV = \$215,000 = \$21,500(1.08)^t; \quad t = \ln(\$215,000/\$21,500)/\ln 1.08 = 29.92 \text{ years}$$

6. To find the length of time for money to double, triple, etc., the present value and future value are irrelevant as long as the future value is twice the present value for doubling, three times as large for tripling, etc. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$$FV = PV(1 + r)^t$$

Solving for t , we get:

$$t = \ln(FV/PV)/\ln(1 + r)$$

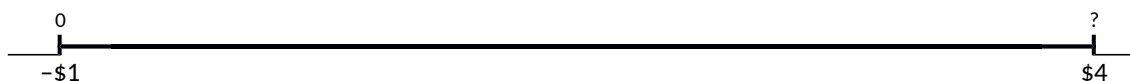
The length of time to double your money is:



$$FV = \$2 = \$1(1.0625)^t$$

$$t = \ln 2/\ln 1.0625 = 11.43 \text{ years}$$

The length of time to quadruple your money is:



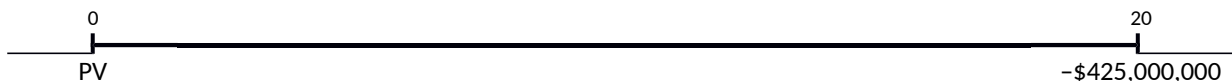
$$FV = \$4 = \$1(1.0625)^t$$

$$t = \ln 4/\ln 1.0625$$

$$t = 22.87 \text{ years}$$

Notice that the length of time to quadruple your money is twice as long as the time needed to double your money (the difference in these answers is due to rounding). This is an important concept of time value of money.

7. The time line is:



To find the PV of a lump sum, we use:

$$PV = FV/(1 + r)^t$$

$$PV = \$425,000,000/(1.059)^{20}$$

$$PV = \$135,042,269.46$$

8. The time line is:



To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$$FV = PV(1 + r)^t$$

Solving for r , we get:

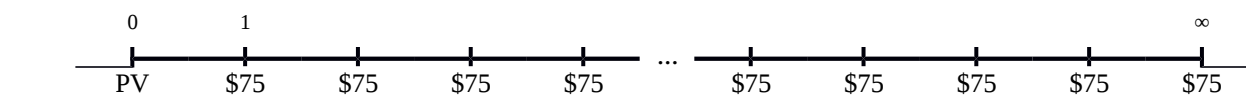
$$r = (FV/PV)^{1/t} - 1$$

$$r = (\$1,100,000/\$1,680,000)^{1/3} - 1$$

$$r = -.1317, \text{ or } -13.17\%$$

Notice that the interest rate is negative. This occurs when the FV is less than the PV.

9. The time line is:



A consol is a perpetuity. To find the PV of a perpetuity, we use the equation:

$$PV = C/r$$

$$PV = \$75/.031$$

$$PV = \$2,419.35$$

10. To find the future value with continuous compounding, we use the equation:

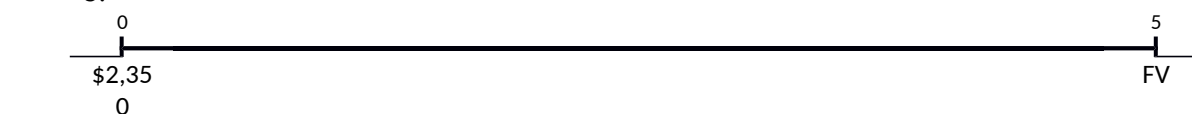
$$FV = PVe^{rt}$$

a.



$$FV = \$2,350e^{.12(9)} = \$6,516.74$$

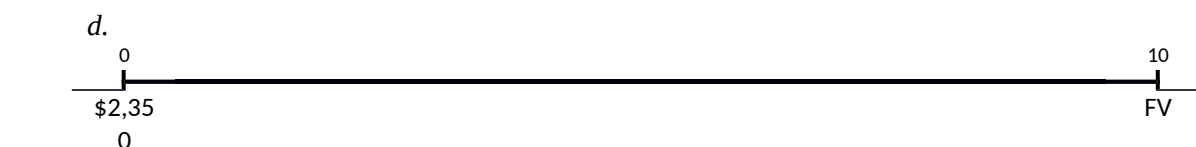
b.



$$FV = \$2,350e^{.08(5)} = \$3,452.92$$

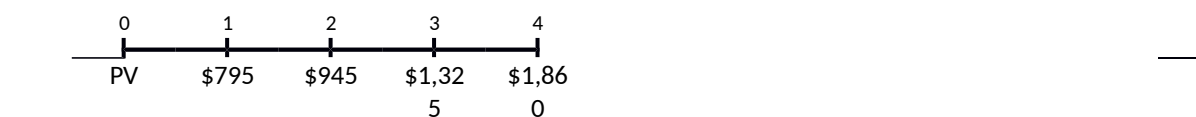


$$FV = \$2,350e^{0.05(17)} = \$5,386.24$$



$$FV = \$2,350e^{0.09(10)} = \$5,563.30$$

11. The time line is:



To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:

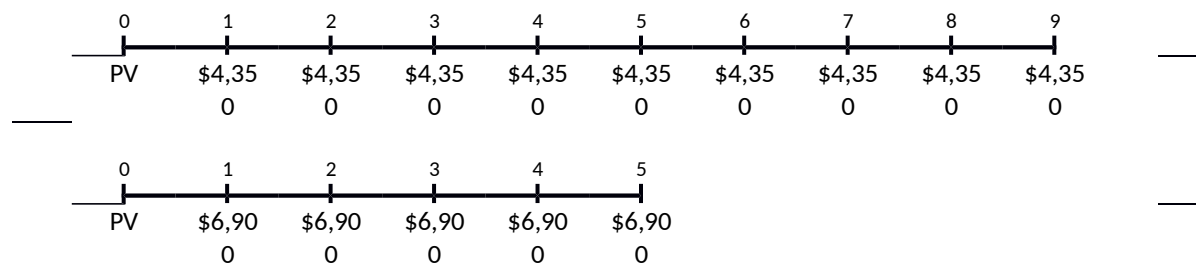
$$PV = FV/(1 + r)^t$$

$$PV@10\% = \$795/1.10 + \$945/1.10^2 + \$1,325/1.10^3 + \$1,860/1.10^4 = \$3,769.62$$

$$PV@18\% = \$795/1.18 + \$945/1.18^2 + \$1,325/1.18^3 + \$1,860/1.18^4 = \$3,118.22$$

$$PV@24\% = \$795/1.24 + \$945/1.24^2 + \$1,325/1.24^3 + \$1,860/1.24^4 = \$2,737.40$$

12. The times lines are:



To find the PVA, we use the equation:

$$PVA = C\{[1 - 1/(1 + r)^t]/r\}$$

At an interest rate of 5 percent:

$$X@5\%: PVA = \$4,350\{[1 - (1/1.05)^9]/.05\} = \$30,919.02$$

$$Y@5\%: PVA = \$6,900\{[1 - (1/1.05)^5]/.05\} = \$29,873.39$$

And at an interest rate of 22 percent:

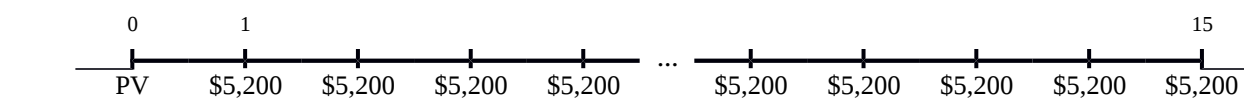
$$X@22\%: PVA = \$4,350\{[1 - (1/1.22)^9]/.22\} = \$16,470.34$$

$$Y@22\%: PVA = \$6,900\{[1 - (1/1.22)^5]/.22\} = \$19,759.11$$

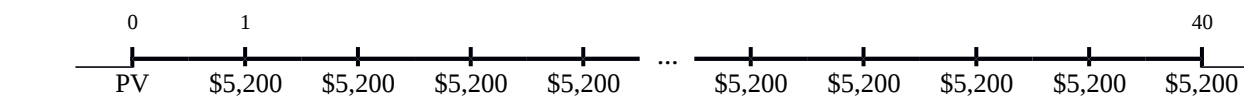
Notice that the PV of Cash flow X has a greater PV than Cash flow Y at an interest rate of 5 percent, but a lower PV at an interest rate of 22 percent. The reason is that X has greater total cash flows. At a lower interest rate, the total cash flow is more important since the cost of waiting (the interest rate) is not as great. At a higher interest rate, Y is more valuable since it has larger cash flows. At a higher interest rate, these bigger cash flows earlier are more important since the cost of waiting (the interest rate) is so much greater.

13. To find the PVA, we use the equation:

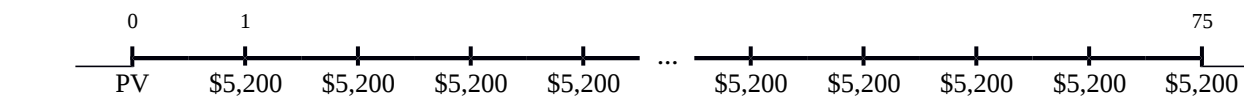
$$PVA = C\{[1 - 1/(1 + r)^t]/r\}$$



$$PVA@15 \text{ years:} \quad PVA = \$5,200\{[1 - (1/1.07)^{15}]/.07\} = \$47,361.15$$



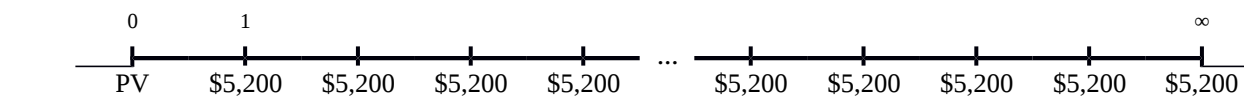
$$PVA@40 \text{ years:} \quad PVA = \$5,200\{[1 - (1/1.07)^{40}]/.07\} = \$69,324.89$$



$$PVA@75 \text{ years:} \quad PVA = \$5,200\{[1 - (1/1.07)^{75}]/.07\} = \$73,821.07$$

To find the PV of a perpetuity, we use the equation:

$$PV = C/r$$

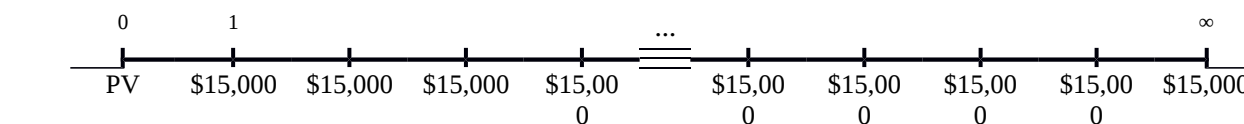


$$PV = \$5,200/.07$$

$$PV = \$74,285.71$$

Notice that as the length of the annuity payments increases, the present value of the annuity approaches the present value of the perpetuity. The present value of the 75-year annuity and the present value of the perpetuity imply that the value today of all perpetuity payments beyond 75 years is only \$464.65.

14. The time line is:



This cash flow is a perpetuity. To find the PV of a perpetuity, we use the equation:

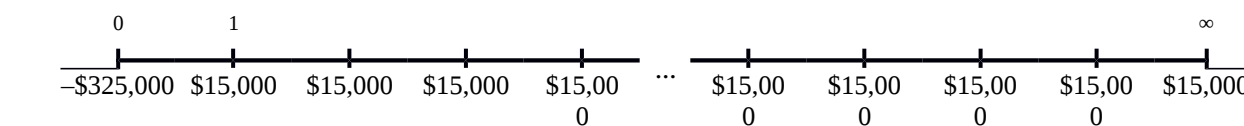
$$PV = C/r$$

$$PV = \$15,000/.038$$

$$PV = \$394,736.84$$

To find the interest rate that equates the perpetuity cash flows with the PV of the cash flows, we can use the PV of a perpetuity equation:

$$PV = C/r$$



$$\$325,000 = \$15,000/r$$

We can now solve for the interest rate as follows:

$$r = \$15,000/\$325,000$$

$$r = .0462, \text{ or } 4.62\%$$

15. For discrete compounding, to find the EAR, we use the equation:

$$EAR = [1 + (APR/m)]^m - 1$$

$$EAR = [1 + (.071/4)]^4 - 1 = .0729, \text{ or } 7.29\%$$

$$EAR = [1 + (.132/12)]^{12} - 1 = .1403, \text{ or } 14.03\%$$

$$EAR = [1 + (.089/365)]^{365} - 1 = .0931, \text{ or } 9.31\%$$

To find the EAR with continuous compounding, we use the equation:

$$EAR = e^r - 1$$

$$EAR = e^{.081} - 1 = .0844, \text{ or } 8.44\%$$

16. Here, we are given the EAR and need to find the APR. Using the equation for discrete compounding:

$$EAR = [1 + (APR/m)]^m - 1$$

We can now solve for the APR. Doing so, we get:

$$APR = m[(1 + EAR)^{1/m} - 1]$$

$$\text{EAR} = .101 = [1 + (\text{APR}/2)]^2 - 1 \quad \text{APR} = 2[(1.101)^{1/2} - 1] = .0986, \text{ or } 9.86\%$$

$$\text{EAR} = .174 = [1 + (\text{APR}/12)]^{12} - 1 \quad \text{APR} = 12[(1.174)^{1/12} - 1] = .1615, \text{ or } 16.15\%$$

$$\text{EAR} = .086 = [1 + (\text{APR}/52)]^{52} - 1 \quad \text{APR} = 52[(1.086)^{1/52} - 1] = .0826, \text{ or } 8.26\%$$

Solving the continuous compounding EAR equation:

$$\text{EAR} = e^r - 1$$

We get:

$$\text{APR} = \ln(1 + \text{EAR})$$

$$\text{APR} = \ln(1 + .113)$$

$$\text{APR} = .1071, \text{ or } 10.71\%$$

17. For discrete compounding, to find the EAR, we use the equation:

$$\text{EAR} = [1 + (\text{APR}/m)]^m - 1$$

So, for each bank, the EAR is:

$$\text{First National: } \text{EAR} = [1 + (.114/12)]^{12} - 1 = .1201, \text{ or } 12.01\%$$

$$\text{First United: } \text{EAR} = [1 + (.116/2)]^2 - 1 = .1194, \text{ or } 11.94\%$$

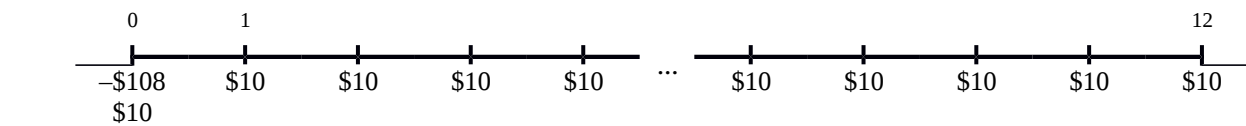
A higher APR does not necessarily mean a higher EAR. The number of compounding periods within a year will also affect the EAR.

18. The cost of a case of wine is 10 percent less than the cost of 12 individual bottles, so the cost of a case will be:

$$\text{Cost of case} = (12)(\$10)(1 - .10)$$

$$\text{Cost of case} = \$108$$

Now, we need to find the interest rate. The cash flows are an annuity due, so:



$$\text{PVA} = (1 + r)C(\{1 - [1/(1 + r)]^t\}/r)$$

$$\$108 = (1 + r)\$10(\{1 - [1/(1 + r)]^{12}\}/r)$$

Solving for the interest rate, we get:

$$r = .0198, \text{ or } 1.98\% \text{ per week}$$

So, the APR of this investment is:

$$\text{APR} = .0198(52)$$

$$\text{APR} = 1.0277, \text{ or } 102.77\%$$

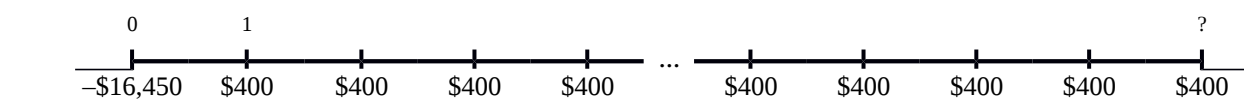
And the EAR is:

$$\text{EAR} = (1 + .0198)^{52} - 1$$

$$\text{EAR} = 1.7668, \text{ or } 176.68\%$$

The analysis appears to be correct. He really can earn about 177 percent buying wine by the case. The only question left is this: Can you really find a fine bottle of Bordeaux for \$10?

19. The time line is:



Here, we need to find the length of an annuity. We know the interest rate, the PV, and the payments. Using the PVA equation:

$$\text{PVA} = C(\{1 - [1/(1 + r)]^t\}/r)$$

$$\$16,450 = \$400\{[1 - (1/1.011)^t]/.011\}$$

Now, we solve for t :

$$1/1.011^t = 1 - [(\$16,450)(.011)/(\$400)]$$

$$1.011^t = 1/.5476 = 1.8261$$

$$t = \ln 1.8261 / \ln 1.011$$

$$t = 55.04 \text{ months}$$

20. The time line is:



Here, we are trying to find the interest rate when we know the PV and FV. Using the FV equation:

$$\text{FV} = \text{PV}(1 + r)$$

$$\$4 = \$3(1 + r)$$

$$r = 4/3 - 1 = 33.33\% \text{ per week}$$

The interest rate is 33.33% per week. To find the APR, we multiply this rate by the number of weeks in a year, so:

$$\text{APR} = (52)33.33\%$$

$$\text{APR} = 1,733.33\%$$

And using the equation to find the EAR:

$$\text{EAR} = [1 + (\text{APR}/m)]^m - 1$$

$$\text{EAR} = [1 + .3333]^{52} - 1$$

$$\text{EAR} = 313,916,515.69\%$$

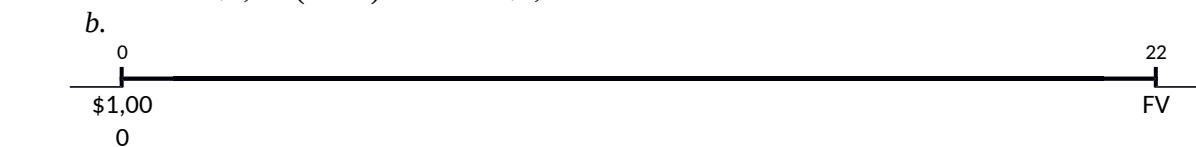
Intermediate

21. To find the FV of a lump sum with discrete compounding, we use:

$$\text{FV} = \text{PV}(1 + r)^t$$



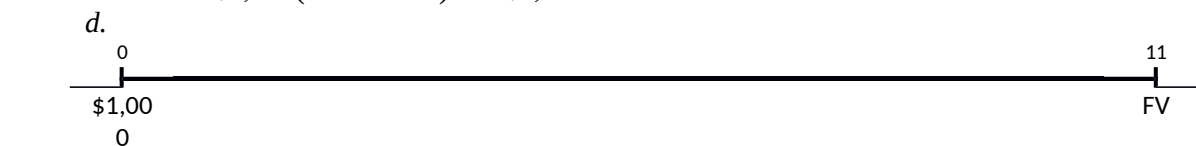
$$\text{FV} = \$1,000(1.089)^{11} = \$2,554.50$$



$$\text{FV} = \$1,000(1 + .089/2)^{22} = \$2,606.07$$



$$\text{FV} = \$1,000(1 + .089/12)^{132} = \$2,652.19$$



To find the future value with continuous compounding, we use the equation:

$$\text{FV} = \text{PV}e^{rt}$$

$$\text{FV} = \$1,000e^{.089(11)} = \$2,661.79$$

- e. The future value increases when the compounding period is shorter because interest is earned on previously accrued interest. The shorter the compounding period, the more frequently interest is earned, and the greater the future value, assuming the same stated interest rate.
22. The total interest paid by First Simple Bank is the interest rate per period times the number of periods. In other words, the interest paid by First Simple Bank over 10 years will be:
- $$.053(10) = .53$$

First Complex Bank pays compound interest, so the interest paid by this bank will be the FV factor of \$1, or:

$$(1 + r)^{10}$$

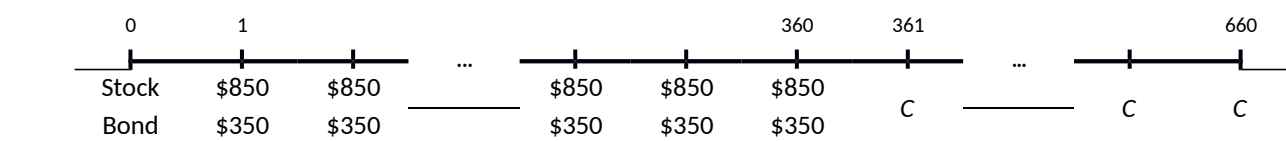
Setting the two equal, we get:

$$(.053)(10) = (1 + r)^{10} - 1$$

$$r = 1.53^{1/10} - 1$$

$$r = .0434, \text{ or } 4.34\%$$

23. Although the stock and bond accounts have different interest rates, we can draw one time line, but we need to remember to apply different interest rates. The time line is:



We need to find the annuity payment in retirement. Our retirement savings end at the same time the retirement withdrawals begin, so the PV of the retirement withdrawals will be the FV of the retirement savings. So, we find the FV of the stock account and the FV of the bond account and add the two FVs.

$$\text{Stock account: FVA} = \$850 \left[\frac{[1 + (.10/12)]^{360} - 1}{(.10/12)} \right] = \$1,921,414.74$$

$$\text{Bond account: FVA} = \$350 \left[\frac{[1 + (.06/12)]^{360} - 1}{(.06/12)} \right] = \$351,580.26$$

So, the total amount saved at retirement is:

$$\$1,921,414.74 + 351,580.26 = \$2,272,995.00$$

Solving for the withdrawal amount in retirement using the PVA equation gives us:

$$\text{PVA} = \$2,272,995 = C \left[1 - \frac{1}{[1 + (.07/12)]^{300}} \right] / (.07/12)$$

$$C = \$2,272,995 / 141.4869$$

$$C = \$16,065.06 \text{ withdrawal per month}$$

24. The time line is:



Since we are looking to quadruple our money, the PV and FV are irrelevant as long as the FV is four times as large as the PV. The number of periods is four, the number of quarters per year. So:

$$\text{FV} = \$4 = \$1(1 + r)^{(12/3)}$$

$$r = .4142, \text{ or } 41.42\%$$

25. Here, we need to find the interest rate for two possible investments. Each investment is a lump sum, so:

G:



$$\begin{aligned} PV &= \$65,000 = \$125,000/(1 + r)^6 \\ (1 + r)^6 &= \$125,000/\$65,000 \\ r &= 1.923^{1/6} - 1 \\ r &= .1151, \text{ or } 11.51\% \end{aligned}$$

H:



$$\begin{aligned} PV &= \$65,000 = \$205,000/(1 + r)^{10} \\ (1 + r)^{10} &= \$205,000/\$65,000 \\ r &= 3.154^{1/10} - 1 \\ r &= .1217, \text{ or } 12.17\% \end{aligned}$$

26. This is a growing perpetuity. The present value of a growing perpetuity is:

$$\begin{aligned} PV &= C/(r - g) \\ PV &= \$175,000/ (.097 - .038) \\ PV &= \$2,966,101.69 \end{aligned}$$

It is important to recognize that when dealing with annuities or perpetuities, the present value equation calculates the present value one period before the first payment. In this case, since the first payment is in two years, we have calculated the present value one year from now. To find the value today, we discount this value as a lump sum. Doing so, we find the value of the cash flow stream today is:

$$\begin{aligned} PV &= FV/(1 + r)^t \\ PV &= \$2,966,101.69/(1 + .097)^1 \\ PV &= \$2,703,830.17 \end{aligned}$$

27. The dividend payments are made quarterly, so we must use the quarterly interest rate. The quarterly interest rate is:

$$\begin{aligned} \text{Quarterly rate} &= \text{Stated rate}/4 \\ \text{Quarterly rate} &= .038/4 \\ \text{Quarterly rate} &= .0095 \end{aligned}$$

The time line is:



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<u> </u>	PV	\$2.25	\$2.25	\$2.25	\$2.25	<u> </u>	\$2.25	\$2.25	\$2.25	\$2.25	\$2.25
<u> </u>						<u> </u>					

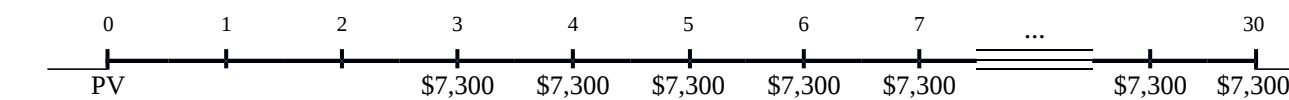
Using the present value equation for a perpetuity, we find the value today of the dividends paid must be:

$$PV = C/r$$

$$PV = \$2.25/.0095$$

$$PV = \$236.84$$

28. The time line is:



We can use the PVA annuity equation to answer this question. The annuity has 28 payments, not 27 payments. Since there is a payment made in Year 3, the annuity actually begins in Year 2. So, the value of the annuity in Year 2 is:

$$PVA = C(\{1 - [1/(1 + r)]^t\}/r)$$

$$PVA = \$7,300(\{1 - [1/(1 + .07)]^{28}\}/.07)$$

$$PVA = \$88,600.91$$

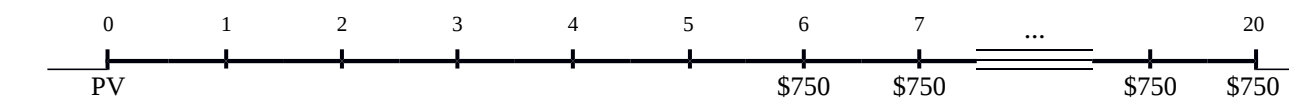
This is the value of the annuity one period before the first payment, or Year 2. So, the value of the cash flows today is:

$$PV = FV/(1 + r)^t$$

$$PV = \$88,600.91/(1 + .07)^2$$

$$PV = \$77,387.47$$

29. The time line is:



We need to find the present value of an annuity. Using the PVA equation, and the 11 percent interest rate, we get:

$$PVA = C(\{1 - [1/(1 + r)]^t\}/r)$$

$$PVA = \$750(\{1 - [1/(1 + .11)]^{15}\}/.11)$$

$$PVA = \$5,393.15$$

This is the value of the annuity in Year 5, one period before the first payment. Finding the value of this amount today, we find:

$$PV = FV/(1 + r)^t$$

$$PV = \$5,393.15/(1 + .08)^5$$

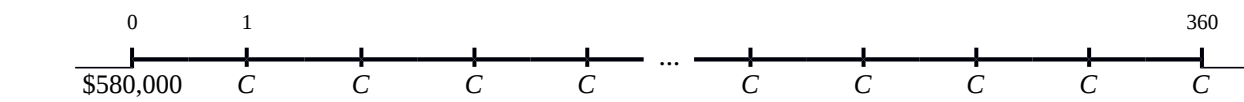
$$PV = \$3,670.49$$

30. The amount borrowed is the value of the home times one minus the down payment, or:

$$\text{Amount borrowed} = \$725,000(1 - .20)$$

$$\text{Amount borrowed} = \$580,000$$

The time line is:

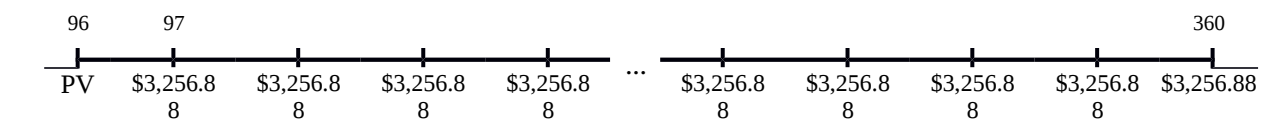


The monthly payments with a balloon payment loan are calculated assuming a longer amortization schedule, in this case, 30 years. The payments based on a 30-year repayment schedule would be:

$$\text{PVA} = \$580,000 = C(\{1 - [1/(1 + .054/12)]^{360}\}/(.054/12))$$

$$C = \$3,256.88$$

Now, at Year 8 (Month 96), we need to find the PV of the payments which have not been made. The time line is:



The balloon payment will be:

$$\text{PVA} = \$3,256.88(\{1 - [1/(1 + .054/12)]^{22(12)}\}/(.054/12))$$

$$\text{PVA} = \$502,540.87$$

31. The time line is:



Here, we need to find the FV of a lump sum, with a changing interest rate. We must do this problem in two parts. After the first six months, the balance will be:

$$\text{FV} = \$12,400[1 + (.0199/12)]^6 = \$12,523.89$$

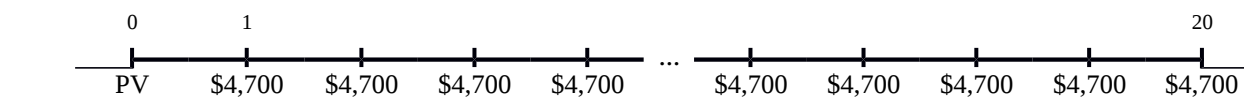
This is the balance in six months. The FV in another six months will be:

$$\text{FV} = \$12,523.89[1 + (.18/12)]^6 = \$13,694.17$$

The problem asks for the interest accrued, so, to find the interest, we subtract the beginning balance from the FV. The interest accrued is:

$$\text{Interest} = \$13,694.17 - 12,400 = \$1,294.17$$

35. The time line is:



The relationship between the PVA and the interest rate is:

PVA falls as r increases, and PVA rises as r decreases.

FVA rises as r increases, and FVA falls as r decreases.

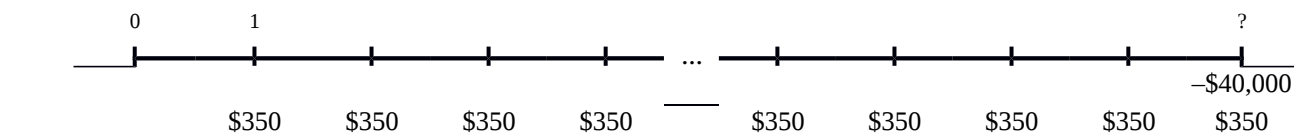
The present values of \$4,700 per year for 20 years at the various interest rates given are:

$$\text{PVA}@10\% = \$4,700\{[1 - (1/1.10)^{20}]/.10\} = \$40,013.75$$

$$\text{PVA}@5\% = \$4,700\{[1 - (1/1.05)^{20}]/.05\} = \$58,572.39$$

$$\text{PVA}@15\% = \$4,700\{[1 - (1/1.15)^{20}]/.15\} = \$29,418.86$$

36. The time line is:



Here, we are given the FVA, the interest rate, and the amount of the annuity. We need to solve for the number of payments. Using the FVA equation:

$$\text{FVA} = \$40,000 = \$350\{[1 + (.10/12)]^t - 1\}/(.10/12)$$

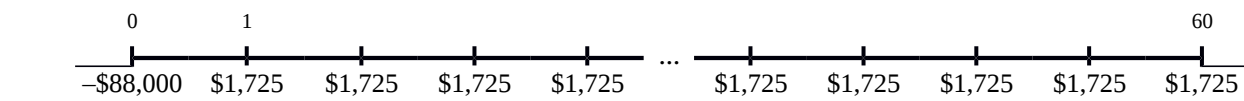
Solving for t , we get:

$$1.00833^t = 1 + [(\$40,000)(.10/12)/\$350]$$

$$t = \ln 1.95238 / \ln 1.00833$$

$$t = 80.62 \text{ payments}$$

37. The time line is:



Here, we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. Using the PVA equation:

$$\text{PVA} = \$88,000 = \$1,725\{[1 - 1/(1 + r)^{60}]/r\}$$

To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate lowers the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

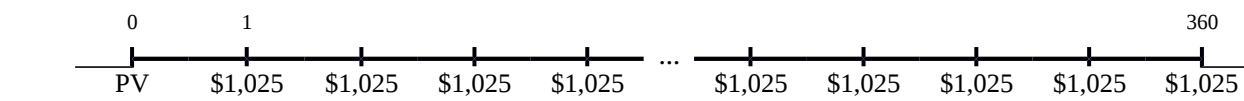
$$r = .548\%$$

The APR is the periodic interest rate times the number of periods in the year, so:

$$\text{APR} = 12(.548\%)$$

$$\text{APR} = 6.58\%$$

38. The time line is:

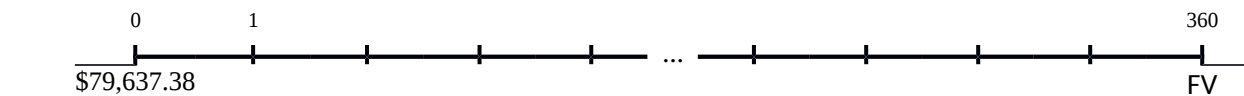


The amount of principal paid on the loan is the PV of the monthly payments you make. So, the present value of the \$1,025 monthly payments is:

$$\text{PVA} = \$1,025[(1 - \{1/[1 + (.048/12)]\}^{360})/(.048/12)] = \$195,362.62$$

The monthly payments of \$1,025 will amount to a principal payment of \$195,362.62. The amount of principal you will still owe is:

$$\$275,000 - \$195,362.62 = \$79,637.38$$

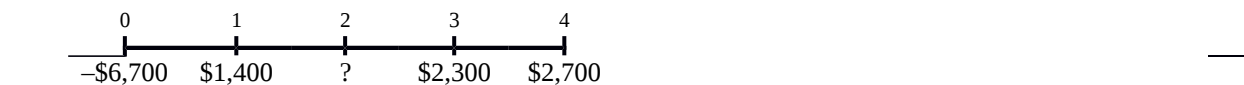


This remaining principal amount will increase at the interest rate on the loan until the end of the loan period. So the balloon payment in 30 years, which is the FV of the remaining principal, will be:

$$\text{Balloon payment} = \$79,637.38[1 + (.048/12)]^{360}$$

$$\text{Balloon payment} = \$335,161.06$$

39. The time line is:



We are given the total PV of all four cash flows. If we find the PV of the three cash flows we know, and subtract them from the total PV, the amount left over must be the PV of the missing cash flow. So, the PV of the cash flows we know is:

$$\text{PV of Year 1 CF: } \$1,400/1.071 = \$1,307.19$$

$$\text{PV of Year 3 CF: } \$2,300/1.071^3 = \$1,872.23$$

$$\text{PV of Year 4 CF: } \$2,700/1.071^4 = \$2,052.13$$

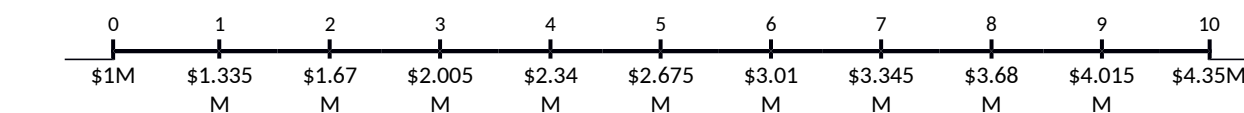
So, the PV of the missing CF is:

$$\$6,700 - 1,307.19 - 1,872.23 - 2,052.13 = \$1,468.44$$

The question asks for the value of the cash flow in Year 2, so we must find the future value of this amount. The value of the missing CF is:

$$\$1,468.44(1.071)^2 = \$1,684.37$$

40. The time line is:



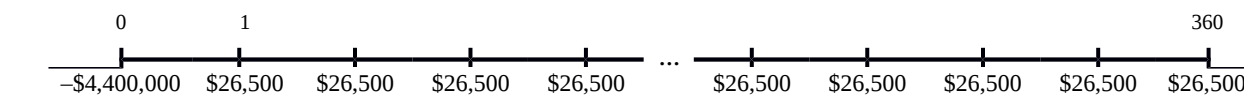
To solve this problem, we need to find the PV of each lump sum and add them together. It is important to note that the first cash flow of \$1 million occurs today, so we do not need to discount that cash flow. The PV of the lottery winnings is:

$$\begin{aligned} & \$1,000,000 + \$1,335,000/1.058 + \$1,670,000/1.058^2 + \$2,005,000/1.058^3 + \$2,340,000/1.058^4 + \\ & \$2,675,000/1.058^5 + \$3,010,000/1.058^6 + \$3,345,000/1.058^7 + \$3,680,000/1.058^8 + \\ & \$4,015,000/1.058^9 + \$4,350,000/1.058^{10} \\ \text{PV} &= \$20,969,067.06 \end{aligned}$$

41. Here, we are finding the interest rate for an annuity cash flow. We are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. We should also note that the PV of the annuity is not the amount borrowed since we are making a down payment on the warehouse. The amount borrowed is:

$$\text{Amount borrowed} = .80(\$5,500,000) = \$4,400,000$$

The time line is:



Using the PVA equation:

$$\text{PVA} = \$4,400,000 = \$26,500[\{1 - [1/(1 + r)]^{360}\}/r]$$

Unfortunately, this equation cannot be solved to find the interest rate using algebra. To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate decreases the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

$$r = .504\%$$

The APR is the monthly interest rate times the number of months in the year, so:

$$\text{APR} = 12(.504\%)$$

$$\text{APR} = 6.04\%$$

And the EAR is:

$$\text{EAR} = (1 + .00504)^{12} - 1$$

$$\text{EAR} = .0621, \text{ or } 6.21\%$$

42. The time line is:



The profit the firm earns is the PV of the sales price minus the cost to produce the asset. We find the PV of the sales price as the PV of a lump sum:

$$\text{PV} = \$165,000/1.13^3$$

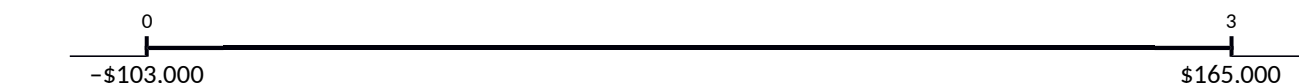
$$\text{PV} = \$114,353.28$$

And the firm's profit is:

$$\text{Profit} = \$114,353.28 - 103,000$$

$$\text{Profit} = \$11,353.28$$

To find the interest rate at which the firm will break even, we need to find the interest rate using the PV (or FV) of a lump sum. Using the PV equation for a lump sum, we get:

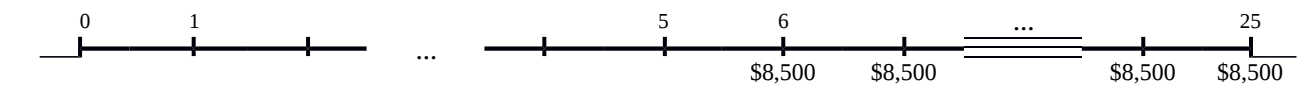


$$\$103,000 = \$165,000/(1 + r)^3$$

$$r = (\$165,000/\$103,000)^{1/3} - 1$$

$$r = .1701, \text{ or } 17.01\%$$

43. The time line is:



We want to find the value of the cash flows today, so we will find the PV of the annuity, and then bring the lump sum PV back to today. The annuity has 20 payments, so the PV of the annuity is:

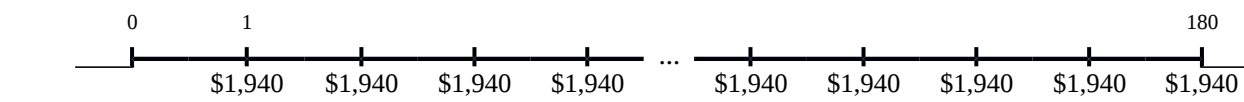
$$\text{PVA} = \$8,500\{[1 - (1/1.067)^{20}]/.067\}$$

$$\text{PVA} = \$92,187.54$$

Since this is an ordinary annuity equation, this is the PV one period before the first payment, so it is the PV at $t = 5$. To find the value today, we find the PV of this lump sum. The value today is:

$$\begin{aligned} PV &= \$92,187.54/1.067^5 \\ PV &= \$66,657.67 \end{aligned}$$

44. The time line for the annuity is:



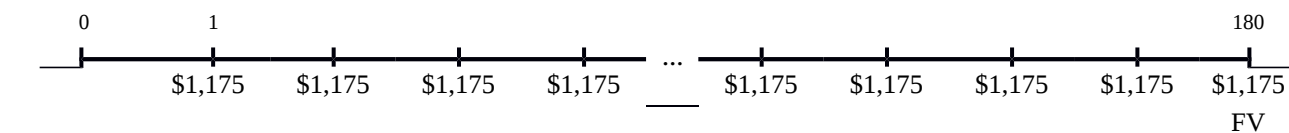
This question is asking for the present value of an annuity, but the interest rate changes during the life of the annuity. We need to find the present value of the cash flows for the last eight years first. The PV of these cash flows is:

$$\begin{aligned} PVA_2 &= \$1,940[1 - 1/[1 + (.06/12)]^{96}]/(.06/12) \\ PVA_2 &= \$147,624.72 \end{aligned}$$

Note that this is the PV of this annuity exactly seven years from today. Now, we can discount this lump sum to today as well as finding the PV of the annuity for the first seven years. The value of this cash flow today is:

$$\begin{aligned} PV &= \$147,624.72/[1 + (.11/12)]^{84} + \$1,940[1 - 1/[1 + (.11/12)]^{84}]/(.11/12) \\ PV &= \$181,893.99 \end{aligned}$$

45. The time line for the annuity is:



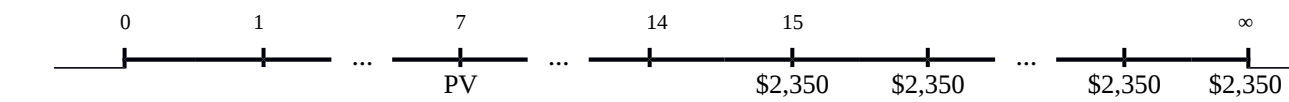
Here, we are trying to find the dollar amount invested today that will equal the FVA with a known interest rate and payments. First, we need to determine how much we would have in the annuity account. Finding the FV of the annuity, we get:

$$\begin{aligned} FVA &= \$1,175[1 + (.064/12)]^{180} - 1]/(.064/12) \\ FVA &= \$353,610.97 \end{aligned}$$

Now, we need to find the PV of a lump sum that will give us the same FV. So, using the FV of a lump sum with continuous compounding, we get:

$$\begin{aligned} FV &= \$353,610.97 = PVe^{.07(15)} \\ PV &= \$353,610.97e^{-1.05} \\ PV &= \$123,741.83 \end{aligned}$$

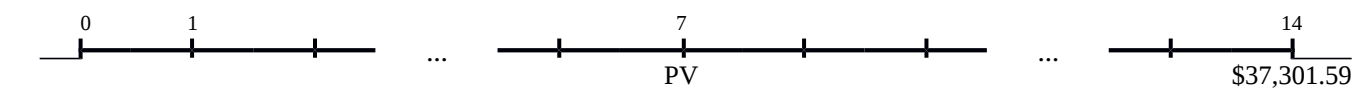
46. The time line is:



To find the value of the perpetuity at $t = 7$, we first need to use the PV of a perpetuity equation. Using this equation we find:

$$PV = \$2,350 / .063$$

$$PV = \$37,301.59$$

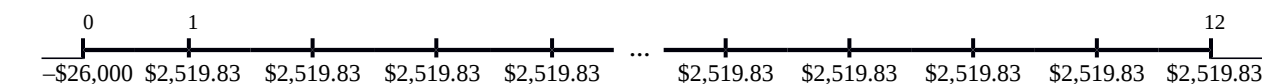


Remember that the PV of a perpetuity (and annuity) equation gives the PV one period before the first payment, so, this is the value of the perpetuity at $t = 14$. To find the value at $t = 7$, we find the PV of this lump sum as:

$$PV = \$37,301.59 / 1.063^7$$

$$PV = \$24,321.73$$

47. The time line is:



To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The interest rate for the cash flows of the loan is:

$$PVA = \$26,000 = \$2,519.83 \{ (1 - [1/(1+r)]^{12}) / r \}$$

Again, we cannot solve this equation for r , so we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. Using a spreadsheet, we find:

$$r = 2.403\% \text{ per month}$$

So the APR is:

$$APR = 12(2.403\%)$$

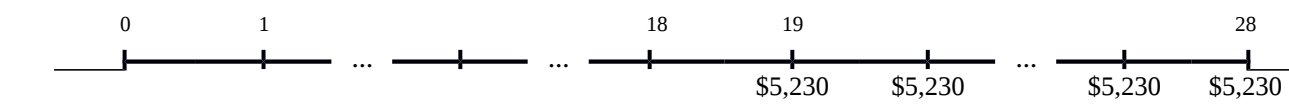
$$APR = 28.84\%$$

And the EAR is:

$$EAR = (1.02403)^{12} - 1$$

$$EAR = 32.97\%$$

48. The time line is:



The cash flows in this problem are semiannual, so we need the effective semiannual rate. The interest rate given is the APR, so the monthly interest rate is:

$$\text{Monthly rate} = .10/12 = .0083$$

To get the semiannual interest rate we can use the EAR equation, but instead of using 12 months as the exponent, we will use 6 months. The effective semiannual rate is:

$$\text{Semiannual rate} = 1.0083^6 - 1 = .0511, \text{ or } 5.11\%$$

We can now use this rate to find the PV of the annuity. The PV of the annuity is:

$$\text{PVA @ } t = 9: \$5,230 \{ [1 - (1/1.0511)^{10}] / .0511 \} = \$40,178.89$$

Note that this is the value one period (six months) before the first payment, so it is the value at $t = 9$. So, the value at the various times the question asked for uses this value 9 years from now.

$$\text{PV @ } t = 5: \$40,178.89 / 1.0511^8 = \$26,977.40$$

Note that you can also calculate this present value (as well as the remaining present values) using the number of years. To do this, you need the EAR. The EAR is:

$$\text{EAR} = (1 + .0083)^{12} - 1 = .1047, \text{ or } 10.47\%$$

So, we can find the PV at $t = 5$ using the following method as well:

$$\text{PV @ } t = 5: \$40,178.89 / 1.1047^4 = \$26,977.40$$

The value of the annuity at the other times in the problem is:

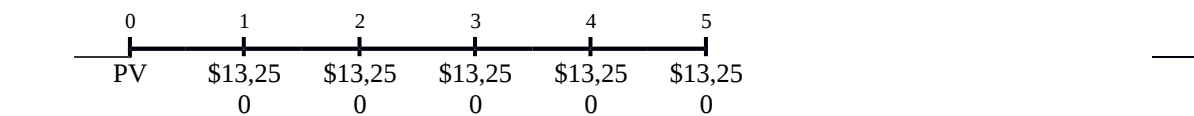
$$\text{PV @ } t = 3: \$40,178.89 / 1.0511^{12} = \$22,105.54$$

$$\text{PV @ } t = 3: \$40,178.89 / 1.1047^6 = \$22,105.54$$

$$\text{PV @ } t = 0: \$40,178.89 / 1.0511^{18} = \$16,396.55$$

$$\text{PV @ } t = 0: \$40,178.89 / 1.1047^9 = \$16,396.55$$

49. a. The time line for the ordinary annuity is:



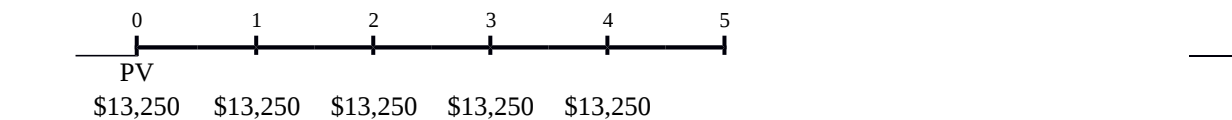
If the payments are in the form of an ordinary annuity, the present value will be:

$$PVA = C(\{1 - [1/(1 + r)^t]\}/r)$$

$$PVA = \$13,250\{1 - [1/(1 + .078)^5]/.078\}$$

$$PVA = \$53,183.45$$

The time line for the annuity due is:



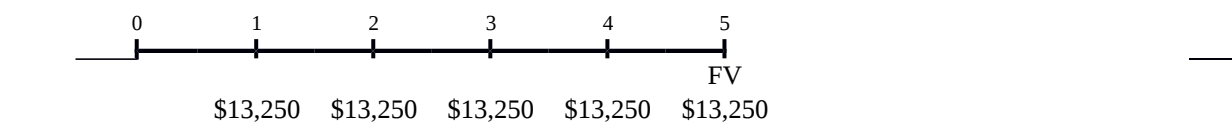
If the payments are an annuity due, the present value will be:

$$PVA_{\text{due}} = (1 + r) PVA$$

$$PVA_{\text{due}} = (1 + .078)\$53,183.45$$

$$PVA_{\text{due}} = \$57,331.76$$

b. The time line for the ordinary annuity is:



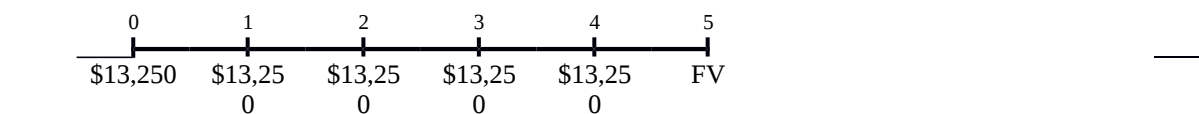
We can find the future value of the ordinary annuity as:

$$FVA = C\{[(1 + r)^t - 1]/r\}$$

$$FVA = \$13,250\{[(1 + .078)^5 - 1]/.078\}$$

$$FVA = \$77,423.06$$

The time line for the annuity due is:



If the payments are an annuity due, the future value will be:

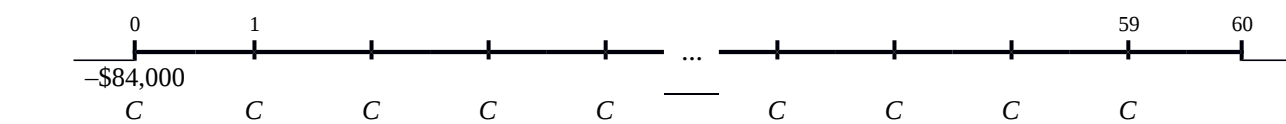
$$FVA_{\text{due}} = (1 + r) FVA$$

$$FVA_{\text{due}} = (1 + .075)\$77,423.06$$

$$FVA_{\text{due}} = \$83,462.06$$

- c. Assuming a positive interest rate, the present value of an annuity due will always be larger than the present value of an ordinary annuity. Each cash flow in an annuity due is received one period earlier, which means there is one period less to discount each cash flow. Assuming a positive interest rate, the future value of an ordinary due will always be higher than the future value of an ordinary annuity. Since each cash flow is made one period sooner, each cash flow receives one extra period of compounding.

50. The time line is:



We need to use the PVA due equation, that is:

$$PVA_{\text{due}} = (1 + r)PVA$$

Using this equation:

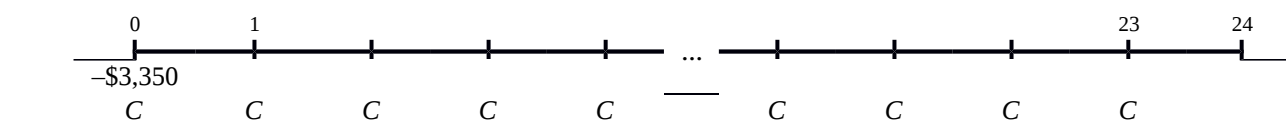
$$PVA_{\text{due}} = \$84,000 = [1 + (.0608/12)] \times C \left[\frac{1 - 1/[1 + (.0608/12)]^{60}}{(.0608/12)} \right]$$

$$C = \$1,618.88$$

Notice, to find the payment for the PVA due we compound the payment for an ordinary annuity forward one period.

Challenge

51. The time line is:



The monthly interest rate is the annual interest rate divided by 12, or:

$$\text{Monthly interest rate} = .107/12$$

$$\text{Monthly interest rate} = .00892$$

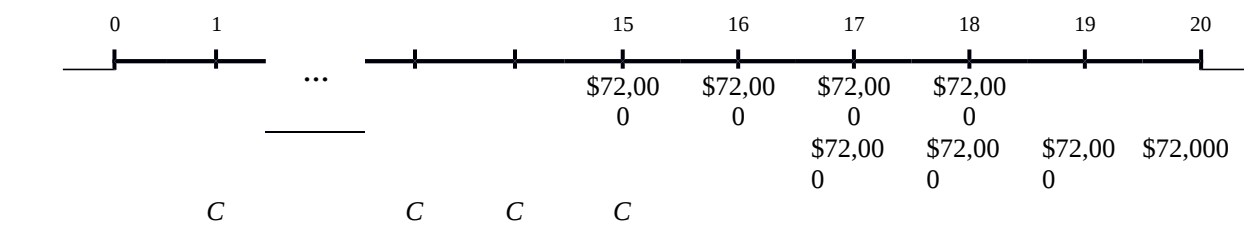
Now we can set the present value of the lease payments equal to the cost of the equipment, or \$3,350. The lease payments are in the form of an annuity due, so:

$$PVA_{\text{due}} = (1 + r)C \left(\frac{1 - [1/(1 + r)]^t}{r} \right)$$

$$\$3,350 = (1 + .00892)C \left(\frac{1 - [1/(1 + .00892)]^{24}}{.00892} \right)$$

$$C = \$154.29$$

52. The time line is:



First, we will calculate the present value of the college expenses for each child. The expenses are an annuity, so the present value of the college expenses is:

$$\begin{aligned}PVA &= C(\{1 - [1/(1 + r)]^t\}/r) \\PVA &= \$72,000(\{1 - [1/(1 + .079)]^4\}/.079) \\PVA &= \$239,004.91\end{aligned}$$

This is the cost of each child's college expenses one year before they enter college. So, the cost of the oldest child's college expenses today will be:

$$\begin{aligned}PV &= FV/(1 + r)^t \\PV &= \$239,004.91/(1 + .079)^{14} \\PV &= \$82,434.04\end{aligned}$$

And the cost of the youngest child's college expenses today will be:

$$\begin{aligned}PV &= FV/(1 + r)^t \\PV &= \$239,004.91/(1 + .079)^{16} \\PV &= \$70,804.96\end{aligned}$$

Therefore, the total cost today of your children's college expenses is:

$$\begin{aligned}\text{Cost today} &= \$82,434.04 + 70,804.96 \\ \text{Cost today} &= \$153,239.01\end{aligned}$$

This is the present value of your annual savings, which are an annuity. So, the amount you must save each year will be:

$$\begin{aligned}PVA &= C(\{1 - [1/(1 + r)]^t\}/r) \\ \$153,239.01 &= C(\{1 - [1/(1 + .079)]^{15}\}/.079) \\ C &= \$17,793.68\end{aligned}$$

53. The salary is a growing annuity, so we use the equation for the present value of a growing annuity. The salary growth rate is 3.8 percent and the discount rate is 7.1 percent, so the value of the salary offer today is:

$$\begin{aligned}PV &= C \{ [1/(r - g)] - [1/(r - g)] \times [(1 + g)/(1 + r)]^t \} \\ PV &= \$71,000 \{ [1/ (.071 - .038)] - [1/ (.071 - .038)] \times [(1 + .038)/(1 + .071)]^{25} \} \\ PV &= \$1,167,636.64\end{aligned}$$

The yearly bonuses are 10 percent of the annual salary. This means that next year's bonus will be:

$$\begin{aligned}\text{Next year's bonus} &= .10(\$71,000) \\ \text{Next year's bonus} &= \$7,100\end{aligned}$$

Since the salary grows at 3.8 percent, the bonus will grow at 3.8 percent as well. Using the growing annuity equation, with a 3.8 percent growth rate and a 7.1 percent discount rate, the present value of the annual bonuses is:

$$\begin{aligned}PV &= C \{ [1/(r - g)] - [1/(r - g)] \times [(1 + g)/(1 + r)]^t \} \\ PV &= \$7,100 \{ [1/ (.071 - .038)] - [1/ (.071 - .038)] \times [(1 + .038)/(1 + .071)]^{25} \} \\ PV &= \$116,763.66\end{aligned}$$

Notice the present value of the bonus is 10 percent of the present value of the salary. The present value of the bonus will always be the same percentage of the present value of the salary as the bonus percentage. So, the total value of the offer is:

$$PV = PV(\text{Salary}) + PV(\text{Bonus}) + \text{Bonus paid today}$$

$$PV = \$1,167,636.64 + 116,763.66 + 10,000$$

$$PV = \$1,294,400.30$$

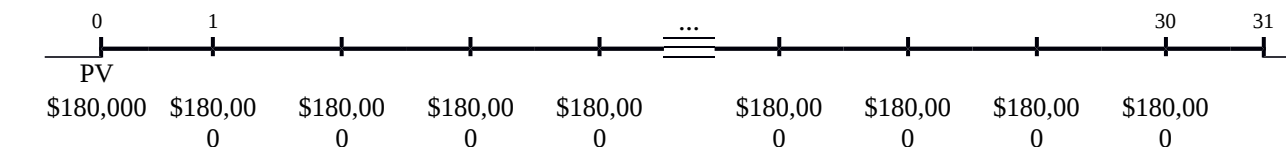
54. Here, we need to compare two options. In order to do so, we must get the value of the two cash flow streams to the same time, so we will find the value of each today. We must also make sure to use the aftertax cash flows, since they are more relevant. For Option A, the aftertax cash flows are:

$$\text{Aftertax cash flows} = \text{Pretax cash flows}(1 - \text{tax rate})$$

$$\text{Aftertax cash flows} = \$250,000(1 - .28)$$

$$\text{Aftertax cash flows} = \$180,000$$

So, the cash flows are:



The aftertax cash flows from Option A are in the form of an annuity due, so the present value of the cash flow today is:

$$PVA_{\text{due}} = (1 + r)C(\{1 - [1/(1 + r)]^t\}/r)$$

$$PVA_{\text{due}} = (1 + .0585)\$180,000(\{1 - [1/(1 + .0585)]^{31}\}/.0585)$$

$$PVA_{\text{due}} = \$2,697,950.16$$

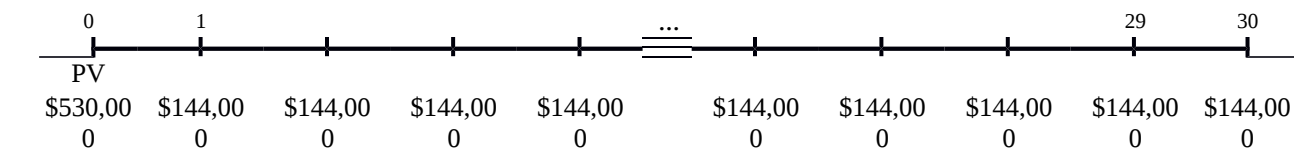
For Option B, the aftertax cash flows are:

$$\text{Aftertax cash flows} = \text{Pretax cash flows}(1 - \text{tax rate})$$

$$\text{Aftertax cash flows} = \$200,000(1 - .28)$$

$$\text{Aftertax cash flows} = \$144,000$$

The cash flows are:



The aftertax cash flows from Option B are an ordinary annuity, plus the cash flow today, so the present value is:

$$PV = C(\{1 - [1/(1 + r)]^t\}/r) + CF_0$$

$$PV = \$144,000\{1 - [1/(1 + .0585)]^{30}\}/.0585 + \$530,000$$

$$PV = \$2,544,360.13$$

You should choose Option A because it has a higher present value on an aftertax basis.

55. We need to find the first payment into the retirement account. The present value of the desired amount at retirement is:

$$\begin{aligned} PV &= FV/(1 + r)^t \\ PV &= \$2,500,000/(1 + .094)^{30} \\ PV &= \$168,818.62 \end{aligned}$$

This is the value today. Since the savings are in the form of a growing annuity, we can use the growing annuity equation and solve for the payment. Doing so, we get:

$$\begin{aligned} PV &= C \{ [1/(r - g)] - [1/(r - g)] \times [(1 + g)/(1 + r)]^t \} \\ \$168,818.62 &= C \{ [1/(.094 - .03)] - [1/(.094 - .03)] \times [(1 + .03)/(1 + .094)]^{30} \} \\ C &= \$12,922.47 \end{aligned}$$

This is the amount you need to save next year. So, the percentage of your salary is:

$$\begin{aligned} \text{Percentage of salary} &= \$12,922.47/\$86,000 \\ \text{Percentage of salary} &= .1503, \text{ or } 15.03\% \end{aligned}$$

Note that this is the percentage of your salary you must save each year. Since your salary is increasing at 3 percent, and the savings are increasing at 3 percent, the percentage of salary will remain constant.

56. Since she put \$1,500 down, the amount borrowed will be:

$$\begin{aligned} \text{Amount borrowed} &= \$17,000 - 1,500 \\ \text{Amount borrowed} &= \$15,500 \end{aligned}$$

So, the monthly payments will be:

$$\begin{aligned} PVA &= C \{ [1 - 1/(1 + r)]^t / r \} \\ \$15,500 &= C \{ [1 - 1/(1 + .068/12)]^{60} / (.068/12) \} \\ C &= \$305.46 \end{aligned}$$

The amount remaining on the loan is the present value of the remaining payments. Since the first payment was made on October 1, 2018, and she made a payment on October 1, 2020, there are 35 payments remaining, with the first payment due immediately. So, we can find the present value of the remaining 34 payments after November 1, 2020, and add the payment made on this date. So the remaining principal owed on the loan is:

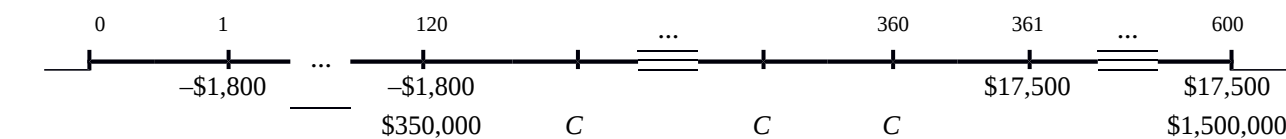
$$\begin{aligned} PV &= C \{ [1 - 1/(1 + r)]^t / r \} + C_0 \\ PV &= \$305.46 \{ [1 - 1/(1 + .068/12)]^{34} / (.068/12) \} \\ C &= \$9,422.19 \end{aligned}$$

She must also pay a one percent prepayment penalty and the payment is due on November 1, 2020, so the total amount of the payment is:

$$\begin{aligned} \text{Total payment} &= \text{Balloon amount}(1 + \text{Prepayment penalty}) + \text{Current payment} \\ \text{Total payment} &= \$9,422.19(1 + .01) + \$305.46 \end{aligned}$$

Total payment = \$9,821.87

57. The time line is:



The cash flows for this problem occur monthly, and the interest rate given is the EAR. Since the cash flows occur monthly, we must get the effective monthly rate. One way to do this is to find the APR based on monthly compounding, and then divide by 12. So, the pre-retirement APR is:

$$\text{EAR} = .11 = [1 + (\text{APR}/12)]^{12} - 1; \text{APR} = 12(1.11^{1/12} - 1) = .1048, \text{ or } 10.48\%$$

And the post-retirement APR is:

$$\text{EAR} = .08 = [1 + (\text{APR}/12)]^{12} - 1; \text{APR} = 12(1.08^{1/12} - 1) = .0772, \text{ or } 7.72\%$$

First, we will calculate how much he needs at retirement. The amount needed at retirement is the PV of the monthly spending plus the PV of the inheritance. The PV of these two cash flows is:

$$\begin{aligned} \text{PVA} &= \$17,500 \{ 1 - [1/(1 + .0772/12)^{12(20)}] \} / (.0772/12) \\ \text{PVA} &= \$2,136,360.28 \end{aligned}$$

$$\begin{aligned} \text{PV} &= \$1,500,000 / (1 + .08)^{20} \\ \text{PV} &= \$321,822.31 \end{aligned}$$

So, at retirement, he needs:

$$\$2,136,360.28 + 321,822.31 = \$2,458,182.59$$

He will be saving \$1,800 per month for the next 10 years until he purchases the cabin. The value of his savings after 10 years will be:

$$\begin{aligned} \text{FVA} &= \$1,800 [\{ 1 + (.1048/12) \}^{12(10)} - 1] / (.1048/12) \\ \text{FVA} &= \$379,062.59 \end{aligned}$$

After he purchases the cabin, the amount he will have left is:

$$\$379,062.59 - 350,000 = \$29,062.59$$

He still has 20 years until retirement. When he is ready to retire, this amount will have grown to:

$$\begin{aligned} \text{FV} &= \$29,062.59 [1 + (.1048/12)]^{12(20)} \\ \text{FV} &= \$234,311.67 \end{aligned}$$

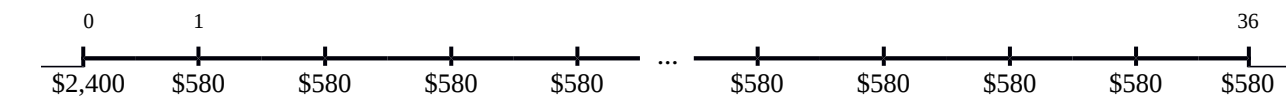
So, when he is ready to retire, based on his current savings, he will be short:

$$\$2,458,182.59 - 234,311.67 = \$2,223,870.92$$

This amount is the FV of the monthly savings he must make between Years 10 and 30. So, finding the annuity payment using the FVA equation, we find his monthly savings will need to be:

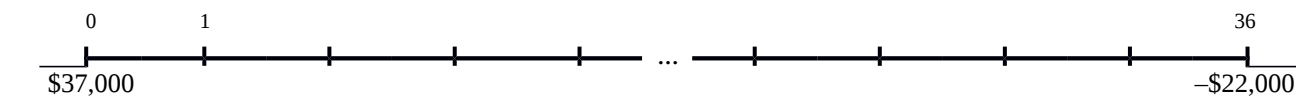
$$\begin{aligned} FVA &= \$2,223,870.92 = C \left[\frac{1 + (.1048/12)^{12(20)} - 1}{(.1048/12)} \right] \\ C &= \$2,750.46 \end{aligned}$$

58. To answer this question, we should find the PV of both options, and compare them. Since we are purchasing the car, the lowest PV is the best option. The PV of the leasing option is the PV of the lease payments, plus the \$2,400. The interest rate we would use for the leasing option is the same as the interest rate of the loan. The PV of leasing is:



$$\begin{aligned} PV &= \$2,400 + \$580 \left\{ 1 - \frac{1}{(1 + .06/12)^{12(3)}} \right\} / (.06/12) \\ PV &= \$21,465.19 \end{aligned}$$

The PV of purchasing the car is the current price of the car minus the PV of the resale price. The PV of the resale price is:



$$\begin{aligned} PV &= \$22,000 / [1 + (.06/12)^{12(3)}] \\ PV &= \$18,384.19 \end{aligned}$$

The PV of the decision to purchase is:

$$\$37,000 - \$18,384.19 = \$18,615.81$$

In this case, it is cheaper to buy the car than to lease it since the PV of the leasing cash flows is lower. To find the break-even resale price, we need to find the resale price that makes the PV of the two options the same. In other words, the PV of the decision to buy should be:

$$\begin{aligned} \$37,000 - \text{PV of resale price} &= \$21,465.19 \\ \text{PV of resale price} &= \$15,534.81 \end{aligned}$$

The resale price that would make the PV of the lease versus buy decision equal is the FV of this value, so:

$$\begin{aligned} \text{Break-even resale price} &= \$15,534.81 [1 + (.06/12)^{12(3)}] \\ \text{Break-even resale price} &= \$18,590.21 \end{aligned}$$

59. To find the quarterly salary for the player, we first need to find the PV of the current contract. The cash flows for the contract are annual, and we are given a daily interest rate. We need to find the EAR so the interest compounding is the same as the timing of the cash flows. The EAR is:

$$\begin{aligned} \text{EAR} &= [1 + (.057/365)]^{365} - 1 \\ \text{EAR} &= .0587, \text{ or } 5.87\% \end{aligned}$$

The PV of the current contract offer is the sum of the PV of the cash flows. So, the PV is:

$$\begin{aligned} \text{PV} &= \$7,900,000 + \$4,300,000/1.0587 + \$4,900,000/1.0587^2 + \$5,700,000/1.0587^3 \\ &\quad + \$6,700,000/1.0587^4 + \$7,300,000/1.0587^5 + \$8,400,000/1.0587^6 \\ \text{PV} &= \$37,929,060.53 \end{aligned}$$

The player wants the contract value to be increased by \$3,500,000, so the PV of the new contract will be:

$$\begin{aligned} \text{PV} &= \$37,929,060.53 + 3,500,000 \\ \text{PV} &= \$41,429,060.53 \end{aligned}$$

The player has also requested a signing bonus payable today in the amount of \$10 million. We can subtract this amount from the PV of the new contract. The remaining amount will be the PV of the future quarterly paychecks.

$$\$41,429,060.53 - 10,000,000 = \$31,429,060.53$$

To find the quarterly payments, first realize that the interest rate we need is the effective quarterly rate. Using the daily interest rate, we can find the quarterly interest rate using the EAR equation, with the number of days being 91.25, the number of days in a quarter (365/4). The effective quarterly rate is:

$$\begin{aligned} \text{Effective quarterly rate} &= [1 + (.057/365)]^{91.25} - 1 \\ \text{Effective quarterly rate} &= .01435 \text{ or } 1.435\% \end{aligned}$$

Now, we have the interest rate, the length of the annuity, and the PV. Using the PVA equation and solving for the payment, we get:

$$\begin{aligned} \text{PVA} &= \$31,429,060.53 = C\{[1 - (1/1.01435)^{24}]/.01435\} \\ C &= \$1,557,264.39 \end{aligned}$$

60. The time line for the cash flows is:

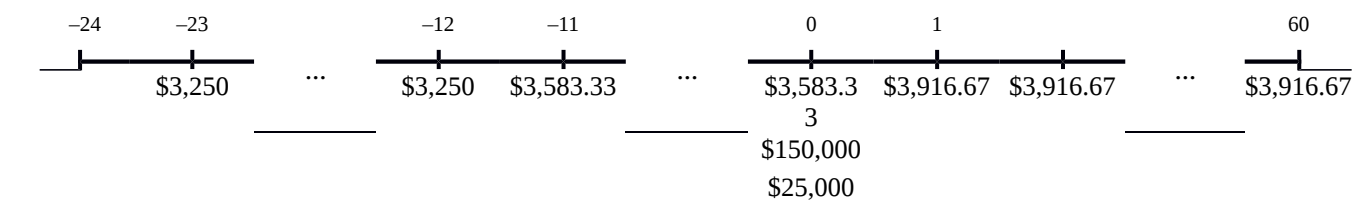


To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The cash flows of the loan are the \$20,000 you must repay in one year, and the \$16,720 you borrow today. The interest rate of the loan is:

$$\begin{aligned} \$20,000 &= \$16,720(1 + r) \\ r &= (\$20,000/\$16,720) - 1 \\ r &= .1962, \text{ or } 19.62\% \end{aligned}$$

Because of the discount, you only get the use of \$16,720, and the interest you pay on that amount is 19.62 percent, not 16.4 percent.

61. The time line is:



Here, we have cash flows that would have occurred in the past and cash flows that would occur in the future. We need to bring both cash flows to today. Before we calculate the value of the cash flows today, we must adjust the interest rate, so we have the effective monthly interest rate. Finding the APR with monthly compounding and dividing by 12 will give us the effective monthly rate. The APR with monthly compounding is:

$$\text{APR} = 12[(1.074)^{1/12} - 1]$$

$$\text{APR} = .0716, \text{ or } 7.16\%$$

To find the value today of the back pay from two years ago, we will find the FV of the annuity (salary), and then find the FV of the lump sum value of the salary. Doing so gives us:

$$\text{FV} = (\$39,000/12)[\{[1 + (.0716/12)]^{12} - 1\}/(.0716/12)](1 + .074)$$

$$\text{FV} = \$43,288.32$$

Notice we found the FV of the annuity with the effective monthly rate, and then found the FV of the lump sum with the EAR. Alternatively, we could have found the FV of the lump sum with the effective monthly rate as long as we used 12 periods. The answer would be the same either way.

Now, we need to find the value today of last year's back pay:

$$\text{FVA} = (\$43,000/12)[\{[1 + (.0716/12)]^{12} - 1\}/(.0716/12)]$$

$$\text{FVA} = \$44,439.62$$

Next, we find the value today of the five year's future salary:

$$\text{PVA} = (\$47,000/12)[\{1 - [1 + (.0716/12)]^{12(5)}\}/(.0716/12)]$$

$$\text{PVA} = \$197,046.10$$

The value today of the jury award is the sum of salaries, plus the compensation for pain and suffering, and court costs. The award should be for the amount of:

$$\text{Award} = \$43,288.32 + 44,439.62 + 197,046.10 + 150,000 + 25,000$$

$$\text{Award} = \$459,774.05$$

As the plaintiff, you would prefer a lower rate. In this problem, we are calculating both the PV and FV of annuities. A lower rate will decrease the FVA, but increase the PVA. So, by using a lower rate, we are lowering the value of the back pay. But, we are also increasing the PV of the future salary. Since the future salary is larger and has a longer time period, this is the more important cash flow to the plaintiff.

62. To find the interest rate of a loan, we need to look at the cash flows of the loan. Since this loan is in the form of a lump sum, the amount you will repay is the FV of the principal amount, which will be:

$$\text{Loan repayment amount} = \$10,000(1.08)$$

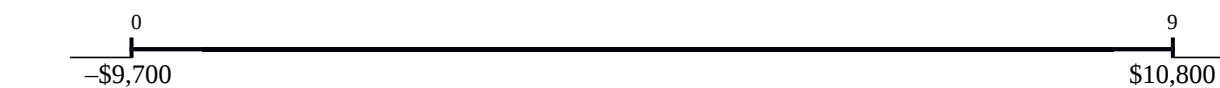
$$\text{Loan repayment amount} = \$10,800$$

The amount you will receive today is the principal amount of the loan times one minus the points.

$$\text{Amount received} = \$10,000(1 - .03)$$

$$\text{Amount received} = \$9,700$$

So, the time line is:



Now, we find the interest rate for this PV and FV.

$$\$10,800 = \$9,700(1 + r)$$

$$r = (\$10,800/\$9,700) - 1$$

$$r = .1134, \text{ or } 11.34\%$$

With a quoted interest rate of 11 percent and two points, the EAR is:

$$\text{Loan repayment amount} = \$10,000(1.11)$$

$$\text{Loan repayment amount} = \$11,100$$

$$\text{Amount received} = \$10,000(1 - .02)$$

$$\text{Amount received} = \$9,800$$

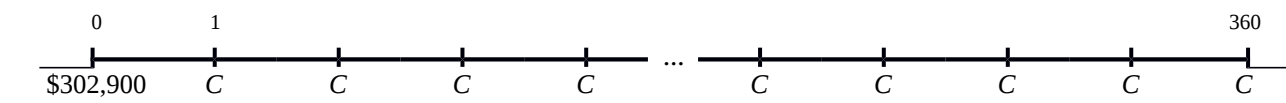
$$\$11,100 = \$9,800(1 + r)$$

$$r = (\$11,100/\$9,800) - 1$$

$$r = .1327, \text{ or } 13.27\%$$

The effective rate is not affected by the loan amount, since it drops out when solving for r .

63. First, we will find the APR and EAR for the loan with the refundable fee. Remember, we need to use the actual cash flows of the loan to find the interest rate. With the \$2,900 application fee, you will need to borrow \$302,900 to have \$300,000 after deducting the fee. The time line is:

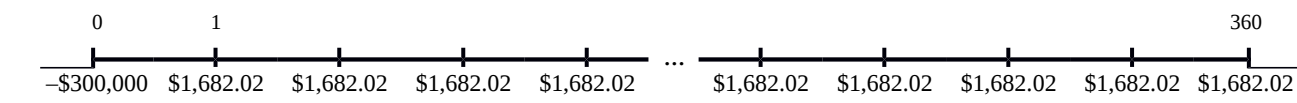


Solving for the payment under these circumstances, we get:

$$\text{PVA} = \$302,900 = C\{[1 - 1/(\.053/12)^{360}]/(\.053/12)\}$$

$$C = \$1,682.02$$

We can now use this amount in the PVA equation with the original amount we wished to borrow, \$300,000.



Solving for r , we find:

$$PVA = \$300,000 = \$1,682.02 \left[\frac{1 - [1/(1 + r)]^{360}}{r} \right]$$

Solving for r with a spreadsheet, on a financial calculator, or by trial and error, gives:

$$r = .4489\% \text{ per month}$$

$$APR = 12(.4489\%)$$

$$APR = 5.39\%$$

$$EAR = (1 + .004489)^{12} - 1$$

$$EAR = .0552, \text{ or } 5.52\%$$

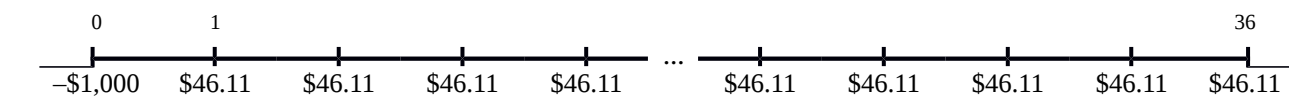
With the nonrefundable fee, the APR of the loan is the quoted APR since the fee is not considered part of the loan. So:

$$APR = 5.30\%$$

$$EAR = [1 + (.053/12)]^{12} - 1$$

$$EAR = .0543, \text{ or } 5.43\%$$

64. The time line is:



Be careful of interest rate quotations. The actual interest rate of a loan is determined by the cash flows. Here, we are told that the PV of the loan is \$1,000, and the payments are \$46.11 per month for three years, so the interest rate on the loan is:

$$PVA = \$1,000 = \$46.11 \left[\frac{1 - [1/(1 + r)]^{36}}{r} \right]$$

Solving for r with a spreadsheet, on a financial calculator, or by trial and error, gives:

$$r = 3.04\% \text{ per month}$$

$$APR = 12(3.04\%)$$

$$APR = 36.54\%$$

$$\text{EAR} = (1 + .03040)^{12} - 1$$

$$\text{EAR} = .4332, \text{ or } 43.32\%$$

It's called add-on interest because the interest amount of the loan is added to the principal amount of the loan before the loan payments are calculated.

65. We will calculate the number of periods necessary to repay the balance with no fee first. We need to use the PVA equation and solve for the number of payments.

Without fee and annual rate = 18.6%:

$$\text{PVA} = \$12,000 = \$250\{[1 - (1/1.0155)^t]/.0155\} \text{ where } .0155 = .186/12$$

Solving for t , we get:

$$t = \ln\{1/[1 - (\$12,000/\$250)(.0155)]\}/\ln(1.0155)$$

$$t = \ln 3.90625/\ln 1.0155$$

$$t = 88.59 \text{ months}$$

Without fee and annual rate = 8.2%:

$$\text{PVA} = \$12,000 = \$250\{[1 - (1/1.006833)^t]/.006833\} \text{ where } .006833 = .082/12$$

Solving for t , we get:

$$t = \ln\{1/[1 - (\$12,000/\$250)(.006833)]\}/\ln(1.006833)$$

$$t = \ln 1.4881/\ln 1.006833$$

$$t = 58.37 \text{ months}$$

You will pay off your account:

$$88.59 - 58.37 = 30.22 \text{ months quicker}$$

Note that we do not need to calculate the time necessary to repay your current credit card with a fee since no fee will be incurred. The time to repay the new card with a transfer fee is:

With fee and annual rate = 8.20%:

$$\text{PVA} = \$12,240 = \$250\{[1 - (1/1.006833)^t]/.006833\} \text{ where } .006833 = .082/12$$

Solving for t , we get:

$$t = \ln\{1/[1 - (\$12,240/\$250)(.006833)]\}/\ln(1.006833)$$

$$t = \ln 1.5028/\ln 1.006833$$

$$t = 59.81 \text{ months}$$

You will pay off your account:

$$88.59 - 59.81 = 28.78 \text{ months quicker}$$

66. We need to find the FV of the premiums to compare with the cash payment promised at age 65. We have to find the value of the premiums at Year 6 first since the interest rate changes at that time. So:

$$FV_1 = \$500(1.11)^5 = \$842.53$$

$$FV_2 = \$600(1.11)^4 = \$910.84$$

$$FV_3 = \$700(1.11)^3 = \$957.34$$

$$FV_4 = \$800(1.11)^2 = \$985.68$$

$$FV_5 = \$900(1.11)^1 = \$999.00$$

$$\text{Value at Year 6} = \$842.53 + 910.84 + 957.34 + 985.68 + 999 + 1,000$$

$$\text{Value at Year 6} = \$5,695.39$$

Finding the FV of this lump sum at the child's 65th birthday:

$$FV = \$5,695.39(1.07)^{59}$$

$$FV = \$308,437.08$$

The policy is not worth buying; the future value of the policy is \$308,437.08, but the policy contract will pay off \$300,000. The premiums are worth \$8,437.08 more than the policy payoff.

Note, we could also compare the PV of the two cash flows. The PV of the premiums is:

$$PV = \$500/1.11 + \$600/1.11^2 + \$700/1.11^3 + \$800/1.11^4 + \$900/1.11^5 + \$1,000/1.11^6$$

$$PV = \$3,044.99$$

And the value today of the \$300,000 at age 65 is:

$$PV = \$300,000/1.07^{59}$$

$$PV = \$5,539.60$$

$$PV = \$5,539.60/1.11^6$$

$$PV = \$2,961.70$$

The premiums still have the higher cash flow. At Year 0, the difference is \$83.29. When you are comparing two or more cash flow streams, the cash flow with the highest value at one time will have the highest value at any other time.

Here is a question for you: Suppose you invest \$83.29, the difference in the cash flows at time zero, for 6 years at an 11 percent interest rate, and then for 59 years at a 7 percent interest rate. How much will it be worth? Without doing calculations, you know it will be worth \$8,437.08, the difference in the cash flows at Year 65!

67. Since the payments occur at six month intervals, we need to get the effective six-month interest rate. We can calculate the daily interest rate since we have an APR compounded daily, so the effective six-month interest rate is:

$$\text{Effective six-month rate} = (1 + \text{Daily rate})^{180} - 1$$

$$\text{Effective six-month rate} = (1 + .09/360)^{180} - 1$$

$$\text{Effective six-month rate} = .0460, \text{ or } 4.60\%$$

Now, we can use the PVA equation to find the present value of the semi-annual payments. Doing so, we find:

$$\text{PVA} = C(\{1 - [1/(1 + r)]^t\}/r)$$

$$\text{PVA} = \$1,900,000(\{1 - [1/(1 + .0460)]^{40}\}/.0460)$$

$$\text{PVA} = \$34,458,785.87$$

This is the value six months from today, which is one period (six months) prior to the first payment. So, the value today is:

$$\text{PV} = \$34,458,785.87/(1 + .0460)$$

$$\text{PV} = \$32,942,697.79$$

This means the total value of the lottery winnings today is:

$$\text{Value of winnings today} = \$32,942,697.79 + 5,500,000$$

$$\text{Value of winnings today} = \$38,442,697.79$$

You should not take the offer since the value of the offer is less than the present value of the payments.

68. Here, we need to find the interest rate that makes the PVA, the college costs, equal to the FVA, the savings. The PV of the college costs is:

$$\text{PVA} = \$20,000[\{1 - [1/(1 + r)]^4\}/r]$$

And the FV of the savings is:

$$\text{FVA} = \$9,000\{[(1 + r)^6 - 1]/r\}$$

Setting these two equations equal to each other, we get:

$$\$20,000[\{1 - [1/(1 + r)]^4\}/r] = \$9,000\{[(1 + r)^6 - 1]/r\}$$

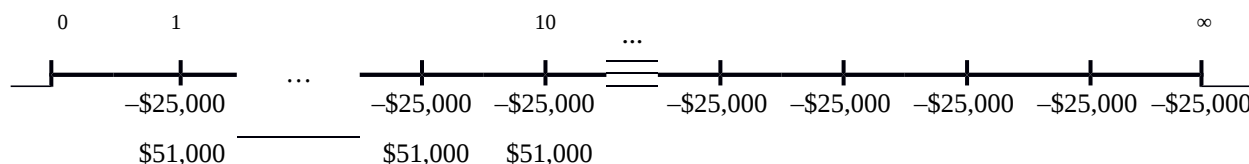
Reducing the equation gives us:

$$9(1 + r)^{10} - 29(1 + r)^4 + 20 = 0$$

Now, we need to find the root of this equation. We can solve using trial and error, a root-solving calculator routine, or a spreadsheet. Using a spreadsheet, we find:

$$r = 8.07\%$$

69. The time line is:



Here, we need to find the interest rate that makes us indifferent between an annuity and a perpetuity. To solve this problem, we need to find the PV of the two options and set them equal to each other. The PV of the perpetuity is:

$$PV = \$25,000/r$$

And the PV of the annuity is:

$$PVA = \$51,000[\{1 - [1/(1 + r)]^{10}\}/r]$$

Setting them equal and solving for r , we get:

$$\$25,000/r = \$51,000[\{1 - [1/(1 + r)]^{10}\}/r]$$

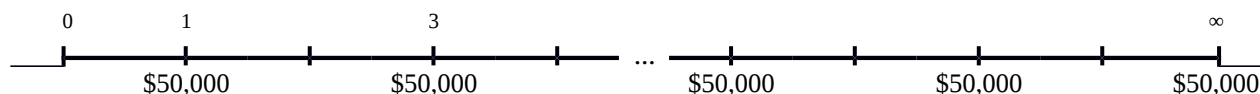
$$\$25,000/\$51,000 = 1 - [1/(1 + r)]^{10}$$

$$.4902^{1/10} = 1/(1 + r)$$

$$r = 1/.5098^{1/10} - 1$$

$$r = .0697, \text{ or } 6.97\%$$

70. The time line is:



The cash flows in this problem occur every two years, so we need to find the effective two year rate. One way to find the effective two year rate is to use an equation similar to the EAR, except use the number of days in two years as the exponent. (We use the number of days in two years since it is daily compounding; if monthly compounding was assumed, we would use the number of months in two years.) So, the effective two-year interest rate is:

$$\text{Effective 2-year rate} = [1 + (.09/365)]^{365(2)} - 1$$

$$\text{Effective 2-year rate} = .1972, \text{ or } 19.72\%$$

We can use this interest rate to find the PV of the perpetuity. Doing so, we find:

$$PV = \$50,000/.1972$$

$$PV = \$253,561.53$$

This is an important point: Remember that the PV equation for a perpetuity (and an ordinary annuity) tells you the PV one period before the first cash flow. In this problem, since the cash flows are two years apart, we have found the value of the perpetuity one period (two years) before the first payment, which is one year ago. We need to compound this value for one year to find the value today. The value of the cash flows today is:

$$\begin{aligned} FV &= \$253,561.53(1 + .09/365)^{365} \\ FV &= \$277,437.42 \end{aligned}$$

The second part of the question assumes the perpetuity cash flows begin in four years. In this case, when we use the PV of a perpetuity equation, we find the value of the perpetuity two years from today. So, the value of these cash flows today is:

$$\begin{aligned} PV &= \$253,561.53/(1 + .09/365)^{2(365)} \\ PV &= \$211,797.09 \end{aligned}$$

71. To solve for the PVA due:

$$\begin{aligned} PVA &= \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \dots + \frac{C}{(1+r)^t} \\ PVA_{\text{due}} &= C + \frac{C}{(1+r)} + \dots + \frac{C}{(1+r)^{t-1}} \\ PVA_{\text{due}} &= (1+r) \left(\frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \dots + \frac{C}{(1+r)^t} \right) \\ PVA_{\text{due}} &= (1+r)PVA \end{aligned}$$

And the FVA due is:

$$\begin{aligned} FVA &= C + C(1+r) + C(1+r)^2 + \dots + C(1+r)^{t-1} \\ FVA_{\text{due}} &= C(1+r) + C(1+r)^2 + \dots + C(1+r)^t \\ FVA_{\text{due}} &= (1+r)[C + C(1+r) + \dots + C(1+r)^{t-1}] \\ FVA_{\text{due}} &= (1+r)FVA \end{aligned}$$

72. a. The APR is the interest rate per week times 52 weeks in a year, so:

$$\begin{aligned} APR &= 52(7.1\%) \\ APR &= 369.20\% \\ EAR &= (1 + .071)^{52} - 1 \\ EAR &= 34.4040, \text{ or } 3,440.40\% \end{aligned}$$

- b. In a discount loan, the amount you receive is lowered by the discount, and you repay the full principal. With a discount of 7.1 percent, you would receive \$9.29 for every \$10 in principal, so the weekly interest rate would be:

$$\begin{aligned} \$10 &= \$9.29(1+r) \\ r &= (\$10/\$9.29) - 1 \\ r &= .0764, \text{ or } 7.64\% \end{aligned}$$

Note the dollar amount we use is irrelevant. In other words, we could use \$.929 and \$1, \$92.90 and \$100, or any other combination and we would get the same interest rate. Now we can find the APR and the EAR:

$$\text{APR} = 52(7.64\%)$$

$$\text{APR} = 397.42\%$$

$$\text{EAR} = (1 + .0764)^{52} - 1$$

$$\text{EAR} = 45.0450, \text{ or } 4,504.50\%$$

- c. Using the cash flows from the loan, we have the PVA and the annuity payments and need to find the interest rate, so:

$$\text{PVA} = \$68.43 = \$25[\{1 - [1/(1 + r)]^4\}/r]$$

Using a spreadsheet, trial and error, or a financial calculator, we find:

$$r = 17.11\% \text{ per week}$$

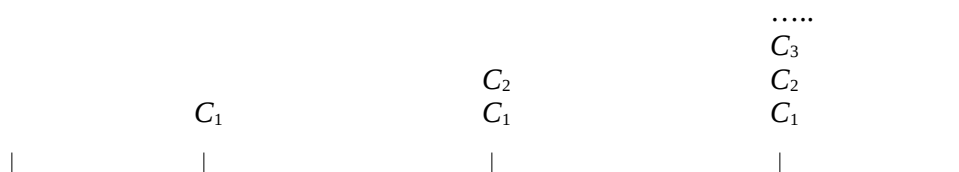
$$\text{APR} = 52(17.11\%)$$

$$\text{APR} = 889.82\%$$

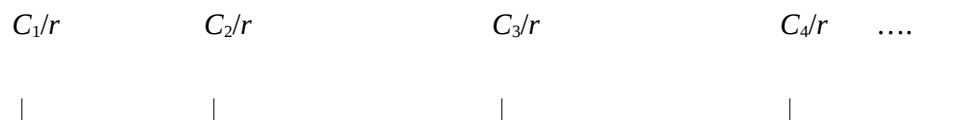
$$\text{EAR} = 1.1711^{52} - 1$$

$$\text{EAR} = 3,690.8773, \text{ or } 369,087.73\%$$

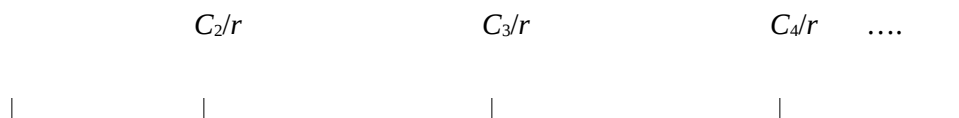
73. To answer this, we can diagram the perpetuity cash flows, which are: (Note, the subscripts are only to differentiate when the cash flows begin. The cash flows are all the same amount.)



Thus, each of the increased cash flows is a perpetuity in itself. So, we can write the cash flows stream as:



So, we can write the cash flows as the present value of a perpetuity with a perpetuity payment of:



The present value of this perpetuity is:

$$PV = (C/r)/r = C/r^2$$

So, the present value equation of a perpetuity that increases by C each period is:

$$PV = C/r + C/r^2$$

74. Since it is only an approximation, we know the Rule of 72 is exact for only one interest rate. Using the basic future value equation for an amount that doubles in value and solving for t , we find:

$$FV = PV(1 + r)^t$$

$$\$2 = \$1(1 + r)^t$$

$$\ln(2) = t \ln(1 + r)$$

$$t = \ln(2)/\ln(1 + r)$$

We also know the Rule of 72 approximation is:

$$t = 72/r$$

We can set these two equations equal to each other and solve for r . We also need to remember that the exact future value equation uses decimals, so the equation becomes:

$$.72/r = \ln(2)/\ln(1 + r)$$

$$0 = (.72/r)/[\ln(2)/\ln(1 + r)]$$

It is not possible to solve this equation directly for r , but using Solver, we find the interest rate for which the Rule of 72 is exact is 7.846894 percent.

75. We are only concerned with the time it takes money to double, so the dollar amounts are irrelevant. So, we can write the future value of a lump sum with continuously compounded interest as:

$$\$2 = \$1e^{rt}$$

$$2 = e^{rt}$$

$$rt = \ln(2)$$

$$rt = .693147$$

$$t = .693147/r$$

Since we are using percentage interest rates while the equation uses decimal form, to make the equation correct with percentages, we can multiply by 100:

$$t = 69.3147/r$$

Calculator Solutions**1.**

Enter	10	7.7%	\$7,800		
	N	I/Y	PV	PMT	FV
Solve for					\$16,377.65

$$\$16,377.65 - 13,806 = \$2,571.65$$

2.

Enter	10	5%	\$1,250		
	N	I/Y	PV	PMT	FV
Solve for					\$2,036.12

Enter	10	10%	\$1,250		
	N	I/Y	PV	PMT	FV
Solve for					\$3,242.18

Enter	20	5%	\$1,250		
	N	I/Y	PV	PMT	FV
Solve for					\$3,316.62

3.

Enter	6	7%			\$13,827
	N	I/Y	PV	PMT	FV
Solve for			\$9,213.51		

Enter	11	15%			\$43,852
	N	I/Y	PV	PMT	FV
Solve for			\$9,425.69		

Enter	19	11%			\$725,380
	N	I/Y	PV	PMT	FV
Solve for			\$99,868.60		

Enter	29	18%			\$590,710
	N	I/Y	PV	PMT	FV
Solve for			\$4,861.79		

4.

Enter	4		\$189		±\$287
	N	I/Y	PV	PMT	FV
Solve for		11.01%			

Enter	8		\$410		$\pm \$887$
	N	I/Y	PV	PMT	FV
Solve for		10.13%			

Enter	14		\$51,700		$\pm \$152,184$
	N	I/Y	PV	PMT	FV
Solve for		8.02%			

Enter	27		\$21,400		$\pm \$538,600$
	N	I/Y	PV	PMT	FV
Solve for		12.69%			

5.

Enter		7%	\$625		$\pm \$1,104$
	N	I/Y	PV	PMT	FV
Solve for	8.41				

Enter		12%	\$810		$\pm \$5,275$
	N	I/Y	PV	PMT	FV
Solve for	16.53				

Enter		17%	\$16,500		$\pm \$245,830$
	N	I/Y	PV	PMT	FV
Solve for	17.21				

Enter		8%	\$21,500		$\pm \$215,000$
	N	I/Y	PV	PMT	FV
Solve for	29.92				

6.

Enter		6.25%	\$1		$\pm \$2$
	N	I/Y	PV	PMT	FV
Solve for	11.43				

Enter		6.25%	\$1		$\pm \$4$
	N	I/Y	PV	PMT	FV
Solve for	22.87				

7.

Enter	20	5.9%			\$425,000,000
	N	I/Y	PV	PMT	FV
Solve for			\$135,042,269.46		

8.

Enter	3		±\$1,680,000		\$1,100,000
	N	I/Y	PV	PMT	FV
Solve for		-13.17%			

11.

CFo	\$0	CFo	\$0	CFo	\$0
C01	\$795	C01	\$795	C01	\$795
F01	1	F01	1	F01	1
C02	\$945	C02	\$945	C02	\$945
F02	1	F02	1	F02	1
C03	\$1,325	C03	\$1,325	C03	\$1,325
F03	1	F03	1	F03	1
C04	\$1,860	C04	\$1,860	C04	\$1,860
F04	1	F04	1	F04	1
I = 10		I = 18		I = 24	
NPV CPT		NPV CPT		NPV CPT	
\$3,769.62		\$3,118.22		\$2,737.40	

12.

Enter	9	5%		\$4,350	
	N	I/Y	PV	PMT	FV
Solve for			\$30,919.02		

Enter	9	22%		\$4,350	
	N	I/Y	PV	PMT	FV
Solve for			\$16,470.34		

Enter	5	5%		\$6,900	
	N	I/Y	PV	PMT	FV
Solve for			\$29,873.39		

Enter	5	22%		\$6,900	
	N	I/Y	PV	PMT	FV
Solve for			\$19,759.11		

13.

Enter	15	7%		\$5,200	
	N	I/Y	PV	PMT	FV
Solve for			\$47,361.15		

Enter	40	7%		\$5,200	
	N	I/Y	PV	PMT	FV
Solve for			\$69,324.89		

Enter	75	7%		\$5,200	
	N	I/Y	PV	PMT	FV
Solve for			\$73,821.07		

15.

Enter	7.1%		4
	NOM	EFF	C/Y
Solve for		7.29%	

Enter	13.2%		12
	NOM	EFF	C/Y
Solve for		14.03%	

Enter	8.9%		365
	NOM	EFF	C/Y
Solve for		9.31%	

16.

Enter		10.1%	2
	NOM	EFF	C/Y
Solve for	9.86%		

Enter		17.4%	12
	NOM	EFF	C/Y
Solve for	16.15%		

Enter		8.6%	52
	NOM	EFF	C/Y
Solve for	8.26%		

17.

Enter	11.4%		12
	NOM	EFF	C/Y
Solve for		12.01%	

Enter	11.6%		2
	NOM	EFF	C/Y
Solve for		11.94%	

18. 2nd BGN 2nd SET

Enter	12		\$108	±\$10	
	N	I/Y	PV	PMT	FV
Solve for		1.98%			

$$\text{APR} = 1.98\% \times 52 = 102.77\%$$

Enter 102.77%
NOM **EFF** 52
C/Y
 Solve for 176.68%

19.

Enter 1.1%
N **I/Y** \$16,450 \pm \$400
PV **PMT** **FV**
 Solve for 55.04

20.

Enter 1,733.33%
NOM **EFF** 52
C/Y
 Solve for 313,916,515.69%

21.

Enter 11 8.9% \$1,000
N **I/Y** **PV** **PMT** **FV**
 Solve for \$2,554.50

Enter 11×2 8.9%/2 \$1,000
N **I/Y** **PV** **PMT** **FV**
 Solve for \$2,606.07

Enter 11×12 8.9%/12 \$1,000
N **I/Y** **PV** **PMT** **FV**
 Solve for \$2,652.19

23. Stock account:

Enter 360 10%/12 \$850
N **I/Y** **PV** **PMT** **FV**
 Solve for \$1,921,414.74

Bond account:

Enter 360 6%/12 \$350
N **I/Y** **PV** **PMT** **FV**
 Solve for \$351,580.26

Savings at retirement = \$1,921,414.74 + 351,580.26 = \$2,272,995.00

Enter 300 7%/12 \$2,272,995
N **I/Y** **PV** **PMT** **FV**
 Solve for \$16,065.06

24.

Enter	12/3		±\$1		\$4
	N	I/Y	PV	PMT	FV
Solve for		41.42%			

25.

Enter	6		±\$65,000		\$125,000
	N	I/Y	PV	PMT	FV
Solve for		11.51%			

Enter	10		±65,000		\$205,000
	N	I/Y	PV	PMT	FV
Solve for		12.17%			

28.

Enter	28	7%		\$7,300	
	N	I/Y	PV	PMT	FV
Solve for			\$88,600.91		

Enter	2	7%			\$88,600.91
	N	I/Y	PV	PMT	FV
Solve for			\$77,387.47		

29.

Enter	15	11%		\$750	
	N	I/Y	PV	PMT	FV
Solve for			\$5,393.15		

Enter	5	8%			\$5,393.15
	N	I/Y	PV	PMT	FV
Solve for			\$3,670.49		

30.

Enter	360	5.4%/12	.80(\$725,000)		
	N	I/Y	PV	PMT	FV
Solve for				\$3,256.88	

Enter	22 × 12	5.4%/12		\$3,256.88	
	N	I/Y	PV	PMT	FV
Solve for			\$502,540.87		

31.

Enter	6	1.99%/12	\$12,400		
	N	I/Y	PV	PMT	FV
Solve for					\$12,523.89

39.

CF₀	\$0
C01	\$1,400
F01	1
C02	\$0
F02	1
C03	\$2,300
F03	1
C04	\$2,700
F04	1

I = 7.1%

NPV CPT

\$5,231.56

PV of missing CF = \$6,700 – 5,231.56 = \$1,468.44

Value of missing CF:

Enter	2	7.1%	\$1,468.44		
	N	I/Y	PV	PMT	FV
Solve for					\$1,684.37

40.

CF₀	\$1,000,000
C01	\$1,335,000
F01	1
C02	\$1,670,000
F02	1
C03	\$2,005,000
F03	1
C04	\$2,340,000
F04	1
C05	\$2,675,000
F05	1
C06	\$3,010,000
F06	1
C07	\$3,345,000
F07	1
C08	\$3,680,000
F08	1
C09	\$4,015,000
F09	1
C010	\$4,350,000

I = 5.8%

NPV CPT

\$20,969,067.06

41.

Enter	360		.80(\$5,500,000)	±\$26,500	
	N	I/Y	PV	PMT	FV
Solve for		.504%			

$$\text{APR} = .504\% \times 12 = 6.04\%$$

Enter	6.04%		12
	NOM	EFF	C/Y
Solve for		6.21%	

42.

Enter	3	13%			\$165,000
	N	I/Y	PV	PMT	FV
Solve for			\$114,353.28		

$$\text{Profit} = \$114,353.28 - 103,000 = \$11,353.28$$

Enter	3		±\$103,000		\$165,000
	N	I/Y	PV	PMT	FV
Solve for		17.01%			

43.

Enter	20	6.7%		\$8,500	
	N	I/Y	PV	PMT	FV
Solve for			\$92,187.54		

Enter	5	6.7%			\$92,187.54
	N	I/Y	PV	PMT	FV
Solve for			\$66,657.67		

44.

Enter	96	6%/12		\$1,940	
	N	I/Y	PV	PMT	FV
Solve for			\$147,624.72		

Enter	84	11%/12		±\$1,940	\$147,624.72
	N	I/Y	PV	PMT	FV
Solve for			\$181,893.99		

45.

Enter	15 × 12	6.4%/12		\$1,175	
	N	I/Y	PV	PMT	FV
Solve for					\$353,610.97

$$FV = \$353,610.97 = PVe^{.07(15)}; PV = \$353,610.97e^{-1.05} = \$123,741.83$$

46. PV@ $t = 14$: $\$2,350/.063 = \$37,301.59$

Enter	7	6.3%			\$37,301.59
	N	I/Y	PV	PMT	FV
Solve for			\$24,321.73		

47.

Enter	12		\$26,000	$\pm \$2,519.83$	
	N	I/Y	PV	PMT	FV
Solve for		2.403%			

$$APR = 2.403\% \times 12 = 28.84\%$$

Enter	28.84%		12	
	NOM	EFF	C/Y	
Solve for		32.97%		

48. Monthly rate = $.10/12 = .0083$; semiannual rate = $(1.0083)^6 - 1 = 5.11\%$

Enter	10	5.11%		\$5,230	
	N	I/Y	PV	PMT	FV
Solve for			\$40,178.89		

Enter	8	5.11%			\$40,178.89
	N	I/Y	PV	PMT	FV
Solve for			\$26,977.40		

Enter	12	5.11%			\$40,178.89
	N	I/Y	PV	PMT	FV
Solve for			\$22,105.54		

Enter	18	5.11%			\$40,178.89
	N	I/Y	PV	PMT	FV
Solve for			\$16,396.55		

49.*a.*

Enter	5	7.8%		\$13,250	
	N	I/Y	PV	PMT	FV
Solve for			\$53,183.45		

2nd BGN 2nd SET

Enter	5	7.8%		\$13,250	
	N	I/Y	PV	PMT	FV
Solve for			\$57,331.76		

b.

Enter	5	7.8%		\$13,250	
	N	I/Y	PV	PMT	FV
Solve for					\$77,423.06

2nd BGN 2nd SET

Enter	5	7.8%		\$13,250	
	N	I/Y	PV	PMT	FV
Solve for					\$83,462.06

50. 2nd BGN 2nd SET

Enter	60	6.08%/12	\$84,000		
	N	I/Y	PV	PMT	FV
Solve for				\$1,618.88	

51. 2nd BGN 2nd SET

Enter	24	10.7%/12	\$3,350		
	N	I/Y	PV	PMT	FV
Solve for				\$154.29	

52. PV of college expenses:

Enter	4	7.9%		\$72,000	
	N	I/Y	PV	PMT	FV
Solve for			\$239,004.91		

Cost today of oldest child's expenses:

Enter	14	7.9%			\$239,004.91
	N	I/Y	PV	PMT	FV
Solve for			\$82,434.04		

Cost today of youngest child's expenses:

Enter	16	7.9%			\$239,004.91
	N	I/Y	PV	PMT	FV
Solve for			\$70,804.96		

Total cost today = \$82,434.04 + 70,804.96 = \$153,239.01

Enter	15	7.9%	\$153,239.01		
	N	I/Y	PV	PMT	FV
Solve for				\$17,793.68	

54. Option A:

Aftertax cash flows = Pretax cash flows(1 – tax rate)

Aftertax cash flows = \$250,000(1 – .28)

Aftertax cash flows = \$180,000

2ND BGN 2nd SET

Enter	31	5.85%		\$180,000	
	N	I/Y	PV	PMT	FV
Solve for			\$2,697,950.16		

Option B:

Aftertax cash flows = Pretax cash flows(1 – tax rate)

Aftertax cash flows = \$200,000(1 – .28)

Aftertax cash flows = \$144,000

Enter	30	5.85%		\$144,000	
	N	I/Y	PV	PMT	FV
Solve for			\$2,014,360.13		

\$2,014,360.13 + 530,000 = \$2,544,360.13

56.

Enter	5 × 12	6.8%/12	\$17,000		
	N	I/Y	PV	PMT	FV
Solve for				\$305.46	

Enter	34	6.8%/12		\$305.46	
	N	I/Y	PV	PMT	FV
Solve for			\$9,422.19		

Total payment = Amount due(1 + Prepayment penalty) + Last payment

Total payment = \$9,422.19(1 + .01) + \$305.46

Total payment = \$9,821.87

57. Pre-retirement APR:

Enter		11%	12
	NOM	EFF	C/Y
Solve for	10.48%		

Post-retirement APR:

Enter		8%	12
	NOM	EFF	C/Y
Solve for	7.72%		

At retirement, he needs:

Enter	240	7.72%/12		±\$17,500	\$1,500,000
	N	I/Y	PV	PMT	FV
Solve for			\$2,458,182.59		

In 10 years, his savings will be worth:

Enter	120	10.48%/12		\$1,800	
	N	I/Y	PV	PMT	FV
Solve for					\$379,062.59

After purchasing the cabin, he will have: $\$379,062.59 - \$350,000 = \$29,062.59$

Each month between Years 10 and 30, he needs to save:

Enter	240	10.48%/12	±\$29,062.59		\$2,458,182.59
	N	I/Y	PV	PMT	FV
Solve for				-\$2,750.46	

58. PV of purchase:

Enter	36	6%/12			\$22,000
	N	I/Y	PV	PMT	FV
Solve for			\$18,384.19		

$\$37,000 - \$18,384.19 = \$18,615.81$

PV of lease:

Enter	36	6%/12		\$580	
	N	I/Y	PV	PMT	FV
Solve for			\$19,065.19		

$\$19,065.19 + \$2,400 = \$21,465.19$

Buy the car.

You would be indifferent when the PV of the two cash flows are equal. The present value of the purchase decision must be \$21,465.19. Since the difference in the two cash flows is \$37,000 – 21,465.19 = \$15,534.81, this must be the present value of the future resale price of the car. The break-even resale price of the car is:

Enter	36	6%/12	\$15,534.81		
	N	I/Y	PV	PMT	FV
Solve for					\$18,590.21

59.

Enter	5.7%		365
	NOM	EFF	C/Y
Solve for		5.87%	

CF₀	\$7,900,000
C01	\$4,300,000
F01	1
C02	\$4,900,000
F02	1
C03	\$5,700,000
F03	1
C04	\$6,700,000
F04	1
C05	\$7,300,000
F05	1
C06	\$8,400,000
F06	1

I = 5.87%

NPV CPT

\$37,929,060.53

New contract value = \$37,929,060.53 + 3,500,000 = \$41,429,060.53

PV of payments = \$41,429,060.53 – 10,000,000 = \$31,429,060.53

Effective quarterly rate = $[1 + (.057/365)]^{91.25} - 1 = 1.435\%$

Enter	24	1.435%	\$31,429,060.53		
	N	I/Y	PV	PMT	FV
Solve for				\$1,557,264.39	

60.

Enter	1		\$16,720		±\$20,000
	N	I/Y	PV	PMT	FV
Solve for		19.62%			

61.

Enter		7.4%	12
	NOM	EFF	C/Y
Solve for	7.16%		

Enter	12	7.16%/12		\$39,000/12	
	N	I/Y	PV	PMT	FV
Solve for					\$40,305.70

Enter	1	7.4%	\$40,305.70		
	N	I/Y	PV	PMT	FV
Solve for					\$43,288.32

Enter	12	7.16%/12		\$43,000/12	
	N	I/Y	PV	PMT	FV
Solve for					\$44,439.62

Enter	60	7.16%/12		\$47,000/12	
	N	I/Y	PV	PMT	FV
Solve for			\$197,046.10		

$$\text{Award} = \$43,288.32 + \$44,439.62 + \$197,046.10 + \$150,000 + \$25,000 = \$459,774.05$$

62.

Enter	1		\$9,700		±\$10,800
	N	I/Y	PV	PMT	FV
Solve for		11.34%			

Enter	1		\$9,800		±\$11,100
	N	I/Y	PV	PMT	FV
Solve for		13.27%			

63. Refundable fee: With the \$2,900 application fee, you will need to borrow \$302,900 to have \$300,000 after deducting the fee. Solve for the payment under these circumstances.

Enter	30×12	5.30%/12	\$302,900		
	N	I/Y	PV	PMT	FV
Solve for				\$1,682.02	

Enter	30×12		\$300,000	±\$1,682.02	
	N	I/Y	PV	PMT	FV
Solve for		.4489%			

APR = .4489% \times 12 = 5.39%

Enter	5.39%		12
	NOM	EFF	C/Y
Solve for		5.52%	

Without refundable fee: $\text{APR} = 5.30\%$

Enter	5.30%		12
	NOM	EFF	C/Y
Solve for		5.43%	

64.

Enter	36		\$1,000	$\pm \$46.11$	
	N	I/Y	PV	PMT	FV
Solve for		3.04%			

$\text{APR} = 3.04\% \times 12 = 36.54\%$

Enter	36.54%		12
	NOM	EFF	C/Y
Solve for		43.32%	

65. Without fee:

Enter		18.6%/12	\$12,000	$\pm \$250$	
	N	I/Y	PV	PMT	FV
Solve for	88.59				

Enter		8.2%/12	\$12,000	$\pm \$250$	
	N	I/Y	PV	PMT	FV
Solve for	58.37				

With fee:

Enter		8.2%/12	\$12,240	$\pm \$250$	
	N	I/Y	PV	PMT	FV
Solve for	59.81				

66. Value at Year 6:

Enter	5	11%	\$500		
	N	I/Y	PV	PMT	FV
Solve for					\$842.53

Enter	4	11%	\$600		
	N	I/Y	PV	PMT	FV
Solve for					\$910.84

Enter	3	11%	\$700		
	N	I/Y	PV	PMT	FV
Solve for					\$957.34

Enter	2	11%	\$800		
	N	I/Y	PV	PMT	FV
Solve for					\$985.68

Enter	1	11%	\$900		
	N	I/Y	PV	PMT	FV
Solve for					\$999.00

So, at Year 6, the value is: $\$842.53 + 910.84 + 957.34 + 985.68 + 999.00$
 $+ 1,000 = \$5,695.39$

At Year 65, the value is:

Enter	59	7%	\$5,695.39		
	N	I/Y	PV	PMT	FV
Solve for					\$308,437.08

The policy is not worth buying; the future value of the payments is \$308,437.08 but the policy contract will pay off \$300,000.

67. Effective six-month rate = $(1 + \text{Daily rate})^{180} - 1$
 Effective six-month rate = $(1 + .09/360)^{180} - 1$
 Effective six-month rate = .0460 or 4.60%

Enter	40	4.60%		\$1,900,000	
	N	I/Y	PV	PMT	FV
Solve for			\$34,458,785.87		

Enter	1	4.60%			\$34,458,785.87
	N	I/Y	PV	PMT	FV
Solve for			\$32,942,697.79		

Value of winnings today = $\$32,942,697.79 + 5,500,000$
 Value of winnings today = \$38,442,697.79

68.

CFo	±\$9,000
C01	±\$9,000
F01	5
C02	\$20,000
F02	4
IRR CPT	
8.07%	

72.

a. $\text{APR} = 7.1\% \times 52 = 369.20\%$

Enter	369.20%		52
	NOM	EFF	C/Y
Solve for		3,440.40%	

b.

Enter	1		\$92.90		$\pm \$10.00$
	N	I/Y	PV	PMT	FV
Solve for		7.64%			

$$\text{APR} = 7.64\% \times 52 = 397.42\%$$

Enter	397.42%		52
	NOM	EFF	C/Y
Solve for		4,504.50%	

c.

Enter	4		\$68.43	$\pm \$25$	
	N	I/Y	PV	PMT	FV
Solve for		17.11%			

$$\text{APR} = 17.11\% \times 52 = 889.82\%$$

Enter	889.82%		52
	NOM	EFF	C/Y
Solve for		369,087.73%	

CHAPTER 4, APPENDIX

NET PRESENT VALUE: FIRST

PRINCIPLES OF FINANCE

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

1. The potential consumption for a borrower next year is the salary during the year, minus the repayment of the loan and interest to fund the current consumption. The amount that must be borrowed to fund this year's consumption is:

$$\text{Amount to borrow} = \$100,000 - 80,000 = \$20,000$$

Interest will be charged on the amount borrowed, so the repayment of this loan next year will be:

$$\text{Loan repayment} = \$20,000(1.10) = \$22,000$$

So, the consumption potential next year is the salary minus the loan repayment, or:

$$\text{Consumption potential} = \$90,000 - 22,000 = \$68,000$$

2. The potential consumption for a saver next year is the salary during the year, plus the savings from the current year and the interest earned. The amount saved this year is:

$$\text{Amount saved} = \$50,000 - 35,000 = \$15,000$$

The saver will earn interest over the year, so the value of the savings next year will be:

$$\text{Savings value in one year} = \$15,000(1.12) = \$16,800$$

So, the consumption potential next year is the salary plus the value of the savings, or:

$$\text{Consumption potential} = \$60,000 + 16,800 = \$76,800$$

3. Financial markets arise to facilitate borrowing and lending between individuals. By borrowing and lending, people can adjust their pattern of consumption over time to fit their particular preferences. This allows corporations to accept all positive NPV projects, regardless of the inter-temporal consumption preferences of the shareholders.

4. a. The present value of labor income is the total of the maximum current consumption. So, solving for the interest rate, we find:

$$\$86 = \$40 + \$50/(1 + r)$$

$$r = .0870, \text{ or } 8.70\%$$

- b. The NPV of the investment is the difference between the new maximum current consumption minus the old maximum current consumption, or:

$$\text{NPV} = \$98 - 86 = \$12$$

- c. The total maximum current consumption amount must be the present value of the equal annual consumption amount. If C is the equal annual consumption amount, we find:

$$\$98 = C + C/(1 + r)$$

$$\$98 = C + C/(1.0870)$$

$$C = \$51.04$$

5. a. The market interest rate must be the increase in the maximum current consumption to the maximum consumption next year, which is:

$$\text{Market interest rate} = \$90,000/\$80,000 - 1 = .1250, \text{ or } 12.50\%$$

- b. Harry will invest \$10,000 in financial assets and \$30,000 in productive assets today.

- c. $\text{NPV} = -\$30,000 + \$56,250/1.125$

$$\text{NPV} = \$20,000$$

CHAPTER 5

NET PRESENT VALUE AND OTHER INVESTMENT RULES

Answers to Concepts Review and Critical Thinking Questions

1. Assuming conventional cash flows, a payback period less than the project's life means that the NPV is positive for a zero discount rate, but nothing more definitive can be said. For discount rates greater than zero, the payback period will still be less than the project's life, but the NPV may be positive, zero, or negative, depending on whether the discount rate is less than, equal to, or greater than the IRR. The discounted payback includes the effect of the relevant discount rate. If a project's discounted payback period is less than the project's life, it must be the case that NPV is positive.
2. Assuming conventional cash flows, if a project has a positive NPV for a certain discount rate, then it will also have a positive NPV for a zero discount rate; thus, the payback period must be less than the project life. Since discounted payback is calculated at the same discount rate as is NPV, if NPV is positive, the discounted payback period must be less than the project's life. If NPV is positive, then the present value of future cash inflows is greater than the initial investment cost; thus, PI must be greater than 1. If NPV is positive for a certain discount rate R , then it will be zero for some larger discount rate R^* ; thus, the IRR must be greater than the required return.
3.
 - a. Payback period is the accounting break-even point of a series of cash flows. To actually compute the payback period, it is assumed that any cash flow occurring during a given period is realized continuously throughout the period, and not at a single point in time. The payback is then the point in time for the series of cash flows when the initial cash outlays are fully recovered. Given some predetermined cutoff for the payback period, the decision rule is to accept projects that pay back before this cutoff, and reject projects that take longer to pay back. The worst problem associated with the payback period is that it ignores the time value of money. In addition, the selection of a hurdle point for the payback period is an arbitrary exercise that lacks any steadfast rule or method. The payback period is biased towards short-term projects; it fully ignores any cash flows that occur after the cutoff point.
 - b. The IRR is the discount rate that causes the NPV of a series of cash flows to be identically zero. IRR can thus be interpreted as a financial break-even rate of return; at the IRR discount rate, the net value of the project is zero. The acceptance and rejection criteria are:
 - If $C_0 < 0$ and all future cash flows are positive, accept the project if the internal rate of return is greater than or equal to the discount rate.
 - If $C_0 < 0$ and all future cash flows are positive, reject the project if the internal rate of return is less than the discount rate.
 - If $C_0 > 0$ and all future cash flows are negative, accept the project if the internal rate of return is less than or equal to the discount rate.
 - If $C_0 > 0$ and all future cash flows are negative, reject the project if the internal rate of return is greater than the discount rate.

IRR is the discount rate that causes NPV for a series of cash flows to be zero. NPV is preferred in all situations to IRR; IRR can lead to ambiguous results if there are non-conventional cash flows, and it also may ambiguously rank some mutually exclusive projects. However, for stand-alone projects with conventional cash flows, IRR and NPV are interchangeable techniques.

- c. The profitability index is the present value of cash inflows relative to the project cost. As such, it is a benefit/cost ratio, providing a measure of the relative profitability of a project. The profitability index decision rule is to accept projects with a PI greater than one, and to reject projects with a PI less than one. The profitability index can be expressed as: $PI = (NPV + \text{cost})/\text{cost} = 1 + (NPV/\text{cost})$. If a firm has a basket of positive NPV projects and is subject to capital rationing, PI may provide a good ranking measure of the projects, indicating the “bang for the buck” of each particular project.
 - d. NPV is the present value of a project’s cash flows, including the initial outlay. NPV specifically measures, after considering the time value of money, the net increase or decrease in firm wealth due to the project. The decision rule is to accept projects that have a positive NPV, and reject projects with a negative NPV. NPV is superior to the other methods of analysis presented in the text because it has no serious flaws. The method unambiguously ranks mutually exclusive projects, and it can differentiate between projects of different scale and with different time horizons. The only drawback to NPV is that it relies on cash flow and discount rate values that are often estimates and thus not certain, but this is a problem shared by the other performance criteria as well. A project with $NPV = \$2,500$ implies that the total shareholder wealth of the firm will increase by \$2,500 if the project is accepted.
4. For a project with future cash flows that are an annuity:

$$\text{Payback} = I/C$$

And the IRR is:

$$0 = -I + C/IRR$$

Solving the IRR equation for IRR, we get:

$$IRR = C/I$$

Notice this is just the reciprocal of the payback. So:

$$IRR = 1/PB$$

For long-lived projects with relatively constant cash flows, the sooner the project pays back, the greater is the IRR, and the IRR is approximately equal to the reciprocal of the payback period.

5. There are a number of reasons. Two of the most important have to do with transportation costs and exchange rates. Manufacturing in the U.S. places the finished product much closer to the point of sale, resulting in significant savings in transportation costs. It also reduces inventories because goods spend less time in transit. Higher labor costs tend to offset these savings to some degree, at least compared to other possible manufacturing locations. Of great importance is the fact that manufacturing in the U.S. means that a much higher proportion of the costs are paid in dollars. Since sales are in dollars, the net effect is to immunize profits to a large extent against fluctuations in exchange rates. This issue is discussed in greater detail in the chapter on international finance.

6. The single biggest difficulty, by far, is coming up with reliable cash flow estimates. Determining an appropriate discount rate is also not a simple task. These issues are discussed in greater depth in the next several chapters. The payback approach is probably the simplest, followed by the AAR, but even these require revenue and cost projections. The discounted cash flow measures (discounted payback, NPV, IRR, and profitability index) are really only slightly more difficult in practice.
7. Yes, they are. Such entities generally need to allocate available capital efficiently, just as for-profits do. However, it is frequently the case that the “revenues” from not-for-profit ventures are intangible. For example, charitable giving has real opportunity costs, but the benefits are generally hard to measure. To the extent that benefits are measurable, the question of an appropriate required return remains. Payback rules are commonly used in such cases. Finally, realistic cost/benefit analysis along the lines indicated should definitely be used by the U.S. government and would go a long way toward balancing the budget!
8. The statement is false. If the cash flows of Project B occur early and the cash flows of Project A occur late, then, for a low discount rate, the NPV of A can exceed the NPV of B. Observe the following example.

	C_0	C_1	C_2	IRR	NPV @ 0%
Project A	–\$1,000,000	\$0	\$1,440,000	20%	\$440,000
Project B	–\$2,000,000	\$2,400,000	\$0	20%	400,000

However, in one particular case, the statement is true for equally risky projects. If the lives of the two projects are equal and the cash flows of Project B are twice the cash flows of Project A in every time period, the NPV of Project B will be twice the NPV of Project A.

9. Although the profitability index (PI) is higher for Project B than for Project A, Project A should be chosen because it has the greater NPV. Confusion arises because Project B requires a smaller investment than Project A. Since the denominator of the PI ratio is lower for Project B than for Project A, B can have a higher PI yet have a lower NPV. Only in the case of capital rationing could the company’s decision have been incorrect.
10.
 - a. Project A would have a higher IRR since the initial investment for Project A is less than that of Project B, if the cash flows for the two projects are identical.
 - b. Yes, since both the cash flows as well as the initial investment are twice that of Project B.
11. Project B’s NPV would be more sensitive to changes in the discount rate. The reason is the time value of money. Cash flows that occur further out in the future are always more sensitive to changes in the interest rate. This sensitivity is similar to the interest rate risk of a bond.
12. The MIRR is calculated by finding the present value of all cash outflows, the future value of all cash inflows to the end of the project, and then calculating the IRR of the two cash flows. As a result, the cash flows have been discounted or compounded by one interest rate (the required return), and then the interest rate between the two remaining cash flows is calculated. As such, the MIRR is not a true interest rate. In contrast, consider the IRR. If you take the initial investment, and calculate the future value at the IRR, you can replicate the future cash flows of the project exactly.

13. The statement is incorrect. It is true that if you calculate the future value of all intermediate cash flows to the end of the project at the required return, then calculate the NPV of this future value and the initial investment, you will get the same NPV. However, NPV says nothing about reinvestment of intermediate cash flows. The NPV is the present value of the project cash flows. What is actually done with those cash flows once they are generated is irrelevant. Put differently, the value of a project depends on the cash flows generated by the project, not on the future value of those cash flows. The fact that the reinvestment “works” only if you use the required return as the reinvestment rate is also irrelevant because reinvestment is not relevant to the value of the project in the first place.

One caveat: Our discussion here assumes that the cash flows are truly available once they are generated, meaning that it is up to firm management to decide what to do with the cash flows. In certain cases, there may be a requirement that the cash flows be reinvested. For example, in international investing, a company may be required to reinvest the cash flows in the country in which they are generated and not “repatriate” the money. Such funds are said to be “blocked” and reinvestment becomes relevant because the cash flows are not truly available.

14. The statement is incorrect. It is true that if you calculate the future value of all intermediate cash flows to the end of the project at the IRR, then calculate the IRR of this future value and the initial investment, you will get the same IRR. However, as in the previous question, what is done with the cash flows once they are generated does not affect the IRR. Consider the following example:

	C_0	C_1	C_2	IRR
Project A	-\$100	\$10	\$110	10%

Suppose this \$100 is a deposit into a bank account. The IRR of the cash flows is 10 percent. Does the IRR change if the Year 1 cash flow is reinvested in the account, or if it is withdrawn and spent on pizza? No. Finally, consider the yield to maturity calculation on a bond. If you think about it, the YTM is the IRR on the bond, but no mention of a reinvestment assumption for the bond coupons is suggested. The reason is that reinvestment is irrelevant to the YTM calculation; in the same way, reinvestment is irrelevant in the IRR calculation. Our caveat about blocked funds applies here as well.

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. a. The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

Project A:

$$\begin{aligned} \text{Cumulative cash flows Year 1} &= \$10,400 &&= \$10,400 \\ \text{Cumulative cash flows Year 2} &= \$10,400 + 5,900 &&= \$16,300 \end{aligned}$$

Companies can calculate a more precise value using fractional years. To calculate the fractional payback period, find the fraction of Year 2's cash flows that is needed for the company to have cumulative undiscounted cash flows of \$15,000. Divide the difference between the initial investment and the cumulative undiscounted cash flows as of Year 1 by the undiscounted cash flow of Year 2.

$$\text{Payback period} = 1 + (\$15,000 - 10,400)/\$5,900$$

$$\text{Payback period} = 1.780 \text{ years}$$

Project B:

$$\text{Cumulative cash flows Year 1} = \$12,700 = \$12,700$$

$$\text{Cumulative cash flows Year 2} = \$12,700 + 6,100 = \$18,800$$

$$\text{Cumulative cash flows Year 3} = \$12,700 + 6,100 + 5,300 = \$24,100$$

To calculate the fractional payback period, find the fraction of Year 3's cash flow that is needed for the company to have cumulative undiscounted cash flows of \$19,000. Divide the difference between the initial investment and the cumulative undiscounted cash flows as of Year 2 by the undiscounted cash flow of Year 3.

$$\text{Payback period} = 2 + (\$19,000 - 12,700 - 6,100)/\$5,300$$

$$\text{Payback period} = 2.038 \text{ years}$$

Since Project A has a shorter payback period than Project B has, the company should choose Project A.

- b. Discount each project's cash flows at 15 percent. Choose the project with the highest NPV.

Project A:

$$\text{NPV} = -\$15,000 + \$10,400/1.15 + \$5,900/1.15^2 + \$2,100/1.15^3$$

$$\text{NPV} = -\$114.49$$

Project B:

$$\text{NPV} = -\$19,000 + \$12,700/1.15 + \$6,100/1.15^2 + \$5,300/1.15^3$$

$$\text{NPV} = \$140.79$$

The firm should choose Project B since it has a higher NPV than Project A.

2. To calculate the payback period, we need to find the time that the project has taken to recover its initial investment. The cash flows in this problem are an annuity, so the calculation is simpler. If the initial cost is \$2,700, the payback period is:

$$\text{Payback} = 4 + (\$140/\$640) = 4.22 \text{ years}$$

There is a shortcut to calculate the payback period if the future cash flows are an annuity. Divide the initial cost by the annual cash flow. For the \$2,700 cost, the payback period is:

$$\text{Payback} = \$2,700/\$640 = 4.22 \text{ years}$$

For an initial cost of \$3,900, the payback period is:

$$\text{Payback} = \$3,900/\$640 = 6.09 \text{ years}$$

The payback period for an initial cost of \$6,800 is a little trickier. Notice that the total cash inflows after eight years will be:

$$\text{Total cash inflows} = 8(\$640) = \$5,120$$

If the initial cost is \$6,800, the project never pays back. Notice that if you use the shortcut for annuity cash flows, you get:

$$\text{Payback} = \$6,800/\$640 = 10.63 \text{ years}$$

This answer does not make sense since the cash flows stop after eight years, so there is no payback period.

3. When we use discounted payback, we need to find the value of all cash flows today. The value today of the project cash flows for the first four years is:

$$\text{Value today of Year 1 cash flow} = \$5,000/1.11 = \$4,504.50$$

$$\text{Value today of Year 2 cash flow} = \$5,500/1.11^2 = \$4,463.92$$

$$\text{Value today of Year 3 cash flow} = \$6,000/1.11^3 = \$4,387.15$$

$$\text{Value today of Year 4 cash flow} = \$7,000/1.11^4 = \$4,611.12$$

To find the discounted payback, we use these values to find the payback period. The discounted first year cash flow is \$4,504.50, so the discounted payback for an initial cost of \$8,000 is:

$$\text{Discounted payback} = 1 + (\$8,000 - 4,504.50)/\$4,463.92 = 1.78 \text{ years}$$

For an initial cost of \$12,000, the discounted payback is:

$$\text{Discounted payback} = 2 + (\$12,000 - 4,504.50 - 4,463.92)/\$4,387.15 = 2.69 \text{ years}$$

Notice the calculation of discounted payback. We know the payback period is between two and three years, so we subtract the discounted values of the Year 1 and Year 2 cash flows from the initial cost. This is the numerator, which is the discounted amount we still need to make to recover our initial investment. We divide this amount by the discounted amount we will earn in Year 3 to get the fractional portion of the discounted payback.

If the initial cost is \$16,000, the discounted payback is:

$$\text{Discounted payback} = 3 + (\$16,000 - 4,504.50 - 4,463.92 - 4,387.15)/\$4,611.12 = 3.57 \text{ years}$$

4. To calculate the discounted payback, discount all future cash flows back to the present, and use these discounted cash flows to calculate the payback period. To find the fractional year, we divide the amount we need to make in the last year to pay back the project by the amount we will make. Doing so, we find:

$$r = 0\%: \quad 3 + (\$3,400/\$3,900) = 3.87 \text{ years}$$

$$\text{Discounted payback} = \text{Regular payback} = 3.87 \text{ years}$$

$$r = 10\% \quad \$3,900/1.10 + \$3,900/1.10^2 + \$3,900/1.10^3 + \$3,900/1.10^4 + \$3,900/1.10^5 = \$14,784.07$$

$$\$3,900/1.10^6 = \$2,201.45$$

$$\text{Discounted payback} = 5 + (\$15,100 - \$14,784.07)/\$2,201.45 = 5.14 \text{ years}$$

$$r = 17\%: \$3,900/1.17 + \$3,900/1.17^2 + \$3,900/1.17^3 + \$3,900/1.17^4 + \$3,900/1.17^5$$

$$+ \$3,900/1.17^6 = \$13,997.82$$

The project never pays back.

5. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this project is:

$$0 = C_0 + C_1/(1 + \text{IRR}) + C_2/(1 + \text{IRR})^2 + C_3/(1 + \text{IRR})^3$$

$$0 = -\$27,000 + \$13,100/(1 + \text{IRR}) + \$17,200/(1 + \text{IRR})^2 + \$8,400/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 21.80\%$$

Since the IRR is greater than the required return we would accept the project.

6. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for Project A is:

$$0 = C_0 + C_1/(1 + \text{IRR}) + C_2/(1 + \text{IRR})^2 + C_3/(1 + \text{IRR})^3$$

$$0 = -\$7,300 + \$3,940/(1 + \text{IRR}) + \$3,450/(1 + \text{IRR})^2 + \$2,480/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 18.24\%$$

And the IRR for Project B is:

$$0 = C_0 + C_1/(1 + \text{IRR}) + C_2/(1 + \text{IRR})^2 + C_3/(1 + \text{IRR})^3$$

$$0 = -\$4,390 + \$2,170/(1 + \text{IRR}) + \$2,210/(1 + \text{IRR})^2 + \$1,730/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 19.31\%$$

7. The profitability index is defined as the PV of the future cash flows divided by the PV of the initial cost. The cash flows from this project are an annuity, so the equation for the profitability index is:

$$\text{PI} = C(\text{PVIFA}_{R,t})/C_0$$

$$\text{PI} = \$67,000(\text{PVIFA}_{13\%,7})/\$325,000$$

$$\text{PI} = .912$$

8. a. The profitability index is the present value of the future cash flows divided by the initial cost. So, for Project Alpha, the profitability index is:

$$PI_{\text{Alpha}} = [\$1,500/1.085 + \$1,300/1.085^2 + \$1,100/1.085^3]/\$2,700 = 1.240$$

And for Project Beta the profitability index is:

$$PI_{\text{Beta}} = [\$900/1.085 + \$2,600/1.085^2 + \$3,200/1.085^3]/\$4,100 = 1.352$$

- b. According to the profitability index, you would accept Project Beta. However, remember the profitability index rule can lead to an incorrect decision when ranking mutually exclusive projects.

Intermediate

9. a. To have a payback equal to the project's life, given C is a constant cash flow for N years:

$$C = I/N$$

- b. To have a positive NPV, $I < C (PVIFA_{r\%, N})$. Thus, $C > I/(PVIFA_{r\%, N})$.

- c. Benefit = $C(PVIFA_{r\%, N}) = 2 \times \text{costs} = 2I$
 $C = 2I/(PVIFA_{r\%, N})$

10. a. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this project is:

$$\begin{aligned} 0 &= C_0 + C_1/(1 + \text{IRR}) + C_2/(1 + \text{IRR})^2 + C_3/(1 + \text{IRR})^3 + C_4/(1 + \text{IRR})^4 \\ 0 &= \$8,700 - \$3,900/(1 + \text{IRR}) - \$2,900/(1 + \text{IRR})^2 - \$2,300/(1 + \text{IRR})^3 \\ &\quad - \$1,800/(1 + \text{IRR})^4 \end{aligned}$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 11.21\%$$

- b. This problem differs since the initial cash flow is positive and all future cash flows are negative. In other words, **this is a financing-type project**. For **financing projects, accept the project when the IRR is less than the discount rate**. Reject the project when the IRR is greater than the discount rate.

$$\text{IRR} = 11.21\%$$

$$\text{Discount Rate} = 10\%$$

$$\text{IRR} > \text{Discount Rate}$$

Reject the offer when the discount rate is less than the IRR.

- c. Using the same reason as part *b*, we would accept the project if the discount rate is 20 percent.

$$\text{IRR} = 11.21\%$$

$$\text{Discount Rate} = 20\%$$

$$\text{IRR} < \text{Discount Rate}$$

Accept the offer when the discount rate is greater than the IRR.

- d. The NPV is the sum of the present value of all cash flows, so the NPV of the project if the discount rate is 10 percent will be:

$$\text{NPV} = \$8,700 - \$3,900/1.1 - \$2,900/1.1^2 - \$2,300/1.1^3 - \$1,800/1.1^4$$

$$\text{NPV} = -\$199.60$$

When the discount rate is 10 percent, the NPV of the offer is $-\$199.60$. Reject the offer.

And the NPV of the project if the discount rate is 20 percent will be:

$$\text{NPV} = \$8,700 - \$3,900/1.2 - \$2,900/1.2^2 - \$2,300/1.2^3 - \$1,800/1.2^4$$

$$\text{NPV} = \$1,237.04$$

When the discount rate is 20 percent, the NPV of the offer is $\$1,237.04$. Accept the offer.

- e. Yes, the decisions under the NPV rule are consistent with the choices made under the IRR rule since the signs of the cash flows change only once.

11. a. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR for each project is:

Deepwater Fishing IRR:

$$0 = C_0 + C_1/(1 + \text{IRR}) + C_2/(1 + \text{IRR})^2 + C_3/(1 + \text{IRR})^3$$

$$0 = -\$725,000 + \$270,000/(1 + \text{IRR}) + \$420,000/(1 + \text{IRR})^2 + \$380,000/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 20.96\%$$

Submarine Ride IRR:

$$0 = C_0 + C_1/(1 + \text{IRR}) + C_2/(1 + \text{IRR})^2 + C_3/(1 + \text{IRR})^3$$

$$0 = -\$1,450,000 + \$820,000/(1 + \text{IRR}) + \$650,000/(1 + \text{IRR})^2 + \$540,000/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 19.87\%$$

Based on the IRR rule, the deepwater fishing project should be chosen because it has the higher IRR.

- b. To calculate the incremental IRR, we subtract the smaller project's cash flows from the larger project's cash flows. In this case, we subtract the deepwater fishing cash flows from the submarine ride cash flows. The incremental IRR is the IRR of these incremental cash flows. So, the incremental cash flows of the submarine ride are:

	Year 0	Year 1	Year 2	Year 3
Submarine Ride	-\$1,450,000	\$820,000	\$650,000	\$540,000
Deepwater Fishing	-725,000	270,000	420,000	380,000
Submarine – Fishing	-\$725,000	\$550,000	\$230,000	\$160,000

Setting the present value of these incremental cash flows equal to zero, we find the incremental IRR is:

$$0 = C_0 + C_1/(1 + \text{IRR}) + C_2/(1 + \text{IRR})^2 + C_3/(1 + \text{IRR})^3$$

$$0 = -\$725,000 + \$550,000/(1 + \text{IRR}) + \$230,000/(1 + \text{IRR})^2 + \$160,000/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{Incremental IRR} = 18.40\%$$

For investing-type projects, accept the larger project when the incremental IRR is greater than the discount rate. Since the incremental IRR, 18.40 percent, is greater than the required rate of return of 14 percent, choose the submarine ride project. Note that this is not the choice when evaluating only the IRR of each project. The IRR decision rule is flawed because there is a scale problem. That is, the submarine ride has a greater initial investment than does the deepwater fishing project. This problem is corrected by calculating the IRR of the incremental cash flows, or by evaluating the NPV of each project.

- c. The NPV is the sum of the present value of the cash flows from the project, so the NPV of each project will be:

Deepwater Fishing:

$$\text{NPV} = -\$725,000 + \$270,000/1.14 + \$420,000/1.14^2 + \$380,000/1.14^3$$

$$\text{NPV} = \$91,507.64$$

Submarine Ride:

$$\text{NPV} = -\$1,450,000 + \$820,000/1.14 + \$650,000/1.14^2 + \$540,000/1.14^3$$

$$\text{NPV} = \$133,936.76$$

Since the NPV of the submarine ride project is greater than the NPV of the deepwater fishing project, choose the submarine ride project. The incremental IRR rule is always consistent with the NPV rule.

12. a. The profitability index is the PV of the future cash flows divided by the initial investment. The cash flows for both projects are an annuity, so:

$$PI_I = \$23,200(PVIFA_{10\%,3})/\$45,000 = 1.282$$

$$PI_{II} = \$11,700(PVIFA_{10\%,3})/\$19,800 = 1.470$$

The profitability index decision rule implies that we accept Project II, since PI_{II} is greater than PI_I .

- b. The NPV of each project is:

$$NPV_I = -\$45,000 + \$23,200(PVIFA_{10\%,3}) = \$12,694.97$$

$$NPV_{II} = -\$19,800 + \$11,700(PVIFA_{10\%,3}) = \$9,296.17$$

The NPV decision rule implies accepting Project I, since NPV_I is greater than NPV_{II} .

- c. Using the profitability index to compare mutually exclusive projects can be ambiguous when the magnitudes of the cash flows for the two projects are of different scales. In this problem, Project I is more than twice as large as Project II and produces a larger NPV, yet the profitability index criterion implies that Project II is more acceptable.

13. a. The equation for the NPV of the project is:

$$NPV = -\$65,000,000 + \$92,000,000/1.1 - \$11,000,000/1.1^2$$

$$NPV = \$9,545,454.55$$

The NPV is greater than 0, so we would accept the project.

- b. The equation for the IRR of the project is:

$$0 = -\$65,000,000 + \$92,000,000/(1 + IRR) - \$11,000,000/(1 + IRR)^2$$

From Descartes' rule of signs, we know there are two IRRs since the cash flows change signs twice. From trial and error, the two IRRs are:

$$IRR = 28.35\%, -86.82\%$$

When there are multiple IRRs, the IRR decision rule is ambiguous. Both IRRs are correct; that is, both discount rates make the NPV of the project equal to zero. If we are evaluating whether or not to accept this project, we would not want to use the IRR to make our decision.

14. a. The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

Board game:

$$\text{Cumulative cash flows Year 1} = \$670 = \$670$$

$$\text{Cumulative cash flows Year 2} = \$670 + 510 = \$1,180$$

$$\text{Payback period} = 1 + (\$850 - 670)/\$510 = 1.35 \text{ years}$$

DVD:

$$\text{Cumulative cash flows Year 1} = \$1,300 = \$1,300$$

$$\text{Cumulative cash flows Year 2} = \$1,300 + 750 = \$2,050$$

$$\text{Payback period} = 1 + (\$1,700 - 1,300)/\$750$$

$$\text{Payback period} = 1.53 \text{ years}$$

Since the board game has a shorter payback period than the DVD project, the company should choose the board game.

- b. The NPV is the sum of the present value of the cash flows from the project, so the NPV of each project will be:

Board game:

$$\text{NPV} = -\$850 + \$670/1.10 + \$510/1.10^2 + \$90/1.10^3$$

$$\text{NPV} = \$248.20$$

DVD:

$$\text{NPV} = -\$1,700 + \$1,300/1.10 + \$750/1.10^2 + \$350/1.10^3$$

$$\text{NPV} = \$364.61$$

Since the NPV of the DVD is greater than the NPV of the board game, choose the DVD.

- c. The IRR is the interest rate that makes the NPV of a project equal to zero. So, the IRR of each project is:

Board game:

$$0 = -\$850 + \$670/(1 + \text{IRR}) + \$510/(1 + \text{IRR})^2 + \$90/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 30.86\%$$

DVD:

$$0 = -\$1,700 + \$1,300/(1 + \text{IRR}) + \$750/(1 + \text{IRR})^2 + \$350/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 24.96\%$$

Since the IRR of the board game is greater than the IRR of the DVD, IRR implies we choose the board game. Note that this is the choice when evaluating only the IRR of each project. The IRR decision rule is flawed because there is a scale problem. That is, the DVD has a greater initial investment than does the board game. This problem is corrected by calculating the IRR of the incremental cash flows, or by evaluating the NPV of each project.

- d. To calculate the incremental IRR, we subtract the smaller project's cash flows from the larger project's cash flows. In this case, we subtract the board game cash flows from the DVD cash flows. The incremental IRR is the IRR of these incremental cash flows. So, the incremental cash flows of the DVD are:

	Year 0	Year 1	Year 2	Year 3
DVD	−\$1,700	\$1,300	\$750	\$350
Board game	−850	670	510	90
DVD – Board game	−\$850	\$630	\$240	\$260

Setting the present value of these incremental cash flows equal to zero, we find the incremental IRR is:

$$0 = C_0 + C_1/(1 + \text{IRR}) + C_2/(1 + \text{IRR})^2 + C_3/(1 + \text{IRR})^3$$

$$0 = -\$850 + \$630/(1 + \text{IRR}) + \$240/(1 + \text{IRR})^2 + \$260/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{Incremental IRR} = 19.29\%$$

For investing-type projects, accept the larger project when the incremental IRR is greater than the discount rate. Since the incremental IRR, 19.29 percent, is greater than the required rate of return of 10 percent, choose the DVD project.

15. a. The profitability index is the PV of the future cash flows divided by the initial investment. The profitability index for each project is:

$$\text{PI}_{\text{CDMA}} = [\$23,000,000/1.10 + \$16,000,000/1.10^2 + \$6,000,000/1.10^3]/\$18,000,000 = 2.15$$

$$\text{PI}_{\text{G4}} = [\$21,000,000/1.10 + \$51,000,000/1.10^2 + \$41,000,000/1.10^3]/\$25,000,000 = 3.68$$

$$\text{PI}_{\text{Wi-Fi}} = [\$39,000,000/1.10 + \$66,000,000/1.10^2 + \$42,000,000/1.10^3]/\$43,000,000 = 2.83$$

The profitability index implies we accept the G4 project. Remember this is not necessarily correct because the profitability index does not necessarily rank projects with different initial investments correctly.

- b. The NPV of each project is:

$$\begin{aligned} \text{NPV}_{\text{CDMA}} &= -\$18,000,000 + \$23,000,000/1.10 + \$16,000,000/1.10^2 + \$6,000,000/1.10^3 \\ \text{NPV}_{\text{CDMA}} &= \$20,640,120.21 \end{aligned}$$

$$\begin{aligned} \text{NPV}_{\text{G4}} &= -\$25,000,000 + \$21,000,000/1.10 + \$51,000,000/1.10^2 + \$41,000,000/1.10^3 \\ \text{NPV}_{\text{G4}} &= \$67,043,576.26 \end{aligned}$$

$$\begin{aligned} \text{NPV}_{\text{Wi-Fi}} &= -\$43,000,000 + \$39,000,000/1.10 + \$66,000,000/1.10^2 + \$42,000,000/1.10^3 \\ \text{NPV}_{\text{Wi-Fi}} &= \$78,555,221.64 \end{aligned}$$

NPV implies we accept the Wi-Fi project since it has the highest NPV. This is the correct decision if the projects are mutually exclusive.

- c. We would like to invest in all three projects since each has a positive NPV. If the budget is limited to \$43 million, we can only accept the CDMA project and the G4 project, or the Wi-Fi project. NPV is additive across projects and the company. The total NPV of the CDMA project and the G4 project is:

$$\begin{aligned} \text{NPV}_{\text{CDMA and G4}} &= \$20,640,120.21 + 67,043,576.26 \\ \text{NPV}_{\text{CDMA and G4}} &= \$87,683,696.47 \end{aligned}$$

This is greater than the Wi-Fi project, so we should accept the CDMA project and the G4 project.

16. a. The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

AZM Mini-SUV:

$$\begin{aligned} \text{Cumulative cash flows Year 1} &= \$373,000 &= \$373,000 \\ \text{Cumulative cash flows Year 2} &= \$373,000 + 219,000 &= \$592,000 \end{aligned}$$

$$\text{Payback period} = 1 + \$202,000/\$219,000 = 1.92 \text{ years}$$

AZF Full-SUV:

$$\begin{aligned} \text{Cumulative cash flows Year 1} &= \$395,000 &= \$395,000 \\ \text{Cumulative cash flows Year 2} &= \$395,000 + 477,000 &= \$872,000 \\ \text{Cumulative cash flows Year 3} &= \$395,000 + 477,000 + 339,000 &= \$1,211,000 \end{aligned}$$

$$\text{Payback period} = 2 + \$108,000/\$339,000 = 2.32 \text{ years}$$

Since the AZM has a shorter payback period than the AZF, the company should choose the AZM. Remember the payback period does not necessarily rank projects correctly.

- b. The NPV of each project is:

$$\begin{aligned} \text{NPV}_{\text{AZM}} &= -\$575,000 + \$373,000/1.10 + \$219,000/1.10^2 + \$185,000/1.10^3 \\ \text{NPV}_{\text{AZM}} &= \$84,075.88 \end{aligned}$$

$$\begin{aligned}\text{NPV}_{\text{AZF}} &= -\$980,000 + \$395,000/1.10 + \$477,000/1.10^2 + \$339,000/1.10^3 \\ \text{NPV}_{\text{AZF}} &= \$28,001.50\end{aligned}$$

The NPV criteria implies we accept the AZM because it has the highest NPV.

- c. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR of the AZM is:

$$0 = -\$575,000 + \$373,000/(1 + \text{IRR}) + \$219,000/(1 + \text{IRR})^2 + \$185,000/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR}_{\text{AZM}} = 19.36\%$$

And the IRR of the AZF is:

$$0 = -\$980,000 + \$395,000/(1 + \text{IRR}) + \$477,000/(1 + \text{IRR})^2 + \$339,000/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR}_{\text{AZF}} = 11.65\%$$

The IRR criteria implies we accept the AZM because it has the highest IRR. Remember the IRR does not necessarily rank projects correctly.

- d. Incremental IRR analysis is not necessary. The AZM has the smallest initial investment, and the largest NPV, so it should be accepted.
17. a. The profitability index is the PV of the future cash flows divided by the initial investment. The profitability index for each project is:

$$\text{PI}_A = [\$165,000/1.12 + \$165,000/1.12^2]/\$225,000 = 1.24$$

$$\text{PI}_B = [\$300,000/1.12 + \$300,000/1.12^2]/\$450,000 = 1.13$$

$$\text{PI}_C = [\$180,000/1.12 + \$135,000/1.12^2]/\$225,000 = 1.19$$

- b. The NPV of each project is:

$$\begin{aligned}\text{NPV}_A &= -\$225,000 + \$165,000/1.12 + \$165,000/1.12^2 \\ \text{NPV}_A &= \$53,858.42\end{aligned}$$

$$\begin{aligned}\text{NPV}_B &= -\$450,000 + \$300,000/1.12 + \$300,000/1.12^2 \\ \text{NPV}_B &= \$57,015.31\end{aligned}$$

$$\begin{aligned}\text{NPV}_C &= -\$225,000 + \$180,000/1.12 + \$135,000/1.12^2 \\ \text{NPV}_C &= \$43,335.46\end{aligned}$$

- c. Accept Projects A, B, and C. Since the projects are independent, accept all three projects because the respective profitability index of each is greater than 1.
- d. Accept Project B. Since the Projects are mutually exclusive, choose the Project with the highest PI, while taking into account the scale of the Project. Because Projects A and C have the same initial investment, the problem of scale does not arise when comparing the profitability indexes. Based on the profitability index rule, Project C can be eliminated because its PI is less than the PI of Project A. Because of the problem of scale, we cannot compare the PIs of Projects A and B. However, we can calculate the PI of the incremental cash flows of the two projects, which are:

Project	C_0	C_1	C_2
B – A	–\$225,000	\$135,000	\$135,000

When calculating incremental cash flows, remember to subtract the cash flows of the project with the smaller initial cash outflow from those of the project with the larger initial cash outflow. This procedure insures that the incremental initial cash outflow will be negative. The incremental PI calculation is:

$$\text{PI}(B - A) = [\$135,000/1.12 + \$135,000/1.12^2]/\$225,000$$

$$\text{PI}(B - A) = 1.014$$

The company should accept Project B since the PI of the incremental cash flows is greater than 1.

- e. Remember that the NPV is additive across projects. Since we can spend \$450,000, we could take Project B or Project A and Project C. Since the combined NPV of Projects A and C is higher than Project B, we should take both Projects A and C.
18. a. The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

Dry Prepeg:

$$\begin{aligned} \text{Cumulative cash flows Year 1} &= \$900,000 &= \$900,000 \\ \text{Cumulative cash flows Year 2} &= \$900,000 + 700,000 &= \$1,600,000 \end{aligned}$$

$$\text{Payback period} = 1 + (\$600,000/\$700,000) = 1.86 \text{ years}$$

Solvent Prepeg:

$$\begin{aligned} \text{Cumulative cash flows Year 1} &= \$345,000 &= \$345,000 \\ \text{Cumulative cash flows Year 2} &= \$345,000 + 570,000 &= \$915,000 \end{aligned}$$

$$\text{Payback period} = 1 + (\$305,000/\$570,000) = 1.54 \text{ years}$$

Since the solvent prepeg has a shorter payback period than the dry prepeg, the company should choose the solvent prepeg. Remember the payback period does not necessarily rank projects correctly.

- b. The NPV of each project is:

$$\begin{aligned}\text{NPV}_{\text{Dry prepeg}} &= -\$1,500,000 + \$900,000/1.08 + \$700,000/1.08^2 + \$725,000/1.08^3 \\ \text{NPV}_{\text{Dry prepeg}} &= \$508,998.88\end{aligned}$$

$$\begin{aligned}\text{NPV}_{\text{Solvent prepeg}} &= -\$650,000 + \$345,000/1.08 + \$570,000/1.08^2 + \$360,000/1.08^3 \\ \text{NPV}_{\text{Solvent prepeg}} &= \$443,907.18\end{aligned}$$

The NPV criteria implies accepting the dry prepeg because it has the highest NPV.

- c. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR of the dry prepeg is:

$$0 = -\$1,500,000 + \$900,000/(1 + \text{IRR}) + \$700,000/(1 + \text{IRR})^2 + \$725,000/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR}_{\text{Dry prepeg}} = 26.84\%$$

And the IRR of the solvent prepeg is:

$$0 = -\$650,000 + \$345,000/(1 + \text{IRR}) + \$570,000/(1 + \text{IRR})^2 + \$360,000/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR}_{\text{Solvent prepeg}} = 42.16\%$$

The IRR criteria implies accepting the solvent prepeg because it has the highest IRR. Remember the IRR does not necessarily rank projects correctly.

- d. Incremental IRR analysis is necessary. The solvent prepeg has a higher IRR, but is relatively smaller in terms of investment and NPV. In calculating the incremental cash flows, we subtract the cash flows from the project with the smaller initial investment from the cash flows of the project with the large initial investment, so the incremental cash flows are:

	Year 0	Year 1	Year 2	Year 3
Dry prepeg	-\$1,500,000	\$900,000	\$700,000	\$725,000
Solvent prepeg	-650,000	345,000	570,000	360,000
Dry prepeg - Solvent prepeg	-\$ 850,000	\$555,000	\$130,000	\$365,000

Setting the present value of these incremental cash flows equal to zero, we find the incremental IRR is:

$$0 = -\$850,000 + \$555,000/(1 + \text{IRR}) + \$130,000/(1 + \text{IRR})^2 + \$365,000/(1 + \text{IRR})^3$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{Incremental IRR} = 12.68\%$$

3

For investing-type projects, we accept the larger project when the incremental IRR is greater than the discount rate. Since the incremental IRR, 12.68%, is greater than the required rate of return of 8 percent, we choose the dry prepeg.

19. a. The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

NP-30:

Cumulative cash flows Year 1 = \$239,000	= \$239,000
Cumulative cash flows Year 2 = \$239,000 + 239,000	= \$478,000
Cumulative cash flows Year 3 = \$239,000 + 239,000 + 239,000	= \$717,000

$$\text{Payback period} = 3 + (\$18,000/\$239,000) = 3.08 \text{ years}$$

NX-20:

Cumulative cash flows Year 1 = \$130,000	= \$130,000
Cumulative cash flows Year 2 = \$130,000 + 143,000	= \$273,000
Cumulative cash flows Year 3 = \$130,000 + 143,000 + 157,300	= \$430,300
Cumulative cash flows Year 4 = \$130,000 + 143,000 + 157,300 + 173,030	= \$603,330

$$\text{Payback period} = 3 + (\$29,700/\$173,030) = 3.17 \text{ years}$$

Since the NP-30 has a shorter payback period than the NX-20, the company should choose the NP-30. Remember the payback period does not necessarily rank projects correctly.

- b. The IRR is the interest rate that makes the NPV of the project equal to zero, so the IRR of each project is:

NP-30:

$$0 = -\$735,000 + \$239,000(\{1 - [1/(1 + \text{IRR})^5]\}/\text{IRR})$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR}_{\text{NP-30}} = 18.74\%$$

And the IRR of the NX-20 is:

$$0 = -\$460,000 + \$130,000/(1 + \text{IRR}) + \$143,000/(1 + \text{IRR})^2 + \$157,300/(1 + \text{IRR})^3 + \$173,030/(1 + \text{IRR})^4 + \$190,333/(1 + \text{IRR})^5$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR}_{\text{NX-20}} = 19.87\%$$

The IRR criteria implies accepting the NX-20.

- c. The profitability index is the present value of all subsequent cash flows, divided by the initial investment, so the profitability index of each project is:

$$\begin{aligned} \text{PI}_{\text{NP-30}} &= (\$239,000\{[1 - (1/1.15)^5]/.15\})/\$735,000 \\ \text{PI}_{\text{NP-30}} &= 1.090 \end{aligned}$$

$$\begin{aligned} \text{PI}_{\text{NX-20}} &= [\$130,000/1.15 + \$143,000/1.15^2 + \$157,300/1.15^3 + \$173,030/1.15^4 \\ &\quad + \$190,333/1.15^5]/\$460,000 \\ \text{PI}_{\text{NX-20}} &= 1.126 \end{aligned}$$

The PI criteria implies accepting the NX-20.

- d. The NPV of each project is:

$$\begin{aligned} \text{NPV}_{\text{NP-30}} &= -\$735,000 + \$239,000\{[1 - (1/1.15)^5]/.15\} \\ \text{NPV}_{\text{NP-30}} &= \$66,165.07 \end{aligned}$$

$$\begin{aligned} \text{NPV}_{\text{NX-20}} &= -\$460,000 + \$130,000/1.15 + \$143,000/1.15^2 + \$157,300/1.15^3 \\ &\quad + \$173,030/1.15^4 + \$190,333/1.15^5 \\ \text{NPV}_{\text{NX-20}} &= \$58,158.93 \end{aligned}$$

The NPV criteria implies accepting the NP-30.

20. The MIRRs for the project with all three approaches is:

Discounting approach:

In the discounting approach, we find the value of all negative cash outflows at Time 0, while any positive cash inflows remain at the time at which they occur. So, discounting the cash outflows to Time 0, we find:

$$\begin{aligned} \text{Time 0 cash flow} &= -\$47,000 - \$9,500/1.10^5 \\ \text{Time 0 cash flow} &= -\$52,898.75 \end{aligned}$$

So, the MIRR using the discounting approach is:

$$\begin{aligned} 0 &= -\$52,898.75 + \$16,900/(1 + \text{MIRR}) + \$20,300/(1 + \text{MIRR})^2 + \$25,800/(1 + \text{MIRR})^3 \\ &\quad + \$19,600/(1 + \text{MIRR})^4 \end{aligned}$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find:

$$\text{MIRR} = 19.68\%$$

Reinvestment approach:

In the reinvestment approach, we find the future value of all cash except the initial cash flow at the end of the project. So, reinvesting the cash flows to Time 5, we find:

$$\begin{aligned} \text{Time 5 cash flow} &= \$16,900(1.10^4) + \$20,300(1.10^3) + \$25,800(1.10^2) + \$19,600(1.10) - \$9,500 \\ \text{Time 5 cash flow} &= \$95,040.59 \end{aligned}$$

So, the MIRR using the reinvestment approach is:

$$0 = -\$47,000 + \$95,040.59/(1 + \text{MIRR})^5$$

$$\$95,040.59/\$47,000 = (1 + \text{MIRR})^5$$

$$\text{MIRR} = (\$95,040.59/\$47,000)^{1/5} - 1$$

$$\text{MIRR} = .1512, \text{ or } 15.12\%$$

Combination approach:

In the combination approach, we find the value of all cash outflows at Time 0, and the value of all cash inflows at the end of the project. So, the value of the cash flows is:

$$\text{Time 0 cash flow} = -\$47,000 - \$9,500/1.10^5$$

$$\text{Time 0 cash flow} = -\$52,898.75$$

$$\text{Time 5 cash flow} = \$16,900(1.10^4) + \$20,300(1.10^3) + \$25,800(1.10^2) + \$19,600(1.10)$$

$$\text{Time 5 cash flow} = \$104,540.59$$

So, the MIRR using the combination approach is:

$$0 = -\$52,898.75 + \$104,540.59/(1 + \text{MIRR})^5$$

$$\$104,540.59/\$52,898.75 = (1 + \text{MIRR})^5$$

$$\text{MIRR} = (\$104,540.59/\$52,898.75)^{1/5} - 1$$

$$\text{MIRR} = .1460, \text{ or } 14.60\%$$

Intermediate

21. With different discounting and reinvestment rates, we need to make sure to use the appropriate interest rates. The MIRRs for the project with all three approaches are:

Discounting approach:

In the discounting approach, we find the value of all cash outflows at Time 0 at the discount rate, while any cash inflows remain at the time at which they occur. So, discounting the cash outflows to Time 0, we find:

$$\text{Time 0 cash flow} = -\$47,000 - \$9,500/1.11^5$$

$$\text{Time 0 cash flow} = -\$52,637.79$$

So, the MIRR using the discounting approach is:

$$0 = -\$52,637.79 + \$16,900/(1 + \text{MIRR}) + \$20,300/(1 + \text{MIRR})^2 + \$25,800/(1 + \text{MIRR})^3 + \$19,600/(1 + \text{MIRR})^4$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{MIRR} = 19.93\%$$

Reinvestment approach:

In the reinvestment approach, we find the future value of all cash flows except the initial cash flow at the end of the project using the reinvestment rate. So, reinvesting the cash flows to Time 5, we find:

$$\begin{aligned}\text{Time 5 cash flow} &= \$16,900(1.08^4) + \$20,300(1.08^3) + \$25,800(1.08^2) + \$19,600(1.08) - \$9,500 \\ \text{Time 5 cash flow} &= \$90,325.54\end{aligned}$$

So, the MIRR using the discounting approach is:

$$\begin{aligned}0 &= -\$47,000 + \$90,325.54/(1 + \text{MIRR})^5 \\ \$90,325.54/\$47,000 &= (1 + \text{MIRR})^5 \\ \text{MIRR} &= (\$90,325.54/\$47,000)^{1/5} - 1 \\ \text{MIRR} &= .1396, \text{ or } 13.96\%\end{aligned}$$

Combination approach:

In the combination approach, we find the value of all cash outflows at Time 0 using the discount rate, and the value of all cash inflows at the end of the project using the reinvestment rate. So, the value of the cash flows is:

$$\begin{aligned}\text{Time 0 cash flow} &= -\$47,000 - \$9,500/1.11^5 \\ \text{Time 0 cash flow} &= -\$52,637.79\end{aligned}$$

$$\begin{aligned}\text{Time 5 cash flow} &= \$16,900(1.08^4) + \$20,300(1.08^3) + \$25,800(1.08^2) + \$19,600(1.08) \\ \text{Time 5 cash flow} &= \$99,825.54\end{aligned}$$

So, the MIRR using the discounting approach is:

$$\begin{aligned}0 &= -\$52,637.79 + \$99,825.54/(1 + \text{MIRR})^5 \\ \$99,825.54/\$52,637.79 &= (1 + \text{MIRR})^5 \\ \text{MIRR} &= (\$99,825.54/\$52,637.79)^{1/5} - 1 \\ \text{MIRR} &= .1366, \text{ or } 13.66\%\end{aligned}$$

Challenge

22. The equation for the IRR of the project is:

$$0 = -\$75,000 + \$155,000/(1 + \text{IRR}) - \$65,000/(1 + \text{IRR})^2$$

From Descartes' Rule of Signs, we know there are either zero IRRs or two IRRs since the cash flows change signs twice. We can rewrite this equation as:

$$\begin{aligned}0 &= -\$75,000 + \$155,000X - \$65,000X^2 \\ \text{where } X &= 1/(1 + \text{IRR})\end{aligned}$$

This is a quadratic equation. We can solve for the roots of this equation with the quadratic formula:

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Remember that the quadratic formula is written as:

$$0 = aX^2 + bX + c$$

In this case, the equation is:

$$0 = -\$65,000X^2 + \$155,000X - \$75,000$$

$$X = \frac{-155,000 \pm \sqrt{(155,000)^2 - 4(-75,000)(-65,000)}}{2(-65,000)}$$

$$X = \frac{-155,000 \pm \sqrt{4,525,000,000}}{2(-65,000)}$$

$$X = \frac{-155,000 \pm 67,268.12}{-130,000}$$

Solving the quadratic equation, we find two Xs:

$$X = .6749, 1.7098$$

Since:

$$X = 1/(1 + \text{IRR})$$

$$1.7098 = 1/(1 + \text{IRR})$$

$$\text{IRR} = -.4151, \text{ or } -41.51\%$$

And:

$$X = 1/(1 + \text{IRR})$$

$$.6749 = 1/(1 + \text{IRR})$$

$$\text{IRR} = .4818, \text{ or } 48.18\%$$

To find the maximum (or minimum) of a function, we find the derivative and set it equal to zero. The derivative of this IRR function is:

$$0 = -\$155,000(1 + \text{IRR})^{-2} + \$130,000(1 + \text{IRR})^{-3}$$

$$-\$155,000(1 + \text{IRR})^{-2} = \$130,000(1 + \text{IRR})^{-3}$$

$$-\$155,000(1 + \text{IRR})^3 = \$130,000(1 + \text{IRR})^2$$

$$-\$155,000(1 + \text{IRR}) = \$130,000$$

$$\text{IRR} = \$130,000/\$155,000 - 1$$

$$\text{IRR} = -.1613, \text{ or } -16.13\%$$

To determine if this is a maximum or minimum, we can find the second derivative of the IRR function. If the second derivative is positive, we have found a minimum and if the second derivative is negative we have found a maximum. Using the reduced equation above, that is:

$$-\$155,000(1 + \text{IRR}) = \$130,000$$

The second derivative is $-\$262,722.18$, therefore we have a maximum.

23. Given the six-year payback, the worst case is that the payback occurs at the end of the sixth year. Thus, the worst case:

$$\begin{aligned}\text{NPV} &= -\$573,000 + \$573,000/1.11^6 \\ \text{NPV} &= -\$266,650.80\end{aligned}$$

The best case has infinite cash flows beyond the payback point. Thus, the best-case NPV is infinite.

24. The equation for the IRR of the project is:

$$\begin{aligned}0 &= -\$6,048 + \$34,344/(1 + \text{IRR}) - \$72,840/(1 + \text{IRR})^2 + \$68,400/(1 + \text{IRR})^3 \\ &\quad - \$24,000/(1 + \text{IRR})^4\end{aligned}$$

Using Descartes' rule of signs, from looking at the cash flows we know there are four IRRs for this project. Even with most computer spreadsheets, we have to do some trial and error. From trial and error, IRRs of 25%, 33.33%, 42.86%, and 66.67% are found.

We would accept the project when the NPV is greater than zero. See for yourself that the NPV is greater than zero for required returns between 25% and 33.33% or between 42.86% and 66.67%.

25. a. Here the cash inflows of the project go on forever, which is a perpetuity. Unlike ordinary perpetuity cash flows, the cash flows here grow at a constant rate forever, which is a growing perpetuity. The PV of the future cash flows from the project is:

$$\begin{aligned}\text{PV of cash inflows} &= C_I/(R - g) \\ \text{PV of cash inflows} &= \$415,000/ (.11 - .038) \\ \text{PV of cash inflows} &= \$5,763,888.89\end{aligned}$$

NPV is the PV of the outflows minus the PV of the inflows, so the NPV is:

$$\begin{aligned}\text{NPV} &= -\$4,700,000 + 5,763,888.89 \\ \text{NPV} &= \$1,063,888.89\end{aligned}$$

The NPV is positive, so we would accept the project.

- b. Here we want to know the minimum growth rate in cash flows necessary to accept the project. The minimum growth rate is the growth rate at which we would have a zero NPV. The equation for a zero NPV, using the equation for the PV of a growing perpetuity is:

$$0 = -\$4,700,000 + \$415,000/ (.11 - g)$$

Solving for g , we get:

$$g = .0217, \text{ or } 2.17\%$$

26. a. The project involves three cash flows: the initial investment, the annual cash inflows, and the abandonment costs. The mine will generate cash inflows over its 11-year economic life. To express the PV of the annual cash inflows, apply the growing annuity formula, discounted at the IRR and growing at eight percent.

$$\begin{aligned} \text{PV}(\text{Cash Inflows}) &= C \left\{ \frac{1}{(r - g)} - \frac{1}{(r - g)} \times \frac{(1 + g)}{(1 + r)^t} \right\} \\ \text{PV}(\text{Cash Inflows}) &= \$575,000 \left\{ \frac{1}{(\text{IRR} - .08)} - \frac{1}{(\text{IRR} - .08)} \times \frac{(1 + .08)}{(1 + \text{IRR})^{11}} \right\} \end{aligned}$$

At the end of 11 years, the company will abandon the mine, incurring a \$450,000 charge. Discounting the abandonment costs back 11 years at the IRR to express its present value, we get:

$$\begin{aligned} \text{PV}(\text{Abandonment}) &= C_{11}/(1 + \text{IRR})^{11} \\ \text{PV}(\text{Abandonment}) &= -\$450,000/(1 + \text{IRR})^{11} \end{aligned}$$

So, the IRR equation for this project is:

$$0 = -\$3,400,000 + \$575,000 \left\{ \frac{1}{(\text{IRR} - .08)} - \frac{1}{(\text{IRR} - .08)} \times \frac{(1 + .08)}{(1 + \text{IRR})^{11}} \right\} - \$450,000/(1 + \text{IRR})^{11}$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 18.73\%$$

- b. Yes. Since the mine's IRR exceeds the required return of 13 percent, the mine should be opened. The correct decision rule for an investment-type project is to accept the project if the IRR is greater than the discount rate. Although it appears there is a sign change at the end of the project because of the abandonment costs, the last cash flow is actually positive because of the operating cash flow in the last year.
27. First, we need to find the future value of the cash flows for the one year in which they are blocked by the government. So, reinvesting each cash inflow for one year, we find:

$$\begin{aligned} \text{Year 2 cash flow} &= \$295,000(1.04) = \$306,800 \\ \text{Year 3 cash flow} &= \$325,000(1.04) = \$338,000 \\ \text{Year 4 cash flow} &= \$413,000(1.04) = \$429,520 \\ \text{Year 5 cash flow} &= \$240,000(1.04) = \$249,600 \end{aligned}$$

So, the NPV of the project is:

$$\begin{aligned} \text{NPV} &= -\$945,000 + \$306,800/1.11^2 + \$338,000/1.11^3 + \$429,520/1.11^4 + \$249,600/1.11^5 \\ \text{NPV} &= -\$17,787.97 \end{aligned}$$

And the IRR of the project is:

$$0 = -\$945,000 + \$306,800/(1 + \text{IRR})^2 + \$338,000/(1 + \text{IRR})^3 + \$429,520/(1 + \text{IRR})^4 + \$249,600/(1 + \text{IRR})^5$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 10.37\%$$

While this may look like an MIRR calculation, it is not an MIRR, rather it is a standard IRR calculation. Since the cash inflows are blocked by the government, they are not available to the company for a period of one year. Thus, all we are doing is calculating the IRR based on when the cash flows actually occur for the company.

28. a. We can apply the growing perpetuity formula to find the PV of Stream A. The perpetuity formula values the stream as of one year before the first payment. Therefore, the growing perpetuity formula values the stream of cash flows as of Year 2. Next, discount the PV as of the end of Year 2 back two years to find the PV as of today, Year 0. Doing so, we find:

$$\begin{aligned} \text{PV(A)} &= [C_3/(r - g)]/(1 + r)^2 \\ \text{PV(A)} &= [\$11,600/(\.12 - .04)]/(1.12)^2 \\ \text{PV(A)} &= \$115,593.11 \end{aligned}$$

We can apply the perpetuity formula to find the PV of Stream B. The perpetuity formula discounts the stream back to Year 1, one period prior to the first cash flow. Discount the PV as of the end of Year 1 back one year to find the PV as of today, Year 0. Doing so, we find:

$$\begin{aligned} \text{PV(B)} &= [C_2/r]/(1 + r) \\ \text{PV(B)} &= [-\$13,000/\.12]/(1.12) \\ \text{PV(B)} &= -\$96,726.19 \end{aligned}$$

- b. If we combine the cash flow streams to form Project C, we get:

$$\text{Project A} = [C_3/(r - g)]/(1 + r)^2$$

$$\text{Project B} = [C_2/r]/(1 + r)$$

$$\text{Project C} = \text{Project A} + \text{Project B}$$

$$\text{Project C} = [C_3/(r - g)]/(1 + r)^2 + [C_2/r]/(1 + r)$$

$$0 = [\$11,600/(\text{IRR} - .04)]/(1 + \text{IRR})^2 + [-\$13,000/\text{IRR}]/(1 + \text{IRR})$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$\text{IRR} = 16.90\%$$

- c. The correct decision rule for an investing-type project is to accept the project if the discount rate is below the IRR. Since there is one IRR, a decision can be made. At a point in the future, the cash flows from Project A will be greater than those from Project B. Therefore, although there are many cash flows, there will be only one change in sign. When the sign of the cash

flows changes more than once over the life of the project, there may be multiple internal rates of return. In such cases, there is no correct decision rule for accepting and rejecting projects using the internal rate of return.

29. To answer this question, we need to examine the incremental cash flows. To make the projects equally attractive, Project Billion must have a larger initial investment. We know this because the subsequent cash flows from Project Billion are larger than the subsequent cash flows from Project Million. So, subtracting the Project Million cash flows from the Project Billion cash flows, we find the incremental cash flows are:

<u>Year</u>	<u>Incremental cash flows</u>
0	$-I_0 + \$1,200$
1	240
2	240
3	400

Now we can find the present value of the subsequent incremental cash flows at the discount rate, 12 percent. The present value of the incremental cash flows is:

$$\begin{aligned} PV &= \$1,200 + \$240/1.12 + \$240/1.12^2 + \$400/1.12^3 \\ PV &= \$1,890.32 \end{aligned}$$

So, if I_0 is greater than \$1,890.32, the incremental cash flows will be negative. Since we are subtracting Project Million from Project Billion, this implies that for any value over \$1,890.32 the NPV of Project Billion will be less than that of Project Million, so I_0 must be less than \$1,890.32.

30. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR of the project is:

$$0 = \$20,000 - \$26,000/(1 + \text{IRR}) + \$13,000/(1 + \text{IRR})^2$$

Even though it appears there are two IRRs, a spreadsheet, financial calculator, or trial and error will not give an answer. The reason is that there is no real IRR for this set of cash flows. If you examine the IRR equation, what we are really doing is solving for the roots of the equation. Going back to high school algebra, in this problem we are solving a quadratic equation. In case you don't remember, the quadratic equation is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case, the equation is:

$$x = \frac{-(-26,000) \pm \sqrt{(-26,000)^2 - 4(20,000)(13,000)}}{2(20,000)}$$

The square root term works out to be:

$$676,000,000 - 1,040,000,000 = -364,000,000$$

The square root of a negative number is a complex number, so there is no real number solution, meaning the project has no real IRR.

Calculator Solutions

1. *b.* *Project A*

CF₀	-\$15,000
C01	\$10,400
F01	1
C02	\$5,900
F02	1
C03	\$2,100
F03	1

I = 15%

NPV CPT

-\$114.49

CF₀	-\$19,000
C01	\$12,700
F01	1
C02	\$6,100
F02	1
C03	\$5,300
F03	1

I = 15%

NPV CPT

\$140.79

5.

CF₀	-\$27,000
C01	\$13,100
F01	1
C02	\$17,200
F02	1
C03	\$8,400
F03	1

IRR CPT

21.80%

6.

Project A

CF₀	-\$7,300
C01	\$3,940
F01	1
C02	\$3,450
F02	1
C03	\$2,480
F03	1

IRR CPT

18.24%

Project B

CF₀	-\$4,390
C01	\$2,170
F01	1
C02	\$2,210
F02	1
C03	\$1,730
F03	1

IRR CPT

19.31%

7.

CF₀	0
C01	\$67,000
F01	7

I = 13%

NPV CPT

\$296,314.90

$$PI = \$296,314.90 / \$325,000 = .912$$

10.

CF₀	\$8,700
C01	-\$3,900
F01	1
C02	-\$2,900
F02	1
C03	-\$2,300
F03	1
C04	-\$1,800
F04	1

IRR CPT

11.21%

CF₀	\$8,700
C01	-\$3,900
F01	1
C02	-\$2,900
F02	1
C03	-\$2,300
F03	1
C04	-\$1,800
F04	1

I = 10%

NPV CPT

-\$199.60

CF₀	\$8,700
C01	-\$3,900
F01	1
C02	-\$2,900
F02	1
C03	-\$2,300
F03	1
C04	-\$1,800
F04	1

I = 20%

NPV CPT

\$1,237.04

11. a. *Deepwater fishing*

CF₀	-\$725,000
C01	\$270,000
F01	1
C02	\$420,000
F02	1
C03	\$380,000
F03	1

IRR CPT

20.96%

Submarine ride

CF₀	-\$1,450,000
C01	\$820,000
F01	1
C02	\$650,000
F02	1
C03	\$540,000
F03	1

IRR CPT

19.87%

b.

CF₀	-\$725,000
C₀₁	\$550,000
F₀₁	1
C₀₂	\$230,000
F₀₂	1
C₀₃	\$160,000
F₀₃	1

IRR CPT

18.40%

c.

Deepwater fishing

CF₀	-\$725,000
C₀₁	\$270,000
F₀₁	1
C₀₂	\$420,000
F₀₂	1
C₀₃	\$380,000
F₀₃	1

I = 14%

NPV CPT

\$91,507.64

Submarine ride

CF₀	-\$1,450,000
C₀₁	\$820,000
F₀₁	1
C₀₂	\$650,000
F₀₂	1
C₀₃	\$540,000
F₀₃	1

I = 14%

NPV CPT

\$133,936.76

12.

Project I

CF₀	\$0
C₀₁	\$23,200
F₀₁	3

I = 10%

NPV CPT

\$57,694.97

CF₀	-\$45,000
C₀₁	\$23,200
F₀₁	3

I = 10%

NPV CPT

\$12,694.97

$$PI = \$57,694.97 / \$45,000 = 1.282$$

Project II

CF₀	\$0
C₀₁	\$11,700
F₀₁	3

I = 10%

NPV CPT

\$29,096.17

CF₀	-\$19,800
C₀₁	\$11,700
F₀₁	3

I = 10%

NPV CPT

\$9,296.17

$$PI = \$29,096.17 / \$19,800 = 1.470$$

13.

CF₀	–\$65,000,000	CF₀	–\$65,000,000
C01	\$92,000,000	C01	\$92,000,000
F01	1	F01	1
C02	–\$11,000,000	C02	–\$11,000,000
F02	1	F02	1
I = 10%		IRR CPT	
NPV CPT		28.35%	
\$9,545,454.55			

Financial calculators will only give you one IRR, even if there are multiple IRRs. Using trial and error, or a root solving calculator, the other IRR is –86.82%.

14. b.

<i>Board game</i>		<i>DVD</i>	
CF₀	–\$850	CF₀	–\$1,700
C01	\$670	C01	\$1,300
F01	1	F01	1
C02	\$510	C02	\$750
F02	1	F02	1
C03	\$90	C03	\$350
F03	1	F03	1
I = 10%		I = 10%	
NPV CPT		NPV CPT	
\$248.20		\$364.61	

c.

<i>Board game</i>		<i>DVD</i>	
CF₀	–\$850	CF₀	–\$1,700
C01	\$670	C01	\$1,300
F01	1	F01	1
C02	\$510	C02	\$750
F02	1	F02	1
C03	\$90	C03	\$350
F03	1	F03	1
IRR CPT		IRR CPT	
30.86%		24.96%	

d.

CF₀	–\$850
C01	\$630
F01	1
C02	\$240
F02	1
C03	\$260
F03	1
IRR CPT	
19.29%	

15. a.

CDMA

CF₀	0
C01	\$23,000,000
F01	1
C02	\$16,000,000
F02	1
C03	\$6,000,000
F03	1

I = 10%

NPV CPT

\$38,640,120.21

G4

CF₀	0
C01	\$21,000,000
F01	1
C02	\$51,000,000
F02	1
C03	\$41,000,000
F03	1

I = 10%

NPV CPT

\$92,043,576.26

Wi-Fi

CF₀	0
C01	\$39,000,000
F01	1
C02	\$66,000,000
F02	1
C03	\$42,000,000
F03	1

I = 10%

NPV CPT

\$121,555,221.64

$$PI_{CDMA} = \$38,640,120.21 / \$18,000,000 = 2.15$$

$$PI_{G4} = \$92,043,576.26 / \$25,000,000 = 3.68$$

$$PI_{Wi-Fi} = \$121,555,221.64 / \$43,000,000 = 2.83$$

b.

CDMA

CF₀	-\$18,000,000
C01	\$23,000,000
F01	1
C02	\$16,000,000
F02	1
C03	\$6,000,000
F03	1

I = 10%

NPV CPT

\$20,640,120.21

G4

CF₀	-\$25,000,000
C01	\$21,000,000
F01	1
C02	\$51,000,000
F02	1
C03	\$41,000,000
F03	1

I = 10%

NPV CPT

\$67,043,576.26

Wi-Fi

CF₀	-\$43,000,000
C01	\$39,000,000
F01	1
C02	\$66,000,000
F02	1
C03	\$42,000,000
F03	1

I = 10%

NPV CPT

\$78,555,221.64

16. b.

AZM

CF₀	-\$575,000
C01	\$373,000
F01	1
C02	\$219,000
F02	1
C03	\$185,000
F03	1

I = 10%

NPV CPT

\$84,075.88

AZF

CF₀	-\$980,000
C01	\$395,000
F01	1
C02	\$477,000
F02	1
C03	\$339,000
F03	1

I = 10%

NPV CPT

\$28,001.50

c.

AZM

CF₀	-\$575,000
C01	\$373,000
F01	1
C02	\$219,000
F02	1
C03	\$185,000
F03	1

IRR CPT

19.36%

AZF

CF₀	-\$980,000
C01	\$395,000
F01	1
C02	\$477,000
F02	1
C03	\$339,000
F03	1

IRR CPT

11.65%

17. a.

Project A

CF₀	0
C01	\$165,000
F01	1
C02	\$165,000
F02	1

I = 12%
NPV CPT
\$278,858.42

Project B

CF₀	0
C01	\$300,000
F01	1
C02	\$300,000
F02	1

I = 12%
NPV CPT
\$507,015.31

Project C

CF₀	0
C01	\$180,000
F01	1
C02	\$135,000
F02	1

I = 12%
NPV CPT
\$268,335.46

$$PI_A = \$278,858.42 / \$225,000 = 1.24$$

$$PI_B = \$507,015.31 / \$450,000 = 1.13$$

$$PI_C = \$268,335.46 / \$225,000 = 1.19$$

b.

Project A

CF₀	-\$225,000
C01	\$165,000
F01	1
C02	\$165,000
F02	1

I = 12%
NPV CPT
\$53,858.42

Project B

CF₀	-\$450,000
C01	\$300,000
F01	1
C02	\$300,000
F02	1

I = 12%
NPV CPT
\$57,015.31

Project C

CF₀	-\$225,000
C01	\$180,000
F01	1
C02	\$135,000
F02	1

I = 12%
NPV CPT
\$43,335.46

18. b.

Dry prepeg

CF₀	-\$1,500,000
C01	\$900,000
F01	1
C02	\$700,000
F02	1
C03	\$725,000
F03	1

I = 8%
NPV CPT
\$508,998.88

Solvent prepeg

CF₀	-\$650,000
C01	\$345,000
F01	1
C02	\$570,000
F02	1
C03	\$360,000
F03	1

I = 8%
NPV CPT
\$443,907.18

c.

Dry prepeg

CF₀	-\$1,500,000
C01	\$900,000
F01	1
C02	\$700,000
F02	1
C03	\$725,000
F03	1

IRR CPT
26.84%

Solvent prepeg

CF₀	-\$650,000
C01	\$345,000
F01	1
C02	\$570,000
F02	1
C03	\$360,000
F03	1

IRR CPT
42.16%

d.

CF₀	-\$850,000
C01	\$555,000
F01	1
C02	\$130,000
F02	1
C03	\$365,000
F03	1

IRR CPT
12.68%

19. b.

NP-30

CF₀	-\$735,000
C01	\$239,000
F01	5
C02	
F02	
C03	
F03	
C04	
F04	
C05	
F05	

IRR CPT
18.74%

NX-20

CF₀	-\$460,000
C01	\$130,000
F01	1
C02	\$143,000
F02	1
C03	\$157,300
F03	1
C04	\$173,030
F04	1
C05	\$190,333
F05	1

IRR CPT
19.87%

c.

NP-30

CF₀	0
C01	\$239,000
F01	5
C02	
F02	
C03	
F03	
C04	
F04	
C05	
F05	

I = 15%
NPV CPT
\$801,165.07

NX-20

CF₀	0
C01	\$130,000
F01	1
C02	\$143,000
F02	1
C03	\$157,300
F03	1
C04	\$170,030
F04	1
C05	\$190,333
F05	1

I = 15%
NPV CPT
\$518,158.93

$$PI_{NP-30} = \$801,165.07 / \$735,000 = 1.090$$

$$PI_{NX-20} = \$518,158.93 / \$460,000 = 1.126$$

d.

NP-30

CF₀	-\$735,000
C01	\$239,000
F01	5
C02	
F02	
C03	
F03	
C04	
F04	
C05	
F05	

I = 15%

NPV CPT

\$66,165.07

NX-20

CF₀	-\$460,000
C01	\$130,000
F01	1
C02	\$143,000
F02	1
C03	\$157,300
F03	1
C04	\$173,030
F04	1
C05	\$190,333
F05	1

I = 15%

NPV CPT

\$58,158.93

30.

CF₀	\$20,000
C01	-\$26,000
F01	1
C02	\$13,000
F02	1

IRR CPT

ERROR 7

CHAPTER 6

MAKING CAPITAL INVESTMENT DECISIONS

Answers to Concepts Review and Critical Thinking Questions

1. In this context, an opportunity cost refers to the value of an asset or other input that will be used in a project. The relevant cost is what the asset or input is actually worth today, not, for example, what it cost to acquire it.
2.
 - a. Yes, the reduction in the sales of the company's other products, referred to as erosion, should be treated as an incremental cash flow. These lost sales are included because they are a cost (a revenue reduction) that the firm must bear if it chooses to produce the new product.
 - b. Yes, expenditures on plant and equipment should be treated as incremental cash flows. These are costs of the new product line. However, if these expenditures have already occurred (and cannot be recaptured through a sale of the plant and equipment), they are sunk costs and are not included as incremental cash flows.
 - c. No, the research and development costs should not be treated as incremental cash flows. The costs of research and development undertaken on the product during the past three years are sunk costs and should not be included in the evaluation of the project. Decisions made and costs incurred in the past cannot be changed. They should not affect the decision to accept or reject the project.
 - d. Yes, the annual depreciation expense must be taken into account when calculating the cash flows related to a given project. While depreciation is not a cash expense that directly affects cash flow, it decreases a firm's net income and hence lowers its tax bill for the year. Because of this depreciation tax shield, the firm has more cash on hand at the end of the year than it would have had without expensing depreciation.
 - e. No, dividend payments should not be treated as incremental cash flows. A firm's decision to pay or not pay dividends is independent of the decision to accept or reject any given investment project. For this reason, dividends are not an incremental cash flow to a given project. Dividend policy is discussed in more detail in later chapters.
 - f. Yes, the resale value of plant and equipment at the end of a project's life should be treated as an incremental cash flow. The price at which the firm sells the equipment is a cash inflow, and any difference between the book value of the equipment and its sale price will create accounting gains or losses that result in either a tax credit or liability.
 - g. Yes, salary and medical costs for production employees hired for a project should be treated as incremental cash flows. The salaries of all personnel connected to the project must be included as costs of that project.

3. Item (a) is a relevant cost because the opportunity to sell the land is lost if the new golf club is produced. Item (b) is also relevant because the firm must take into account the erosion of sales of existing products when a new product is introduced. If the firm produces the new club, the earnings from the existing clubs will decrease, effectively creating a cost that must be included in the decision. Item (c) is irrelevant because the costs of research and development are sunk costs. Decisions made in the past cannot be changed. They are not relevant to the production of the new club.
4. For tax purposes, a firm would choose MACRS because it provides for larger depreciation deductions earlier. These larger deductions reduce taxes, but have no other cash consequences. Notice that the choice between MACRS and straight-line is purely a time value issue; the total depreciation is the same, only the timing differs.
5. It's probably only a mild over-simplification. Current liabilities will all be paid, presumably. The cash portion of current assets will be retrieved. Some receivables won't be collected, and some inventory will not be sold, of course. Counterbalancing these losses is the fact that inventory sold above cost (and not replaced at the end of the project's life) acts to increase working capital. These effects tend to offset one another.
6. Management's discretion to set the firm's capital structure is applicable at the firm level. Since any one particular project could be financed entirely with equity, another project could be financed with debt, and the firm's overall capital structure would remain unchanged. Financing costs are irrelevant in the analysis of a project's incremental cash flows according to the stand-alone principle.
7. The EAC approach is appropriate when comparing mutually exclusive projects with different lives that will be replaced when they wear out. This type of analysis is necessary so that the projects have a common life span over which they can be compared. For example, if one project has a three-year life and the other has a five-year life, then a 15-year horizon is the minimum necessary to place the two projects on an equal footing, implying that one project will be repeated five times and the other will be repeated three times. Note the shortest common life may be quite long when there are more than two alternatives and/or when the individual project lives are relatively long. Assuming this type of analysis is valid implies that the project cash flows remain the same over the common life, thus ignoring the possible effects of, among other things: (1) inflation, (2) changing economic conditions, (3) the increasing unreliability of cash flow estimates that occur far into the future, and (4) the possible effects of future technology improvement that could alter the project cash flows.
8. Depreciation is a non-cash expense, but it is tax-deductible on the income statement. Thus depreciation causes taxes paid, an actual cash outflow, to be reduced by an amount equal to the depreciation tax shield, $T_C D$. A reduction in taxes that would otherwise be paid is the same thing as a cash inflow, so the effects of the depreciation tax shield must be added in to get the total incremental aftertax cash flows.
9. There are two particularly important considerations. The first is erosion. Will the "essentialized" book displace copies of the existing book that would have otherwise been sold? This is of special concern given the lower price. The second consideration is competition. Will other publishers step in and produce such a product? If so, then any erosion is much less relevant. A particular concern to book publishers (and producers of a variety of other product types) is that the publisher only makes money from the sale of new books. Thus, it is important to examine whether the new book would displace sales of used books (good from the publisher's perspective) or new books (not good). The concern arises any time there is an active market for the used product.

10. Definitely. The damage to Porsche's reputation is a factor the company needed to consider. If the reputation were to be damaged, the company would have lost sales of its existing car lines.
11. One company may be able to produce at lower incremental cost or market better. Also, of course, one of the two may have made a mistake!
12. Porsche would recognize that the outsized profits would dwindle as more products come to market and competition becomes more intense.

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. Using the tax shield approach to calculating OCF, we get:

$$\begin{aligned}\text{OCF} &= (\text{Sales} - \text{Costs})(1 - T_c) + T_c \text{Depreciation} \\ \text{OCF} &= [(\$4.95 \times 1,400) - (\$1.97 \times 1,400)](1 - .21) + .21(\$9,300/5) \\ \text{OCF} &= \$3,686.48\end{aligned}$$

So, the NPV of the project is:

$$\begin{aligned}\text{NPV} &= -\$9,300 + \$3,686.48(\text{PVIFA}_{14\%,5}) \\ \text{NPV} &= \$3,355.98\end{aligned}$$

2. We will use the bottom-up approach to calculate the operating cash flow for each year. We also must be sure to include the net working capital cash flows each year. So, the net income and total cash flow each year will be:

	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>
Sales	\$13,400	\$15,000	\$16,400	\$12,900
Costs	2,900	3,100	4,200	2,800
Depreciation	6,575	6,575	6,575	6,575
EBT	\$3,925	\$5,325	\$5,625	\$3,525
Tax	864	1,172	1,238	776
Net income	\$3,062	\$4,154	\$4,388	\$2,750
OCF	\$9,637	\$10,729	\$10,963	\$9,325
Capital spending	-\$26,300			
NWC	-300	-200	-150	875
Incremental cash flow	-\$26,600	\$9,437	\$10,504	\$10,200

The NPV for the project is:

$$\begin{aligned}\text{NPV} &= -\$26,600 + \$9,437/1.12 + \$10,504/1.12^2 + \$10,813/1.12^3 + \$10,200/1.12^4 \\ \text{NPV} &= \$4,376.86\end{aligned}$$

3. Using the tax shield approach to calculating OCF, we get:

$$\begin{aligned}\text{OCF} &= (\text{Sales} - \text{Costs})(1 - T_c) + T_c \text{Depreciation} \\ \text{OCF} &= (\$1,090,000 - 475,000)(1 - .25) + .25(\$1,420,000/3) \\ \text{OCF} &= \$579,583.33\end{aligned}$$

So, the NPV of the project is:

$$\begin{aligned}\text{NPV} &= -\$1,420,000 + \$579,583.33(\text{PVIFA}_{12\%,3}) \\ \text{NPV} &= -\$27,938.63\end{aligned}$$

4. The cash outflow at the beginning of the project will increase because of the spending on NWC. At the end of the project, the company will recover the NWC, so it will be a cash inflow. The sale of the equipment will result in a cash inflow, but we also must account for the taxes which will be paid on this sale. So, the cash flows for each year of the project will be:

<u>Year</u>	<u>Cash Flow</u>	
0	-\$1,670,000	=\$-1,420,000 - 250,000
1	579,583.33	
2	579,583.33	
3	1,002,083.33	=\$579,583.33 + 250,000 + 230,000 + (0 - 230,000)(.25)

And the NPV of the project is:

$$\begin{aligned}\text{NPV} &= -\$1,670,000 + \$579,583.33(\text{PVIFA}_{12\%,2}) + (\$1,002,083.33/1.12^3) \\ \text{NPV} &= \$22,788.53\end{aligned}$$

5. First, we will calculate the annual depreciation for the equipment necessary for the project. The depreciation amount each year will be:

$$\begin{aligned}\text{Year 1 depreciation} &= \$1,420,000(.3333) = \$473,286 \\ \text{Year 2 depreciation} &= \$1,420,000(.4445) = \$631,190 \\ \text{Year 3 depreciation} &= \$1,420,000(.1481) = \$210,302\end{aligned}$$

So, the book value of the equipment at the end of three years, which will be the initial investment minus the accumulated depreciation, is:

$$\begin{aligned}\text{Book value in 3 years} &= \$1,420,000 - (\$473,286 + 631,190 + 210,302) \\ \text{Book value in 3 years} &= \$105,222\end{aligned}$$

The asset is sold at a gain to book value, so this gain is taxable.

$$\begin{aligned}\text{Aftertax salvage value} &= \$230,000 + (\$105,222 - 230,000)(.25) \\ \text{Aftertax salvage value} &= \$198,805.50\end{aligned}$$

To calculate the OCF, we will use the tax shield approach, so the cash flow each year is:

$$\text{OCF} = (\text{Sales} - \text{Costs})(1 - T_c) + T_c \text{Depreciation}$$

<u>Year</u>	<u>Cash Flow</u>	
0	-\$1,670,000	= -\$1,420,000 - 250,000
1	579,571.50	= (\$615,000)(.75) + .25(\$473,286)
2	619,047.50	= (\$615,000)(.75) + .25(\$631,190)
3	962,631.00	= (\$615,000)(.75) + .25(\$210,302) + \$198,805.50 + 250,000

Remember to include the NWC cost in Year 0, and the recovery of the NWC at the end of the project. The NPV of the project with these assumptions is:

$$\text{NPV} = -\$1,670,000 + \$579,571.50/1.12 + \$619,047.50/1.12^2 + \$962,631.00/1.12^3$$

$$\text{NPV} = \$26,157.16$$

6. The book value of the asset is zero, so the gain on the sale is taxable.

$$\text{Aftertax salvage value} = \$230,000 + (\$0 - 230,000)(.25)$$

$$\text{Aftertax salvage value} = \$172,500$$

To calculate the OCF, we will use the tax shield approach, so the cash flow each year is:

$$\text{OCF} = (\text{Sales} - \text{Costs})(1 - T_c) + T_c \text{Depreciation}$$

<u>Year</u>	<u>Cash Flow</u>	
0	-\$1,670,000	= -\$1,420,000 - 250,000
1	816,250	= (\$615,000)(.75) + .25(\$1,420,000)
2	461,250	= (\$615,000)(.75)
3	883,750	= (\$615,000)(.75) + \$172,500 + 250,000

Remember to include the NWC cost in Year 0, and the recovery of the NWC at the end of the project. The NPV of the project with these assumptions is:

$$\text{NPV} = -\$1,670,000 + \$816,250/1.12 + \$461,250/1.12^2 + \$883,750/1.12^3$$

$$\text{NPV} = \$55,536.11$$

7. First, we will calculate the annual depreciation of the new equipment. It will be:

$$\text{Annual depreciation charge} = \$575,000/5$$

$$\text{Annual depreciation charge} = \$115,000$$

The aftertax salvage value of the equipment is:

$$\text{Aftertax salvage value} = \$60,000(1 - .23)$$

$$\text{Aftertax salvage value} = \$46,200$$

Using the tax shield approach, the OCF is:

$$\text{OCF} = \$176,000(1 - .23) + .23(\$115,000)$$

$$\text{OCF} = \$161,970$$

Now we can find the project IRR. There is an unusual feature that is a part of this project. Accepting this project means that we will reduce NWC. This reduction in NWC is a cash inflow at Year 0. This reduction in NWC implies that when the project ends, we will have to increase NWC. So, at the end of the project, we will have a cash outflow to restore the NWC to its level before the project. We also must include the aftertax salvage value at the end of the project. The IRR of the project is:

$$\text{NPV} = 0 = -\$575,000 + 80,000 + \$161,970(\text{PVIFA}_{\text{IRR}\%,4}) + [(\$161,970 + 46,200 - 80,000)/(1 + \text{IRR})^5]$$

$$\text{IRR} = 17.70\%$$

8. First, we will calculate the annual depreciation of the new equipment. It will be:

$$\begin{aligned}\text{Annual depreciation} &= \$375,000/5 \\ \text{Annual depreciation} &= \$75,000\end{aligned}$$

Now, we calculate the aftertax salvage value. The aftertax salvage value is the market price minus (or plus) the taxes on the sale of the equipment, so:

$$\text{Aftertax salvage value} = \text{MV} + (\text{BV} - \text{MV})T_c$$

Very often, the book value of the equipment is zero as it is in this case. If the book value is zero, the equation for the aftertax salvage value becomes:

$$\begin{aligned}\text{Aftertax salvage value} &= \text{MV} + (0 - \text{MV})T_c \\ \text{Aftertax salvage value} &= \text{MV}(1 - T_c)\end{aligned}$$

We will use this equation to find the aftertax salvage value since we know the book value is zero. So, the aftertax salvage value is:

$$\begin{aligned}\text{Aftertax salvage value} &= \$25,000(1 - .24) \\ \text{Aftertax salvage value} &= \$19,000\end{aligned}$$

Using the tax shield approach, we find the OCF for the project is:

$$\begin{aligned}\text{OCF} &= \$95,000(1 - .24) + .24(\$75,000) \\ \text{OCF} &= \$90,200\end{aligned}$$

Now we can find the project NPV. Notice that we include the NWC in the initial cash outlay. The recovery of the NWC occurs in Year 5, along with the aftertax salvage value.

$$\begin{aligned}\text{NPV} &= -\$375,000 - 15,000 + \$90,200(\text{PVIFA}_{10\%,5}) + (\$19,000 + 15,000)/1.10^5 \\ \text{NPV} &= -\$26,959.71\end{aligned}$$

9. The book value of the asset will be zero at the end of the project, so the aftertax salvage value is:

$$\begin{aligned}\text{Aftertax salvage value} &= \$25,000(1 - .24) \\ \text{Aftertax salvage value} &= \$19,000\end{aligned}$$

Using the tax shield approach, we find the OCF for the first year of the project is:

$$\begin{aligned}\text{OCF} &= \$95,000(1 - .24) + .24(\$375,000) \\ \text{OCF} &= \$162,200\end{aligned}$$

And the OCF for Years 2 to 5 is:

$$\begin{aligned}\text{OCF} &= \$95,000(1 - .24) \\ \text{OCF} &= \$72,200\end{aligned}$$

Now we can find the project NPV. Notice that we include the NWC in the initial cash outlay. The recovery of the NWC occurs in Year 5, along with the aftertax salvage value.

$$\begin{aligned}\text{NPV} &= -\$375,000 - 15,000 + \$162,200/1.10 + \$72,200/1.10^2 + \$72,200/1.10^3 + \$72,200/1.10^4 \\ &\quad + (\$72,200 + 19,000 + 15,000)/1.10^5 \\ \text{NPV} &= -\$13,375.69\end{aligned}$$

10. To find the book value at the end of four years, we need to find the accumulated depreciation for the first four years. We could calculate a table with the depreciation each year, but an easier way is to add the MACRS depreciation amounts for each of the first four years and multiply this percentage times the cost of the asset. We can then subtract this from the asset cost. Doing so, we get:

$$\begin{aligned}\text{BV}_4 &= \$7,600,000 - 7,600,000(.2000 + .3200 + .1920 + .1152) \\ \text{BV}_4 &= \$1,313,280\end{aligned}$$

The asset is sold at a gain to book value, so this gain is taxable.

$$\begin{aligned}\text{Aftertax salvage value} &= \$1,400,000 + (\$1,313,280 - 1,400,000)(.21) \\ \text{Aftertax salvage value} &= \$1,381,789\end{aligned}$$

11. We will begin by calculating the initial cash outlay, that is, the cash flow at Time 0. To undertake the project, we will have to purchase the equipment and increase net working capital. So, the cash outlay today for the project will be:

Equipment	-\$4,100,000
NWC	<u>-150,000</u>
Total	-\$4,250,000

Using the bottom-up approach to calculating the operating cash flow, we find the operating cash flow each year will be:

Sales	\$2,350,000
Costs	587,500
Depreciation	<u>1,025,000</u>
EBT	\$737,500
Tax	<u>184,375</u>
Net income	<u><u>\$553,125</u></u>

The operating cash flow is:

$$\text{OCF} = \text{Net income} + \text{Depreciation}$$

$$\text{OCF} = \$553,125 + 1,025,000$$

$$\text{OCF} = \$1,578,125$$

To find the NPV of the project, we add the present value of the project cash flows. We must be sure to add back the net working capital at the end of the project life, since we are assuming the net working capital will be recovered. So, the project NPV is:

$$\text{NPV} = -\$4,250,000 + \$1,578,125(\text{PVIFA}_{13\%,4}) + \$150,000/1.13^4$$

$$\text{NPV} = \$536,085.37$$

12. We will need the aftertax salvage value of the equipment to compute the EAC. Even though the equipment for each product has a different initial cost, both have the same salvage value. The aftertax salvage value for both is:

$$\text{Aftertax salvage value} = \$25,000(1 - .21)$$

$$\text{Aftertax salvage value} = \$19,750$$

To calculate the EAC, we first need the OCF and PV of costs of each option. The OCF and PV of costs for Techron I is:

$$\text{OCF} = -\$41,000(1 - .21) + .21(\$265,000/3)$$

$$\text{OCF} = -\$13,840$$

$$\text{PV of costs} = -\$265,000 + -\$13,840(\text{PVIFA}_{9\%,3}) + (\$19,750/1.09^3)$$

$$\text{PV of costs} = -\$284,782.49$$

$$\text{EAC} = -\$284,782.49/(\text{PVIFA}_{9\%,3})$$

$$\text{EAC} = -\$112,504.68$$

And the OCF and PV of costs for Techron II is:

$$\text{OCF} = -\$52,000(1 - .21) + .21(\$330,000/5)$$

$$\text{OCF} = -\$27,220$$

$$\text{PV of costs} = -\$330,000 - \$27,220(\text{PVIFA}_{9\%,5}) + (\$19,750/1.09^5)$$

$$\text{PV of costs} = -\$423,040.16$$

$$\text{EAC} = -\$423,040.16/(\text{PVIFA}_{9\%,5})$$

$$\text{EAC} = -\$108,760.43$$

The two milling machines have unequal lives, so they can only be compared by expressing both on an equivalent annual basis, which is what the EAC method does. Thus, you prefer the Techron II because it has the lower (less negative) annual cost.

Intermediate

13. First, we will calculate the depreciation each year, which will be:

$$D_1 = \$670,000(.2000) = \$134,000$$

$$D_2 = \$670,000(.3200) = \$214,400$$

$$D_3 = \$670,000(.1920) = \$128,640$$

$$D_4 = \$670,000(.1152) = \$77,184$$

The book value of the equipment at the end of the project is:

$$BV_4 = \$670,000 - (\$134,000 + 214,400 + 128,640 + 77,184)$$

$$BV_4 = \$115,776$$

The asset is sold at a loss to book value, so this creates a tax refund. The aftertax salvage value will be:

$$\text{Aftertax salvage value} = \$55,000 + (\$115,776 - 55,000)(.23)$$

$$\text{Aftertax salvage value} = \$68,978.48$$

So, the OCF for each year will be:

$$OCF_1 = \$245,000(1 - .23) + .23(\$134,000) = \$219,470.00$$

$$OCF_2 = \$245,000(1 - .23) + .23(\$214,400) = \$237,962.00$$

$$OCF_3 = \$245,000(1 - .23) + .23(\$128,640) = \$218,237.20$$

$$OCF_4 = \$245,000(1 - .23) + .23(\$77,184) = \$206,402.32$$

Now we have all the necessary information to calculate the project NPV. We need to be careful with the NWC in this project. Notice the project requires \$20,000 of NWC at the beginning, and \$2,500 more in NWC each successive year. We will subtract the \$20,000 from the initial cash flow and subtract \$2,500 each year from the OCF to account for this spending. In Year 4, we will add back the total spent on NWC, which is \$27,500. The \$2,500 spent on NWC capital during Year 4 is irrelevant. Why? Well, during this year the project required an additional \$2,500, but we would get the money back immediately. So, the net cash flow for additional NWC would be zero. With all this, the equation for the NPV of the project is:

$$\begin{aligned} \text{NPV} = & -\$670,000 - 20,000 + (\$219,470 - 2,500)/1.08 + (\$237,962 - 2,500)/1.08^2 \\ & + (\$218,237.20 - 2,500)/1.08^3 + (\$206,402.32 + 27,500 + 68,978.48)/1.08^4 \\ \text{NPV} = & \$106,654.44 \end{aligned}$$

14. The book value of the asset is zero, so the aftertax salvage value will be:

$$\text{Aftertax salvage value} = \$55,000 + (\$0 - 55,000)(.23)$$

$$\text{Aftertax salvage value} = \$42,350$$

So, the OCF for each year will be:

$$OCF_1 = \$245,000(1 - .23) + .23(\$670,000) = \$342,750$$

$$OCF_2 = \$245,000(1 - .23) = \$188,650$$

$$OCF_3 = \$245,000(1 - .23) = \$188,650$$

$$OCF_4 = \$245,000(1 - .23) = \$188,650$$

Now we have all the necessary information to calculate the project NPV. We need to be careful with the NWC in this project. Notice the project requires \$20,000 of NWC at the beginning, and \$2,500 more in NWC each successive year. We will subtract the \$20,000 from the initial cash flow and subtract \$2,500 each year from the OCF to account for this spending. In Year 4, we will add back the total spent on NWC, which is \$27,500. The \$2,500 spent on NWC capital during Year 4 is irrelevant. Why? Well, during this year the project required an additional \$2,500, but we would get the money back immediately. So, the net cash flow for additional NWC would be zero. With all this, the equation for the NPV of the project is:

$$\begin{aligned}\text{NPV} &= -\$675,000 - 20,000 + (\$342,750 - 2,500)/1.08 + (\$188,650 - 2,500)/1.08^2 \\ &\quad + (\$188,650 - 2,500)/1.08^3 + (\$188,650 + 27,500 + 42,350)/1.08^4 \\ \text{NPV} &= \$122,417.01\end{aligned}$$

15. If we are trying to decide between two projects that will not be replaced when they wear out, the proper capital budgeting method to use is NPV. Both projects only have costs associated with them, not sales, so we will use these to calculate the NPV of each project. Using the tax shield approach to calculate the OCF, the NPV of System A is:

$$\begin{aligned}\text{OCF}_A &= -\$73,000(1 - .23) + .23(\$265,000/4) \\ \text{OCF}_A &= -\$40,973\end{aligned}$$

$$\begin{aligned}\text{NPV}_A &= -\$265,000 - \$40,973(\text{PVIFA}_{7.5\%,4}) \\ \text{NPV}_A &= -\$402,230.27\end{aligned}$$

And the NPV of System B is:

$$\begin{aligned}\text{OCF}_B &= -\$64,000(1 - .23) + .23(\$380,000/6) \\ \text{OCF}_B &= -\$34,713\end{aligned}$$

$$\begin{aligned}\text{NPV}_B &= -\$380,000 - \$34,713(\text{PVIFA}_{7.5\%,6}) \\ \text{NPV}_B &= -\$542,939.06\end{aligned}$$

If the system will not be replaced when it wears out, then System A should be chosen, because it has the less negative NPV.

16. If the equipment will be replaced at the end of its useful life, the correct capital budgeting technique is EAC. Using the NPVs we calculated in the previous problem, the EAC for each system is:

$$\begin{aligned}\text{EAC}_A &= -\$402,230.27/(\text{PVIFA}_{7.5\%,4}) \\ \text{EAC}_A &= -\$120,092.89\end{aligned}$$

$$\begin{aligned}\text{EAC}_B &= -\$542,939.06/(\text{PVIFA}_{7.5\%,6}) \\ \text{EAC}_B &= -\$115,670.39\end{aligned}$$

If the conveyor belt system will be continually replaced, we should choose System B since it has the less negative EAC.

17. The aftertax salvage value is:

$$\text{Aftertax salvage value} = \$45,000(1 - .22)$$

$$\text{Aftertax salvage value} = \$35,100$$

And the OCF each year will be:

	<u>Year 1</u>	<u>Years 2-5</u>
Sales	\$275,000	\$275,000
Variable costs	96,250	96,250
Fixed costs	47,000	47,000
Depreciation	425,000	0
EBIT	-\$293,250	\$131,750
Tax	-64,515	28,985
Net income	-\$228,735	\$102,765
+ Depreciation	425,000	0
OCF	\$196,265	\$102,765

Now we have all the necessary information to calculate the project NPV. The project NPV is:

$$\text{NPV} = -\$425,000 - 25,000 + \$196,265/1.09 + \$102,765/1.09^2 + \$102,765/1.09^3 + \$102,765/1.09^4 + (\$102,765 + 35,100 + 25,000)/1.09^5$$

$$\text{NPV} = \$74,560.70$$

18. Since we need to calculate the EAC for each machine, sales are irrelevant. EAC only uses the costs of operating the equipment, not the sales. Using the bottom up approach, or net income plus depreciation, method to calculate OCF, we get:

	<u>Machine A</u>	<u>Machine B</u>
Variable costs	-\$4,340,000	-\$3,720,000
Fixed costs	-187,000	-145,000
Depreciation	-330,000	-600,000
EBT	-\$4,857,000	-\$4,465,000
Tax	1,019,970	937,650
Net income	-\$3,837,030	-\$3,527,350
+ Depreciation	330,000	600,000
OCF	-\$3,507,030	-\$2,927,350

The PV of costs and EAC for Machine A is:

$$\text{PV of costs}_A = -\$1,980,000 - \$3,507,030(\text{PVIFA}_{10\%,6})$$

$$\text{PV of costs}_A = -\$17,254,029.93$$

$$\text{EAC}_A = -\$17,254,029.93/(\text{PVIFA}_{10\%,6})$$

$$\text{EAC}_A = -\$3,961,652.61$$

And the PV of costs and EAC for Machine B is:

$$\begin{aligned}\text{PV of costs}_B &= -\$5,400,000 - 2,927,350(\text{PVIFA}_{10\%,9}) \\ \text{PV of costs}_B &= -\$22,258,678.37\end{aligned}$$

$$\begin{aligned}\text{EAC}_B &= -\$22,258,678.37/(\text{PVIFA}_{10\%,9}) \\ \text{EAC}_B &= -\$3,865,008.91\end{aligned}$$

You should choose Machine B since it has a less negative EAC.

19. When we are dealing with nominal cash flows, we must be careful to discount cash flows at the nominal interest rate, and we must discount real cash flows using the real interest rate. Project A's cash flows are in real terms, so we need to find the real interest rate. Using the Fisher equation, the real interest rate is:

$$\begin{aligned}1 + R &= (1 + r)(1 + h) \\ 1.11 &= (1 + r)(1 + .04) \\ r &= .0673, \text{ or } 6.73\%\end{aligned}$$

So, the NPV of Project A's real cash flows, discounting at the real interest rate, is:

$$\begin{aligned}\text{NPV} &= -\$42,000 + \$21,000/1.0673 + \$19,000/1.0673^2 + \$17,000/1.0673^3 \\ \text{NPV} &= \$8,337.19\end{aligned}$$

Project B's cash flow are in nominal terms, so the NPV discounted at the nominal interest rate is:

$$\begin{aligned}\text{NPV} &= -\$53,000 + \$24,000/1.11 + \$25,000/1.11^2 + \$23,000/1.11^3 \\ \text{NPV} &= \$5,729.58\end{aligned}$$

We should accept Project A if the projects are mutually exclusive since it has the highest NPV.

20. To determine the value of a firm, we can find the present value of the firm's future cash flows. No depreciation is given, so we can assume depreciation is zero. Using the tax shield approach, we can find the present value of the aftertax revenues, and the present value of the aftertax costs. The required return, growth rates, price, and costs are all given in real terms. Subtracting the costs from the revenues will give us the value of the firm's cash flows. We must calculate the present value of each separately since each is growing at a different rate. First, we will find the present value of the revenues. The revenues in Year 1 will be the number of bottles sold, times the price per bottle, or:

$$\begin{aligned}\text{Aftertax revenue in Year 1 in real terms} &= (3,400,000 \times \$1.43)(1 - .21) \\ \text{Aftertax revenue in Year 1 in real terms} &= \$3,840,980\end{aligned}$$

Revenues will grow at 6 percent per year in real terms forever. Apply the growing perpetuity formula, we find the present value of the revenues is:

$$\begin{aligned}\text{PV of revenues} &= C_1/(R - g) \\ \text{PV of revenues} &= \$3,840,980/ (.06 - .013) \\ \text{PV of revenues} &= \$81,722,979\end{aligned}$$

The real aftertax costs in Year 1 will be:

$$\text{Aftertax costs in year 1 in real terms} = (3,400,000 \times \$0.85)(1 - .21)$$

$$\text{Aftertax costs in year 1 in real terms} = \$2,283,100$$

Costs will grow at .9 percent per year in real terms forever. Applying the growing perpetuity formula, we find the present value of the costs is:

$$\text{PV of costs} = C_1/(R - g)$$

$$\text{PV of costs} = \$2,283,100/(\text{.06} - \text{.009})$$

$$\text{PV of costs} = \$44,766,667$$

Now we can find the value of the firm, which is:

$$\text{Value of the firm} = \text{PV of revenues} - \text{PV of costs}$$

$$\text{Value of the firm} = \$81,722,979 - 44,766,667$$

$$\text{Value of the firm} = \$36,956,312$$

21. To calculate the nominal cash flows, we increase each item in the income statement by the inflation rate, except for depreciation. Depreciation is a nominal cash flow, so it does not need to be adjusted for inflation in nominal cash flow analysis. Since the resale value is given in nominal terms as of the end of Year 5, it does not need to be adjusted for inflation. Also, no inflation adjustment is needed for net working capital since it is already expressed in nominal terms. Note that an increase in required net working capital is a negative cash flow whereas a decrease in required net working capital is a positive cash flow. We first need to calculate the taxes on the salvage value. Remember, to calculate the taxes paid (or tax credit) on the salvage value, we take the book value minus the market value, times the tax rate, which, in this case, would be:

$$\text{Taxes on salvage value} = (BV - MV)t_c$$

$$\text{Taxes on salvage value} = (\$0 - 55,000)(.21)$$

$$\text{Taxes on salvage value} = -\$11,550$$

So, the nominal aftertax salvage value is:

Market price	\$55,000
Tax on sale	<u>-11,550</u>
Aftertax salvage value	\$43,450

Now we can find the nominal cash flows each year using the income statement. Doing so, we find:

	<u>Year 0</u>	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>	<u>Year 5</u>
Sales		\$275,000	\$283,250	\$291,748	\$300,500	\$309,515
Expenses		87,000	89,610	92,298	95,067	97,919
Depreciation		87,000	87,000	87,000	87,000	87,000
EBT		\$101,000	\$106,640	\$112,449	\$118,433	\$124,596
Tax		21,210	22,394	23,614	24,871	26,165
Net income		\$79,790	\$84,246	\$88,835	\$93,562	\$98,431
OCF		\$166,790	\$171,246	\$175,835	\$180,562	\$185,431
Capital spending	-\$435,000					43,450
NWC	-15,000					15,000
Total cash flow	-\$450,000	\$166,790	\$171,246	\$175,835	\$180,562	\$243,881

22. The present value of the company is the present value of the future cash flows generated by the company. Here we have real cash flows, a real interest rate, and a real growth rate. The cash flows are a growing perpetuity, with a negative growth rate. Using the growing perpetuity equation, the present value of the cash flows is:

$$PV = C_1 / (R - g)$$

$$PV = \$345,000 / [.04 - (-.03)]$$

$$PV = \$4,928,571.43$$

23. To find the EAC, we first need to calculate the PV of costs of the incremental cash flows. We will begin with the aftertax salvage value, which is:

$$\text{Taxes on salvage value} = (BV - MV)T_c$$

$$\text{Taxes on salvage value} = (\$0 - 7,500)(.23)$$

$$\text{Taxes on salvage value} = -\$1,725$$

Market price	\$7,500
Tax on sale	<u>-1,725</u>
Aftertax salvage value	\$5,775

Now we can find the operating cash flows. Using the tax shield approach, the operating cash flow each year will be:

$$OCF = -\$10,600(1 - .23) + .23(\$96,000/3)$$

$$OCF = -\$725.33$$

So, the PV of costs of the cost of the decision to buy is:

$$PV \text{ of costs} = -\$97,000 - \$725.33(PVIFA_{12\%,3}) + (\$5,775/1.12^3)$$

$$PV \text{ of costs} = -\$94,631.60$$

In order to calculate the equivalent annual cost, set the PV of costs of the equipment equal to an annuity with the same economic life. Since the project has an economic life of 3 years and is discounted at 12 percent, set the PV of costs equal to a 3-year annuity, discounted at 12 percent.

$$EAC = -\$94,631.60 / (PVIFA_{12\%,3})$$

$$EAC = -\$39,399.77$$

24. We will calculate the aftertax salvage value first. The aftertax salvage value of the equipment will be:

$$\text{Taxes on salvage value} = (BV - MV)T_c$$

$$\text{Taxes on salvage value} = (\$0 - 30,000)(.22)$$

$$\text{Taxes on salvage value} = -\$6,600$$

Market price	\$30,000
Tax on sale	<u>-6,600</u>
Aftertax salvage value	\$23,400

Next, we will calculate the initial cash outlay, that is, the cash flow at Time 0. To undertake the project, we will have to purchase the equipment. The new project will decrease the net working capital, so this is a cash inflow at the beginning of the project. So, the cash outlay today for the project will be:

Equipment	-\$385,000
NWC	<u>70,000</u>
Total	-\$315,000

Now we can calculate the operating cash flow each year for the project. Using the bottom up approach, the operating cash flow will be:

Saved salaries	\$90,000
Depreciation	<u>77,000</u>
EBT	\$13,000
Taxes	<u>2,860</u>
Net income	\$10,140

And the OCF will be:

$$OCF = \$10,140 + 77,000$$

$$OCF = \$87,140$$

Now we can find the NPV of the project. In Year 5, we must replace the saved NWC, so:

$$NPV = -\$315,000 + \$87,140(PVIFA_{9\%,5}) + (\$23,400 - 70,000)/1.09^5$$

$$NPV = -\$6,342.59$$

25. Replacement decision analysis is the same as the analysis of two competing projects, in this case, keep the current equipment, or purchase the new equipment. We will consider the purchase of the new machine first.

Purchase new machine:

The initial cash outlay for the new machine is the cost of the new machine, plus the increased net working capital. So, the initial cash outlay will be:

Purchase new machine	–\$15,600,000
Net working capital	<u>–250,000</u>
Total	–\$15,850,000

Next, we can calculate the operating cash flow created if the company purchases the new machine. The saved operating expense is an incremental cash flow. Additionally, the reduced operating expense is a cash inflow, so it should be treated as such in the income statement. So, the annual operating cash flow created by purchasing the new machine, will be:

Operating expense	\$6,300,000
Depreciation	<u>3,900,000</u>
EBT	\$2,400,000
Taxes	<u>504,000</u>
Net income	<u>\$1,896,000</u>
OCF	\$5,796,000

So, the NPV of purchasing the new machine, including the recovery of the net working capital, is:

$$\text{NPV} = -\$15,850,000 + \$5,796,000(\text{PVIFA}_{10\%,4}) + \$250,000/1.10^4$$

$$\text{NPV} = \$2,693,293.49$$

And the IRR is:

$$0 = -\$15,850,000 + \$5,796,000(\text{PVIFA}_{\text{IRR},4}) + \$250,000/(1 + \text{IRR})^4$$

Using a spreadsheet or financial calculator, we find the IRR is:

$$\text{IRR} = 17.58\%$$

Now we can calculate the decision to keep the old machine:

Keep old machine:

The initial cash outlay for the old machine is the market value of the old machine, including any potential tax consequence. The decision to keep the old machine has an opportunity cost, namely, the company could sell the old machine. Also, if the company sells the old machine at its current value, it will receive a tax benefit. Both of these cash flows need to be included in the analysis. So, the initial cash flow of keeping the old machine will be:

Keep machine	-\$4,100,000
Taxes	<u>-273,000</u>
Total	-\$4,373,000

Next, we can calculate the operating cash flow created if the company keeps the old machine. There are no incremental cash flows from keeping the old machine, but we need to account for the cash flow effects of depreciation. The income statement, adding depreciation to net income to calculate the operating cash flow, will be:

Depreciation	<u>\$1,350,000</u>
EBT	-\$1,350,000
Taxes	<u>-283,500</u>
Net income	<u><u>-\$1,066,500</u></u>
OCF	\$283,500

So, the NPV of the decision to keep the old machine will be:

$$\text{NPV} = -\$4,373,000 + \$283,500(\text{PVIFA}_{10\%,4})$$

$$\text{NPV} = -\$3,474,343.15$$

And the IRR is:

$$0 = -\$4,373,000 + \$283,500(\text{PVIFA}_{\text{IRR},4})$$

Using a spreadsheet or financial calculator, we find the IRR is:

$$\text{IRR} = -38.33\%$$

There is another way to analyze a replacement decision that is often used. It is an incremental cash flow analysis of the change in cash flows from the existing machine to the new machine, assuming the new machine is purchased. In this type of analysis, the initial cash outlay would be the cost of the new machine, the increased NWC, and the cash inflow (including any applicable taxes) of selling the old machine. In this case, the initial cash flow under this method would be:

Purchase new machine	–\$15,600,000
Net working capital	–250,000
Sell old machine	4,100,000
Taxes on old machine	<u>273,000</u>
Total	–\$11,477,000

The cash flows from purchasing the new machine would be the saved operating expenses. We would also need to include the change in depreciation. The old machine has a depreciation of \$1.35 million per year, and the new machine has depreciation of \$3.9 million per year, so the increased depreciation will be \$2.55 million per year. The pro forma income statement and operating cash flow under this approach will be:

Operating expense savings	\$6,300,000
Depreciation	<u>2,550,000</u>
EBT	\$3,750,000
Taxes	<u>787,500</u>
Net income	<u>\$2,962,500</u>
OCF	\$5,512,500

The NPV under this method is:

$$\text{NPV} = -\$11,477,000 + \$5,512,500(\text{PVIFA}_{10\%,4}) + \$250,000/1.10^4$$

$$\text{NPV} = \$6,167,636.64$$

And the IRR is:

$$0 = -\$11,477,000 + \$5,512,500(\text{PVIFA}_{\text{IRR},4}) + \$250,000/(1 + \text{IRR})^4$$

Using a spreadsheet or financial calculator, we find the IRR is:

$$\text{IRR} = 32.83\%$$

So, this analysis still tells us the company should purchase the new machine. This is really the same type of analysis we originally performed. Consider this: Subtract the NPV of the decision to keep the old machine from the NPV of the decision to purchase the new machine. You will get:

$$\text{Differential NPV} = \$2,693,293.49 - (-\$3,474,343.15) = \$6,167,636.64$$

This is the exact same NPV we calculated when using the second method.

26. We can find the NPV of a project using nominal cash flows or real cash flows. Either method will result in the same NPV. For this problem, we will calculate the NPV using both nominal and real cash flows. The initial investment in either case is \$650,000 since it will be spent today. We will begin with the nominal cash flows. The revenues and production costs increase at different rates, so we must be careful to increase each at the appropriate growth rate. The nominal cash flows for each year will be:

	<u>Year 0</u>	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>
			\$593,250.0	
Revenues		\$565,000.00	0	\$622,912.50
Costs		\$345,000.00	358,800.00	373,152.00
Depreciation		92,857.14	92,857.14	92,857.14
			\$141,592.8	
EBT		\$127,142.86	6	\$156,903.36
Taxes		30,514.29	33,982.29	37,656.81
			\$107,610.5	
Net income		\$96,628.57	7	\$119,246.55
			\$200,467.7	
OCF		\$189,485.71	1	\$212,103.69
Capital spending	-\$650,000			
			\$200,467.7	
Total cash flow	-\$650,000	\$189,485.71	1	\$212,103.69
	<u>Year 4</u>	<u>Year 5</u>	<u>Year 6</u>	<u>Year 7</u>
Revenues	\$654,058.13	\$686,761.03	\$721,099.08	\$757,154.04
Costs	388,078.08	403,601.20	419,745.25	436,535.06
Depreciation	92,857.14	92,857.14	92,857.14	92,857.14
EBT	\$173,122.90	\$190,302.69	\$208,496.69	\$227,761.83
Taxes	41,549.50	45,672.64	50,039.21	54,662.84
Net income	\$131,573.41	\$144,630.04	\$158,457.48	\$173,098.99
OCF	\$224,430.55	\$237,487.18	\$251,314.63	\$265,956.14
Capital spending				
Total cash flow	\$224,430.55	\$237,487.18	\$251,314.63	\$265,956.14

Now that we have the nominal cash flows, we can find the NPV. We must use the nominal required return with nominal cash flows. Using the Fisher equation to find the nominal required return, we get:

$$(1 + R) = (1 + r)(1 + h)$$

$$(1 + R) = (1 + .07)(1 + .05)$$

$$R = .1235, \text{ or } 12.35\%$$

So, the NPV of the project using nominal cash flows is:

$$\begin{aligned}\text{NPV} &= -\$650,000 + \$189,485.71/1.1235 + \$200,467.71/1.1235^2 + \$212,103.69/1.1235^3 \\ &\quad + \$224,430.55/1.1235^4 + \$237,487.18/1.1235^5 + \$251,314.63/1.1235^6 \\ &\quad + \$265,956.14/1.1235^7 \\ \text{NPV} &= \$343,238.38\end{aligned}$$

We can also find the NPV using real cash flows. Since the revenues and costs are given in nominal terms, we must find the real value for each by discounting at the inflation rate. So:

$$\begin{aligned}\text{Real revenues} &= \$565,000/(1 + .05) \\ \text{Real revenues} &= \$538,095.24\end{aligned}$$

$$\begin{aligned}\text{Real costs} &= \$345,000/(1 + .05) \\ \text{Real costs} &= \$328,571.43\end{aligned}$$

Also, we must note that the costs are increasing at a different rate than the general inflation rate. As a result, we must find the real cost increase, which is:

$$\begin{aligned}\text{Real cost increase} &= [(1 + .04)/(1 + .05)] - 1 \\ \text{Real cost increase} &= -.0095, \text{ or } -.95\%\end{aligned}$$

In real terms, the cost will actually decrease by .95 percent per year. So, the real cash flows will be:

	<u>Year 0</u>	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>
Revenues		\$538,095.24	\$538,095.24	\$538,095.24
Costs		\$328,571.43	325,442.18	322,342.73
Depreciation		88,435.37	84,224.17	80,213.49
			\$128,428.9	
EBT		\$121,088.44	0	\$135,539.02
Taxes		29,061.22	30,822.93	32,529.36
Net income		\$92,027.21	\$97,605.96	\$103,009.65
			\$181,830.1	
OCF		\$180,462.59	3	\$183,223.15
Capital spending	-\$650,000			
			\$181,830.1	
Total cash flow	-\$650,000	\$180,462.59	3	\$183,223.15

	<u>Year 4</u>	<u>Year 5</u>	<u>Year 6</u>	<u>Year 7</u>
Revenues	\$538,095.24	\$538,095.24	\$538,095.24	\$538,095.24
Costs	319,272.80	316,232.10	313,220.37	310,237.32
Depreciation	76,393.80	72,756.00	69,291.43	65,991.84
EBT	\$142,428.64	\$149,107.13	\$155,583.44	\$161,866.08
Taxes	34,182.87	35,785.71	37,340.03	38,847.86
Net income	\$108,245.77	\$113,321.42	\$118,243.41	\$123,018.22
OCF	\$184,639.57	\$186,077.42	\$187,534.84	\$189,010.06

Capital spending

Total cash flow \$184,639.57 \$186,077.42 \$187,534.84 \$189,010.06

So, the NPV of the project using real cash flows is:

$$\begin{aligned} \text{NPV} &= -\$650,000 + \$180,462.59/1.07 + \$181,830.13/1.07^2 + \$183,223.15/1.07^3 \\ &\quad + \$184,639.57/1.07^4 + \$186,077.42/1.07^5 + \$187,534.84/1.07^6 \\ &\quad + \$189,010.06/1.07^7 \\ \text{NPV} &= \$343,238.38 \end{aligned}$$

We can also find the NPV using real cash flows and the nominal required return. This will allow us to find the operating cash flow using the tax shield approach. Both the revenues and expenses are growing annuities, but growing at different rates. This means we must find the present value of each separately. We also need to account for the effect of taxes, so we will multiply by one minus the tax rate. So, the present value of the aftertax revenues using the growing annuity equation is:

$$\begin{aligned} \text{PV of aftertax revenues} &= C\{[1/(r-g)] - [1/(r-g)] \times [(1+g)/(1+r)]^t\}(1-T_c) \\ \text{PV of aftertax revenues} &= \$565,000\{[1/(\cdot.1235 - .05)] - [1/(\cdot.1235 - .05)] \\ &\quad \times [(1+.05)/(1+.1235)]^7\}(1-.24) \\ \text{PV of aftertax revenues} &= \$2,203,962.73 \end{aligned}$$

And the present value of the aftertax costs will be:

$$\begin{aligned} \text{PV of aftertax costs} &= C\{[1/(r-g)] - [1/(r-g)] \times [(1+g)/(1+r)]^t\}(1-T_c) \\ \text{PV of aftertax costs} &= \$345,000\{[1/(\cdot.1235 - .04)] - [1/(\cdot.1235 - .04)] \\ &\quad \times [(1+.04)/(1+.1235)]^7\}(1-.24) \\ \text{PV of aftertax costs} &= \$1,311,312.03 \end{aligned}$$

Now we need to find the present value of the depreciation tax shield. The depreciation amount in the first year is a nominal value, so we can find the present value of the depreciation tax shield as an ordinary annuity using the nominal required return. So, the present value of the depreciation tax shield will be:

$$\begin{aligned} \text{PV of depreciation tax shield} &= (\$650,000/7)(.24)(\text{PVIFA}_{12.35\%,7}) \\ \text{PV of depreciation tax shield} &= \$100,587.68 \end{aligned}$$

Using the present value of the real cash flows to find the NPV, we get:

$$\text{NPV} = \text{Initial cost} + \text{PV of revenues} - \text{PV of costs} + \text{PV of depreciation tax shield}$$

$$\text{NPV} = -\$650,000 + \$2,203,962.73 - 1,311,312.03 + 100,587.68$$

$$\text{NPV} = \$343,238.38$$

Notice, the NPVs using nominal cash flows and real cash flows are identical, which is what we would expect.

27. Here we have a project in which the quantity sold each year increases. First, we need to calculate the quantity sold each year by increasing the current year's quantity by the growth rate. So, the quantity sold each year will be:

$$\text{Year 1 quantity} = 10,400$$

$$\text{Year 2 quantity} = 10,400(1 + .08) = 11,232$$

$$\text{Year 3 quantity} = 11,232(1 + .08) = 12,131$$

$$\text{Year 4 quantity} = 12,131(1 + .08) = 13,101$$

$$\text{Year 5 quantity} = 13,101(1 + .08) = 14,149$$

Now we can calculate the sales revenue and variable costs each year. The pro forma income statements and operating cash flow each year will be:

	<u>Year 0</u>	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>	<u>Year 5</u>
Revenues		\$634,400.00	\$685,152.00	\$739,964.16	\$799,161.29	\$863,094.20
Fixed costs		125,000.00	125,000.00	125,000.00	125,000.00	125,000.00
Variable costs		197,600.00	213,408.00	230,480.64	248,919.09	268,832.62
Depreciation		115,000.00	115,000.00	115,000.00	115,000.00	115,000.00
EBT		\$196,800.00	\$231,744.00	\$269,483.52	\$310,242.20	\$354,261.58
Taxes		41,328.00	48,666.24	56,591.54	65,150.86	74,394.93
Net income		<u>\$155,472.00</u>	<u>\$183,077.76</u>	<u>\$212,891.98</u>	<u>\$245,091.34</u>	<u>\$279,866.65</u>
OCF		\$270,472.00	\$298,077.76	\$327,891.98	\$360,091.34	\$394,866.65

Capital spending -\$575,000

NWC -45,000 45,000

Total cash flow -\$620,000 \$270,472.00 \$298,077.76 \$327,891.98 \$360,091.34 \$439,866.65

So, the NPV of the project is:

$$\text{NPV} = -\$620,000 + \$270,472/1.18 + \$298,077.76/1.18^2 + \$327,891.98/1.18^3$$

$$+ \$360,091.34/1.18^4 + \$439,866.65/1.18^5$$

$$\text{NPV} = \$400,854.42$$

We could also calculate the cash flows using the tax shield approach, with growing annuities and ordinary annuities. The sales and variable costs increase at the same rate as sales, so both are growing annuities. The fixed costs and depreciation are both ordinary annuities. Using the growing annuity equation, the present value of the revenues is:

$$\begin{aligned}\text{PV of revenues} &= C\{[1/(r-g)] - [1/(r-g)] \times [(1+g)/(1+r)]^t\} \\ \text{PV of revenues} &= \$634,400\{[1/(.18-.08)] - [1/(.18-.08)] \times [(1+.08)/(1+.18)]^5\} \\ \text{PV of revenues} &= \$2,269,522.58\end{aligned}$$

And the present value of the variable costs will be:

$$\begin{aligned}\text{PV of variable costs} &= C\{[1/(r-g)] - [1/(r-g)] \times [(1+g)/(1+r)]^t\} \\ \text{PV of variable costs} &= \$197,600\{[1/(.18-.08)] - [1/(.18-.08)] \times [(1+.08)/(1+.18)]^5\} \\ \text{PV of variable costs} &= \$706,900.48\end{aligned}$$

The fixed costs and depreciation are both ordinary annuities. The present value of each is:

$$\begin{aligned}\text{PV of fixed costs} &= C\{[1 - 1/(1+r)]/r\} \\ \text{PV of fixed costs} &= \$125,000(\text{PVIFA}_{18\%,5}) \\ \text{PV of fixed costs} &= \$390,896.38\end{aligned}$$

$$\begin{aligned}\text{PV of depreciation} &= C\{[1 - 1/(1+r)]/r\} \\ \text{PV of depreciation} &= \$115,000(\text{PVIFA}_{18\%,5}) \\ \text{PV of depreciation} &= \$359,624.67\end{aligned}$$

Now, we can use the depreciation tax shield approach to find the NPV of the project, which is:

$$\begin{aligned}\text{NPV} &= -\$620,000 + (\$2,269,522.58 - 706,900.48 - 390,896.38)(1 - .21) + (\$359,624.67)(.21) \\ &\quad + \$45,000/1.18^5 \\ \text{NPV} &= \$400,854.42\end{aligned}$$

28. We will begin by calculating the aftertax salvage value of the equipment at the end of the project's life. The aftertax salvage value is the market value of the equipment minus any taxes paid (or refunded), so the aftertax salvage value in four years will be:

$$\begin{aligned}\text{Taxes on salvage value} &= (\text{BV} - \text{MV})t_c \\ \text{Taxes on salvage value} &= (\$0 - 310,000)(.23) \\ \text{Taxes on salvage value} &= -\$71,300\end{aligned}$$

Market price	\$310,000
Tax on sale	<u>-71,300</u>
Aftertax salvage value	\$238,700

Now we need to calculate the operating cash flow each year. Note, we assume that the net working capital cash flow occurs immediately. Using the bottom up approach to calculating operating cash flow, we find:

	<u>Year 0</u>	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>
Revenues		\$2,165,500	\$2,409,500	\$2,806,000	\$1,860,500
Fixed costs		375,000	375,000	375,000	375,000
Variable costs		324,825	361,425	420,900	279,075
Depreciation		1,133,220	1,511,300	503,540	251,940
EBT		\$332,455	\$161,775	\$1,506,560	\$954,485
Taxes		76,465	37,208	346,509	219,532
Net income		\$255,990	\$124,567	\$1,160,051	\$734,953
OCF		\$1,389,210	\$1,635,867	\$1,663,591	\$986,893
Capital spending	-\$3,400,000				238,700
Land	-900,000				1,200,000
NWC	-225,000				225,000
Total cash flow	-\$4,525,000	\$1,389,210	\$1,635,867	\$1,663,591	\$2,650,593

Notice the calculation of the cash flow at Time 0. The capital spending on equipment and investment in net working capital are cash outflows. The aftertax selling price of the land is also a cash outflow. Even though no cash is actually spent on the land because the company already owns it, the aftertax cash flow from selling the land is an opportunity cost, so we need to include it in the analysis. With all the project cash flows, we can calculate the NPV, which is:

$$\begin{aligned} \text{NPV} &= -\$4,525,000 + \$1,389,210/1.13 + \$1,635,867/1.13^2 + \$1,663,591/1.13^3 \\ &\quad + \$2,650,593/1.13^4 \\ \text{NPV} &= \$764,124.06 \end{aligned}$$

The company should accept the new product line.

29. The aftertax salvage value will be the same as the previous problem, \$238,700. We need to calculate the operating cash flow each year. Note, we assume that the net working capital cash flow occurs immediately. Using the bottom up approach to calculating operating cash flow, we find:

	<u>Year 0</u>	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>
Revenues		\$2,165,500	\$2,409,500	\$2,806,000	\$1,860,500
Fixed costs		375,000	375,000	375,000	375,000
Variable costs		324,825	361,425	420,900	279,075
Depreciation		3,400,000	0	0	0
EBT		-\$1,934,325	\$1,673,075	\$2,010,100	\$1,206,425
Taxes		-444,895	384,807	462,323	277,478
Net income		-\$1,489,430	\$1,288,268	\$1,547,777	\$928,947
OCF		\$1,910,570	\$1,288,268	\$1,547,777	\$928,947
Capital spending	-\$3,400,000				238,700
Land	-900,000				1,200,000
NWC	-225,000				225,000
Total cash flow	-\$4,525,000	\$1,910,570	\$1,288,268	\$1,547,777	\$2,592,647

Notice the calculation of the cash flow at Time 0. The capital spending on equipment and investment in net working capital are cash outflows. The aftertax selling price of the land is also a cash outflow. Even though no cash is actually spent on the land because the company already owns it, the aftertax cash flow from selling the land is an opportunity cost, so we need to include it in the analysis. With all the project cash flows, we can calculate the NPV, which is:

$$\begin{aligned} \text{NPV} &= -\$4,525,000 + \$1,910,570/1.13 + \$1,288,268/1.13^2 + \$1,547,777/1.13^3 \\ &\quad + \$2,592,647/1.13^4 \\ \text{NPV} &= \$837,478.52 \end{aligned}$$

30. Replacement decision analysis is the same as the analysis of two competing projects; in this case, keep the current equipment, or purchase the new equipment. We will consider the purchase of the new machine first.

Purchase new machine:

The initial cash outlay for the new machine is the cost of the new machine. We can calculate the operating cash flow created if the company purchases the new machine. The maintenance cost is an incremental cash flow, so using the pro forma income statement, and adding depreciation to net income, the operating cash flow created each year by purchasing the new machine will be:

Maintenance cost	\$350,000
Depreciation	<u>900,000</u>
EBT	−\$1,250,000
Taxes	<u>−262,500</u>
Net income	−\$987,500
OCF	−\$87,500

Notice the taxes are negative, implying a tax credit. The new machine also has a salvage value at the end of five years, so we need to include this in the cash flows analysis. The aftertax salvage value will be:

Sell machine	\$900,000
Taxes	<u>−189,000</u>
Total	\$711,000

The NPV of purchasing the new machine is:

$$\text{NPV} = -\$4,500,000 - \$87,500(\text{PVIFA}_{8\%,5}) + \$711,000/1.08^5$$

$$\text{NPV} = -\$4,365,467.48$$

Notice the NPV is negative. This does not necessarily mean we should not purchase the new machine. In this analysis, we are only dealing with costs, so we would expect a negative NPV. The revenue is not included in the analysis since it is not incremental to the machine. Similar to an EAC analysis, we will use the machine with the least negative NPV. Now we can calculate the decision to keep the old machine.

Keep old machine:

The initial cash outlay for keeping the old machine is the market value of the old machine, including any potential tax. The decision to keep the old machine has an opportunity cost, namely, the company could sell the old machine. Also, if the company sells the old machine at its current value, it will incur taxes. Both of these cash flows need to be included in the analysis. So, the initial cash flow of keeping the old machine will be:

Keep machine	−\$2,800,000
Taxes	<u>252,000</u>
Total	−\$2,548,000

Next, we can calculate the operating cash flow created if the company keeps the old machine. We need to account for the cost of maintenance, as well as the cash flow effects of depreciation. The pro forma income statement, adding depreciation to net income to calculate the operating cash flow will be:

Maintenance cost	\$855,000
Depreciation	<u>320,000</u>
EBT	-\$1,175,000
Taxes	<u>-246,750</u>
Net income	<u>-\$928,250</u>
OCF	-\$608,250

The old machine also has a salvage value at the end of five years, so we need to include this in the cash flows analysis. The aftertax salvage value will be:

Sell machine	\$140,000
Taxes	<u>-29,400</u>
Total	\$110,600

So, the NPV of the decision to keep the old machine will be:

$$\text{NPV} = -\$2,548,000 - \$608,250(\text{PVIFA}_{8\%,5}) + \$110,600/1.08^5$$

$$\text{NPV} = -\$4,901,293.38$$

The company should buy the new machine since it has a greater NPV.

There is another way to analyze a replacement decision that is often used. It is an incremental cash flow analysis of the change in cash flows from the existing machine to the new machine, assuming the new machine is purchased. In this type of analysis, the initial cash outlay would be the cost of the new machine, and the cash inflow (including any applicable taxes) of selling the old machine. In this case, the initial cash flow under this method would be:

Purchase new machine	-\$4,500,000
Sell old machine	2,800,000
Taxes on old machine	<u>-252,000</u>
Total	-\$1,952,000

The cash flows from purchasing the new machine would be the difference in the operating expenses. We would also need to include the change in depreciation. The old machine has a depreciation of \$320,000 per year, and the new machine has a depreciation of \$900,000 per year, so the increased depreciation will be \$580,000 per year. The pro forma income statement and operating cash flow under this approach will be:

Maintenance cost	–\$505,000
Depreciation	<u>580,000</u>
EBT	–\$75,000
Taxes	<u>–15,750</u>
Net income	<u>–\$59,250</u>
OCF	\$520,750

The salvage value of the differential cash flow approach is more complicated. The company will sell the new machine, and incur taxes on the sale in five years. However, we must also include the lost sale of the old machine. Since we assumed we sold the old machine in the initial cash outlay, we lose the ability to sell the machine in five years. This is an opportunity loss that must be accounted for. So, the salvage value is:

Sell machine	\$900,000
Taxes	–189,000
Lost sale of old	–140,000
Taxes on lost sale of old	<u>29,400</u>
Total	\$600,400

The NPV under this method is:

$$\text{NPV} = -\$1,952,000 + \$520,750(\text{PVIFA}_{8\%,5}) + \$600,400/1.08^5$$

$$\text{NPV} = \$535,825.90$$

So, this analysis still tells us the company should purchase the new machine. This is really the same type of analysis we originally did. Consider this: Subtract the NPV of the decision to keep the old machine from the NPV of the decision to purchase the new machine. You will get:

$$\text{Differential NPV} = -\$4,365,467.48 - (-4,901,293.38)$$

$$\text{Differential NPV} = \$535,825.90$$

This is the exact same NPV we calculated when using the second analysis method.

31. The project has a sales price that increases at 3 percent per year, and a variable cost per unit that increases at 4 percent per year. First, we need to find the sales price and variable cost for each year. The table below shows the price per unit and the variable cost per unit each year.

	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>	<u>Year 5</u>
Sales price	\$47.00	\$48.41	\$49.86	\$51.36	\$52.90
Cost per unit	\$17.00	\$17.68	\$18.39	\$19.12	\$19.89

Using the sales price and variable cost, we can now construct the pro forma income statement for each year. We can use this income statement to calculate the cash flow each year. We must also make sure to include the net working capital outlay at the beginning of the project, and the recovery of the net working capital at the end of the project. The pro forma income statement and cash flows for each year will be:

	<u>Year 0</u>	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>	<u>Year 5</u>
Revenues		\$1,175,000.00	\$1,210,250.00	\$1,246,557.50	\$1,283,954.23	\$1,322,472.85
Fixed costs		235,000.00	235,000.00	235,000.00	235,000.00	235,000.00
Var. costs		425,000.00	442,000.00	459,680.00	478,067.20	497,189.89
Depreciation		240,000.00	240,000.00	240,000.00	240,000.00	240,000.00
EBT		\$275,000.00	\$293,250.00	\$311,877.50	\$330,887.03	\$350,282.96
Taxes		57,750.00	61,582.50	65,494.28	69,486.28	73,559.42
Net income		\$217,250.00	\$231,667.50	\$246,383.23	\$261,400.75	\$276,723.54
OCF		\$457,250.00	\$471,667.50	\$486,383.23	\$501,400.75	\$516,723.54
Cap. spend.	-\$1,200,000					
NWC	-200,000					200,000
Total CF	-\$1,400,000	\$457,250.00	\$471,667.50	\$486,383.23	\$501,400.75	\$716,723.54

With these cash flows, the NPV of the project is:

$$\begin{aligned} \text{NPV} &= -\$1,400,000 + \$457,250/1.11 + \$471,667.50/1.11^2 + \$486,383.23/1.11^3 \\ &\quad + \$501,400.75/1.11^4 + \$716,723.54/1.11^5 \\ \text{NPV} &= \$506,020.82 \end{aligned}$$

We could also answer this problem using the depreciation tax shield approach. The revenues and variable costs are growing annuities, growing at different rates. The fixed costs and depreciation are ordinary annuities. Using the growing annuity equation, the present value of the revenues is:

$$\begin{aligned} \text{PV of revenues} &= C \left\{ \frac{1}{(r-g)} - \frac{1}{(r-g)} \times \frac{(1+g)}{(1+r)^t} \right\} \\ \text{PV of revenues} &= \$1,175,000 \left\{ \frac{1}{(.11 - .03)} - \frac{1}{(.11 - .03)} \times \frac{(1 + .03)}{(1 + .11)^5} \right\} \\ \text{PV of revenues} &= \$4,582,900.40 \end{aligned}$$

And the present value of the variable costs will be:

$$\text{PV of variable costs} = C\{[1/(r - g)] - [1/(r - g)] \times [(1 + g)/(1 + r)]^t\}$$

$$\text{PV of variable costs} = \$425,000\{[1/(.11 - .04)] - [1/(.11 - .04)] \times [(1 + .04)/(1 + .11)]^5\}$$

$$\text{PV of variable costs} = \$1,687,709.72$$

The fixed costs and depreciation are both ordinary annuities. The present value of each is:

$$\text{PV of fixed costs} = C\{[1 - 1/(1 + r)^t]/r\}$$

$$\text{PV of fixed costs} = \$235,000\{[1 - 1/(1 + .11)]^5\}/.11$$

$$\text{PV of fixed costs} = \$868,535.80$$

$$\text{PV of depreciation} = C\{[1 - 1/(1 + r)^t]/r\}$$

$$\text{PV of depreciation} = \$240,000\{[1 - 1/(1 + .11)]^5\}/.11$$

$$\text{PV of depreciation} = \$887,015.28$$

Now, we can use the depreciation tax shield approach to find the NPV of the project, which is:

$$\text{NPV} = -\$1,400,000 + (\$4,582,900.40 - 1,687,709.72 - 868,535.80)(1 - .21) + (\$887,015.28)(.21) + \$200,000/1.11^5$$

$$\text{NPV} = \$506,020.82$$

Challenge

32. Probably the easiest OCF calculation for this problem is the bottom up approach, so we will construct an income statement for each year. Beginning with the initial cash flow at time zero, the project will require an investment in equipment. The project will also require an investment in NWC of \$1,500,000. So, the cash flow required for the project today will be:

Capital spending	-\$18,500,000
Change in NWC	<u>-1,500,000</u>
Total cash flow	-\$20,000,000

Now we can begin the remaining calculations. Sales figures are given for each year, along with the price per unit. The variable costs per unit are used to calculate total variable costs, and fixed costs are given at \$3,400,000 per year. To calculate depreciation each year, we use the initial equipment cost of \$18,500,000, times the appropriate MACRS depreciation each year. The remainder of each income statement is calculated below. Notice at the bottom of the income statement we added back depreciation to get the OCF for each year. The section labeled “Net cash flows” will be discussed below:

<u>Year</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
Ending book value	\$15,856,350	\$11,325,700	\$8,090,050	\$5,779,400	\$4,127,350
Sales	\$23,725,000	\$25,675,000	\$27,300,000	\$26,650,000	\$22,100,000
Variable costs	10,585,000	11,455,000	12,180,000	11,890,000	9,860,000
Fixed costs	3,400,000	3,400,000	3,400,000	3,400,000	3,400,000
Depreciation	2,643,650	4,530,650	3,235,650	2,310,650	1,652,050
EBIT	7,096,350	6,289,350	8,484,350	9,049,350	7,187,950
Taxes	1,632,161	1,446,551	1,951,401	2,081,351	1,653,229
Net income	5,464,190	4,842,800	6,532,950	6,968,000	5,534,722
Depreciation	2,643,650	4,530,650	3,235,650	2,310,650	1,652,050
Operating cash flow	\$8,107,840	\$9,373,450	\$9,768,600	\$9,278,650	\$7,186,772
<u>Net cash flows</u>					
Operating cash flow	\$8,107,840	\$9,373,450	\$9,768,600	\$9,278,650	\$7,186,772
Change in NWC	-292,500	-243,750	97,500	682,500	1,256,250
Capital spending					3,798,291
Total cash flow	\$7,815,340	\$9,129,700	\$9,866,100	\$9,961,150	\$12,241,312

After we calculate the OCF for each year, we need to account for any other cash flows. The other cash flows in this case are NWC cash flows and capital spending, which is the aftertax salvage of the equipment. The required NWC is 15 percent of the sales increase in the next year. We will work through the NWC cash flow for Year 1. The total NWC in Year 1 will be 15 percent of the sales increase from Year 1 to Year 2, or:

$$\begin{aligned}\text{Increase in NWC for Year 1} &= .15(\$23,725,000 - 25,675,000) \\ \text{Increase in NWC for Year 1} &= -\$292,500\end{aligned}$$

Notice that the NWC cash flow is negative. Since the sales are increasing, we will have to spend more money to increase NWC. In Year 3, the NWC cash flow becomes positive when sales are declining. And, in Year 5, the NWC cash flow is the recovery of all NWC the company still has in the project.

To calculate the aftertax salvage value, we first need the book value of the equipment. The book value at the end of the five years will be the purchase price, minus the total depreciation. So, the ending book value is:

$$\begin{aligned}\text{Ending book value} &= \$18,500,000 - (\$2,643,650 + 4,530,650 + 3,235,650 + 2,310,650 + 1,652,050) \\ \text{Ending book value} &= \$4,127,350\end{aligned}$$

The market value of the used equipment is 20 percent of the purchase price, or \$3,700,000, so the aftertax salvage value will be:

$$\text{Aftertax salvage value} = \$3,700,000 + (\$4,127,350 - 3,700,000)(.23)$$

Aftertax salvage value = \$3,798,291

The aftertax salvage value is included in the total cash flows as capital spending. Now we have all of the cash flows for the project. The NPV of the project is:

$$\begin{aligned}\text{NPV} &= -\$20,000,000 + \$7,815,340/1.18 + \$9,129,700/1.18^2 + \$9,866,100/1.18^3 \\ &\quad + \$9,961,150/1.18^4 + \$12,241,312/1.18^5 \\ \text{NPV} &= \$9,673,430.24\end{aligned}$$

And the IRR is:

$$\begin{aligned}\text{IRR} = 0 &= -\$20,000,000 + \$7,815,340/(1 + \text{IRR}) + \$9,129,700/(1 + \text{IRR})^2 \\ &\quad + \$9,866,100/(1 + \text{IRR})^3 + \$9,961,150/(1 + \text{IRR})^4 + \$12,241,312/(1 + \text{IRR})^5 \\ \text{IRR} &= 36.39\%\end{aligned}$$

We should accept the project.

- 33.** To find the initial pretax cost savings necessary to buy the new machine, we should use the tax shield approach to find the OCF. We begin by calculating the depreciation each year using the MACRS depreciation schedule. The depreciation each year is:

$$\begin{aligned}D_1 &= \$645,000(.3333) = \$214,978.50 \\ D_2 &= \$645,000(.4445) = \$286,702.50 \\ D_3 &= \$645,000(.1481) = \$95,524.50 \\ D_4 &= \$645,000(.0741) = \$47,794.50\end{aligned}$$

Using the tax shield approach, the OCF each year is:

$$\begin{aligned}\text{OCF}_1 &= (S - C)(1 - .21) + .21(\$214,978.50) \\ \text{OCF}_2 &= (S - C)(1 - .21) + .21(\$286,702.50) \\ \text{OCF}_3 &= (S - C)(1 - .21) + .21(\$95,524.50) \\ \text{OCF}_4 &= (S - C)(1 - .21) + .21(\$47,794.50) \\ \text{OCF}_5 &= (S - C)(1 - .21)\end{aligned}$$

Now we need the aftertax salvage value of the equipment. The aftertax salvage value is:

$$\begin{aligned}\text{Aftertax salvage value} &= \$50,000(1 - .21) \\ \text{Aftertax salvage value} &= \$39,500\end{aligned}$$

To find the necessary cost reduction, we must realize that we can split the cash flows each year. The OCF in any given year is the cost reduction $(S - C)$ times one minus the tax rate, which is an annuity for the project life, and the depreciation tax shield. To calculate the necessary cost reduction, we would require a zero NPV. The equation for the NPV of the project is:

$$\begin{aligned}\text{NPV} = 0 &= -\$645,000 - 55,000 + (S - C)(.79)(\text{PVIFA}_{12\%,5}) + .21(\$214,978.50/1.12 \\ &\quad + \$286,702.50/1.12^2 + \$95,524.50/1.12^3 + \$47,794.50/1.12^4) + (\$55,000 + 39,500)/1.12^5\end{aligned}$$

Solving this equation for the sales minus costs, we get:

$$\begin{aligned}(S - C)(.79)(\text{PVIFA}_{12\%,5}) &= \$537,415.61 \\ (S - C) &= \$188,714.33\end{aligned}$$

34. To find the bid price, we need to calculate all other cash flows for the project, and then solve for the bid price. The aftertax salvage value of the equipment is:

$$\text{Aftertax salvage value} = \$150,000(1 - .21)$$

$$\text{Aftertax salvage value} = \$118,500$$

Now we can solve for the necessary OCF that will give the project a zero NPV. The equation for the NPV of the project is:

$$\text{NPV} = 0 = -\$2,100,000 - 325,000 + \text{OCF}(\text{PVIFA}_{11\%,5}) + [(\$325,000 + 118,500)/1.11^5]$$

Solving for the OCF, we find the OCF that makes the project NPV equal to zero is:

$$\text{OCF} = \$2,161,804.34/\text{PVIFA}_{11\%,5}$$

$$\text{OCF} = \$584,920.07$$

The easiest way to calculate the bid price is the tax shield approach, so:

$$\text{OCF} = \$584,920.07 = [(P - v)Q - FC](1 - T_c) + T_c D$$

$$\$584,920.07 = [(P - \$9.45)(145,000) - \$650,000](1 - .21) + .21(\$2,100,000/5)$$

$$P = \$18.27$$

35. a. This problem is basically the same as the previous problem, except that we are given a sales price. The cash flow at Time 0 for all three parts of this question will be:

Capital spending	-\$2,100,000
Change in NWC	<u>-325,000</u>
Total cash flow	-\$2,425,000

We will use the initial cash flow and the salvage value we already found in that problem. Using the bottom up approach to calculating the OCF, we get:

Assume price per unit = \$20 and units/year = 145,000

Year	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
Sales	\$2,900,000	\$2,900,000	\$2,900,000	\$2,900,000	\$2,900,000
Variable costs	1,370,250	1,370,250	1,370,250	1,370,250	1,370,250
Fixed costs	650,000	650,000	650,000	650,000	650,000
Depreciation	420,000	420,000	420,000	420,000	420,000
EBIT	\$459,750	\$459,750	\$459,750	\$459,750	\$459,750
Taxes	96,548	96,548	96,548	96,548	96,548
Net Income	\$363,203	\$363,203	\$363,203	\$363,203	\$363,203
Depreciation	420,000	420,000	420,000	420,000	420,000
Operating CF	<u>\$783,203</u>	<u>\$783,203</u>	<u>\$783,203</u>	<u>\$783,203</u>	<u>\$783,203</u>

Year	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
Operating CF	\$783,203	\$783,203	\$783,203	\$783,203	\$783,203
Change in NWC	0	0	0	0	325,000
Capital spending	0	0	0	0	118,500
Total CF	<u>\$783,203</u>	<u>\$783,203</u>	<u>\$783,203</u>	<u>\$783,203</u>	<u>\$1,226,703</u>

With these cash flows, the NPV of the project is:

$$\text{NPV} = -\$2,100,000 - 325,000 + \$783,203(\text{PVIFA}_{14\%,5}) + [(\$325,000 + 118,500)/1.11^5]$$

$$\text{NPV} = \$732,831.45$$

If the actual price is above the bid price that results in a zero NPV, the project will have a positive NPV. As for the cartons sold, if the number of cartons sold increases, the NPV will increase, and if the costs increase, the NPV will decrease.

- b. To find the minimum number of cartons sold to still break even, we need to use the tax shield approach to calculating OCF, and solve the problem similar to finding a bid price. Using the initial cash flow and salvage value we already calculated, the equation for a zero NPV of the project is:

$$\text{NPV} = 0 = -\$2,100,000 - 325,000 + \text{OCF}(\text{PVIFA}_{11\%,5}) + [(\$325,000 + 118,500)/1.11^5]$$

So, the necessary OCF for a zero NPV is:

$$\text{OCF} = \$2,161,804.34/\text{PVIFA}_{11\%,5}$$

$$\text{OCF} = \$584,920.07$$

Now we can use the tax shield approach to solve for the minimum quantity as follows:

$$\text{OCF} = \$584,920.07 = [(P - v)Q - \text{FC}](1 - T_c) + T_c D$$

$$\$584,920.07 = [(\$20 - 9.45)Q - 650,000](1 - .21) + .21(\$2,100,000/5)$$

$$Q = 121,209$$

As a check, we can calculate the NPV of the project with this quantity. The calculations are:

Year	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
Sales	\$2,424,189	\$2,424,189	\$2,424,189	\$2,424,189	\$2,424,189
Variable costs	1,145,429	1,145,429	1,145,429	1,145,429	1,145,429
Fixed costs	650,000	650,000	650,000	650,000	650,000
Depreciation	420,000	420,000	420,000	420,000	420,000
EBIT	\$208,760	\$208,760	\$208,760	\$208,760	\$208,760
Taxes	43,840	43,840	43,840	43,840	43,840
Net Income	\$164,920	\$164,920	\$164,920	\$164,920	\$164,920
Depreciation	420,000	420,000	420,000	420,000	420,000
Operating CF	\$584,920	\$584,920	\$584,920	\$584,920	\$584,920

Year	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
Operating CF	\$584,920	\$584,920	\$584,920	\$584,920	\$584,920
Change in NWC	0	0	0	0	325,000
Capital spending	0	0	0	0	118,500
Total CF	\$584,920	\$584,920	\$584,920	\$584,920	\$1,028,420

$$\text{NPV} = -\$2,100,000 - 325,000 + \$584,920(\text{PVIFA}_{11\%,5}) + [(\$325,000 + 118,500)/1.11^5] \approx \$0$$

Note that the NPV is not exactly equal to zero because we had to round the number of cartons sold; you cannot sell one-half of a carton.

- c. To find the highest level of fixed costs and still break even, we need to use the tax shield approach to calculating OCF, and solve the problem similar to finding a bid price. Using the initial cash flow and salvage value we already calculated, the equation for a zero NPV of the project is:

$$\text{NPV} = 0 = -\$2,100,000 - 325,000 + \text{OCF}(\text{PVIFA}_{11\%,5}) + [(\$325,000 + 118,500)/1.11^5]$$

$$\text{OCF} = \$2,161,804.34/\text{PVIFA}_{11\%,5}$$

$$\text{OCF} = \$584,920.07$$

Notice this is the same OCF we calculated in part *b*. Now we can use the tax shield approach to solve for the maximum level of fixed costs as follows:

$$\text{OCF} = \$584,920.07 = [(P - v)Q - \text{FC}](1 - T_c) + T_c D$$

$$\$584,920.07 = [(\$20 - 9.45)(145,000) - \text{FC}](1 - .21) + .21(\$2,100,000/5)$$

$$\text{FC} = \$900,990.42$$

As a check, we can calculate the NPV of the project with this quantity. The calculations are:

Year	1	2	3	4	5
Sales	\$2,900,000	\$2,900,000	\$2,900,000	\$2,900,000	\$2,900,000
Variable costs	1,370,250	1,370,250	1,370,250	1,370,250	1,370,250
Fixed costs	900,990	900,990	900,990	900,990	900,990
Depreciation	420,000	420,000	420,000	420,000	420,000
EBIT	\$208,760	\$208,760	\$208,760	\$208,760	\$208,760
Taxes	43,840	43,840	43,840	43,840	43,840
Net Income	\$164,920	\$164,920	\$164,920	\$164,920	\$164,920
Depreciation	420,000	420,000	420,000	420,000	420,000
Operating CF	\$584,920	\$584,920	\$584,920	\$584,920	\$584,920

Year	1	2	3	4	5
Operating CF	\$584,920	\$584,920	\$584,920	\$584,920	\$584,920
Change in NWC	0	0	0	0	325,000
Capital spending	0	0	0	0	118,500
Total CF	\$584,920	\$584,920	\$584,920	\$584,920	\$1,028,420

$$\text{NPV} = -\$2,100,000 - 325,000 + \$584,920(\text{PVIFA}_{11\%,5}) + [(\$325,000 + 118,500)/1.11^5] \approx \$0$$

36. We need to find the bid price for a project, but the project has extra cash flows. Since we don't already produce the keyboard, the sales of the keyboard outside the contract are relevant cash flows. Since we know the extra sales number and price, we can calculate the cash flows generated by these sales. The cash flow generated from the sale of the keyboard outside the contract is:

	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>
		\$1,620,00		
Sales	\$540,000	0	\$1,890,000	\$945,000
Variable costs	172,000	516,000	602,000	301,000
		\$1,104,00		
EBT	\$368,000	0	\$1,288,000	\$644,000
Tax	88,320	264,960	309,120	154,560
Net income (and OCF)	\$279,680	\$839,040	\$978,880	\$489,440

So, the addition to NPV of these market sales is:

$$\text{NPV of market sales} = \$279,680/1.13 + \$839,040/1.13^2 + \$978,880/1.13^3 + \$489,440/1.13^4$$

$$\text{NPV of market sales} = \$1,883,191.48$$

You may have noticed that we did not include the initial cash outlay, depreciation, or fixed costs in the calculation of cash flows from the market sales. The reason is that it is irrelevant whether or not we include these here. Remember that we are not only trying to determine the bid price, but we are also determining whether or not the project is feasible. In other words, we are trying to calculate the NPV of the project, not just the NPV of the bid price. We will include these cash flows in the bid price calculation. Whether we include these costs in this initial calculation is irrelevant since you will come up with the same bid price if you include these costs in this calculation, or if you include them in the bid price calculation.

Next, we need to calculate the aftertax salvage value, which is:

$$\text{Aftertax salvage value} = \$250,000(1 - .24)$$

$$\text{Aftertax salvage value} = \$190,000$$

Instead of solving for a zero NPV as is usual in setting a bid price, the company president requires an NPV of \$100,000, so we will solve for an NPV of that amount. The NPV equation for this project is (remember to include the NWC cash flow at the beginning of the project, and the NWC recovery at the end):

$$\text{NPV} = \$100,000 = -\$3,600,000 - 125,000 + 1,883,191.48 + \text{OCF}(\text{PVIFA}_{13\%,4}) + [(\$190,000 + 125,000)/1.13^4]$$

Solving for the OCF, we get:

$$\text{OCF} = \$1,748,613.12/\text{PVIFA}_{13\%,4}$$

$$\text{OCF} = \$587,873.59$$

Now we can solve for the bid price as follows:

$$\text{OCF} = \$587,873.59 = [(P - v)Q - FC](1 - T_c) + T_c D$$

$$\$587,873.59 = [(P - \$43)(18,000) - \$775,000](1 - .24) + .24(\$3,600,000/4)$$

$$P = \$113.24$$

37. a. Since the two computers have unequal lives, the correct method to analyze the decision is the EAC. We will begin with the EAC of the new computer. Using the depreciation tax shield approach, the OCF for the new computer system is:

$$OCF = (\$85,000)(1 - .21) + (\$580,000/5)(.21) = \$91,510$$

Notice that the costs are positive, which represents a cash inflow. The costs are positive in this case since the new computer will generate a cost savings. The only initial cash flow for the new computer is the cost of \$580,000. We next need to calculate the aftertax salvage value, which is:

$$\text{Aftertax salvage value} = \$130,000(1 - .21) = \$102,700$$

Now we can calculate the PV of costs of the new computer as:

$$\begin{aligned} \text{PV of costs} &= -\$580,000 + \$91,510(\text{PVIFA}_{14\%,5}) + \$102,700/1.14^5 \\ \text{PV of costs} &= -\$212,499.60 \end{aligned}$$

And the EAC of the new computer is:

$$\begin{aligned} \text{EAC} &= -\$212,499.60/(\text{PVIFA}_{14\%,5}) \\ \text{EAC} &= -\$61,897.64 \end{aligned}$$

Analyzing the old computer, the only OCF is the depreciation tax shield, so:

$$\begin{aligned} \text{OCF} &= \$90,000(.21) \\ \text{OCF} &= \$18,900 \end{aligned}$$

The initial cost of the old computer is a little trickier. You might assume that since we already own the old computer there is no initial cost, but we can sell the old computer, so there is an opportunity cost. We need to account for this opportunity cost. To do so, we will calculate the aftertax salvage value of the old computer today. We need the book value of the old computer to do so. The book value is not given directly, but we are told that the old computer has depreciation of \$90,000 per year for the next three years, so we can assume the book value is the total amount of depreciation over the remaining life of the system, or \$270,000. So, the aftertax salvage value of the old computer is:

$$\begin{aligned} \text{Aftertax salvage value} &= \$230,000 + (\$270,000 - 230,000)(.21) \\ \text{Aftertax salvage value} &= \$238,400 \end{aligned}$$

This is the initial cost of the old computer system today because we are forgoing the opportunity to sell it today. We next need to calculate the aftertax salvage value of the computer system in two years since we are “buying” it today. The aftertax salvage value in two years is:

$$\begin{aligned} \text{Aftertax salvage value} &= \$60,000 + (\$90,000 - 60,000)(.21) \\ \text{Aftertax salvage value} &= \$66,300 \end{aligned}$$

Now we can calculate the PV of costs of the old computer as:

$$\text{PV of costs} = -\$238,400 + \$18,900(\text{PVIFA}_{14\%,2}) + \$66,300/1.14^2$$

$$\text{PV of costs} = -\$156,262.42$$

And the EAC of the old computer is:

$$\text{EAC} = -\$156,262.42/(\text{PVIFA}_{14\%,2})$$

$$\text{EAC} = -\$94,896.56$$

If we are going to replace the system in two years no matter what our decision today, we should instead replace it today since the EAC is less negative.

- b. If we are only concerned with whether or not to replace the machine now, and are not worrying about what will happen in two years, the correct analysis is NPV. To calculate the NPV of the decision on the computer system now, we need the difference in the total cash flows of the old computer system and the new computer system. From our previous calculations, we can say the cash flows for each computer system are:

<i>t</i>	<u>New computer</u>	<u>Old computer</u>	<u>Difference</u>
0	-\$580,000	-\$238,400	-\$341,600
1	91,510	18,900	72,610
2	91,510	85,200	6,310
3	91,510	0	91,510
4	91,510	0	91,510
5	194,210	0	194,210

Since we are only concerned with marginal cash flows, the cash flows of the decision to replace the old computer system with the new computer system are the differential cash flows. The NPV of the decision to replace, ignoring what will happen in two years is:

$$\text{NPV} = -\$341,600 + \$72,610/1.14 + \$6,310/1.14^2 + \$91,510/1.14^3 + \$91,510/1.14^4 + \$194,210/1.14^5$$

$$\text{NPV} = -\$56,237.18$$

If we are not concerned with what will happen in two years, we should not replace the old computer system.

38. To answer this question, we need to compute the NPV of all three alternatives, specifically, continue to rent the building, Project A, or Project B. We would choose the project with the highest NPV. If all three of the projects have a positive NPV, the project that is more favorable is the one with the highest NPV

There are several important cash flows we should not consider in the incremental cash flow analysis. The remaining fraction of the value of the building and depreciation are not incremental and should not be included in the analysis of the two alternatives. The \$1,450,000 purchase price of the building is the same for all three options and should be ignored. In effect, what we are doing is finding the NPV of the future cash flows of each option, so the only cash flow today would be the building modifications needed for Project A and Project B. If we did include these costs, the effect would be to lower the NPV of all three options by the same amount, thereby leading to the same conclusion. The cash flows from renting the building after Year 15 are also irrelevant. No matter what the

company chooses today, it will rent the building after Year 15, so these cash flows are not incremental to any project.

We will begin by calculating the NPV of the decision of continuing to rent the building first.

Continue to rent:

Rent	\$75,000
Taxes	<u>15,750</u>
Net income	\$59,250

Since there is no incremental depreciation, the operating cash flow is the net income. So, the NPV of the decision to continue to rent is:

$$\text{NPV} = \$59,250(\text{PVIFA}_{12\%,15})$$

$$\text{NPV} = \$403,543.72$$

Product A:

Next, we will calculate the NPV of the decision to modify the building to produce Product A. The income statement for this modification is the same for the first 14 years, and in Year 15, the company will have an additional expense to convert the building back to its original form. This will be an expense in Year 15, so the income statement for that year will be slightly different. The cash flow at time zero will be the cost of the equipment, and the cost of the initial building modifications, both of which are depreciable on a straight-line basis. So, the pro forma cash flows for Product A are:

Initial cash outlay:

Building modifications	-\$115,000
Equipment	<u>-340,000</u>
Total cash flow	-\$455,000

	<u>Years 1-14</u>	<u>Year 15</u>
Revenue	\$275,000	\$275,000
Expenditures	115,000	115,000
Depreciation	30,333	30,333
Restoration cost	<u>0</u>	<u>75,000</u>
EBT	\$129,667	\$54,667
Tax	<u>27,230</u>	<u>11,480</u>
NI	<u>\$102,437</u>	<u>\$43,187</u>
OCF	\$132,770	\$73,520

The OCF each year is net income plus depreciation. So, the NPV for modifying the building to manufacture Product A is:

$$\text{NPV} = -\$455,000 + \$132,770(\text{PVIFA}_{12\%,14}) + \$73,520/1.12^{15}$$

$$\text{NPV} = \$438,453.72$$

Product B:

Now we will calculate the NPV of the decision to modify the building to produce Product B. The income statement for this modification is the same for the first 14 years, and in Year 15, the company will have an additional expense to convert the building back to its original form. This will be an expense in Year 15, so the income statement for that year will be slightly different. The cash flow at time zero will be the cost of the equipment, and the cost of the initial building modifications, both of which are depreciable on a straight-line basis. So, the pro forma cash flows for Product B are:

Initial cash outlay:

Building modifications	-\$160,000
Equipment	<u>-345,000</u>
Total cash flow	-\$505,000

	<u>Years 1-14</u>	<u>Year 15</u>
Revenue	\$295,000	\$295,000
Expenditures	130,000	130,000
Depreciation	33,667	33,667
Restoration cost	-	85,000
EBT	<u>\$131,333</u>	<u>\$46,333</u>
Tax	<u>27,580</u>	<u>9,730</u>
NI	<u>\$103,753</u>	<u>\$36,603</u>
OCF	\$137,420	\$70,270

The OCF each year is net income plus depreciation. So, the NPV for modifying the building to manufacture Product B is:

$$\text{NPV} = -\$505,000 + \$137,420(\text{PVIFA}_{12\%,14}) + \$70,270/1.12^{15}$$

$$\text{NPV} = \$418,680.94$$

Since Product A has the highest NPV, the company should choose that option.

We could have also done the analysis as the incremental cash flows between Product A and continuing to rent the building, and the incremental cash flows between Product B and continuing to rent the building. The results of this type of analysis would be:

NPV of differential cash flows between Product A and continuing to rent:

$$\text{NPV} = \text{NPV}_{\text{Product A}} - \text{NPV}_{\text{Rent}}$$

$$\text{NPV} = \$438,453.72 - 403,543.72$$

$$\text{NPV} = \$34,910.00$$

NPV of differential cash flows between Product B and continuing to rent:

$$\text{NPV} = \text{NPV}_{\text{Product B}} - \text{NPV}_{\text{Rent}}$$

$$\text{NPV} = \$418,680.94 - 403,543.72$$

$$\text{NPV} = \$15,137.22$$

Since the differential of the NPV of Product A compared to continuing to rent is the greatest, we should choose Product A, which is the same as our original result.

39. The discount rate is expressed in real terms, and the cash flows are expressed in nominal terms. We can answer this question by converting all of the cash flows to real dollars. We can then use the real interest rate. The real value of each cash flow is the present value of the Year 1 nominal cash flows, discounted back to the present at the inflation rate. So, the real value of the revenue and costs will be:

$$\text{Revenue in real terms} = \$415,000/1.06 = \$391,509.43$$

$$\text{Labor costs in real terms} = \$209,000/1.06 = \$197,169.81$$

$$\text{Other costs in real terms} = \$73,000/1.06 = \$68,867.92$$

$$\text{Lease payment in real terms} = \$160,000/1.06 = \$150,943.40$$

Revenues, labor costs, and other costs are all growing perpetuities. Each has a different growth rate, so we must calculate the present value of each separately. Using the real required return, the present value of each of these is:

$$PV_{\text{Revenue}} = \$391,509.43/(\cdot 10 - \cdot 04) = \$6,525,157.23$$

$$PV_{\text{Labor costs}} = \$197,169.81/(\cdot 10 - \cdot 03) = \$2,816,711.59$$

$$PV_{\text{Other costs}} = \$68,867.92/(\cdot 10 - \cdot 01) = \$765,199.16$$

The lease payments are constant in nominal terms, so they are declining in real terms by the inflation rate. Therefore, the lease payments form a growing perpetuity with a negative growth rate. The real present value of the lease payments is:

$$PV_{\text{Lease payments}} = \$150,943.40/[\cdot 10 - (-\cdot 06)] = \$943,396.23$$

Now we can use the tax shield approach to calculate the net present value. Since there is no investment in equipment, there is no depreciation; therefore, no depreciation tax shield, so we will ignore this in our calculation. This means the cash flows each year are equal to net income. There is also no initial cash outlay, so the NPV is the present value of the future aftertax cash flows. The NPV of the project is:

$$NPV = (PV_{\text{Revenue}} - PV_{\text{Labor costs}} - PV_{\text{Other costs}} - PV_{\text{Lease payments}})(1 - T_C)$$

$$NPV = (\$6,525,157.23 - 2,816,711.59 - 765,199.16 - 943,396.23)(1 - \cdot 22)$$

$$NPV = \$1,559,883.20$$

In general, we can answer capital budgeting problems in nominal terms as well. However, in this case, the computation in nominal terms would have been impossible. The reason is that we are dealing with growing perpetuities. In other problems, when calculating the NPV of nominal cash flows, we could calculate the nominal cash flow each year since the cash flows were finite. Because of the perpetual nature of the cash flows in this problem, we cannot calculate the nominal cash flows each year until the end of the project. When faced with two alternative approaches, where both are equally correct, always choose the simplest one.

40. We are given the real revenue and costs, and the real growth rates, so the simplest way to solve this problem is to calculate the NPV with real values. While we could calculate the NPV using nominal values, we would need to find the nominal growth rates, and convert all values to nominal terms. The real labor costs will increase at a real rate of 2 percent per year, and the real energy costs will increase at a real rate of 3 percent per year, so the real costs each year will be:

	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>
Real labor cost each year	\$15.70	\$16.01	\$16.33	\$16.66
Real energy cost each year	\$4.10	\$4.22	\$4.35	\$4.48

Remember that the depreciation tax shield also affects a firm's aftertax cash flows. The present value of the depreciation tax shield must be added to the present value of a firm's revenues and expenses to find the present value of the cash flows related to the project. The depreciation the firm will recognize each year is:

Annual depreciation = Investment/Economic Life

Annual depreciation = \$115,000,000/4

Annual depreciation = \$28,750,000

Depreciation is a nominal cash flow, so to find the real value of depreciation each year, we discount the real depreciation amount by the inflation rate. Doing so, we find the real depreciation each year is:

Year 1 real depreciation = \$28,750,000/1.05 = \$27,380,952.38

Year 2 real depreciation = \$28,750,000/1.05² = \$26,077,097.51

Year 3 real depreciation = \$28,750,000/1.05³ = \$24,835,330.96

Year 4 real depreciation = \$28,750,000/1.05⁴ = \$23,652,696.15

Now we can calculate the pro forma income statement each year in real terms. We can then add back depreciation to net income to find the operating cash flow each year. Doing so, we find the cash flow of the project each year is:

	<u>Year 0</u>	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>
Revenues		\$60,175,000.00	\$68,475,000.00	\$74,700,000.00	\$64,325,000.00
Labor cost		17,584,000.00	19,216,800.00	22,214,620.80	21,326,035.97
Energy cost		861,000.00	950,175.00	1,109,170.95	1,075,243.37
Depreciation		27,380,952.38	26,077,097.51	24,835,330.96	23,652,696.15
EBT		\$14,349,047.62	\$22,230,927.49	\$26,540,877.29	\$18,271,024.51
Taxes		3,013,300.00	4,668,494.77	5,573,584.23	3,836,915.15
Net income		\$11,335,747.62	\$17,562,432.72	\$20,967,293.06	\$14,434,109.37
OCF		\$38,716,700.00	\$43,639,530.23	\$45,802,624.02	\$38,086,805.52
Cap. spend.	-\$115,000,000				
Total CF	-\$115,000,000	\$38,716,700.00	\$43,639,530.23	\$45,802,624.02	\$38,086,805.52

We can use the total cash flows each year to calculate the NPV, which is:

$$\begin{aligned}\text{NPV} &= -\$115,000,000 + \$38,716,700/1.04 + \$43,639,530.23/1.04^2 + \$45,802,624.02/1.04^3 \\ &\quad + \$38,086,805.52/1.04^4 \\ \text{NPV} &= \$35,849,921.91\end{aligned}$$

42. Since the project requires an initial investment in inventory as a percentage of sales, we will calculate the sales figures for each year first. The incremental sales will include the sales of the new table, but we also need to include the lost sales of the existing model. This is an erosion cost of the new table. The lost sales of the existing table are constant for every year, but the sales of the new table change every year. So, the total incremental sales figure for the five years of the project will be:

	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>	<u>Year 5</u>
New	\$10,620,000	\$12,685,000	\$15,340,000	\$13,865,000	\$12,980,000
Lost sales	<u>-1,075,000</u>	<u>-1,075,000</u>	<u>-1,075,000</u>	<u>-1,075,000</u>	<u>-1,075,000</u>
Total	\$9,545,000	\$11,610,000	\$14,265,000	\$12,790,000	\$11,905,000

Now we will calculate the initial cash outlay that will occur today. The company has the necessary production capacity to manufacture the new table without adding equipment today. So, the equipment will not be purchased today, but rather in two years. The reason is that the existing capacity is not being used. If the existing capacity were being used, the new equipment would be required, so it would be a cash flow today. The old equipment would have an opportunity cost if it could be sold. As there is no discussion that the existing equipment could be sold, we must assume it cannot be sold. The only initial cash flow is the cost of the inventory. The company will have to spend money for inventory with the new table, but will be able to reduce inventory of the existing table. So, the initial cash flow today is:

New table	-\$1,062,000
Old table	<u>107,500</u>
Total	-\$954,500

In Year 2, the company will have a cash outflow to pay for the cost of the new equipment. Since the equipment will be purchased in two years rather than now, the equipment will have a higher salvage value. The book value of the equipment in five years will be the initial cost, minus the accumulated depreciation, or:

$$\begin{aligned}\text{Book value} &= \$16,000,000 - 2,286,400 - 3,918,400 - 2,798,400 \\ \text{Book value} &= \$6,996,800\end{aligned}$$

The taxes on the salvage value will be:

$$\begin{aligned}\text{Taxes on salvage} &= (\$6,996,800 - 4,800,000)(.21) \\ \text{Taxes on salvage} &= \$461,328\end{aligned}$$

So, the aftertax salvage value of the equipment in five years will be:

Sell equipment	\$6,800,000
Taxes	<u>461,328</u>

Salvage value \$5,261,328

Next, we need to calculate the variable costs each year. The variable costs of the lost sales are included as a variable cost savings, so the variable costs will be:

	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>	<u>Year 5</u>
New	\$3,929,400	\$4,693,450	\$5,675,800	\$5,130,050	\$4,802,600
Lost sales	<u>-430,000</u>	<u>-430,000</u>	<u>-430,000</u>	<u>-430,000</u>	<u>-430,000</u>
Variable costs	\$3,499,400	\$4,263,450	\$5,245,800	\$4,700,050	\$4,372,600

Now we can prepare the rest of the pro forma income statements for each year. The project will have no incremental depreciation for the first two years as the equipment is not purchased for two years. Adding back depreciation to net income to calculate the operating cash flow, we get:

	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>	<u>Year 5</u>
Sales	\$9,545,000	\$11,610,000	\$14,265,000	\$12,790,000	\$11,905,000
VC	3,499,400	4,263,450	5,245,800	4,700,050	4,372,600
Fixed costs	2,050,000	2,050,000	2,050,000	2,050,000	2,050,000
Dep.	<u>0</u>	<u>0</u>	<u>2,286,400</u>	<u>3,918,400</u>	<u>2,798,400</u>
EBT	\$3,995,600	\$5,296,550	\$4,682,800	\$2,121,550	\$2,684,000
Tax	<u>839,076</u>	<u>1,112,276</u>	<u>983,388</u>	<u>445,526</u>	<u>563,640</u>
NI	\$3,156,524	\$4,184,275	\$3,699,412	\$1,676,025	\$2,120,360
+Dep.	<u>0</u>	<u>0</u>	<u>2,286,400</u>	<u>3,918,400</u>	<u>2,798,400</u>
OCF	\$3,156,524	\$4,184,275	\$5,985,812	\$5,594,425	\$4,918,760

Next, we need to account for the changes in inventory each year. The inventory is a percentage of sales. The way we will calculate the change in inventory is the beginning of period inventory minus the end of period inventory. The sign of this calculation will tell us whether the inventory change is a cash inflow, or a cash outflow. The inventory each year, and the inventory change, will be:

	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>	<u>Year 5</u>
Beginning	\$1,062,000	\$1,268,500	\$1,534,000	\$1,386,500	\$1,298,000
Ending	1,268,500	1,534,000	1,386,500	1,298,000	0
Old table					-107,500
Change	-\$206,500	-\$265,500	\$147,500	\$88,500	\$1,190,500

Notice that we recover the remaining inventory at the end of the project. We must also spend \$138,000 for inventory since the 250 units per year in sales of the oak table will begin again. The total cash flows for the project will be the sum of the operating cash flow, the capital spending, and the inventory cash flows, so:

	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>	<u>Year 5</u>
OCF	\$3,156,524	\$4,184,275	\$5,985,812	\$5,594,425	\$4,918,760
Equipment	0	-16,000,000	0	0	5,261,328
Inventory	-206,500	-265,500	147,500	88,500	1,190,500
		\$(12,081,226			
Total	\$2,950,024)	\$6,133,312	\$5,682,925	\$11,370,588

The NPV of the project, including the inventory cash flow at the beginning of the project, will be:

$$\begin{aligned}\text{NPV} &= -\$954,500 + \$2,950,024/1.11 - \$12,081,226/1.11^2 + \$6,133,312/1.11^3 \\ &\quad + \$5,682,925/1.11^4 + \$11,370,588/1.11^5 \\ \text{NPV} &= \$6,873,819.46\end{aligned}$$

The company should go ahead with the new table.

- b. You can perform an IRR analysis, and would expect to find three IRRs since the cash flows change signs three times.
- c. The profitability index is intended as a “bang for the buck” measure; that is, it shows how much shareholder wealth is created for every dollar of initial investment. This is usually a good measure of the investment since most projects have conventional cash flows. In this case, the largest investment is not at the beginning of the project, but later in its life, so while the decision rule is the same, it really does not measure the bang for the dollar invested.

CHAPTER 7

RISK ANALYSIS, REAL OPTIONS, AND CAPITAL BUDGETING

Answers to Concepts Review and Critical Thinking Questions

1. Forecasting risk is the risk that a poor decision is made because of errors in projected cash flows. The danger is greatest with a new product because the cash flows are probably harder to predict.
2. With a sensitivity analysis, one variable is examined over a broad range of values. With a scenario analysis, all variables are examined for a limited range of values.
3. It is true that if average revenue is less than average cost, the firm is losing money. This much of the statement is correct. At the margin, however, accepting a project with marginal revenue in excess of its marginal cost clearly acts to increase operating cash flow.
4. From the shareholder perspective, the financial break-even point is the most important. A project can exceed the accounting and cash break-even points but still be below the financial break-even point. This causes a reduction in shareholder (your) wealth.
5. The project will reach the cash break-even first, the accounting break-even next, and finally the financial break-even. For a project with an initial investment and sales afterwards, this ordering will always apply. The cash break-even is achieved first since it excludes depreciation. The accounting break-even is next since it includes depreciation. Finally, the financial break-even, which includes the time value of money, is achieved.
6. Traditional NPV analysis is often too conservative because it ignores profitable options such as the ability to expand the project if it is profitable, or abandon the project if it is unprofitable. The option to alter a project when it has already been accepted has a value, which increases the NPV of the project.
7. The type of option most likely to affect the decision is the option to expand. If the country just liberalized its markets, there is likely the potential for growth. First entry into a market, whether an entirely new market, or with a new product, can give a company name recognition and market share. This may make it more difficult for competitors entering the market.
8. Sensitivity analysis can determine how the financial break-even point changes when some factors (such as fixed costs, variable costs, or revenue) change.
9. There are two sources of value with this decision to wait. The price of the timber can potentially increase, and the amount of timber will almost definitely increase, barring a natural catastrophe or forest fire. The option to wait for a logging company is quite valuable, and companies in the industry have models to estimate the future growth of a forest depending on its age.

10. Option analysis should stop when the additional analysis has a negative NPV. Since the additional analysis is likely to occur almost immediately, then it would have a negative NPV when the benefits of the additional analysis outweigh the costs. The benefits of the additional analysis are the reduction in the possibility of making a bad decision. Of course, the additional benefits are often difficult, if not impossible, to measure, so much of this decision is based on experience.

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. a. To calculate the accounting break-even, we first need to find the depreciation for each year. The depreciation is:

$$\text{Depreciation} = \$604,000/8$$

$$\text{Depreciation} = \$75,500 \text{ per year}$$

And the accounting break-even is:

$$Q_A = (\$685,000 + 75,500)/(\$36 - 17)$$

$$Q_A = 40,026 \text{ units}$$

- b. We will use the tax shield approach to calculate the OCF. The OCF is:

$$\text{OCF}_{\text{base}} = [(P - v)Q - FC](1 - T_C) + T_C D$$

$$\text{OCF}_{\text{base}} = [(\$36 - 17)(55,000) - \$685,000](.79) + .21(\$75,500)$$

$$\text{OCF}_{\text{base}} = \$300,255$$

Now we can calculate the NPV using our base-case projections. There is no salvage value or NWC, so the NPV is:

$$\text{NPV}_{\text{base}} = -\$604,000 + \$300,255(\text{PVIFA}_{15\%,8})$$

$$\text{NPV}_{\text{base}} = \$743,340.720$$

To calculate the sensitivity of the NPV to changes in the quantity sold, we will calculate the NPV at a different quantity. We will use sales of 56,000 units. The OCF at this sales level is:

$$\text{OCF}_{\text{new}} = [(\$36 - 17)(56,000) - \$685,000](.79) + .21(\$75,500)$$

$$\text{OCF}_{\text{new}} = \$315,265$$

And the NPV is:

$$\text{NPV}_{\text{new}} = -\$604,000 + \$315,265(\text{PVIFA}_{15\%,8})$$

$$\text{NPV}_{\text{new}} = \$810,695.42$$

So, the change in NPV for every unit change in sales is:

$$\begin{aligned}\Delta \text{NPV}/\Delta S &= (\$743,340.72 - 810,695.42)/(55,000 - 56,000) \\ \Delta \text{NPV}/\Delta S &= +\$67.355\end{aligned}$$

If sales were to drop by 500 units, then NPV would drop by:

$$\text{NPV drop} = \$67.355(500) = \$33,677.35$$

You may wonder why we chose 56,000 units. Because it doesn't matter! Whatever sales number we use, when we calculate the change in NPV per unit sold, the ratio will be the same.

- c. To find out how sensitive OCF is to a change in variable costs, we will compute the OCF at a variable cost of \$18. Again, the number we choose to use here is irrelevant: We will get the same ratio of OCF to a one dollar change in variable cost no matter what variable cost we use. So, using the tax shield approach, the OCF at a variable cost of \$18 is:

$$\begin{aligned}\text{OCF}_{\text{new}} &= [(\$36 - 18)(55,000) - 685,000](.79) + .21(\$75,500) \\ \text{OCF}_{\text{new}} &= \$256,805\end{aligned}$$

So, the change in OCF for a \$1 change in variable costs is:

$$\begin{aligned}\Delta \text{OCF}/\Delta v &= (\$300,255 - 256,805)/(\$17 - 18) \\ \Delta \text{OCF}/\Delta v &= -\$43,450\end{aligned}$$

If variable costs decrease by \$1 then OCF would increase by \$43,450

2. We will use the tax shield approach to calculate the OCF for the best- and worst-case scenarios. For the best-case scenario, the price and quantity increase by 10 percent, so we will multiply the base case numbers by 1.1, a 10 percent increase. The variable and fixed costs both decrease by 10 percent, so we will multiply the base case numbers by .9, a 10 percent decrease. Doing so, we get:

$$\begin{aligned}\text{OCF}_{\text{best}} &= \{[(\$36)(1.1) - (\$17)(.9)](55,000)(1.1) - \$685,000(.9)\}(.79) + .21(\$75,500) \\ \text{OCF}_{\text{best}} &= \$690,238.50\end{aligned}$$

The best-case NPV is:

$$\begin{aligned}\text{NPV}_{\text{best}} &= -\$604,000 + \$690,238.50(\text{PVIFA}_{15\%,8}) \\ \text{NPV}_{\text{best}} &= \$2,493,322.07\end{aligned}$$

For the worst-case scenario, the price and quantity decrease by 10 percent, so we will multiply the base case numbers by .9, a 10 percent decrease. The variable and fixed costs both increase by 10 percent, so we will multiply the base case numbers by 1.1, a 10 percent increase. Doing so, we get:

$$\begin{aligned}\text{OCF}_{\text{worst}} &= \{[(\$36)(.9) - (\$17)(1.1)](55,000)(.9) - \$685,000(1.1)\}(.79) + .21(\$75,500) \\ \text{OCF}_{\text{worst}} &= -\$43,671.50\end{aligned}$$

The worst-case NPV is:

$$\begin{aligned}\text{NPV}_{\text{worst}} &= -\$604,000 - \$43,671.50(\text{PVIFA}_{15\%,8}) \\ \text{NPV}_{\text{worst}} &= -\$799,968.06\end{aligned}$$

3. We can use the accounting break-even equation:

$$Q_A = (\text{FC} + D)/(\text{P} - v)$$

to solve for the unknown variable in each case. Doing so, we find:

$$\begin{aligned}(1): Q_A &= 95,800 = (\$820,000 + D)/(\$42 - 30) \\ D &= \$329,600\end{aligned}$$

$$\begin{aligned}(2): Q_A &= 143,806 = (\$2,750,000 + 1,150,000)/(\text{P} - \$64) \\ \text{P} &= \$91.12\end{aligned}$$

$$\begin{aligned}(3): Q_A &= 7,835 = (\$245,000 + 105,000)/(\$97 - v) \\ v &= \$52.33\end{aligned}$$

4. When calculating the financial break-even point, we express the initial investment as an equivalent annual cost (EAC). Dividing the initial investment by the 5-year annuity factor, discounted at 12 percent, the EAC of the initial investment is:

$$\begin{aligned}\text{EAC} &= \text{Initial Investment}/\text{PVIFA}_{12\%,5} \\ \text{EAC} &= \$435,000/3.60478 \\ \text{EAC} &= \$120,673.23\end{aligned}$$

Note that this calculation solves for the annuity payment with the initial investment as the present value of the annuity. In other words:

$$\begin{aligned}\text{PVA} &= C(\{1 - [1/(1 + R)]^t\}/R) \\ \$435,000 &= C\{[1 - (1/1.12)^5]/.12\} \\ C &= \$120,673.23\end{aligned}$$

The annual depreciation is the cost of the equipment divided by the economic life, or:

$$\begin{aligned}\text{Annual depreciation} &= \$435,000/5 \\ \text{Annual depreciation} &= \$87,000\end{aligned}$$

Now we can calculate the financial break-even point. The financial break-even point for this project is:

$$\begin{aligned}Q_F &= [\text{EAC} + \text{FC}(1 - T_c) - D(T_c)]/[(\text{P} - \text{VC})(1 - T_c)] \\ Q_F &= [\$120,673.23 + \$295,000(1 - .24) - \$87,000(.24)]/[(\$16 - 5)(1 - .24)] \\ Q_F &= 38,755.17, \text{ or about } 38,755 \text{ units}\end{aligned}$$

5. If we purchase the machine today, the NPV is the cost plus the present value of the increased cash flows, so:

$$\text{NPV}_0 = -\$2,800,000 + \$435,000(\text{PVIFA}_{9\%,10})$$

$$\text{NPV}_0 = -\$8,318.90$$

We should not necessarily purchase the machine today. We would want to purchase the machine when the NPV is the highest. So, we need to calculate the NPV each year. The NPV each year will be the cost plus the present value of the increased cash savings. We must be careful, however. In order to make the correct decision, the NPV for each year must be taken to a common date. We will discount all of the NPVs to today. Doing so, we get:

$$\begin{aligned}\text{Year 1: } NPV_1 &= [-\$2,585,000 + \$435,000(PVIFA_{9\%,9})]/1.09 \\ NPV_1 &= \$21,038.90\end{aligned}$$

$$\begin{aligned}\text{Year 2: } NPV_2 &= [-\$2,370,000 + \$435,000(PVIFA_{9\%,8})]/1.09^2 \\ NPV_2 &= \$31,686.15\end{aligned}$$

$$\begin{aligned}\text{Year 3: } NPV_3 &= [-\$2,155,000 + \$435,000(PVIFA_{9\%,7})]/1.09^3 \\ NPV_3 &= \$26,512.52\end{aligned}$$

$$\begin{aligned}\text{Year 4: } NPV_4 &= [-\$2,155,000 + \$435,000(PVIFA_{9\%,6})]/1.09^4 \\ NPV_4 &= -\$144,253.38\end{aligned}$$

$$\begin{aligned}\text{Year 5: } NPV_5 &= [-\$2,155,000 + \$435,000(PVIFA_{9\%,5})]/1.09^5 \\ NPV_5 &= -\$300,919.34\end{aligned}$$

$$\begin{aligned}\text{Year 6: } NPV_6 &= [-\$2,155,000 + \$435,000(PVIFA_{9\%,4})]/1.09^6 \\ NPV_6 &= -\$444,649.58\end{aligned}$$

The company should purchase the machine in Year 2 when the NPV is the highest.

6. We need to calculate the NPV of the two options: Go directly to market now, or utilize test marketing first. The NPV of going directly to market now is:

$$\begin{aligned}NPV &= C_{\text{Success}} (\text{Prob. of Success}) + C_{\text{Failure}} (\text{Prob. of Failure}) \\ NPV &= \$24,000,000(.50) + \$8,500,000(.50) \\ NPV &= \$16,250,000\end{aligned}$$

Now we can calculate the NPV of test marketing first. Test marketing requires a \$1.2 million cash outlay. Choosing the test marketing option will also delay the launch of the product by one year. Thus, the expected payoff is delayed by one year and must be discounted back to Year 0.

$$\begin{aligned}NPV &= C_0 + \{[C_{\text{Success}} (\text{Prob. of Success})] + [C_{\text{Failure}} (\text{Prob. of Failure})]\}/(1 + R)^t \\ NPV &= -\$1,200,000 + \{[\$24,000,000(.80)] + [\$8,500,000(.20)]\}/1.11 \\ NPV &= \$17,628,828.83\end{aligned}$$

The company should test market first since that option has the highest expected payoff.

7. We need to calculate the NPV of each option, and choose the option with the highest NPV. So, the NPV of going directly to market is:

$$\begin{aligned}NPV &= C_{\text{Success}} (\text{Prob. of Success}) \\ NPV &= \$1,900,000(.50) \\ NPV &= \$950,000\end{aligned}$$

The NPV of the focus group is:

$$\begin{aligned}\text{NPV} &= C_0 + C_{\text{Success}}(\text{Prob. of Success}) \\ \text{NPV} &= -\$155,000 + \$1,900,000(.65) \\ \text{NPV} &= \$1,080,000\end{aligned}$$

And the NPV of using the consulting firm is:

$$\begin{aligned}\text{NPV} &= C_0 + C_{\text{Success}}(\text{Prob. of Success}) \\ \text{NPV} &= -\$345,000 + \$1,900,000(.80) \\ \text{NPV} &= \$1,175,000\end{aligned}$$

The firm should use the consulting firm since that option has the highest NPV.

8. The company should analyze both options, and choose the option with the greatest NPV. So, if the company goes to market immediately, the NPV is:

$$\begin{aligned}\text{NPV} &= C_{\text{Success}}(\text{Prob. of Success}) + C_{\text{Failure}}(\text{Prob. of Failure}) \\ \text{NPV} &= \$16,500,000(.55) + \$7,500,000(.45) \\ \text{NPV} &= \$12,450,000\end{aligned}$$

Customer segment research requires a \$875,000 cash outlay. Choosing the research option will also delay the launch of the product by one year. Thus, the expected payoff is delayed by one year and must be discounted back to Year 0. So, the NPV of the customer segment research is:

$$\begin{aligned}\text{NPV} &= C_0 + \{[C_{\text{Success}}(\text{Prob. of Success})] + [C_{\text{Failure}}(\text{Prob. of Failure})]\}/(1 + R)^t \\ \text{NPV} &= -\$875,000 + \{[\$16,500,000(.70)] + [\$7,500,000(.30)]\}/1.13 \\ \text{NPV} &= \$11,337,389.38\end{aligned}$$

The company should go to market now since it has the largest NPV.

9. a. The accounting break-even is the aftertax sum of the fixed costs and depreciation charge divided by the aftertax contribution margin (selling price minus variable cost). So, the accounting break-even level of sales is:

$$\begin{aligned}Q_A &= [(FC + \text{Depreciation})(1 - T_c)]/[(P - VC)(1 - T_c)] \\ Q_A &= [(\$435,000 + \$910,000/7)(1 - .21)]/[(\$43 - 10.45)(1 - .21)] \\ Q_A &= 17,357.91, \text{ or about } 17,358 \text{ units}\end{aligned}$$

- b. When calculating the financial break-even point, we express the initial investment as an equivalent annual cost (EAC). Dividing the initial investment by the 7-year annuity factor, discounted at 15 percent, the EAC of the initial investment is:

$$\begin{aligned}\text{EAC} &= \text{Initial Investment}/\text{PVIFA}_{15\%,7} \\ \text{EAC} &= \$910,000/4.1604 \\ \text{EAC} &= \$218,727.93\end{aligned}$$

Note that this calculation solves for the annuity payment with the initial investment as the present value of the annuity. In other words:

$$\begin{aligned} PVA &= C\{[1 - 1/(1 + R)]^t/R\} \\ \$910,000 &= C\{[1 - (1/1.15)^7]/.15\} \\ C &= \$218,727.93 \end{aligned}$$

Now we can calculate the financial break-even point. The financial break-even point for this project is:

$$\begin{aligned} Q_F &= [EAC + FC(1 - T_c) - D(T_c)]/[(P - VC)(1 - T_c)] \\ Q_F &= [\$218,727.93 + \$435,000(.79) - (\$910,000/7)(.21)]/[(\$43 - 10.45)(.79)] \\ Q_F &= 20,808.41, \text{ or about } 20,808 \text{ units} \end{aligned}$$

10. When calculating the financial break-even point, we express the initial investment as an equivalent annual cost (EAC). Dividing the initial investment by the 5-year annuity factor, discounted at 8 percent, the EAC of the initial investment is:

$$\begin{aligned} EAC &= \text{Initial Investment}/PVIFA_{8\%,5} \\ EAC &= \$530,000/3.99271 \\ EAC &= \$132,741.92 \end{aligned}$$

Note that this calculation solves for the annuity payment with the initial investment as the present value of the annuity. In other words:

$$\begin{aligned} PVA &= C\{[1 - 1/(1 + R)]^t/R\} \\ \$530,000 &= C\{[1 - (1/1.08)^5]/.08\} \\ C &= \$132,741.92 \end{aligned}$$

The annual depreciation is the cost of the equipment divided by the economic life, or:

$$\begin{aligned} \text{Annual depreciation} &= \$530,000/5 \\ \text{Annual depreciation} &= \$106,000 \end{aligned}$$

Now we can calculate the financial break-even point. The financial break-even point for this project is:

$$\begin{aligned} Q_F &= [EAC + FC(1 - T_c) - D(T_c)]/[(P - VC)(1 - T_c)] \\ Q_F &= [\$132,741.92 + \$235,000(1 - .21) - \$106,000(.21)]/[(\$75 - 27)(1 - .21)] \\ Q_F &= 7,809.39, \text{ or about } 7,809 \text{ units} \end{aligned}$$

Intermediate

11. a. At the accounting break-even, the IRR is zero percent since the project recovers the initial investment. The payback period is N years, the length of the project since the initial investment is exactly recovered over the project life. The NPV at the accounting break-even is:

$$NPV = I[(I/N)(PVIFA_{R\%,N}) - 1]$$

- b. At the cash break-even level, the IRR is –100 percent, the payback period is negative, and the NPV is negative and equal to the initial cash outlay.
- c. The definition of the financial break-even is where the NPV of the project is zero. If this is true, then the IRR of the project is equal to the required return. It is impossible to state the payback period, except to say that the payback period must be less than the length of the project. Since the discounted cash flows are equal to the initial investment, the undiscounted cash flows are greater than the initial investment, so the payback must be less than the project life.

12. Using the tax shield approach, the OCF at 90,000 units will be:

$$\begin{aligned}\text{OCF} &= [(P - v)Q - \text{FC}](1 - T_c) + T_c(D) \\ \text{OCF} &= [(\$41 - 24)(90,000) - 189,000](.77) + .23(\$485,000/4) \\ \text{OCF} &= \$1,060,457.50\end{aligned}$$

We will calculate the OCF at 91,000 units. The choice of the second level of quantity sold is arbitrary and irrelevant. No matter what level of units sold we choose we will still get the same sensitivity. So, the OCF at this level of sales is:

$$\begin{aligned}\text{OCF} &= [(\$41 - 24)(91,000) - 189,000](.77) + .23(\$485,000/4) \\ \text{OCF} &= \$1,073,547.50\end{aligned}$$

The sensitivity of the OCF to changes in the quantity sold is:

$$\begin{aligned}\text{Sensitivity} &= \Delta\text{OCF}/\Delta Q = (\$1,060,457.50 - 1,073,547.50)/(90,000 - 91,000) \\ \Delta\text{OCF}/\Delta Q &= +\$13.09\end{aligned}$$

OCF will increase by \$13.09 for every additional unit sold.

13. a. The base-case, best-case, and worst-case values are shown below. Remember that in the best-case, unit sales increase, while costs decrease. In the worst-case, unit sales decrease and costs increase.

<u>Scenario</u>	<u>Unit sales</u>	<u>Variable cost</u>	<u>Fixed costs</u>
Base	380	\$14,100	\$680,000
Best	418	\$12,690	\$612,000
Worst	342	\$15,510	\$748,000

Using the tax shield approach, the OCF and NPV for the base case estimate are:

$$\begin{aligned}\text{OCF}_{\text{base}} &= [(\$17,400 - 14,100)(380) - \$680,000](.79) + .21(\$720,000/4) \\ \text{OCF}_{\text{base}} &= \$491,260\end{aligned}$$

$$\begin{aligned}\text{NPV}_{\text{base}} &= -\$720,00 + \$491,260(\text{PVIFA}_{15\%,4}) \\ \text{NPV}_{\text{base}} &= \$682,536.67\end{aligned}$$

The OCF and NPV for the worst case estimate are:

$$\begin{aligned}\text{OCF}_{\text{worst}} &= [(\$17,400 - 15,510)(342) - \$748,000](.79) + .21(\$720,000/4) \\ \text{OCF}_{\text{worst}} &= -\$42,479.80\end{aligned}$$

$$\begin{aligned}\text{NPV}_{\text{worst}} &= -\$720,00 - \$42,479.80(\text{PVIFA}_{15\%,4}) \\ \text{NPV}_{\text{worst}} &= -\$841,278.91\end{aligned}$$

And the OCF and NPV for the best case estimate are:

$$\begin{aligned}\text{OCF}_{\text{best}} &= [(\$17,400 - 12,690)(418) - \$612,000](.79) + .21(\$720,00/4) \\ \text{OCF}_{\text{best}} &= \$1,109,656.20\end{aligned}$$

$$\begin{aligned}\text{NPV}_{\text{best}} &= -\$720,00 + \$1,109,656.20(\text{PVIFA}_{15\%,4}) \\ \text{NPV}_{\text{best}} &= \$2,448,044.44\end{aligned}$$

- b. To calculate the sensitivity of the NPV to changes in fixed costs, we choose another level of fixed costs. We will use fixed costs of \$690,000. The OCF using this level of fixed costs and the other base case values with the tax shield approach is:

$$\begin{aligned}\text{OCF} &= [(\$17,400 - 14,100)(380) - \$690,000](.79) + .21(\$720,00/4) \\ \text{OCF} &= \$483,360\end{aligned}$$

And the NPV is:

$$\begin{aligned}\text{NPV} &= -\$720,00 + \$483,360(\text{PVIFA}_{15\%,4}) \\ \text{NPV} &= \$659,982.34\end{aligned}$$

The sensitivity of NPV to changes in fixed costs is:

$$\begin{aligned}\Delta\text{NPV}/\Delta\text{FC} &= (\$682,536.67 - 659,982.34)/(\$680,000 - 690,000) \\ \Delta\text{NPV}/\Delta\text{FC} &= -\$2.255\end{aligned}$$

For every dollar FC increase, NPV falls by \$2.255.

- c. The accounting break-even is:

$$\begin{aligned}Q_A &= (\text{FC} + D)/(\text{P} - v) \\ Q_A &= [\$680,000 + (\$720,00/4)]/(\$17,400 - 14,100) \\ Q_A &= 260.61, \text{ or about } 261 \text{ units}\end{aligned}$$

14. The marketing study and the research and development are both sunk costs and should be ignored. We will calculate the sales and variable costs first. Since we will lose sales of the expensive clubs and gain sales of the cheap clubs, these must be accounted for as erosion. The total sales for the new project will be:

<u>Sales</u>		
New clubs	$\$950 \times 50,000 =$	\$47,500,000
Exp. clubs	$\$1,450 \times (-9,000) =$	-13,050,000
Cheap clubs	$\$475 \times 12,000 =$	<u>5,700,000</u>
		\$40,150,000

For the variable costs, we must include the units gained or lost from the existing clubs. Note that the variable costs of the expensive clubs are an inflow. If we are not producing the sets any more, we will save these variable costs, which is an inflow. So:

<u>Var. costs</u>	
New clubs	$-\$415 \times 50,000 = -\$20,750,000$
Exp. clubs	$-\$590 \times (-9,000) = 5,310,000$
Cheap clubs	$-\$210 \times 12,000 = \underline{-2,520,000}$
	$-\$17,960,000$

The pro forma income statement will be:

Sales	\$40,150,000
Variable costs	17,960,000
Fixed costs	9,400,000
Depreciation	<u>4,200,000</u>
EBT	\$8,590,000
Taxes	<u>2,061,600</u>
Net income	<u>\$6,528,400</u>

Using the bottom up OCF calculation, we get:

$$\begin{aligned}\text{OCF} &= \text{NI} + \text{Depreciation} \\ \text{OCF} &= \$6,528,400 + 4,200,000 \\ \text{OCF} &= \$10,728,400\end{aligned}$$

So, the payback period is:

$$\begin{aligned}\text{Payback period} &= 2 + \$10,343,200/\$10,728,400 \\ \text{Payback period} &= 2.964 \text{ years}\end{aligned}$$

The NPV is:

$$\begin{aligned}\text{NPV} &= -\$29,400,000 - 2,400,000 + \$10,728,400(\text{PVIFA}_{14\%,7}) + \$2,400,000/1.14^7 \\ \text{NPV} &= \$15,165,779.21\end{aligned}$$

And the IRR is:

$$\begin{aligned}\text{IRR} &= -\$29,400,000 - 2,400,000 + \$10,728,400(\text{PVIFA}_{\text{IRR}\%,7}) + \$2,400,000/(1 + \text{IRR})^7 \\ \text{IRR} &= 28.18\%\end{aligned}$$

15. The upper and lower bounds for the variables are:

	<u>Base Case</u>	<u>Best Case</u>	<u>Worst Case</u>
Unit sales (new)	50,000	55,000	45,000
Price (new)	\$950	\$1,045	\$855
VC (new)	\$415	\$374	\$457
Fixed costs	\$9,400,000	\$8,460,000	\$10,340,000
Sales lost (expensive)	9,000	8,100	9,900
Sales gained (cheap)	12,000	13,200	10,800

Best-case

We will calculate the sales and variable costs first. Since we will lose sales of the expensive clubs and gain sales of the cheap clubs, these must be accounted for as erosion. The total sales for the new project will be:

Sales

New clubs	$\$1,045 \times 55,000 =$	\$57,475,000
Exp. clubs	$\$1,450 \times (-8,100) =$	-11,745,000
Cheap clubs	$\$475 \times 13,200 =$	<u>6,270,000</u>
		\$52,000,000

For the variable costs, we must include the units gained or lost from the existing clubs. Note that the variable costs of the expensive clubs are an inflow. If we are not producing the sets any more, we will save these variable costs, which is an inflow. So:

Var. costs

New clubs	$-\$374 \times 55,00 =$	-\$25,542,500
Exp. clubs	$-\$590 \times (-8,100) =$	4,779,000
Cheap clubs	$-\$210 \times 13,200 =$	<u>-2,772,000</u>
		-\$18,535,500

The pro forma income statement will be:

Sales	\$52,000,000
Variable costs	18,535,500
Fixed costs	8,460,000
Depreciation	<u>4,200,000</u>
EBT	\$20,804,500
Taxes	<u>4,993,080</u>
Net income	<u><u>\$15,811,420</u></u>

Using the bottom up OCF calculation, we get:

$$\text{OCF} = \text{Net income} + \text{Depreciation}$$

$$\text{OCF} = \$15,811,420 + 4,200,000$$

$$\text{OCF} = \$20,011,420$$

And the best-case NPV is:

$$\text{NPV} = -\$29,400,000 - 2,400,000 + \$20,011,420(\text{PVIFA}_{14\%,7}) + 2,400,000/1.14^7$$

$$\text{NPV} = \$54,974,198.80$$

Worst-case

We will calculate the sales and variable costs first. Since we will lose sales of the expensive clubs and gain sales of the cheap clubs, these must be accounted for as erosion. The total sales for the new project will be:

Sales

New clubs	$\$855 \times 45,000 = \$38,475,000$
Exp. clubs	$\$1,450 \times (-9,900) = -14,355,000$
Cheap clubs	$\$475 \times 10,800 = \underline{5,130,000}$
	$\$29,250,000$

For the variable costs, we must include the units gained or lost from the existing clubs. Note that the variable costs of the expensive clubs are an inflow. If we are not producing the sets any more, we will save these variable costs, which is an inflow. So:

Var. costs

New clubs	$-\$457 \times 45,000 = -\$20,542,500$
Exp. clubs	$-\$590 \times (-9,900) = 5,841,000$
Cheap clubs	$-\$210 \times 10,800 = \underline{-2,268,000}$
	$-\$16,969,500$

The pro forma income statement will be:

Sales	\$29,250,000	
Variable costs	16,969,500	
Costs	10,340,000	
Depreciation	<u>4,200,000</u>	
EBT	<u>-\$2,259,500</u>	
Taxes	<u>-542,280</u>	*assumes a tax credit
Net income	<u><u>-\$1,717,220</u></u>	

Using the bottom up OCF calculation, we get:

$$\text{OCF} = \text{NI} + \text{Depreciation} = -\$1,717,220 + 4,200,000$$

$$\text{OCF} = \$2,482,780$$

And the worst-case NPV is:

$$\text{NPV} = -\$29,400,000 - 2,400,000 + \$2,482,780(\text{PVIFA}_{14\%,7}) + 2,400,000/1.14^7$$

$$\text{NPV} = -\$20,193,952.94$$

16. To calculate the sensitivity of the NPV to changes in the price of the new club, we need to change the price of the new club. We will choose \$960, but the choice is irrelevant as the sensitivity will be the same no matter what price we choose.

We will calculate the sales and variable costs first. Since we will lose sales of the expensive clubs and gain sales of the cheap clubs, these must be accounted for as erosion. The total sales for the new project will be:

<u>Sales</u>		
New clubs	$\$960 \times 50,000 =$	\$48,000,000
Exp. clubs	$\$1,450 \times (-9,000) =$	-13,050,000
Cheap clubs	$\$475 \times 12,000 =$	<u>5,700,000</u>
		\$40,650,000

For the variable costs, we must include the units gained or lost from the existing clubs. Note that the variable costs of the expensive clubs are an inflow. If we are not producing the sets any more, we will save these variable costs, which is an inflow. So:

<u>Var. costs</u>		
New clubs	$-\$415 \times 50,000 =$	-\$20,750,000
Exp. clubs	$-\$590 \times (-9,000) =$	5,310,000
Cheap clubs	$-\$210 \times 12,000 =$	<u>-2,520,000</u>
		-\$17,960,000

The pro forma income statement will be:

Sales	\$40,650,000
Variable costs	17,960,000
Fixed costs	9,400,000
Depreciation	<u>4,200,000</u>
EBT	\$9,090,000
Taxes	<u>2,181,600</u>
Net income	<u>\$6,908,400</u>

Using the bottom up OCF calculation, we get:

$$\begin{aligned}\text{OCF} &= \text{NI} + \text{Depreciation} \\ \text{OCF} &= \$6,908,400 + 4,200,000 \\ \text{OCF} &= \$11,108,400\end{aligned}$$

And the NPV is:

$$\begin{aligned}\text{NPV} &= -\$29,400,000 - 2,400,000 + \$11,108,400(\text{PVIFA}_{14\%,7}) + 2,400,000/1.14^7 \\ \text{NPV} &= \$16,795,335.05\end{aligned}$$

So, the sensitivity of the NPV to changes in the price of the new club is:

$$\Delta \text{NPV} / \Delta P = (\$15,165,779.21 - 16,795,335.05) / (\$950 - 960)$$

$$\Delta \text{NPV} / \Delta P = \$162,955.58$$

For every dollar increase (decrease) in the price of the clubs, the NPV increases (decreases) by \$162,955.58.

To calculate the sensitivity of the NPV to changes in the quantity sold of the new club, we need to change the quantity sold. We will choose 51,000 units, but the choice is irrelevant as the sensitivity will be the same no matter what quantity we choose.

Sales

New clubs	$\$950 \times 51,000 = \$48,450,000$
Exp. clubs	$\$1,450 \times (-9,000) = -13,050,000$
Cheap clubs	$\$475 \times 12,000 = \underline{5,700,000}$
	$\$41,100,000$

For the variable costs, we must include the units gained or lost from the existing clubs. Note that the variable costs of the expensive clubs are an inflow. If we are not producing the sets any more, we will save these variable costs, which is an inflow. So:

Var. costs

New clubs	$-\$415 \times 51,000 = -\$21,165,000$
Exp. clubs	$-\$590 \times (-9,000) = 5,310,000$
Cheap clubs	$-\$210 \times 12,000 = \underline{-2,520,000}$
	$-\$18,375,000$

The pro forma income statement will be:

Sales	\$41,100,000
Variable costs	18,375,000
Fixed costs	9,400,000
Depreciation	<u>4,200,000</u>
EBT	\$9,125,000
Taxes	<u>2,190,000</u>
Net income	<u><u>\$6,935,000</u></u>

Using the bottom up OCF calculation, we get:

$$\text{OCF} = \text{NI} + \text{Depreciation}$$

$$\text{OCF} = \$6,935,000 + 4,200,000$$

$$\text{OCF} = \$11,135,000$$

The NPV at this quantity is:

$$\begin{aligned}\text{NPV} &= -\$29,400,000 - \$2,400,000 + \$11,135,000(\text{PVIFA}_{14\%,7}) + \$2,400,000/1.14^7 \\ \text{NPV} &= \$16,909,403.96\end{aligned}$$

So, the sensitivity of the NPV to changes in the quantity sold is:

$$\begin{aligned}\Delta\text{NPV}/\Delta Q &= (\$15,165,779.21 - \$16,909,403.96)/(50,000 - 51,000) \\ \Delta\text{NPV}/\Delta Q &= \$1,743.62\end{aligned}$$

For an increase (decrease) of one set of clubs sold per year, the NPV increases (decreases) by \$1,743.62.

17. a. The base-case NPV is:

$$\begin{aligned}\text{NPV} &= -\$980,000 + \$279,500(\text{PVIFA}_{16\%,10}) \\ \text{NPV} &= \$370,887.08\end{aligned}$$

- b. We would abandon the project if the cash flow from selling the equipment is greater than the present value of the future cash flows. We need to find the sale quantity where the two are equal, so:

$$\begin{aligned}\$810,000 &= (\$43)Q(\text{PVIFA}_{16\%,9}) \\ Q &= \$810,000/[\$43(4.6065)] \\ Q &= 4,089\end{aligned}$$

Abandon the project if $Q < 4,089$ units, because the NPV of abandoning the project is greater than the NPV of the future cash flows.

- c. The \$810,000 is the market value of the project. If you continue with the project in one year, you forego the \$810,000 that could have been used for something else.

18. a. If the project is a success, the present value of the future cash flows will be:

$$\begin{aligned}\text{PV future CFs} &= \$43(9,100)(\text{PVIFA}_{16\%,9}) \\ \text{PV future CFs} &= \$1,802,540.62\end{aligned}$$

From the previous question, if the quantity sold is 3,700, we would abandon the project, and the cash flow would be \$810,000. Since the project has an equal likelihood of success or failure in one year, the expected value of the project in one year is the average of the success and failure cash flows, plus the cash flow in one year, so:

$$\begin{aligned}\text{Expected value of project at Year 1} &= [(\$1,802,540.62 + \$810,000)/2] + \$279,500 \\ \text{Expected value of project at Year 1} &= \$1,585,770.31\end{aligned}$$

The NPV is the present value of the expected value in one year plus the cost of the equipment, so:

$$\begin{aligned}\text{NPV} &= -\$980,000 + (\$1,585,770.31)/1.16 \\ \text{NPV} &= \$387,043.37\end{aligned}$$

- b. If we couldn't abandon the project, the present value of the future cash flows when the quantity is 3,800 will be:

$$\begin{aligned}\text{PV future CFs} &= \$43(3,700)(\text{PVIFA}_{16\%,9}) \\ \text{PV future CFs} &= \$732,901.13\end{aligned}$$

The gain from the option to abandon is the abandonment value minus the present value of the cash flows if we cannot abandon the project, so:

$$\begin{aligned}\text{Gain from option to abandon} &= \$810,000 - 732,901.13 \\ \text{Gain from option to abandon} &= \$77,098.87\end{aligned}$$

We need to find the value of the option to abandon times the likelihood of abandonment. So, the value of the option to abandon today is:

$$\begin{aligned}\text{Option value} &= (.50)(\$77,098.87)/1.16 \\ \text{Option value} &= \$33,232.27\end{aligned}$$

19. If the project is a success, the present value of the future cash flows will be:

$$\begin{aligned}\text{PV future CFs} &= \$43(18,200)(\text{PVIFA}_{16\%,9}) \\ \text{PV future CFs} &= \$3,605,081.24\end{aligned}$$

If the sales are only 3,700 units, from Problem #17, we know we will abandon the project, with a value of \$810,000. Since the project has an equal likelihood of success or failure in one year, the expected value of the project in one year is the average of the success and failure cash flows, plus the cash flow in one year, so:

$$\begin{aligned}\text{Expected value of project at Year 1} &= [(\$3,605,081.24 + \$810,000)/2] + \$279,500 \\ \text{Expected value of project at Year 1} &= \$2,487,040.62\end{aligned}$$

The NPV is the present value of the expected value in one year plus the cost of the equipment, so:

$$\begin{aligned}\text{NPV} &= -\$980,000 + \$2,487,040.62/1.16 \\ \text{NPV} &= \$1,164,000.53\end{aligned}$$

The gain from the option to expand is the present value of the cash flows from the additional units sold, so:

$$\begin{aligned}\text{Gain from option to expand} &= \$43(9,100)(\text{PVIFA}_{16\%,9}) \\ \text{Gain from option to expand} &= \$1,802,540.62\end{aligned}$$

We need to find the value of the option to expand times the likelihood of expansion. We also need to find the value of the option to expand today, so:

$$\text{Option value} = (.50)(\$1,802,540.62)/1.16$$

$$\text{Option value} = \$776,957.16$$

20. a. The accounting break-even is the aftertax sum of the fixed costs and depreciation charge divided by the contribution margin (selling price minus variable cost). In this case, there are no fixed costs, and the depreciation is the entire price of the press in the first year. So, the accounting break-even level of sales is:

$$Q_A = [(FC + D)(1 - T_C)] / [(P - VC)(1 - T_C)]$$

$$Q_A = [(\$0 + 7,200)(1 - .21)] / [(\$15 - 3.20)(1 - .21)]$$

$$Q_A = 610.17, \text{ or about 610 units}$$

- b. When calculating the financial break-even point, we express the initial investment as an equivalent annual cost (EAC). The initial investment is the \$20,000 in licensing fees. Dividing the initial investment by the three-year annuity factor, discounted at 12 percent, the EAC of the initial investment is:

$$\text{EAC} = \text{Initial Investment} / \text{PVIFA}_{12\%,3}$$

$$\text{EAC} = \$20,000 / 2.4018$$

$$\text{EAC} = \$8,326.98$$

Note, this calculation solves for the annuity payment with the initial investment as the present value of the annuity, in other words:

$$\text{PVA} = C \{ [1 - 1/(1 + R)^t] / R \}$$

$$\$20,000 = C \{ [1 - (1/1.12)^3] / .12 \}$$

$$C = \$8,326.98$$

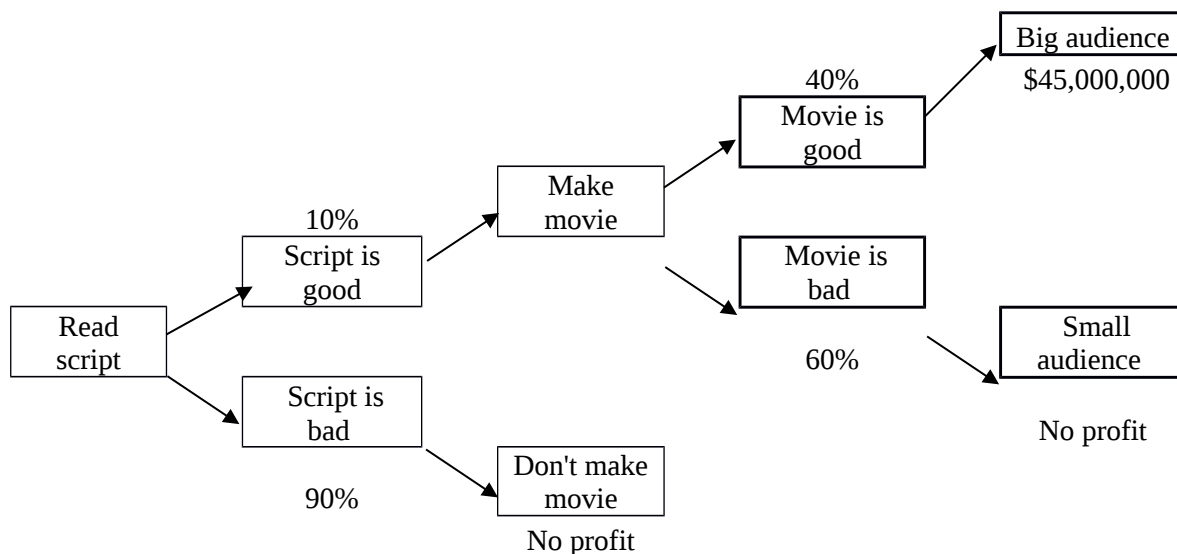
Now we can calculate the financial break-even point. Notice that there are no fixed costs or depreciation. The financial break-even point for this project is:

$$Q_F = [\text{EAC} + FC(1 - T_C) - D(T_C)] / [(P - VC)(1 - T_C)]$$

$$Q_F = (\$8,326.98 + 0 - 0) / [(\$15 - 3.20)(.79)]$$

$$Q_F = 893.26, \text{ or about 893 units}$$

21. The payoff from taking the lump sum is \$25,000, so we need to compare this to the expected payoff from taking 1.25 percent of the profit. The decision tree for the movie project is:



The value of 1.25 percent of the profits is as follows. There is a 40 percent probability the movie is good, and the audience is big, so the expected value of this outcome is:

$$\text{Value} = \$45,000,000 \times .40$$

$$\text{Value} = \$18,000,000$$

The value if the movie is good, and has a big audience, assuming the script is good is:

$$\text{Value} = \$18,000,000 \times .10$$

$$\text{Value} = \$1,800,000$$

This is the expected value for the studio, but the screenwriter will receive 1.25 percent of this amount, so the payment to the screenwriter will be:

$$\text{Payment to screenwriter} = \$1,800,000 \times .0125$$

$$\text{Payment to screenwriter} = \$22,500$$

The screenwriter should take the upfront offer of \$25,000.

22. We can calculate the value of the option to wait as the difference between the NPV of opening the mine today and the NPV of waiting one year to open the mine. The remaining life of the mine is:

$$33,600 \text{ ounces} / 4,200 \text{ ounces per year} = 8 \text{ years}$$

This will be true no matter when you open the mine. The aftertax cash flow per year if opened today is:

$$CF = 4,200(\$900) = \$3,780,000$$

So, the NPV of opening the mine today is:

$$\begin{aligned} NPV &= -\$17,400,000 + \$3,780,000(PVIFA_{12\%,8}) \\ NPV &= \$1,377,678.32 \end{aligned}$$

If you open the mine in one year, the cash flow will be either:

$$\begin{aligned} CF_{Up} &= 4,200(\$1,150) = \$4,830,000 \text{ per year} \\ CF_{Down} &= 4,200(\$700) = \$2,940,000 \text{ per year} \end{aligned}$$

The PV of these cash flows is:

$$\begin{aligned} \text{Price increase CF} &= \$4,830,000(PVIFA_{12\%,8}) = \$23,993,700.07 \\ \text{Price decrease CF} &= \$2,940,000(PVIFA_{12\%,8}) = \$14,604,860.91 \end{aligned}$$

So, the NPV is one year will be:

$$\begin{aligned} NPV &= -\$17,400,000 + [.60(\$23,993,700.07) + .40(\$14,604,860.91)] \\ NPV &= \$2,838,164.41 \end{aligned}$$

And the NPV today is:

$$\begin{aligned} NPV \text{ today} &= \$2,838,164.41 / 1.12 \\ NPV \text{ today} &= \$2,534,075.37 \end{aligned}$$

So, the value of the option to wait is:

$$\begin{aligned} \text{Option value} &= \$2,534,075.37 - 1,377,678.32 \\ \text{Option value} &= \$1,156,397.05 \end{aligned}$$

23. a. The NPV of the project is the sum of the present value of the cash flows generated by the project. The cash flows from this project are an annuity, so the NPV is:

$$\begin{aligned}\text{NPV} &= -\$43,000,000 + \$8,700,000(\text{PVIFA}_{13\%,10}) \\ \text{NPV} &= \$4,208,318.24\end{aligned}$$

- b. The company should abandon the project if the PV of the revised cash flows for the next nine years is less than the project's aftertax salvage value. Since the option to abandon the project occurs in Year 1, discount the revised cash flows to Year 1 as well. To determine the level of expected cash flows below which the company should abandon the project, calculate the equivalent annual cash flows the project must earn to equal the aftertax salvage value. We will solve for C_2 , the revised cash flow beginning in Year 2. So, the revised annual cash flow below which it makes sense to abandon the project is:

$$\begin{aligned}\text{Aftertax salvage value} &= C_2(\text{PVIFA}_{13\%,9}) \\ \$28,000,000 &= C_2(\text{PVIFA}_{13\%,9}) \\ C_2 &= \$28,000,000/\text{PVIFA}_{13\%,9} \\ C_2 &= \$5,456,329.25\end{aligned}$$

24. a. The NPV of the project is sum of the present value of the cash flows generated by the project. The annual cash flow for the project is the number of units sold times the cash flow per unit, which is:

$$\begin{aligned}\text{Annual cash flow} &= 20(\$235,000) \\ \text{Annual cash flow} &= \$4,700,000\end{aligned}$$

The cash flows from this project are an annuity, so the NPV is:

$$\begin{aligned}\text{NPV} &= -\$17,600,000 + \$4,700,000(\text{PVIFA}_{11\%,5}) \\ \text{NPV} &= -\$229,284.02\end{aligned}$$

- b. The company will abandon the project if unit sales are not revised upward. If the unit sales are revised upward, the aftertax cash flows for the project over the last four years will be:

$$\begin{aligned}\text{New annual cash flow} &= 30(\$235,000) \\ \text{New annual cash flow} &= \$7,050,000\end{aligned}$$

The NPV of the project will be the initial cost, plus the expected cash flow in Year 1 based on the 20 unit sales projection, plus the expected value of abandonment, plus the expected value of expansion. We need to remember that the abandonment value occurs in Year 1, and the present value of the expansion cash flows are in Year 1, so each of these must be discounted back to today. So, the project NPV under the abandonment or expansion scenario is:

$$\begin{aligned}\text{NPV} &= -\$17,600,000 + \$4,700,000/1.11 + .50(\$10,400,000)/1.11 \\ &\quad + [.50(\$7,050,000)(\text{PVIFA}_{11\%,4})]/1.11 \\ \text{NPV} &= \$1,171,280.23\end{aligned}$$

25. To calculate the unit sales for each scenario, we multiply the market sales times the company's market share. We can then use the quantity sold to find the revenue each year, and the variable costs each year. After doing these calculations, we will construct the pro forma income statement for each scenario. We can then find the operating cash flow using the bottom up approach, which is net income plus depreciation. Doing so, we find:

	<u>Pessimistic</u>	<u>Expected</u>	<u>Optimistic</u>
Units per year	22,000	27,600	33,000
		\$3,946,800.0	
Revenue	\$3,058,000.00	0	\$4,851,000.00
Variable costs	1,694,000.00	2,014,800.00	2,310,000.00
Fixed costs	975,000.00	920,000.00	890,000.00
Depreciation	383,333.33	358,333.33	325,000.00
EBT	\$5,666.67	\$653,666.67	\$1,326,000.00
Tax	1,190.00	137,270.00	278,460.00
Net income	\$4,476.67	\$516,396.67	\$1,047,540.00
OCF	\$387,810.00	\$874,730.00	\$1,372,540.00

Note that under the pessimistic scenario, the taxable income is negative. We assumed a tax credit in this case. Now we can calculate the NPV under each scenario, which will be:

$$\text{NPV}_{\text{Pessimistic}} = -\$2,300,000 + \$387,810(\text{PVIFA}_{13\%,6})$$

$$\text{NPV} = -\$749,710.22$$

$$\text{NPV}_{\text{Expected}} = -\$2,150,000 + \$874,730(\text{PVIFA}_{13\%,6})$$

$$\text{NPV} = \$1,346,776.73$$

$$\text{NPV}_{\text{Optimistic}} = -\$1,950,000 + \$1,372,540(\text{PVIFA}_{13\%,6})$$

$$\text{NPV} = \$3,536,796.99$$

The NPV under the pessimistic scenario is negative, but the company should probably accept the project.

Challenge

26. a. Using the tax shield approach, the OCF is:

$$\text{OCF} = [(\$375 - 295)(26,000) - \$345,000](.76) + .24(\$2,900,000/5)$$

$$\text{OCF} = \$1,457,800$$

And the NPV is:

$$\text{NPV} = -\$2,900,000 - 500,000 + \$1,457,800(\text{PVIFA}_{13\%,5})$$

$$+ [\$500,000 + 275,000(1 - .24)]/1.13^5$$

$$\text{NPV} = \$2,112,236.53$$

b. In the worst-case, the OCF is:

$$\begin{aligned}\text{OCF}_{\text{worst}} &= \{[(\$375)(.9) - 295](26,000) - \$345,000\}(.76) + .24(\$3,335,000/5) \\ \text{OCF}_{\text{worst}} &= \$737,680\end{aligned}$$

And the worst-case NPV is:

$$\begin{aligned}\text{NPV}_{\text{worst}} &= -\$3,335,000 - 500,000(1.05) + \$737,680(\text{PVIFA}_{13\%,5}) + \\ &\quad [\$500,000(1.05) + 275,000(.85)(1 - .24)]/1.13^5 \\ \text{NPV}_{\text{worst}} &= -\$884,038.57\end{aligned}$$

The best-case OCF is:

$$\begin{aligned}\text{OCF}_{\text{best}} &= \{[\$375(1.1) - 295](26,000) - \$345,000\}(.76) + .24(\$2,465,000/5) \\ \text{OCF}_{\text{best}} &= \$2,177,920\end{aligned}$$

And the best-case NPV is:

$$\begin{aligned}\text{NPV}_{\text{best}} &= -\$2,465,000 - 500,000(.95) + \$2,177,920(\text{PVIFA}_{13\%,5}) + \\ &\quad [\$500,000(.95) + 275,000(1.15)(1 - .24)]/1.13^5 \\ \text{NPV}_{\text{best}} &= \$5,108,511.63\end{aligned}$$

27. To calculate the sensitivity to changes in quantity sold, we will choose a quantity of 27,000. The OCF at this level of sales is:

$$\begin{aligned}\text{OCF} &= [(\$375 - 295)(27,000) - \$345,000](.76) + .24(\$2,900,000/5) \\ \text{OCF} &= \$1,457,800\end{aligned}$$

The sensitivity of changes in the OCF to quantity sold is:

$$\begin{aligned}\Delta\text{OCF}/\Delta Q &= (\$1,457,800 - 1,518,600)/(26,000 - 27,000) \\ \Delta\text{OCF}/\Delta Q &= +\$60.80\end{aligned}$$

The NPV at this level of sales is:

$$\begin{aligned}\text{NPV} &= -\$2,900,000 - 500,000 + \$1,518,600(\text{PVIFA}_{13\%,5}) + [\$500,000 + 275,000(1 - .24)]/1.13^5 \\ \text{NPV} &= \$2,326,084.19\end{aligned}$$

And the sensitivity of NPV to changes in the quantity sold is:

$$\begin{aligned}\Delta\text{NPV}/\Delta Q &= (\$2,112,236.53 - 2,326,084.19)/(26,000 - 27,000) \\ \Delta\text{NPV}/\Delta Q &= +\$213.85\end{aligned}$$

You wouldn't want the quantity to fall below the point where the NPV is zero. We know the NPV changes \$213.85 for every unit sale, so we can divide the NPV for 26,000 units by the sensitivity to get a change in quantity. Doing so, we get:

$$\begin{aligned}\$2,112,236.53 &= \$213.85(\Delta Q) \\ \Delta Q &= 9,877\end{aligned}$$

For a zero NPV, sales would have to decrease 9,877 units, so the minimum quantity is:

$$Q_{\text{Min}} = 26,000 - 9,877$$

$$Q_{\text{Min}} = 16,123$$

28. We will use the bottom up approach to calculate the operating cash flow. Assuming we operate the project for all four years, the cash flows are:

<u>Year</u>	<u>0</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
Sales		\$12,400,000	\$12,400,000	\$12,400,000	\$12,400,000
Operating costs		4,500,000	4,500,000	4,500,000	4,500,000
Depreciation		4,500,000	4,500,000	4,500,000	4,500,000
EBT		\$3,400,000	\$3,400,000	\$3,400,000	\$3,400,000
Tax		714,000	714,000	714,000	714,000
Net income		\$2,686,000	\$2,686,000	\$2,686,000	\$2,686,000
+Depreciation		4,500,000	4,500,000	4,500,000	4,500,000
Operating CF		\$7,186,000	\$7,186,000	\$7,186,000	\$7,186,000
Change in NWC	-\$950,000	0	0	0	\$950,000
Capital spending	-18,000,000	0	0	0	0
Total cash flow	<u>-18,950,000</u>	<u>\$7,186,000</u>	<u>\$7,186,000</u>	<u>\$7,186,000</u>	<u>\$8,136,000</u>

There is no salvage value for the equipment. The NPV is:

$$\text{NPV} = -\$18,950,000 + \$7,186,000(\text{PVIFA}_{13\%,3}) + \$8,136,000/1.13^4$$

$$\text{NPV} = \$3,007,203.74$$

The cash flows if we abandon the project after one year are:

<u>Year</u>	<u>0</u>	<u>1</u>
Sales		\$12,400,000
Operating costs		4,500,000
Depreciation		4,500,000
EBT		\$3,400,000
Tax		714,000
Net income		\$2,686,000
+Depreciation		4,500,000
Operating CF		\$7,186,000
Change in NWC	-\$950,000	\$950,000
Capital spending	-18,000,000	14,685,000
Total cash flow	<u>-18,950,000</u>	<u>\$22,821,000</u>

The book value of the equipment is:

$$\text{Book value} = \$18,000,000 - (1)(\$18,000,000/4)$$

$$\text{Book value} = \$13,500,000$$

So the taxes on the salvage value will be:

$$\text{Taxes} = (\$13,500,000 - 15,000,000)(.21)$$

$$\text{Taxes} = -\$315,000$$

This makes the aftertax salvage value:

$$\text{Aftertax salvage value} = \$15,000,000 - 315,000$$

$$\text{Aftertax salvage value} = \$14,685,000$$

The NPV if we abandon the project after one year is:

$$\text{NPV} = -\$18,950,000 + \$22,821,000/1.13$$

$$\text{NPV} = \$1,245,575.22$$

If we abandon the project after two years, the cash flows are:

Year	<u>0</u>	<u>1</u>	<u>2</u>
Sales		\$12,400,000	\$12,400,000
Operating costs		4,500,000	4,500,000
Depreciation		4,500,000	4,500,000
EBT		\$3,400,000	\$3,400,000
Tax		714,000	714,000
Net income		\$2,686,000	\$2,686,000
+Depreciation		4,500,000	4,500,000
Operating CF		\$7,186,000	\$7,186,000
Change in NWC	-\$950,000	0	950,000
Capital spending	-18,000,000	0	10,580,000
Total cash flow	<u>-18,950,000</u>	<u>\$7,186,000</u>	<u>\$18,716,000</u>

The book value of the equipment is:

$$\text{Book value} = \$18,000,000 - (2)(\$18,000,000/4)$$

$$\text{Book value} = \$9,000,000$$

So the taxes on the salvage value will be:

$$\text{Taxes} = (\$9,000,000 - 11,000,000)(.21)$$

$$\text{Taxes} = -\$420,000$$

This makes the aftertax salvage value:

$$\text{Aftertax salvage value} = \$11,000,000 - 420,000$$

$$\text{Aftertax salvage value} = \$10,580,000$$

The NPV if we abandon the project after two years is:

$$\text{NPV} = -\$18,950,000 + \$7,186,000/1.13 + \$18,716,000/1.13^2$$

$$\text{NPV} = \$2,066,665.36$$

If we abandon the project after three years, the cash flows are:

<u>Year</u>	<u>0</u>	<u>1</u>	<u>2</u>	<u>3</u>
Sales		\$12,400,000	\$12,400,000	\$12,400,000
Operating costs		4,500,000	4,500,000	4,500,000
Depreciation		4,500,000	4,500,000	4,500,000
EBT		\$3,400,000	\$3,400,000	\$3,400,000
Tax		714,000	714,000	714,000
Net income		\$2,686,000	\$2,686,000	\$2,686,000
+Depreciation		4,500,000	4,500,000	4,500,000
Operating CF		\$7,186,000	\$7,186,000	\$7,186,000
Change in NWC	-\$950,000	0	0	950,000
Capital spending	-18,000,000	0	0	7,660,000
Total cash flow	<u><u>-\$18,950,000</u></u>	<u><u>\$7,186,000</u></u>	<u><u>\$7,186,000</u></u>	<u><u>\$15,796,000</u></u>

The book value of the equipment is:

$$\text{Book value} = \$18,000,000 - (3)(\$18,000,000/4)$$

$$\text{Book value} = \$4,500,000$$

So the taxes on the salvage value will be:

$$\text{Taxes} = (\$4,500,000 - 8,500,000)(.21)$$

$$\text{Taxes} = -\$840,000$$

This makes the aftertax salvage value:

$$\text{Aftertax salvage value} = \$8,500,000 - 840,000$$

$$\text{Aftertax salvage value} = \$7,660,000$$

The NPV if we abandon the project after three years is:

$$\begin{aligned}\text{NPV} &= -\$18,950,000 + \$7,186,000(\text{PVIFA}_{13\%,2}) + \$15,796,000/1.13^3 \\ \text{NPV} &= \$3,984,404.47\end{aligned}$$

We should abandon the equipment after three years since the NPV of abandoning the project after three years has the highest NPV.

29. a. The NPV of the project is the sum of the present value of the cash flows generated by the project. The cash flows from this project are an annuity, so the NPV is:

$$\begin{aligned}\text{NPV} &= -\$9,500,000 + \$1,800,000(\text{PVIFA}_{10\%,10}) \\ \text{NPV} &= \$1,560,220.79\end{aligned}$$

- b. The company will abandon the project if the value of abandoning the project is greater than the value of the future cash flows. The present value of the future cash flows if the company revises its sales downward will be:

$$\begin{aligned}\text{PV of downward revision} &= \$395,000(\text{PVIFA}_{10\%,9}) \\ \text{PV of downward revision} &= \$2,274,814.81\end{aligned}$$

Since this is less than the abandonment value, the company should abandon the project if sales are revised downward. So, the revised NPV of the project will be the initial cost, plus the PV of the expected cash flow in Year 1, plus the PV of the expected cash flows based on the upward sales projection, plus the PV of the abandonment value. We need to remember that the abandonment value occurs in Year 1, and the present value of the expansion cash flows are in Year 1, so each of these must be discounted back to today. So, the project NPV under the abandonment or expansion scenario is:

$$\begin{aligned}\text{NPV} &= -\$9,500,000 + \$1,800,000/1.10 + .50(\$3,100,000)/1.10 \\ &\quad + [.50(\$2,950,000)(\text{PVIFA}_{10\%,9})]/1.10 \\ \text{NPV} &= \$1,267,781.94\end{aligned}$$

30. First, determine the cash flow from selling the old harvester. When calculating the salvage value, remember that tax liabilities or credits are generated on the difference between the resale value and the book value of the asset. Using the original purchase price of the old harvester to determine annual depreciation, the annual depreciation for the old harvester is:

$$\begin{aligned}\text{Depreciation}_{\text{Old}} &= \$65,000/15 \\ \text{Depreciation}_{\text{Old}} &= \$4,333.33\end{aligned}$$

Since the machine is five years old, the firm has accumulated five annual depreciation charges, reducing the book value of the machine. The current book value of the machine is equal to the initial purchase price minus the accumulated depreciation, so:

$$\begin{aligned}\text{Book value} &= \text{Initial Purchase Price} - \text{Accumulated Depreciation} \\ \text{Book value} &= \$65,000 - (\$4,333.33 \times 5 \text{ years}) \\ \text{Book value} &= \$43,333.33\end{aligned}$$

Since the firm is able to resell the old harvester for \$21,000, which is less than the \$43,333 book value of the machine, the firm will generate a tax credit on the sale. The aftertax salvage value of the old harvester will be:

$$\text{Aftertax salvage value} = \text{Market value} + T_C(\text{Book value} - \text{Market value})$$

$$\text{Aftertax salvage value} = \$21,000 + .22(\$43,333.33 - 21,000)$$

$$\text{Aftertax salvage value} = \$25,913.33$$

Next, we need to calculate the incremental depreciation. We need to calculate the depreciation tax shield generated by the new harvester less the forgone depreciation tax shield from the old harvester. Let P be the break-even purchase price of the new harvester. So, we find:

$$\text{Depreciation tax shield}_{\text{New}} = (\text{Initial Investment}/\text{Economic Life}) \times T_C$$

$$\text{Depreciation tax shield}_{\text{New}} = (P/10)(.22)$$

And the depreciation tax shield on the old harvester is:

$$\text{Depreciation tax shield}_{\text{Old}} = (\$65,000/15)(.22)$$

$$\text{Depreciation tax shield}_{\text{Old}} = (\$4,333.33)(.22)$$

So, the incremental depreciation tax, which is the depreciation tax shield from the new harvester, minus the depreciation tax shield from the old harvester, is:

$$\text{Incremental depreciation tax shield} = (P/10)(.22) - (\$4,333.33)(.22)$$

$$\text{Incremental depreciation tax shield} = (P/10 - \$4,333.33)(.22)$$

The present value of the incremental depreciation tax shield will be:

$$\text{PV}_{\text{Depreciation tax shield}} = (P/10)(.22)(\text{PVIFA}_{15\%,10}) - \$4,333.33(.22)(\text{PVIFA}_{15\%,10})$$

The new harvester will generate year-end pre-tax cash flow savings of \$13,000 per year for 10 years. We can find the aftertax present value of the cash flows savings as:

$$\text{PV}_{\text{Savings}} = C_1(1 - T_C)(\text{PVIFA}_{15\%,10})$$

$$\text{PV}_{\text{Savings}} = \$13,000(1 - .22)(\text{PVIFA}_{15\%,10})$$

$$\text{PV}_{\text{Savings}} = \$50,890.31$$

The break-even purchase price of the new harvester is the price, P , which makes the NPV of the machine equal to zero.

$$\text{NPV} = -P + \text{Salvage value}_{\text{Old}} + \text{PV}_{\text{Depreciation tax shield}} + \text{PV}_{\text{Savings}}$$

$$\$0 = -P + \$25,913.33 + (P/10)(.22)(\text{PVIFA}_{15\%,10}) - \$4,333.33(.22)(\text{PVIFA}_{15\%,10}) + \$50,890.31$$

$$P - (P/10)(.22)(\text{PVIFA}_{15\%,10}) = \$76,803.65 - \$4,333.33(.22)(\text{PVIFA}_{15\%,10})$$

$$P[1 - (1/10)(.22)(\text{PVIFA}_{15\%,10})] = \$72,019.09$$

$$P = \$80,957.88$$

CHAPTER 8

INTEREST RATES AND BOND VALUATION

Answers to Concept Questions

1. No. As interest rates fluctuate, the value of a Treasury security will fluctuate. Long-term Treasury securities have substantial interest rate risk.
2. All else the same, the Treasury security will have lower coupons because of its lower default risk, so it will have greater interest rate risk.
3. No. If the bid were higher than the ask, the implication would be that a dealer was willing to sell a bond and immediately buy it back at a higher price. How many such transactions would you like to do?
4. Prices and yields move in opposite directions. Since the bid price must be lower, the bid yield must be higher.
5. Bond issuers look at outstanding bonds of similar maturity and risk. The yields on such bonds are used to establish the coupon rate necessary for a particular issue to initially sell for par value. Bond issuers also ask potential purchasers what coupon rate would be necessary to attract them. The coupon rate is fixed and determines what the bond's coupon payments will be. The required return is what investors actually demand on the issue, and it will fluctuate through time. The coupon rate and required return are equal only if the bond sells for exactly par.
6. Yes. Some investors have obligations that are denominated in dollars; i.e., they are nominal. Their primary concern is that an investment provides the needed nominal dollar amounts. Pension funds, for example, often must plan for pension payments many years in the future. If those payments are fixed in dollar terms, then it is the nominal return on an investment that is important.
7. Companies pay to have their bonds rated because unrated bonds can be difficult to sell; many large investors are prohibited from investing in unrated issues.
8. Treasury bonds have no credit risk since they are backed by the U.S. government, so a rating is unnecessary. Junk bonds often are not rated because there would be no point in an issuer paying a rating agency to assign its bonds a low rating (it's like paying someone to kick you!).
9. The term structure is based on pure discount bonds. The yield curve is based on coupon-bearing issues.
10. Bond ratings have a subjective factor to them. Split ratings reflect a difference of opinion among credit agencies.

11. As a general constitutional principle, the federal government cannot tax the states without their consent if doing so would interfere with state government functions. At one time, this principle was thought to provide for the tax-exempt status of municipal interest payments. However, modern court rulings make it clear that Congress can revoke the municipal exemption, so the only basis now appears to be historical precedent. The fact that the states and the federal government do not tax each other's securities is referred to as "reciprocal immunity."
12. Lack of transparency means that a buyer or seller can't see recent transactions, so it is much harder to determine what the best bid and ask prices are at any point in time.
13. When the bonds are initially issued, the coupon rate is set at auction so that the bonds sell at par value. The wide range of coupon rates shows the interest rate when each bond was issued. Notice that interest rates have evidently declined. Why?
14. Companies charge that bond rating agencies are pressuring them to pay for bond ratings. When a company pays for a rating, it has the opportunity to make its case for a particular rating. With an unsolicited rating, the company has no input.
15. A 100-year bond looks like a share of preferred stock. In particular, it is a loan with a life that almost certainly exceeds the life of the lender, assuming that the lender is an individual. With a junk bond, the credit risk can be so high that the borrower is almost certain to default, meaning that the creditors are very likely to end up as part owners of the business. In both cases, the "equity in disguise" has a significant tax advantage.
16.
 - a. The bond price is the present value of the cash flows from a bond. The YTM is the interest rate used in valuing the cash flows from a bond.
 - b. If the coupon rate is higher than the required return on a bond, the bond will sell at a premium, since it provides periodic income in the form of coupon payments in excess of that required by investors on other similar bonds. If the coupon rate is lower than the required return on a bond, the bond will sell at a discount since it provides insufficient coupon payments compared to that required by investors on other similar bonds. For premium bonds, the coupon rate exceeds the YTM; for discount bonds, the YTM exceeds the coupon rate, and for bonds selling at par, the YTM is equal to the coupon rate.
 - c. Current yield is defined as the annual coupon payment divided by the current bond price. For premium bonds, the current yield exceeds the YTM, for discount bonds the current yield is less than the YTM, and for bonds selling at par value, the current yield is equal to the YTM. In all cases, the current yield plus the expected one-period capital gains yield of the bond must be equal to the required return.
17. A long-term bond has more interest rate risk compared to a short-term bond, all else the same. A low coupon bond has more interest rate risk than a high coupon bond, all else the same. When comparing a high coupon, long-term bond to a low coupon, short-term bond, we are unsure which has more interest rate risk. Generally, the maturity of a bond is a more important determinant of the interest rate risk, so the long-term, high coupon bond probably has more interest rate risk. The exception would be if the maturities are close, and the coupon rates are vastly different.

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The price of a pure discount (zero coupon) bond is the present value of the par value. Remember, even though there are no coupon payments, the periods are semiannual to stay consistent with coupon bond payments. So, the price of the bond for each YTM is:

$$a. P = \$1,000 / (1 + .06/2)^{40} = \$306.56$$

$$b. P = \$1,000 / (1 + .08/2)^{40} = \$208.29$$

$$c. P = \$1,000 / (1 + .10/2)^{40} = \$142.05$$

2. The price of any bond is the PV of the interest payments, plus the PV of the par value. Notice this problem assumes a semiannual coupon. The price of the bond at each YTM will be:

$$a. P = \$35 \{ [1 - 1/(1 + .035)^{30}] / .035 \} + \$1,000 [1/(1 + .035)^{30}]$$

$$P = \$1,000.00$$

When the YTM and the coupon rate are equal, the bond will sell at par.

$$b. P = \$35 \{ [1 - 1/(1 + .045)^{30}] / .045 \} + \$1,000 [1/(1 + .045)^{30}]$$

$$P = \$837.11$$

When the YTM is greater than the coupon rate, the bond will sell at a discount.

$$c. P = \$35 \{ [1 - 1/(1 + .025)^{30}] / .025 \} + \$1,000 [1/(1 + .025)^{30}]$$

$$P = \$1,209.30$$

When the YTM is less than the coupon rate, the bond will sell at a premium.

3. Here we are finding the YTM of a semiannual coupon bond. The bond price equation is:

$$P = \$980 = \$31(PVIFA_{R\%,26}) + \$1,000(PVIF_{R\%,26})$$

Since we cannot solve the equation directly for R , using a spreadsheet, a financial calculator, or trial and error, we find:

$$R = 3.215\%$$

Since the coupon payments are semiannual, this is the semiannual interest rate. The YTM is the APR of the bond, so:

$$YTM = 2 \times 3.215\% = 6.43\%$$

4. Here we need to find the coupon rate of the bond. All we need to do is to set up the bond pricing equation and solve for the coupon payment as follows:

$$P = \$1,055 = C(PVIFA_{3.4\%,23}) + \$1,000(PVIF_{3.4\%,23})$$

Solving for the coupon payment, we get:

$$C = \$37.49$$

Since this is the semiannual payment, the annual coupon payment is:

$$2 \times \$37.49 = \$74.97$$

And the coupon rate is the annual coupon payment divided by par value, so:

$$\text{Coupon rate} = \$74.97/\$1,000$$

$$\text{Coupon rate} = .0750, \text{ or } 7.50\%$$

5. The price of any bond is the PV of the interest payments, plus the PV of the par value. The fact that the bond is denominated in euros is irrelevant. Notice this problem assumes an annual coupon. The price of the bond will be:

$$P = \text{€}51(\{1 - [1/(1 + .043)]^{15}\}/.043) + \text{€}1,000[1/(1 + .043)^{15}]$$

$$P = \text{€}1,087.11$$

6. Here we are finding the YTM of an annual coupon bond. The fact that the bond is denominated in yen is irrelevant. The bond price equation is:

$$P = \text{¥}106,000 = \text{¥}2,800(PVIFA_{R\%,17}) + \text{¥}100,000(PVIF_{R\%,17})$$

Since we cannot solve the equation directly for R , using a spreadsheet, a financial calculator, or trial and error, we find:

$$R = 2.37\%$$

Since the coupon payments are annual, this is the yield to maturity.

7. To find the price of a zero coupon bond, we need to find the value of the future cash flows. With a zero coupon bond, the only cash flow is the par value at maturity. We find the present value assuming semiannual compounding to keep the YTM of a zero coupon bond equivalent to the YTM of a coupon bond, so:

$$P = \$10,000(PVIF_{2.25\%,26})$$

$$P = \$5,607.30$$

8. To find the price of this bond, we need to find the present value of the bond's cash flows. So, the price of the bond is:

$$P = \$49(PVIFA_{2.70\%,26}) + \$2,000(PVIF_{2.70\%,26})$$

$$P = \$1,907.45$$

9. To find the price of this bond, we need to find the present value of the bond's cash flows. So, the price of the bond is:

$$P = \$72.50(PVIFA_{1.70\%,24}) + \$5,000(PVIF_{1.70\%,24})$$

$$P = \$4,755.34$$

10. The approximate relationship between nominal interest rates (R), real interest rates (r), and inflation (h) is:

$$R \approx r + h$$

$$\text{Approximate } r = .0415 - .027$$

$$\text{Approximate } r = .0145, \text{ or } 1.45\%$$

The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation is:

$$(1 + R) = (1 + r)(1 + h)$$

$$(1 + .0415) = (1 + r)(1 + .027)$$

$$\text{Exact } r = [(1 + .0415)/(1 + .027)] - 1$$

$$\text{Exact } r = .0141, \text{ or } 1.41\%$$

11. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

$$(1 + R) = (1 + r)(1 + h)$$

$$R = (1 + .0225)(1 + .032) - 1$$

$$R = .0552, \text{ or } 5.52\%$$

12. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

$$(1 + R) = (1 + r)(1 + h)$$

$$h = [(1 + .12)/(1 + .07)] - 1$$

$$h = .0467, \text{ or } 4.67\%$$

13. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

$$(1 + R) = (1 + r)(1 + h)$$

$$r = [(1 + .1208)/(1.046)] - 1$$

$$r = .0715, \text{ or } 7.15\%$$

14. The coupon rate, located in the second column of the quote, is 5.250 percent. The bid price is:

$$\begin{aligned}\text{Bid price} &= 129.1328 = 129.1328\% \\ \text{Bid price} &= (129.1328/100)(\$10,000) \\ \text{Bid price} &= \$12,913.28\end{aligned}$$

The previous day's asked price is found by:

$$\text{Previous day's asked price} = \text{Today's asked price} - \text{Change} = 129.1953 + .1484 = 129.3437$$

The previous day's asked price in dollars was:

$$\begin{aligned}\text{Previous day's asked price} &= 129.3437 = 129.3437\% \\ \text{Previous day's asked price} &= (129.3437/100)(\$10,000) \\ \text{Previous day's asked price} &= \$12,934.37\end{aligned}$$

15. This is a premium bond because it sells for more than 100 percent of face value. The dollar asked price is:

$$\begin{aligned}\text{Price} &= (128.4688/100)(\$10,000) \\ \text{Price} &= \$12,846.88\end{aligned}$$

The current yield is the annual coupon payment divided by the price, so:

$$\begin{aligned}\text{Current yield} &= \text{Annual coupon payment}/\text{Price} \\ \text{Current yield} &= \$437.50/\$12,846.88 \\ \text{Current yield} &= .03405, \text{ or } 3.405\%\end{aligned}$$

The YTM is located under the "Asked Yield" column, so the YTM is 2.665 percent.

The bid-ask spread as a percentage of par is:

$$\begin{aligned}\text{Bid-ask spread} &= 128.4688 - 128.4063 \\ \text{Bid-ask spread} &= .0625\end{aligned}$$

So, in dollars, we get:

$$\begin{aligned}\text{Bid-ask spread} &= (.0625/100)(\$10,000) \\ \text{Bid-ask spread} &= \$6.25\end{aligned}$$

16. Zero coupon bonds are priced with semiannual compounding to correspond with coupon bonds. The price of the bond when purchased was:

$$\begin{aligned}P_0 &= \$1,000/(1 + .0285)^{40} \\ P_0 &= \$324.96\end{aligned}$$

And the price at the end of one year is:

$$\begin{aligned}P_0 &= \$1,000/(1 + .0285)^{38} \\ P_0 &= \$343.75\end{aligned}$$

So, the implied interest, which will be taxable as interest income, is:

$$\text{Implied interest} = \$343.75 - 324.96$$

$$\text{Implied interest} = \$18.79$$

Intermediate

17. Here we are finding the YTM of annual coupon bonds for various maturity lengths. The bond price equation is:

$$P = C(\text{PVIFA}_{R\%,t}) + \$1,000(\text{PVIF}_{R\%,t})$$

Bond Miller:

$$P_0 = \$32.50(\text{PVIFA}_{2.65\%,26}) + \$1,000(\text{PVIF}_{2.65\%,26}) = \$1,111.71$$

$$P_1 = \$32.50(\text{PVIFA}_{2.65\%,24}) + \$1,000(\text{PVIF}_{2.65\%,24}) = \$1,105.55$$

$$P_3 = \$32.50(\text{PVIFA}_{2.65\%,20}) + \$1,000(\text{PVIF}_{2.65\%,20}) = \$1,092.22$$

$$P_8 = \$32.50(\text{PVIFA}_{2.65\%,10}) + \$1,000(\text{PVIF}_{2.65\%,10}) = \$1,052.11$$

$$P_{12} = \$32.50(\text{PVIFA}_{2.65\%,2}) + \$1,000(\text{PVIF}_{2.65\%,2}) = \$1,011.54$$

$$P_{13} = \$1,000$$

Bond Modigliani:

$$P_0 = \$26.50(\text{PVIFA}_{3.25\%,26}) + \$1,000(\text{PVIF}_{3.25\%,26}) = \$895.76$$

$$P_1 = \$26.50(\text{PVIFA}_{3.25\%,24}) + \$1,000(\text{PVIF}_{3.25\%,24}) = \$901.07$$

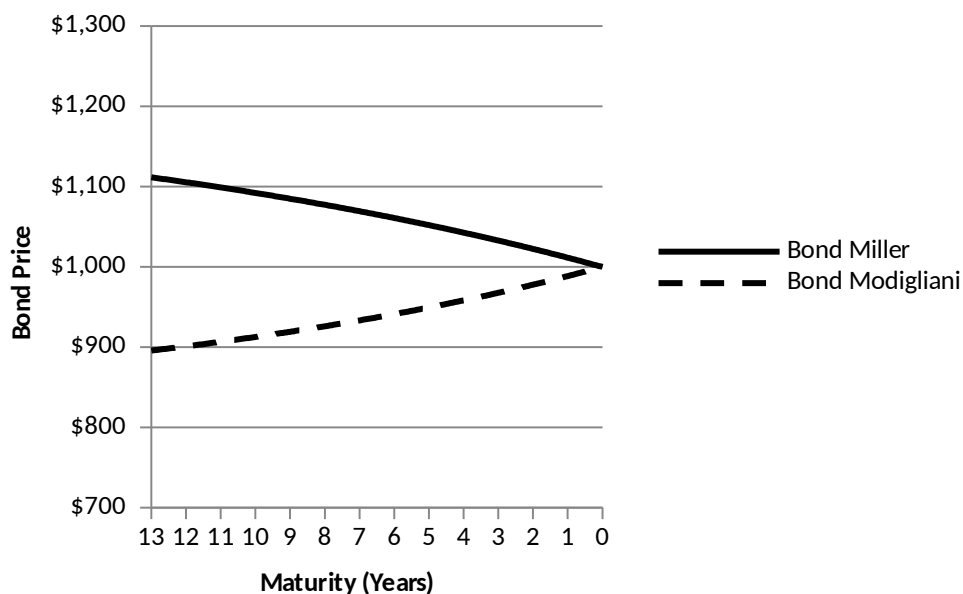
$$P_3 = \$26.50(\text{PVIFA}_{3.25\%,20}) + \$1,000(\text{PVIF}_{3.25\%,20}) = \$912.76$$

$$P_8 = \$26.50(\text{PVIFA}_{3.25\%,10}) + \$1,000(\text{PVIF}_{3.25\%,10}) = \$949.47$$

$$P_{12} = \$26.50(\text{PVIFA}_{3.25\%,2}) + \$1,000(\text{PVIF}_{3.25\%,2}) = \$988.56$$

$$P_{13} = \$1,000$$

Maturity and Bond Price



All else held equal, the premium over par value for a premium bond declines as maturity approaches, and the discount from par value for a discount bond declines as maturity approaches. This is called “pull to par.” In both cases, the largest percentage price changes occur at the shortest maturity lengths.

Also, notice that the price of each bond when no time is left to maturity is the par value, even though the purchaser would receive the par value plus the coupon payment immediately. This is because we calculate the clean price of the bond.

18. Any bond that sells at par has a YTM equal to the coupon rate. Both bonds sell at par, so the initial YTM on both bonds is the coupon rate, 5.8 percent. If the YTM suddenly rises to 7.8 percent:

$$P_{\text{Laurel}} = \$29(\text{PVIFA}_{3.9\%,6}) + \$1,000(\text{PVIF}_{3.9\%,6}) = \$947.41$$

$$P_{\text{Hardy}} = \$29(\text{PVIFA}_{3.9\%,40}) + \$1,000(\text{PVIF}_{3.9\%,40}) = \$799.09$$

The percentage change in price is calculated as:

$$\text{Percentage change in price} = (\text{New price} - \text{Original price}) / \text{Original price}$$

$$\Delta P_{\text{Laurel}} \% = (\$947.41 - 1,000) / \$1,000 = -.0526, \text{ or } -5.26\%$$

$$\Delta P_{\text{Hardy}} \% = (\$799.09 - 1,000) / \$1,000 = -.2009, \text{ or } -20.09\%$$

If the YTM suddenly falls to 3.8 percent:

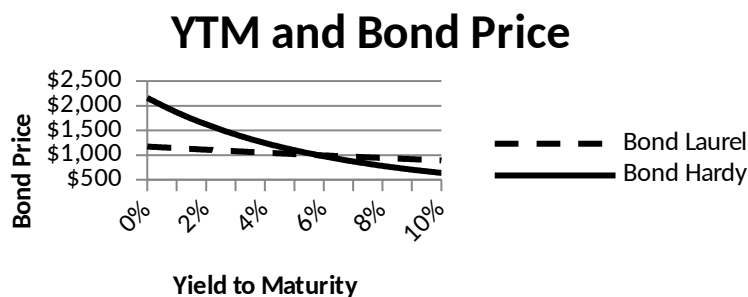
$$P_{\text{Laurel}} = \$29(\text{PVIFA}_{1.9\%,6}) + \$1,000(\text{PVIF}_{1.9\%,6}) = \$1,056.20$$

$$P_{\text{Hardy}} = \$29(\text{PVIFA}_{1.9\%,40}) + \$1,000(\text{PVIF}_{1.9\%,40}) = \$1,278.41$$

$$\Delta P_{\text{Laurel}} \% = (\$1,056.20 - 1,000) / \$1,000 = .0562, \text{ or } 5.62\%$$

$$\Delta P_{\text{Hardy}} \% = (\$1,278.41 - 1,000) / \$1,000 = .2784, \text{ or } 27.84\%$$

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates.



19. Initially, at a YTM of 9 percent, the prices of the two bonds are:

$$P_{\text{Faulk}} = \$35(\text{PVIFA}_{4.5\%,24}) + \$1,000(\text{PVIF}_{4.5\%,24}) = \$855.05$$

$$P_{\text{Yoo}} = \$55(\text{PVIFA}_{4.5\%,24}) + \$1,000(\text{PVIF}_{4.5\%,24}) = \$1,144.95$$

If the YTM rises from 9 percent to 11 percent:

$$P_{\text{Faulk}} = \$35(\text{PVIFA}_{5.5\%,24}) + \$1,000(\text{PVIF}_{5.5\%,24}) = \$736.97$$

$$P_{\text{Yoo}} = \$55(\text{PVIFA}_{5.5\%,24}) + \$1,000(\text{PVIF}_{5.5\%,24}) = \$1,000.00$$

The percentage change in price is calculated as:

$$\text{Percentage change in price} = (\text{New price} - \text{Original price}) / \text{Original price}$$

$$\% \Delta P_{\text{Faulk}} = (\$736.97 - \$855.05) / \$855.05 = -.1381, \text{ or } -13.81\%$$

$$\% \Delta P_{\text{Yoo}} = (\$1,000.00 - \$1,144.95) / \$1,144.95 = -.1266, \text{ or } -12.66\%$$

If the YTM declines from 9 percent to 7 percent:

$$P_{\text{Faulk}} = \$35(\text{PVIFA}_{3.5\%,24}) + \$1,000(\text{PVIF}_{3.5\%,24}) = \$1,000.00$$

$$P_{\text{Yoo}} = \$55(\text{PVIFA}_{3.5\%,24}) + \$1,000(\text{PVIF}_{3.5\%,24}) = \$1,321.17$$

$$\% \Delta P_{\text{Faulk}} = (\$1,000.00 - \$855.05) / \$855.05 = .1695, \text{ or } 16.95\%$$

$$\% \Delta P_{\text{Yoo}} = (\$1,321.17 - \$1,144.95) / \$1,144.95 = .1539, \text{ or } 15.39\%$$

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

20. The bond price equation for this bond is:

$$P_0 = \$1,040 = \$29.50(\text{PVIFA}_{R\%,26}) + \$1,000(\text{PVIF}_{R\%,26})$$

Using a spreadsheet, financial calculator, or trial and error we find:

$$R = 2.733\%$$

This is the semiannual interest rate, so the YTM is:

$$\text{YTM} = 2 \times 2.733\% = 5.47\%$$

The current yield is:

$$\text{Current yield} = \text{Annual coupon payment} / \text{Price}$$

$$\text{Current yield} = \$59 / \$1,040$$

$$\text{Current yield} = .0567, \text{ or } 5.67\%$$

The effective annual yield is the same as the EAR, so using the EAR equation from the previous chapter:

$$\text{Effective annual yield} = (1 + .02733)^2 - 1$$

$$\text{Effective annual yield} = .0554, \text{ or } 5.54\%$$

21. The company should set the coupon rate on its new bonds equal to the required return. The required return can be observed in the market by finding the YTM on outstanding bonds of the company. So, the YTM on the bonds currently sold in the market is:

$$P = \$1,048 = \$28.50(\text{PVIFA}_{R\%,40}) + \$1,000(\text{PVIF}_{R\%,40})$$

Using a spreadsheet, financial calculator, or trial and error we find:

$$R = 2.654\%$$

This is the semiannual interest rate, so the YTM is:

$$\text{YTM} = 2 \times 2.654\% = 5.31\%$$

22. Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are two months until the next coupon payment, so four months have passed since the last coupon payment. The accrued interest for the bond is:

$$\text{Accrued interest} = \$64/2 \times 4/6$$

$$\text{Accrued interest} = \$21.33$$

And we calculate the clean price as:

$$\text{Clean price} = \text{Dirty price} - \text{Accrued interest}$$

$$\text{Clean price} = \$943 - 21.33$$

$$\text{Clean price} = \$921.67$$

23. Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are four months until the next coupon payment, so two months have passed since the last coupon payment. The accrued interest for the bond is:

$$\text{Accrued interest} = \$71/2 \times 2/6$$

$$\text{Accrued interest} = \$11.83$$

And we calculate the dirty price as:

$$\text{Dirty price} = \text{Clean price} + \text{Accrued interest}$$

$$\text{Dirty price} = \$992 + 11.83$$

$$\text{Dirty price} = \$1,003.83$$

24. To find the number of years to maturity for the bond, we need to find the price of the bond. Since we already have the coupon rate, we can use the bond price equation, and solve for the number of years to maturity. We are given the current yield of the bond, so we can calculate the price as:

$$\begin{aligned}\text{Current yield} &= .0842 = \$90/P_0 \\ P_0 &= \$90/.0842 = \$1,068.88\end{aligned}$$

Now that we have the price of the bond, the bond price equation is:

$$P = \$1,068.88 = \$90\{[(1 - (1/1.0781)^t]/.0781\} + \$1,000/1.0781^t$$

We can solve this equation for t as follows:

$$\begin{aligned}\$1,068.88(1.0781)^t &= \$1,152.37(1.0781)^t - 1,152.37 + 1,000 \\ 152.37 &= 83.49(1.0781)^t \\ 1.8251 &= 1.0781^t \\ t &= \log 1.8251 / \log 1.0781 = 8.0004 \approx 8 \text{ years}\end{aligned}$$

The bond has 8 years to maturity.

25. The bond has 12 years to maturity, so the bond price equation is:

$$P = \$964.12 = \$36.20(\text{PVIFA}_{R\%,24}) + \$1,000(\text{PVIF}_{R\%,24})$$

Using a spreadsheet, financial calculator, or trial and error we find:

$$R = 3.852\%$$

This is the semiannual interest rate, so the YTM is:

$$\text{YTM} = 2 \times 3.852\% = 7.70\%$$

The current yield is the annual coupon payment divided by the bond price, so:

$$\begin{aligned}\text{Current yield} &= \$72.40/\$964.12 \\ \text{Current yield} &= .0751, \text{ or } 7.51\%\end{aligned}$$

26. We found the maturity of a bond in Problem 24. However, in this case, the maturity is indeterminate. A bond selling at par can have any length of maturity. In other words, when we solve the bond pricing equation as we did in Problem 24, the number of periods can be any positive number.
27. The price of a zero coupon bond is the PV of the par value, so:

$$\begin{aligned}a. \quad P_0 &= \$1,000/1.0295^{50} \\ P_0 &= \$233.71\end{aligned}$$

- b. In one year, the bond will have 24 years to maturity, so the price will be:

$$\begin{aligned}P_1 &= \$1,000/1.0295^{48} \\ P_1 &= \$247.71\end{aligned}$$

The interest deduction is the price of the bond at the end of the year, minus the price at the beginning of the year, so:

$$\text{Year 1 interest deduction} = \$247.71 - 233.71$$

$$\text{Year 1 interest deduction} = \$13.99$$

The price of the bond when it has one year left to maturity will be:

$$P_{24} = \$1,000/1.0295^2$$

$$P_{24} = \$943.51$$

$$\text{Year 25 interest deduction} = \$1,000 - 943.51$$

$$\text{Year 25 interest deduction} = \$56.49$$

- c. Previous IRS regulations required a straight-line calculation of interest. The total interest received by the bondholder is:

$$\text{Total interest} = \$1,000 - 233.71$$

$$\text{Total interest} = \$766.29$$

The annual interest deduction is the total interest divided by the maturity of the bond, so the straight-line deduction is:

$$\text{Annual interest deduction} = \$766.29/25$$

$$\text{Annual interest deduction} = \$30.65$$

- d. The company will prefer straight-line method when allowed because the valuable interest deductions occur earlier in the life of the bond.

28. a. The coupon bonds have a 7 percent coupon which matches the 7 percent required return, so they will sell at par. The number of bonds that must be sold is the amount needed divided by the bond price, so:

$$\text{Number of coupon bonds to sell} = \$50,000,000/\$1,000$$

$$\text{Number of coupon bonds to sell} = 50,000$$

The number of zero coupon bonds to sell would be:

$$\text{Price of zero coupon bonds} = \$1,000/1.035^{60}$$

$$\text{Price of zero coupon bonds} = \$126.93$$

$$\text{Number of zero coupon bonds to sell} = \$50,000,000/\$126.93$$

$$\text{Number of zero coupon bonds to sell} = 393,905$$

- b. The repayment of the coupon bond will be the par value plus the last coupon payment times the number of bonds issued. So:

$$\text{Coupon bonds repayment} = 50,000(\$1,035)$$

$$\text{Coupon bonds repayment} = \$51,750,000$$

The repayment of the zero coupon bond will be the par value times the number of bonds issued, so:

$$\text{Zeroes repayment} = 393,905(\$1,000)$$

$$\text{Zeroes repayment} = \$393,904,545$$

- c. The total coupon payment for the coupon bonds will be the number of bonds times the coupon payment. For the cash flow of the coupon bonds, we need to account for the tax deductibility of the interest payments. To do this, we will multiply the total coupon payment times one minus the tax rate. So:

$$\text{Coupon bonds} = (50,000)(\$70)(1 - .21)$$

$$\text{Coupon bonds} = \$2,765,000 \text{ cash outflow}$$

Note that this is a cash outflow since the company is making the interest payment.

For the zero coupon bonds, the first year interest payment is the difference in the price of the zero at the end of the year and the beginning of the year. The price of the zeroes in one year will be:

$$P_1 = \$1,000/1.035^{58}$$

$$P_1 = \$135.98$$

The Year 1 interest deduction per bond will be this price minus the price at the beginning of the year, which we found in part *b*, so:

$$\text{Year 1 interest deduction per bond} = \$135.98 - 126.93$$

$$\text{Year 1 interest deduction per bond} = \$9.04$$

The total cash flow for the zeroes will be the interest deduction for the year times the number of zeroes sold, times the tax rate. The cash flow for the zeroes in Year 1 will be:

$$\text{Cash flows for zeroes in Year 1} = (393,905)(\$9.04)(.21)$$

$$\text{Cash flows for zeroes in Year 1} = \$747,862.50$$

Notice the cash flow for the zeroes is a cash inflow. This is because of the tax deductibility of the imputed interest expense. That is, the company gets to write off the interest expense for the year even though the company did not have a cash flow for the interest expense. This reduces the company's tax liability, which is a cash inflow.

During the life of the bond, the zero generates cash inflows to the firm in the form of the interest tax shield of debt. We should note an important point here: If you find the PV of the cash flows from the coupon bond and the zero coupon bond, they will be the same. This is because of the much larger repayment amount for the zeroes.

Challenge

29. To find the capital gains yield and the current yield, we need to find the price of the bond. The current price of Bond P and the price of Bond P in one year is:

$$P: P_0 = \$80(\text{PVIFA}_{6.5\%,10}) + \$1,000(\text{PVIF}_{6.5\%,10}) = \$1,107.83$$

$$P_1 = \$80(\text{PVIFA}_{6.5\%,9}) + \$1,000(\text{PVIF}_{6.5\%,9}) = \$1,099.84$$

$$\text{Current yield} = \$80/\$1,107.83$$

$$\text{Current yield} = .0722, \text{ or } 7.22\%$$

The capital gains yield is:

$$\text{Capital gains yield} = (\text{New price} - \text{Original price})/\text{Original price}$$

$$\text{Capital gains yield} = (\$1,099.84 - \$1,107.83)/\$1,107.83$$

$$\text{Capital gains yield} = -.0072, \text{ or } -.72\%$$

The current price of Bond D and the price of Bond D in one year is:

$$D: P_0 = \$50(\text{PVIFA}_{6.5\%,10}) + \$1,000(\text{PVIF}_{6.5\%,10}) = \$892.17$$

$$P_1 = \$50(\text{PVIFA}_{6.5\%,9}) + \$1,000(\text{PVIF}_{6.5\%,9}) = \$900.16$$

$$\text{Current yield} = \$50/\$892.17$$

$$\text{Current yield} = .0560, \text{ or } 5.60\%$$

$$\text{Capital gains yield} = (\$900.16 - \$892.17)/\$892.17$$

$$\text{Capital gains yield} = .0090, \text{ or } .90\%$$

All else held constant, premium bonds pay a high current income while having price depreciation as maturity nears; discount bonds pay a lower current income but have price appreciation as maturity nears. For either bond, the total return is still 6.5 percent, but this return is distributed differently between current income and capital gains.

30. a. The rate of return you expect to earn if you purchase a bond and hold it until maturity is the YTM. The bond price equation for this bond is:

$$P_0 = \$940 = \$54(\text{PVIFA}_{R\%,10}) + \$1,000(\text{PVIF}_{R\%,10})$$

Using a spreadsheet, financial calculator, or trial and error we find:

$$R = \text{YTM} = 6.22\%$$

- b. To find our HPY, we need to find the price of the bond in two years. The price of the bond in two years, at the new interest rate, will be:

$$P_2 = \$54(\text{PVIFA}_{5.22\%,8}) + \$1,000(\text{PVIF}_{5.42\%,8})$$

$$P_2 = \$1,011.28$$

To calculate the HPY, we need to find the interest rate that equates the price we paid for the bond with the cash flows we received. The cash flows we received were \$54 each year for two years, and the price of the bond when we sold it. The equation to find our HPY is:

$$P_0 = \$940 = \$54(PVIFA_{R\%,2}) + \$1,011.28(PVIF_{R\%,2})$$

Solving for R , we get:

$$R = \text{HPY} = 9.37\%$$

The realized HPY is greater than the expected YTM when the bond was bought because interest rates dropped by 1 percent; bond prices rise when yields fall.

31. The price of any bond (or financial instrument) is the PV of the future cash flows. Even though Bond M makes different coupons payments, to find the price of the bond, we find the PV of the cash flows. The PV of the cash flows for Bond M is:

$$P_M = \$800(PVIFA_{2.8\%,16})(PVIF_{2.8\%,12}) + \$1,000(PVIFA_{2.8\%,12})(PVIF_{2.8\%,28}) + \$30,000(PVIF_{2.8\%,40})$$

$$P_M = \$21,915.52$$

Notice that for the coupon payments of \$800 and \$1,000, we found the PVA for the coupon payments, and then discounted the lump sums back to today.

Bond N is a zero coupon bond with a \$30,000 par value; therefore, the price of the bond is the PV of the par value, or:

$$P_N = \$30,000(PVIF_{2.8\%,40})$$

$$P_N = \$9,940.24$$

32. In general, this is not likely to happen, although it can (and did). The reason this bond has a negative YTM is that it is a callable U.S. Treasury bond. Market participants know this. Given the high coupon rate of the bond, it is extremely likely to be called, which means the bondholder will not receive all the cash flows promised. A better measure of the return on a callable bond is the yield to call (YTC). The YTC calculation is basically the same as the YTM calculation, but the number of periods is the number of periods until the call date. If the YTC were calculated on this bond, it would be positive. However, as we mentioned in the chapter opener, in recent years, interest rates did become negative on sovereign debt and a few select corporate issues.
33. To find the present value, we need to find the real weekly interest rate. To find the real return, we need to use the effective annual rates in the Fisher equation. So, we find the real EAR is:

$$(1 + R) = (1 + r)(1 + h)$$

$$1 + .075 = (1 + r)(1 + .032)$$

$$r = .0417, \text{ or } 4.17\%$$

Now, to find the weekly interest rate, we need to find the APR. Using the equation for discrete compounding:

$$\text{EAR} = [1 + (\text{APR}/m)]^m - 1$$

We can solve for the APR. Doing so, we get:

$$\begin{aligned} \text{APR} &= m[(1 + \text{EAR})^{1/m} - 1] \\ \text{APR} &= 52[(1 + .0417)^{1/52} - 1] \\ \text{APR} &= .0408, \text{ or } 4.08\% \end{aligned}$$

So, the weekly interest rate is:

$$\begin{aligned} \text{Weekly rate} &= \text{APR}/52 \\ \text{Weekly rate} &= .0408/52 \\ \text{Weekly rate} &= .0008, \text{ or } .08\% \end{aligned}$$

Now we can find the present value of the cost of the roses. The real cash flows are an ordinary annuity, discounted at the real interest rate. So, the present value of the cost of the roses is:

$$\begin{aligned} \text{PVA} &= C(\{1 - [1/(1 + r)]^t\}/r) \\ \text{PVA} &= \$8(\{1 - [1/(1 + .0008)]^{30(52)}\}/.0008) \\ \text{PVA} &= \$7,193.18 \end{aligned}$$

34. To answer this question, we need to find the monthly interest rate, which is the APR divided by 12. We also must be careful to use the real interest rate. The Fisher equation uses the effective annual rate, so, the real effective annual interest rates and the monthly interest rates for each account are:

Stock account:

$$\begin{aligned} (1 + R) &= (1 + r)(1 + h) \\ 1 + .12 &= (1 + r)(1 + .04) \\ r &= .0769, \text{ or } 7.69\% \end{aligned}$$

$$\begin{aligned} \text{APR} &= m[(1 + \text{EAR})^{1/m} - 1] \\ \text{APR} &= 12[(1 + .0769)^{1/12} - 1] \\ \text{APR} &= .0743, \text{ or } 7.43\% \end{aligned}$$

$$\begin{aligned} \text{Monthly rate} &= \text{APR}/12 \\ \text{Monthly rate} &= .0743/12 \\ \text{Monthly rate} &= .0062, \text{ or } .62\% \end{aligned}$$

Bond account:

$$\begin{aligned} (1 + R) &= (1 + r)(1 + h) \\ 1 + .07 &= (1 + r)(1 + .04) \\ r &= .0288, \text{ or } 2.88\% \end{aligned}$$

$$\begin{aligned} \text{APR} &= m[(1 + \text{EAR})^{1/m} - 1] \\ \text{APR} &= 12[(1 + .0288)^{1/12} - 1] \\ \text{APR} &= .0285, \text{ or } 2.85\% \end{aligned}$$

$$\begin{aligned} \text{Monthly rate} &= \text{APR}/12 \\ \text{Monthly rate} &= .0285/12 \\ \text{Monthly rate} &= .0024, \text{ or } .24\% \end{aligned}$$

Now we can find the future value of the retirement account in real terms. The future value of each account will be:

Stock account:

$$FVA = C \{ (1 + r)^t - 1 \} / r$$

$$FVA = \$900 \{ [(1 + .0062)^{360} - 1] / .0062 \}$$

$$FVA = \$1,196,731.96$$

Bond account:

$$FVA = C \{ (1 + r)^t - 1 \} / r$$

$$FVA = \$300 \{ [(1 + .0024)^{360} - 1] / .0024 \}$$

$$FVA = \$170,316.78$$

The total future value of the retirement account will be the sum of the two accounts, or:

$$\text{Account value} = \$1,196,731.96 + 170,316.78$$

$$\text{Account value} = \$1,367,048.74$$

Now we need to find the monthly interest rate in retirement. We can use the same procedure that we used to find the monthly interest rates for the stock and bond accounts, so:

$$(1 + R) = (1 + r)(1 + h)$$

$$1 + .08 = (1 + r)(1 + .04)$$

$$r = .0385, \text{ or } 3.85\%$$

$$APR = m[(1 + EAR)^{1/m} - 1]$$

$$APR = 12[(1 + .0385)^{1/12} - 1]$$

$$APR = .0378, \text{ or } 3.78\%$$

$$\text{Monthly rate} = APR/12$$

$$\text{Monthly rate} = .0378/12$$

$$\text{Monthly rate} = .00315, \text{ or } .315\%$$

Now we can find the real monthly withdrawal in retirement. Using the present value of an annuity equation and solving for the payment, we find:

$$PVA = C(\{1 - [1/(1 + r)]^t\} / r)$$

$$\$1,367,048.74 = C(\{1 - [1/(1 + .00315)]^{300}\} / .00315)$$

$$C = \$7,050.75$$

This is the real dollar amount of the monthly withdrawals. The nominal monthly withdrawals will increase by the inflation rate each month. To find the nominal dollar amount of the last withdrawal, we can increase the real dollar withdrawal by the inflation rate. We can increase the real withdrawal by the effective annual inflation rate since we are only interested in the nominal amount of the last withdrawal. So, the last withdrawal in nominal terms will be:

$$FV = PV(1 + r)^t$$

$$FV = \$7,050.75(1 + .04)^{(30 + 25)}$$

$$FV = \$60,963.34$$

35. In this problem, we need to calculate the future value of the annual savings after the five years of operations. The savings are the revenues minus the costs, or:

$$\text{Savings} = \text{Revenue} - \text{Costs}$$

Since the annual fee and the number of members are increasing, we need to calculate the effective growth rate for revenues, which is:

$$\text{Effective growth rate} = (1 + .06)(1 + .03) - 1$$

$$\text{Effective growth rate} = .0918, \text{ or } 9.18\%$$

The revenue for the current year is the number of members times the annual fee, or:

$$\text{Current revenue} = 700(\$400)$$

$$\text{Current revenue} = \$280,000$$

The revenue will grow at 9.18 percent, and the costs will grow at 2 percent, so the savings each year for the next five years will be:

<u>Year</u>	<u>Revenue</u>	<u>Costs</u>	<u>Savings</u>
1	\$305,704.00	\$127,500.00	\$178,204.00
2	333,767.63	130,050.00	203,717.63
3	364,407.50	132,651.00	231,756.50
4	397,860.10	135,304.02	262,556.08
5	434,383.66	138,010.10	296,373.56

Now we can find the value of each year's savings using the future value of a lump sum equation, so:

$$FV = PV(1 + r)^t$$

<u>Year</u>		<u>Future Value</u>
1	$\$178,204.00(1 + .09)^4 =$	\$251,549.49
2	$\$203,717.63(1 + .09)^3 =$	263,820.24
3	$\$231,756.50(1 + .09)^2 =$	275,349.89
4	$\$262,556.08(1 + .09)^1 =$	286,186.13
5		296,373.56

$$\text{Total future value of savings} = \underline{\$1,373,279.31}$$

He will spend \$500,000 on a luxury boat, so the value of his account will be:

$$\text{Value of account} = \$1,373,279.31 - 500,000$$

$$\text{Value of account} = \$873,279.31$$

Now we can use the present value of an annuity equation to find the payment. Doing so, we find:

$$PVA = C(\{1 - [1/(1 + r)]^t\}/r)$$

$$\$873,279.31 = C(\{1 - [1/(1 + .09)]^{25}\}/.09)$$

$$C = \$88,905.29$$

Calculator Solutions**1.***a.*

Enter	40	3%			\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$306.56		

b.

Enter	40	4%			\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$208.29		

c.

Enter	40	5%			\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$142.05		

2.*a.*

Enter	30	3.5%		\$35	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,000.00		

b.

Enter	30	4.5%		\$35	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$837.11		

c.

Enter	30	2.5%		\$35	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,209.30		

3.

Enter	26		±\$980	\$31	\$1,000
	N	I/Y	PV	PMT	FV
Solve for		3.215%			

$3.215\% \times 2 = 6.43\%$

4.

Enter	23	3.4%	±\$1,055		\$1,000
	N	I/Y	PV	PMT	FV
Solve for				\$37.49	

$\$37.49 \times 2 = \74.97
 $\$74.97/\$1,000 = 7.50\%$

5.

Enter	15	4.30%		€51	€1,000
	N	I/Y	PV	PMT	FV
Solve for			€1,087.11		

6.

Enter	17		±¥106,000	¥2,800	¥100,000
	N	I/Y	PV	PMT	FV
Solve for		2.37%			

16.

P_0					
Enter	40	2.85%			\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$324.96		

P_1					
Enter	38	2.85%			\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$343.75		
$\$343.75 - \$324.96 = \$18.79$					

17. Miller Corporation

P_0					
Enter	26	2.65%		\$32.50	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,111.71		

P_1					
Enter	24	2.65%		\$32.50	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,105.55		

P_3					
Enter	20	2.65%		\$32.50	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,092.22		

P_8					
Enter	10	2.65%		\$32.50	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,052.11		

P_{12}					
Enter	2	2.65%		\$32.50	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,011.54		

Modigliani Company

P_0					
Enter	26	3.25%		\$26.50	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$895.76		
P_1					
Enter	24	3.25%		\$26.50	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$901.07		
P_3					
Enter	20	3.25%		\$26.50	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$912.76		
P_8					
Enter	10	3.25%		\$26.50	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$949.47		
P_{12}					
Enter	2	3.25%		\$26.50	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$988.56		

- 18.** If both bonds sell at par, the initial YTM on both bonds is the coupon rate, 5.8 percent. If the YTM suddenly rises to 7.8 percent:

P_{Laurel}					
Enter	6	3.90%		\$29	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$947.41		
$\Delta P_{\text{Laurel}}\% = (\$947.41 - \$1,000)/\$1,000 = -5.26\%$					

P_{Hardy}					
Enter	40	3.90%		\$29	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$799.09		
$\Delta P_{\text{Hardy}}\% = (\$799.09 - \$1,000)/\$1,000 = -20.09\%$					

If the YTM suddenly falls to 3.8 percent:

P_{Laurel}					
Enter	6	1.90%		\$29	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,056.20		
$\Delta P_{\text{Laurel}}\% = (\$1,056.20 - \$1,000)/\$1,000 = +5.62\%$					

P_{Hardy}					
Enter	40	1.90%		\$29	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,278.41		
$\Delta P_{\text{Hardy}} \% = (\$1,278.41 - \$1,000)/\$1,000 = +27.84\%$					

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates.

19. Initially, at a YTM of 9 percent, the prices of the two bonds are:

P_{Faulk}					
Enter	24	4.5%		\$35	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$855.05		

P_{Yoo}					
Enter	24	4.5%		\$55	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,144.95		

If the YTM rises from 9 percent to 11 percent:

P_{Faulk}					
Enter	24	5.5%		\$35	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$736.97		
$\Delta P_{\text{Faulk}} \% = (\$736.97 - \$855.05)/\$855.05 = -13.81\%$					

P_{Yoo}					
Enter	24	5.5%		\$55	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,000.00		
$\Delta P_{\text{Yoo}} \% = (\$1,000.00 - \$1,144.95)/\$1,144.95 = -12.66\%$					

If the YTM declines from 9 percent to 7 percent:

P_{Faulk}					
Enter	24	3.5%		\$35	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,000.00		
$\Delta P_{\text{Faulk}} \% = (\$1,000.00 - \$855.05)/\$855.05 = +16.95\%$					

P_{Yoo}					
Enter	24	3.5%		\$55	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,321.17		
$\Delta P_{\text{Yoo}} \% = (\$1,321.17 - \$1,144.95)/\$1,144.95 = +15.39\%$					

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

20.

Enter	26		±\$1,040	\$29.50	\$1,000
	N	I/Y	PV	PMT	FV
Solve for		2.733%			
YTM = 2.733% × 2 = 5.47%					

21. The company should set the coupon rate on its new bonds equal to the required return; the required return can be observed in the market by finding the YTM on outstanding bonds of the company.

Enter	40		±\$1,048	\$28.50	\$1,000
	N	I/Y	PV	PMT	FV
Solve for		2.654%			
2.654% × 2 = 5.31%					

24. Current yield = .0842 = \$90/P₀ ; P₀ = \$1,068.88

Enter		7.81%	±\$1,068.88	\$90	\$1,000
	N	I/Y	PV	PMT	FV
Solve for	8.0004				
8 years					

25.

Enter	24		±\$964.12	\$36.20	\$1,000
	N	I/Y	PV	PMT	FV
Solve for		3.852%			
3.852% × 2 = 7.70%					

27.*a.* P₀

Enter	50	5.9%/2			\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$233.71		

b. P₁

Enter	48	5.9%/2			\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$247.71		
Year 1 interest deduction = \$247.71 – 233.71 = \$13.99					

P₂₄

Enter	2	5.9%/2%			\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$943.51		
Year 25 interest deduction = \$1,000 – 943.51 = \$56.49					

- c. Total interest = $\$1,000 - 233.71 = \766.29
 Annual interest deduction = $\$766.29/25 = \30.65
- d. The company will prefer the straight-line method when allowed because the valuable interest deductions occur earlier in the life of the bond.

28. a. The coupon bonds have a 7% coupon rate, which matches the 7% required return, so they will sell at par; # of bonds = $\$50,000,000/\$1,000 = 50,000$.

For the zeroes:

Enter	60	7%/2			\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$126.93		

$\$50,000,000/\$126.93 = 393,905$ will be issued.

- b. Coupon bonds: repayment = $50,000(\$1,035) = \$51,750,000$
 Zeroes: repayment = $393,905(\$1,000) = \$393,904,545$
- c. Coupon bonds: $(50,000)(\$70)(1 - .21) = \$2,765,000$ cash outflow
 Zeroes:

Enter	58	7%/2			\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$135.98		

Year 1 interest deduction = $\$135.98 - 126.93 = \9.04
 $(393,905)(\$9.04)(.21) = \$747,862.50$ cash inflow
 During the life of the bond, the zero generates cash inflows to the firm in the form of the interest tax shield of debt.

29.

Bond P

P_0

Enter	10	6.5%		\$80	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,107.83		

P_1

Enter	9	6.5%		\$80	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,099.84		

Current yield = $\$80/\$1,107.83 = 7.22\%$

Capital gains yield = $(\$1,099.84 - 1,107.83)/\$1,107.83 = -.72\%$

Bond D

P_0

Enter	10	6.5%		\$50	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$892.17		

P_1

Enter	9	6.5%		\$50	\$1,000
	N	I/Y	PV	PMT	FV

Solve for \$900.16

Current yield = $\$50/\$892.17 = 5.60\%$ Capital gains yield = $(\$900.16 - 892.17)/\$892.17 = +.90\%$

All else held constant, premium bonds pay a higher current income while having price depreciation as maturity nears; discount bonds pay a lower current income but have price appreciation as maturity nears. For either bond, the total return is still 6.5 percent, but this return is distributed differently between current income and capital gains.

30.*a.*

Enter	10		$\pm\$940$	\$54	\$1,000
	N	I/Y	PV	PMT	FV

Solve for 6.22%

This is the rate of return you expect to earn on your investment when you purchase the bond.

b.

Enter	8	5.22%		\$54	\$1,000
	N	I/Y	PV	PMT	FV

Solve for \$1,011.28

The HPY is:

Enter	2		$\pm\$940$	\$54	\$1,011.28
	N	I/Y	PV	PMT	FV

Solve for 9.37%

The realized HPY is greater than the expected YTM when the bond was bought because interest rates dropped by 1 percent; bond prices rise when yields fall.

31. P_M

CF₀	\$0
C01	\$0
F01	12
C02	\$800
F02	16
C03	\$1,000
F03	11
C04	\$30,000
F04	1

I = 2.8%

NPV CPT

\$21,915.52

$$P_N$$

Enter	40	2.8%			\$30,000
	N	I/Y	PV	PMT	FV
Solve for			\$9,940.24		

34.

Real return for stock account: $1 + .12 = (1 + r)(1 + .04)$; $r = 7.69\%$

Enter		7.69%	12
	NOM	EFF	C/Y
Solve for	7.43%		

Real return for bond account: $1 + .07 = (1 + r)(1 + .04)$; $r = 2.88\%$

Enter		2.88%	12
	NOM	EFF	C/Y
Solve for	2.85%		

Real return post-retirement: $1 + .08 = (1 + r)(1 + .04)$; $r = 3.85\%$

Enter		3.85%	12
	NOM	EFF	C/Y
Solve for	3.78%		

Stock portfolio value:

Enter	12 × 30	7.43%/12		\$900	
	N	I/Y	PV	PMT	FV
Solve for					\$1,196,731.96

Bond portfolio value:

Enter	12 × 30	2.85%/12		\$300	
	N	I/Y	PV	PMT	FV
Solve for					\$170,316.78

Retirement value = \$1,196,731.96 + 170,316.78 = \$1,367,048.74

Retirement withdrawal:

Enter	25 × 12	3.78%/12	\$1,367,048.74		
	N	I/Y	PV	PMT	FV
Solve for				\$7,050.75	

The last withdrawal in real terms is:

Enter	30 + 25	4%	\$7,050.75		
	N	I/Y	PV	PMT	FV
Solve for					\$60,963.34

35.

Future value of savings:

Year 1:

Enter	4	9%	\$178,204		
	N	I/Y	PV	PMT	FV
Solve for					\$251,549.49

Year 2:

Enter	3	9%	\$203,717.63		
	N	I/Y	PV	PMT	FV
Solve for					\$263,820.24

Year 3:

Enter	2	9%	\$231,756.50		
	N	I/Y	PV	PMT	FV
Solve for					\$275,349.89

Year 4:

Enter	1	9%	\$262,556.08		
	N	I/Y	PV	PMT	FV
Solve for					\$286,186.13

$$\text{Future value} = \$251,549.49 + 263,820.24 + 275,349.89 + 286,186.13 + 296,373.56$$

$$\text{Future value} = \$1,373,279.31$$

He will spend \$500,000 on a luxury boat, so the value of his account will be:

$$\text{Value of account} = \$1,373,279.31 - 500,000$$

$$\text{Value of account} = \$873,279.31$$

Enter	25	9%	\$873,279.31		
	N	I/Y	PV	PMT	FV
Solve for				\$88,905.29	

CHAPTER 9

STOCK VALUATION

Answers to Concept Questions

1. The value of any investment depends on the present value of its cash flows; i.e., what investors will actually receive. The cash flows from a share of stock are the dividends.
2. Investors believe the company will eventually start paying dividends (or be sold to another company).
3. In general, companies that need the cash will often forgo dividends since dividends are a cash expense. Young, growing companies with profitable investment opportunities are one example; another example is a company in financial distress. This question is examined in depth in a later chapter.
4. The general method for valuing a share of stock is to find the present value of all expected future dividends. The dividend growth model presented in the text is only valid (i) if dividends are expected to occur forever; that is, the stock provides dividends in perpetuity, and (ii) if a constant growth rate of dividends occurs forever. A violation of the first assumption might be a company that is expected to cease operations and dissolve itself some finite number of years from now. The stock of such a company would be valued by applying the general method of valuation explained in this chapter. A violation of the second assumption might be a start-up firm that isn't currently paying any dividends, but is expected to eventually start making dividend payments some number of years from now. This stock would also be valued by the general dividend valuation method explained in this chapter.
5. The common stock probably has a higher price because the dividend can grow, whereas it is fixed on the preferred. However, the preferred is less risky because of the dividend and liquidation preference, so it is possible the preferred could be worth more, depending on the circumstances.
6. The two components are the dividend yield and the capital gains yield. For most companies, the capital gains yield is larger. This is easy to see for companies that pay no dividends. For companies that do pay dividends, the dividend yields are rarely over five percent and are often much less.
7. Yes. If the dividend grows at a steady rate, so does the stock price. In other words, the dividend growth rate and the capital gains yield are the same.
8. The three factors are: 1) The company's future growth opportunities. 2) The company's level of risk, which determines the interest rate used to discount cash flows. 3) The accounting method used.
9. It wouldn't seem to be. Investors who don't like the voting features of a particular class of stock are under no obligation to buy it.
10. Presumably, the current stock value reflects the risk, timing, and magnitude of all future cash flows, both short-term and long-term. If this is correct, then the statement is false.

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The constant dividend growth model is:

$$P_t = D_t \times (1 + g)/(R - g)$$

So, the price of the stock today is:

$$P_0 = D_0(1 + g)/(R - g) = \$2.07(1.043)/(.11 - .043)$$

$$P_0 = \$32.22$$

The dividend at Year 4 is the dividend today times the FVIF for the growth rate in dividends and 4 years, so:

$$P_3 = D_3(1 + g)/(R - g) = D_0(1 + g)^4/(R - g)$$

$$P_3 = \$2.07(1.043)^4/(\cdot 11 - \cdot 043)$$

$$P_3 = \$36.56$$

We can do the same thing to find the dividend in Year 16, which we can use to find the price in Year 15, so:

$$P_{15} = D_{15}(1 + g)/(R - g) = D_0(1 + g)^{16}/(R - g)$$

$$P_{15} = \$2.07(1.043)^{16}/(\cdot 11 - \cdot 043)$$

$$P_{15} = \$60.60$$

There is another feature of the constant dividend growth model: The stock price grows at the dividend growth rate. If we know the stock price today, we can find the future value for any time in the future we want to calculate the stock price. In this problem, we want to know the stock price in 3 years, and we have already calculated the stock price today. The stock price in 3 years will be:

$$P_3 = P_0(1 + g)^3$$

$$P_3 = \$32.22(1 + .043)^3$$

$$P_3 = \$36.56$$

And the stock price in 15 years will be:

$$P_{15} = P_0(1 + g)^{15}$$

$$P_{15} = \$32.22(1 + .043)^{15}$$

$$P_{15} = \$60.60$$

2. We need to find the required return of the stock. Using the constant growth model, we can solve the equation for R . Doing so, we find:

$$\begin{aligned} R &= (D_1/P_0) + g \\ R &= (\$2.95/\$53.10) + .048 \\ R &= .1036, \text{ or } 10.36\% \end{aligned}$$

3. The dividend yield is the dividend next year divided by the current price, so the dividend yield is:

$$\begin{aligned} \text{Dividend yield} &= D_1/P_0 \\ \text{Dividend yield} &= \$2.95/\$53.10 \\ \text{Dividend yield} &= .0556, \text{ or } 5.56\% \end{aligned}$$

The capital gains yield, or percentage increase in the stock price, is the same as the dividend growth rate, so:

$$\text{Capital gains yield} = 4.8\%$$

4. Using the constant growth model, we find the price of the stock today is:

$$\begin{aligned} P_0 &= D_1/(R - g) \\ P_0 &= \$3.25/ (.105 - .05) \\ P_0 &= \$59.09 \end{aligned}$$

5. The required return of a stock is made up of two parts: The dividend yield and the capital gains yield. So, the required return of this stock is:

$$\begin{aligned} R &= \text{Dividend yield} + \text{Capital gains yield} \\ R &= .049 + .052 \\ R &= .1010, \text{ or } 10.10\% \end{aligned}$$

6. We know the stock has a required return of 9.9 percent, and the dividend and capital gains yield are equal, so:

$$\text{Dividend yield} = 1/2(.099) = .0495 = \text{Capital gains yield}$$

Now we know both the dividend yield and capital gains yield. The dividend is the stock price times the dividend yield, so:

$$D_1 = .0495(\$74) = \$3.66$$

This is the dividend next year. The question asks for the dividend this year. Using the relationship between the dividend this year and the dividend next year:

$$D_1 = D_0(1 + g)$$

We can solve for the dividend that was just paid:

$$\begin{aligned} \$3.66 &= D_0(1 + .0495) \\ D_0 &= \$3.66/1.0495 \\ D_0 &= \$3.49 \end{aligned}$$

7. The price of any financial instrument is the PV of the future cash flows. The future dividends of this stock are an annuity for 11 years, so the price of the stock is the PVA, which will be:

$$P_0 = \$8.50(\text{PVIFA}_{9.5\%,11})$$

$$P_0 = \$56.50$$

8. The price of a share of preferred stock is the dividend divided by the required return. This is the same equation as the constant growth model, with a dividend growth rate of zero percent. Remember that most preferred stock pays a fixed dividend, so the growth rate is zero. Using this equation, we find the price per share of the preferred stock is:

$$R = D/P_0$$

$$R = \$3.75/\$81$$

$$R = .0463, \text{ or } 4.63\%$$

9. The growth rate of earnings is the return on equity times the retention ratio, so:

$$g = \text{ROE} \times b$$

$$g = .13(.80)$$

$$g = .1040, \text{ or } 10.40\%$$

To find next year's earnings, we multiply the current earnings times one plus the growth rate, so:

$$\text{Next year's earnings} = \text{Current earnings}(1 + g)$$

$$\text{Next year's earnings} = \$17,500,000(1 + .1040)$$

$$\text{Next year's earnings} = \$19,320,000$$

10. Using the equation to calculate the price of a share of stock with the PE ratio:

$$P = \text{Benchmark PE ratio} \times \text{EPS}$$

So, with a PE ratio of 18, we find:

$$P = 18(\$3.41)$$

$$P = \$61.38$$

And with a PE ratio of 21, we find:

$$P = 21(\$3.41)$$

$$P = \$71.61$$

Intermediate

11. This stock has a constant growth rate of dividends, but the required return changes twice. To find the value of the stock today, we will begin by finding the price of the stock at Year 6, when both the dividend growth rate and the required return are stable forever. The price of the stock in Year 6 will be the dividend in Year 7, divided by the required return minus the growth rate in dividends. So:

$$P_6 = D_6(1 + g)/(R - g) = D_0(1 + g)^7/(R - g)$$

$$P_6 = \$3.15(1.04)^7/(\.11 - .04)$$

$$P_6 = \$59.22$$

Now we can find the price of the stock in Year 3. We need to find the price here since the required return changes at that time. The price of the stock in Year 3 is the PV of the dividends in Years 4, 5, and 6, plus the PV of the stock price in Year 6. The price of the stock in Year 3 is:

$$P_3 = \$3.15(1.04)^4/1.13 + \$3.15(1.04)^5/1.13^2 + \$3.15(1.04)^6/1.13^3 + \$59.22/1.13^3$$

$$P_3 = \$50.07$$

Finally, we can find the price of the stock today. The price today will be the PV of the dividends in Years 1, 2, and 3, plus the PV of the stock in Year 3. The price of the stock today is:

$$P_0 = \$3.15(1.04)/1.15 + \$3.15(1.04)^2/(1.15)^2 + \$3.15(1.04)^3/(1.15)^3 + \$50.07/(1.15)^3$$

$$P_0 = \$40.67$$

12. Here we have a stock that pays no dividends for 9 years. Once the stock begins paying dividends, it will have a constant growth rate of dividends. We can use the constant growth model at that point. It is important to remember that the general form of the constant dividend growth formula is:

$$P_t = [D_t \times (1 + g)]/(R - g)$$

This means that since we will use the dividend in Year 10, we will be finding the stock price in Year 9. The dividend growth model is similar to the PVA and the PV of a perpetuity: The equation gives you the PV one period before the first payment. So, the price of the stock in Year 9 will be:

$$P_9 = D_{10}/(R - g)$$

$$P_9 = \$15.75/(\.12 - .048)$$

$$P_9 = \$218.75$$

The price of the stock today is the PV of the stock price in the future. We discount the future stock price at the required return. The price of the stock today will be:

$$P_0 = \$218.75/1.12^9$$

$$P_0 = \$78.88$$

13. The price of a stock is the PV of the future dividends. This stock is paying five dividends, so the price of the stock is the PV of these dividends using the required return. The price of the stock is:

$$P_0 = \$18/1.11 + \$21/1.11^2 + \$24/1.11^3 + \$27/1.11^4 + \$30/1.11^5$$

$$P_0 = \$86.40$$

14. With differential dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the PV of the future stock price, plus the PV of all dividends during the differential growth period. The stock begins constant growth in Year 5, so we can find the price of the stock in Year 4, one year before the constant dividend growth begins, as:

$$P_4 = D_4(1 + g)/(R - g) = \$2.55(1.045)/(.10 - .045)$$

$$P_4 = \$48.45$$

The price of the stock today is the PV of the first four dividends, plus the PV of the Year 4 stock price. So, the price of the stock today will be:

$$P_0 = \$9/1.10 + \$7/1.10^2 + \$5.75/1.10^3 + (\$2.55 + 48.45)/1.10^4$$

$$P_0 = \$53.12$$

15. With differential dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the PV of the future stock price, plus the PV of all dividends during the differential growth period. The stock begins constant growth in Year 4, so we can find the price of the stock in Year 3, one year before the constant dividend growth begins as:

$$P_3 = D_3(1 + g)/(R - g) = D_0(1 + g_1)^3(1 + g_2)/(R - g_2)$$

$$P_3 = \$3.24(1.20)^3(1.05)/(.11 - .05)$$

$$P_3 = \$97.98$$

The price of the stock today is the PV of the first three dividends, plus the PV of the Year 3 stock price. The price of the stock today will be:

$$P_0 = \$3.24(1.20)/1.11 + \$3.24(1.20)^2/1.11^2 + \$3.24(1.20)^3/1.11^3 + \$97.98/1.11^3$$

$$P_0 = \$83.02$$

16. Here we need to find the dividend next year for a stock experiencing differential growth. We know the stock price, the dividend growth rates, and the required return, but not the dividend. First, we need to realize that the dividend in Year 3 is the current dividend times the FVIF. The dividend in Year 3 will be:

$$D_3 = D_0(1.25)^3$$

And the dividend in Year 4 will be the dividend in Year 3 times one plus the growth rate, or:

$$D_4 = D_0(1.25)^3(1.17)$$

The stock begins constant growth after the 4th dividend is paid, so we can find the price of the stock in Year 4 as the dividend in Year 5, divided by the required return minus the growth rate. The equation for the price of the stock in Year 4 is:

$$P_4 = D_4(1 + g)/(R - g)$$

Now we can substitute the previous dividend in Year 4 into this equation as follows:

$$P_4 = D_0(1 + g_1)^3(1 + g_2)(1 + g_3)/(R - g_3)$$

$$P_4 = D_0(1.25)^3(1.17)(1.05)/(.11 - .05)$$

$$P_4 = 39.99D_0$$

When we solve this equation, we find that the stock price in Year 4 is 39.99 times as large as the dividend today. Now we need to find the equation for the stock price today. The stock price today is the PV of the dividends in Years 1, 2, 3, and 4, plus the PV of the Year 4 price. So:

$$P_0 = D_0(1.25)/1.11 + D_0(1.25)^2/1.11^2 + D_0(1.25)^3/1.11^3 + D_0(1.25)^3(1.17)/1.11^4 + 39.99D_0/1.11^4$$

We can factor out D_0 in the equation, and combine the last two terms. Doing so, we get:

$$P_0 = \$65.00 = D_0\{1.25/1.11 + 1.25^2/1.11^2 + 1.25^3/1.11^3 + [(1.25)^3(1.17) + 39.99]/1.11^4\}$$

Reducing the equation even further by solving all of the terms in the braces, we get:

$$\$65 = \$31.67D_0$$

$$D_0 = \$65/\$31.67$$

$$D_0 = \$2.05$$

This is the dividend today, so the projected dividend for the next year will be:

$$D_1 = \$2.05(1.25)$$

$$D_1 = \$2.57$$

17. The constant growth model can be applied even if the dividends are declining by a constant percentage, just make sure to recognize the negative growth. So, the price of the stock today will be:

$$P_0 = D_0 (1 + g)/(R - g)$$

$$P_0 = \$12(1 - .04)/[.09 - (-.04)]$$

$$P_0 = \$88.62$$

18. We are given the stock price, the dividend growth rate, and the required return, and are asked to find the dividend. Using the constant dividend growth model, we get:

$$P_0 = \$74.32 = D_0(1 + g)/(R - g)$$

Solving this equation for the dividend gives us:

$$D_0 = \$74.32(.105 - .04)/(1.04)$$

$$D_0 = \$4.65$$

19. The price of a share of preferred stock is the dividend payment divided by the required return. We know the dividend payment in Year 5, so we can find the price of the stock in Year 4, one year before the first dividend payment. Doing so, we get:

$$P_4 = \$4.00/.043$$

$$P_4 = \$93.02$$

The price of the stock today is the PV of the stock price in the future, so the price today will be:

$$P_0 = \$93.02/(1.043)^4$$

$$P_0 = \$78.61$$

20. The dividend yield is the annual dividend divided by the stock price, so:

$$\text{Dividend yield} = \text{Dividend}/\text{Stock price}$$

$$.019 = \text{Dividend}/\$45.18$$

$$\text{Dividend} = \$.86$$

The “Net Chg” of the stock shows the stock decreased by \$.13 on this day, so the closing stock price yesterday was:

$$\text{Yesterday's closing price} = \$45.18 - (-.13)$$

$$\text{Yesterday's closing price} = \$45.31$$

To find the net income, we need to find the EPS. The stock quote tells us the PE ratio for the stock is 23. Since we know the stock price as well, we can use the PE ratio to solve for EPS as follows:

$$\text{PE} = \text{Stock price}/\text{EPS}$$

$$23 = \$45.18/\text{EPS}$$

$$\text{EPS} = \$45.18/23$$

$$\text{EPS} = \$1.964$$

We know that EPS is the total net income divided by the number of shares outstanding, so:

$$\text{EPS} = \text{Net income}/\text{Shares}$$

$$\$1.964 = \text{Net income}/30,000,000$$

$$\text{Net income} = \$1.964(30,000,000)$$

$$\text{Net income} = \$58,930,435$$

21. Here we have a stock paying a constant dividend for a fixed period, and an increasing dividend thereafter. We need to find the present value of the two different cash flows using the appropriate quarterly interest rate. The constant dividend is an annuity, so the present value of these dividends is:

$$\text{PVA} = C(\text{PVIFA}_{R,t})$$

$$\text{PVA} = \$.85(\text{PVIFA}_{2.5\%,12})$$

$$\text{PVA} = \$8.72$$

Now we can find the present value of the dividends beyond the constant dividend phase. Using the present value of a growing annuity equation, we find:

$$P_{12} = D_{13}/(R - g)$$

$$P_{12} = \$.85(1 + .01)/(.025 - .01)$$

$$P_{12} = \$57.23$$

This is the price of the stock immediately after it has paid the last constant dividend. So, the present value of the future price is:

$$\text{PV} = \$57.23/(1 + .025)^{12}$$

$$\text{PV} = \$42.56$$

The price today is the sum of the present value of the two cash flows, so:

$$P_0 = \$8.72 + 42.56$$

$$P_0 = \$51.28$$

22. Here we need to find the dividend next year for a stock with irregular growth in dividends. We know the stock price, the dividend growth rate, and the required return, but not the dividend. First, we need to realize that the dividend in Year 3 is the constant dividend times the FVIF. The dividend in Year 3 will be:

$$D_3 = D(1.035)$$

The equation for the stock price will be the present value of the constant dividends, plus the present value of the future stock price, or:

$$P_0 = D/1.11 + D/1.11^2 + D[(1.035)/(.11 - .035)]/1.11^2$$

$$\$59 = D/1.11 + D/1.11^2 + D[(1.035)/(.11 - .035)]/1.11^2$$

We can factor out D_0 in the equation. Doing so, we get:

$$\$59 = D\{1/1.11 + 1/1.11^2 + [(1.035)/(.11 - .035)]/1.11^2\}$$

Reducing the equation even further by solving all of the terms in the braces, we get:

$$\$59 = D(12.9129)$$

$$D = \$59/12.9129$$

$$D = \$4.57$$

23. The required return of a stock consists of two components, the capital gains yield and the dividend yield. In the constant dividend growth model (growing perpetuity equation), the capital gains yield is the same as the dividend growth rate, or algebraically:

$$R = D_1/P_0 + g$$

We can find the dividend growth rate by the growth rate equation, or:

$$g = ROE \times b$$

$$g = .11 \times .70$$

$$g = .0770, \text{ or } 7.70\%$$

This is also the growth rate in dividends. To find the current dividend, we can use the information provided about the net income, shares outstanding, and payout ratio. The total dividends paid is the net income times the payout ratio. To find the dividend per share, we can divide the total dividends paid by the number of shares outstanding. So:

$$\text{Dividend per share} = (\text{Net income} \times \text{Payout ratio})/\text{Shares outstanding}$$

$$\text{Dividend per share} = (\$29,000,000 \times .30)/2,600,000$$

$$\text{Dividend per share} = \$3.35$$

Now we can use the initial equation for the required return. We must remember that the equation uses the dividend in one year, so:

$$R = D_1/P_0 + g$$

$$R = \$3.35(1 + .0770)/\$105 + .0770$$

$$R = .1113, \text{ or } 11.13\%$$

24. First, we need to find the annual dividend growth rate over the past four years. To do this, we can use the future value of a lump sum equation, and solve for the interest rate. Doing so, we find the dividend growth rate over the past four years was:

$$FV = PV(1 + R)^t$$

$$\$2.10 = \$1.65(1 + R)^4$$

$$R = (\$2.10/\$1.65)^{1/4} - 1$$

$$R = .0621, \text{ or } 6.21\%$$

We know the dividend will grow at this rate for five years before slowing to a constant rate indefinitely. So, the dividend amount in seven years will be:

$$D_7 = D_0(1 + g_1)^5(1 + g_2)^2$$

$$D_7 = \$2.10(1 + .0621)^5(1 + .05)^2$$

$$D_7 = \$3.13$$

25. a. We can find the price of all the outstanding company stock by using the dividends the same way we would value an individual share. Since earnings are equal to dividends, and there is no growth, the value of the company's stock today is the present value of a perpetuity, so:

$$P = D/R$$

$$P = \$720,000/.11$$

$$P = \$6,545,454.55$$

The price-earnings ratio is the stock price divided by the current earnings, so the price-earnings ratio of each company with no growth is:

$$PE = \text{Price/Earnings}$$

$$PE = \$6,545,454.55/\$720,000$$

$$PE = 9.09 \text{ times}$$

- b. Since the earnings have increased, the price of the stock will increase. The new price of the outstanding company stock is:

$$P = D/R$$

$$P = (\$720,000 + 150,000)/.11$$

$$P = \$7,909,090.91$$

The price-earnings ratio is the stock price divided by the current earnings, so the price-earnings with the increased earnings is:

$$PE = \text{Price/Earnings}$$

$$PE = \$7,909,090.91/\$720,000$$

$$PE = 10.98 \text{ times}$$

- c. Since the earnings have increased, the price of the stock will increase. The new price of the outstanding company stock is:

$$P = D/R$$

$$P = (\$720,000 + 300,000)/.11$$

$$P = \$9,272,727.27$$

The price-earnings ratio is the stock price divided by the current earnings, so the price-earnings ratio with the increased earnings is:

$$PE = \text{Price/Earnings}$$

$$PE = \$9,272,727.27/\$720,000$$

$$PE = 12.88 \text{ times}$$

26. a. Using the equation to calculate the price of a share of stock with the PE ratio:

$$P = \text{Benchmark PE ratio} \times \text{EPS}$$

So, with a PE ratio of 19, we find:

$$P = 19(\$3.47)$$

$$P = \$65.93$$

- b. First, we need to find the earnings per share next year, which will be:

$$EPS_1 = EPS_0(1 + g)$$

$$EPS_1 = \$3.47(1 + .06)$$

$$EPS_1 = \$3.68$$

Using the equation to calculate the price of a share of stock with the PE ratio:

$$P_1 = \text{Benchmark PE ratio} \times EPS_1$$

$$P_1 = 19(\$3.68)$$

$$P_1 = \$69.89$$

- c. To find the implied return over the next year, we calculate the return as:

$$R = (P_1 - P_0)/P_0$$

$$R = (\$69.89 - 65.93)/\$65.93$$

$$R = .06, \text{ or } 6\%$$

Notice that the return is the same as the growth rate in earnings. Assuming a stock pays no dividends and the ratio is constant, this will always be true when using price ratios to evaluate the price of a share of stock.

27. We need to find the enterprise value of the company. We can calculate EBITDA as sales minus costs, so:

$$\begin{aligned}\text{EBITDA} &= \text{Sales} - \text{Costs} \\ \text{EBITDA} &= \$48,000,000 - 15,000,000 \\ \text{EBITDA} &= \$33,000,000\end{aligned}$$

Solving the EV/EBITDA multiple for enterprise value, we find:

$$\begin{aligned}\text{Enterprise value} &= \$33,000,000(6.4) \\ \text{Enterprise value} &= \$211,200,000\end{aligned}$$

The total value of equity is the enterprise value minus any outstanding debt, plus cash, so:

$$\begin{aligned}\text{Equity value} &= \text{Enterprise value} - \text{Debt} + \text{Cash} \\ \text{Equity value} &= \$211,200,000 - 64,000,000 + 23,000,000 \\ \text{Equity value} &= \$170,200,000\end{aligned}$$

So, the price per share is:

$$\begin{aligned}\text{Stock price} &= \$170,200,000/1,950,000 \\ \text{Stock price} &= \$87.28\end{aligned}$$

28. a. To value the stock today, we first need to calculate the cash flows for the next 6 years. The sales, costs, and net investment all grow by the same rate, namely 14 percent, 12 percent, 10 percent, and 8 percent, respectively, for the following 4 years, then 6 percent indefinitely. So, the cash flows for each year will be:

	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>	<u>Year 5</u>	<u>Year 6</u>
Sales	\$115,000,000	\$131,100,000	\$146,832,000	\$161,515,200	\$174,436,416	\$184,902,601
Costs	67,000,000	76,380,000	85,545,600	94,100,160	101,628,173	107,725,863
EBT	\$48,000,000	\$54,720,000	\$61,286,400	\$67,415,040	\$72,808,243	\$77,176,738
Taxes	10,080,000	11,491,200	12,870,144	14,157,158	15,289,731	16,207,115
Net income	\$37,920,000	\$43,228,800	\$48,416,256	\$53,257,882	\$57,518,512	\$60,969,623
Investment	12,000,000	13,680,000	15,321,600	16,853,760	18,202,061	19,294,184
Cash flow	\$25,920,000	\$29,548,800	\$33,094,656	\$36,404,122	\$39,316,451	\$41,675,438

To find the terminal value of the company in Year 6, we can discount the Year 7 cash flows as a growing perpetuity, which will be:

$$\begin{aligned}\text{Terminal value} &= \$41,675,438(1 + .06)/(.13 - .06) \\ \text{Terminal value} &= \$631,085,210\end{aligned}$$

So, the value of the company today is:

$$\begin{aligned}\text{Company value today} &= \$25,920,000/1.13 + \$29,548,800/1.13^2 + \$33,094,656/1.13^3 \\ &\quad + \$36,404,122/1.13^4 + \$39,316,451/1.13^5 + (\$41,675,438 + 631,085,210)/1.13^6 \\ \text{Company value today} &= \$435,821,483\end{aligned}$$

Dividing the company value by the shares outstanding to get the share price, we get:

$$\text{Share price} = \$435,821,483/5,500,000$$

$$\text{Share price} = \$79.24$$

- b. In this case, we are going to use the PE multiple to find the terminal value. All of the cash flows from part a will remain the same. So, the terminal value in Year 6 is:

$$\text{Terminal value} = 11(\$60,969,623)$$

$$\text{Terminal value} = \$670,665,851$$

Under this assumption for the terminal value, the value of the company today is:

$$\begin{aligned} \text{Company value today} &= \$25,920,000/1.13 + \$29,548,800/1.13^2 + \$33,094,656/1.13^3 \\ &\quad + \$36,404,122/1.13^4 + \$39,316,451/1.13^5 + (\$41,675,438 + \$670,665,851)/1.13^6 \\ \text{Company value today} &= \$454,832,798 \end{aligned}$$

Dividing the company value by the shares outstanding to get the share price, we get:

$$\text{Share price} = \$454,832,798/5,500,000$$

$$\text{Share price} = \$82.70$$

Challenge

29. We are asked to find the dividend yield and capital gains yield for each of the stocks. All of the stocks have a 15 percent required return, which is the sum of the dividend yield and the capital gains yield. To find the components of the total return, we need to find the stock price for each stock. Using this stock price and the dividend, we can calculate the dividend yield. The capital gains yield for the stock will be the total return (required return) minus the dividend yield.

$$\text{W: } P_0 = D_0(1 + g)/(R - g) = \$3.25(1.075)/(.15 - .075) = \$46.58$$

$$\text{Dividend yield} = D_1/P_0 = \$3.25(1.075)/\$46.58 = .075, \text{ or } 7.5\%$$

$$\text{Capital gains yield} = .15 - .075 = .075, \text{ or } 7.5\%$$

$$\text{X: } P_0 = D_0(1 + g)/(R - g) = \$3.25/ (.15 - 0) = \$21.67$$

$$\text{Dividend yield} = D_1/P_0 = \$3.25/\$21.67 = .15, \text{ or } 15\%$$

$$\text{Capital gains yield} = .15 - .15 = 0\%$$

$$\text{Y: } P_0 = D_0(1 + g)/(R - g) = \$3.25(1 - .05)/(.15 + .05) = \$15.44$$

$$\text{Dividend yield} = D_1/P_0 = \$3.25(.95)/\$15.44 = .20, \text{ or } 20\%$$

$$\text{Capital gains yield} = .15 - .20 = -.05, \text{ or } -5\%$$

$$\begin{aligned} Z: P_2 &= D_2(1+g)/(R-g) = D_0(1+g_1)^2(1+g_2)/(R-g_2) = \$3.25(1.30)^2(1.08)/(.15-.08) \\ P_2 &= \$84.74 \end{aligned}$$

$$\begin{aligned} P_0 &= \$3.25(1.30)/(1.15) + \$3.25(1.30)^2/(1.15)^2 + \$84.74/(1.15)^2 \\ P_0 &= \$71.90 \end{aligned}$$

$$\text{Dividend yield} = D_1/P_0 = \$3.25(1.30)/\$71.90 = .0588, \text{ or } 5.88\%$$

$$\text{Capital gains yield} = .15 - .0588 = .0912, \text{ or } 9.12\%$$

In all cases, the required return is 15 percent, but the return is distributed differently between current income and capital gains. High-growth stocks have an appreciable capital gains component but a relatively small current income yield; conversely, mature, negative-growth stocks provide a high current income but also, price depreciation, over time.

30. a. Using the constant growth model, the price of the stock paying annual dividends will be:

$$\begin{aligned} P_0 &= D_0(1+g)/(R-g) \\ P_0 &= \$3.80(1.045)/(.11-.045) \\ P_0 &= \$61.09 \end{aligned}$$

- b. If the company pays quarterly dividends instead of annual dividends, the quarterly dividend will be one-fourth of the annual dividend, or:

$$\begin{aligned} \text{Quarterly dividend} &: \$3.80(1.045)/4 \\ \text{Quarterly dividend} &= \$.9928 \end{aligned}$$

To find the equivalent annual dividend, we must assume that the quarterly dividends are reinvested at the required return. We can then use this interest rate to find the equivalent annual dividend. In other words, when we receive the quarterly dividend, we reinvest it at the required return on the stock. So, the effective quarterly rate is:

$$\begin{aligned} \text{Effective quarterly rate} &= 1.11^{.25} - 1 \\ \text{Effective quarterly rate} &= .0264, \text{ or } 2.64\% \end{aligned}$$

The effective annual dividend will be the FVA of the quarterly dividend payments at the effective quarterly required return. In this case, the effective annual dividend will be:

$$\begin{aligned} \text{Effective } D_1 &= \$.9928(\text{FVIFA}_{2.64\%,4}) \\ \text{Effective } D_1 &= \$4.13 \end{aligned}$$

Now, we can use the constant growth model to find the current stock price as:

$$\begin{aligned} P_0 &= \$4.13/ (.11 - .045) \\ P_0 &= \$63.56 \end{aligned}$$

Note that we cannot use the quarterly effective required return and growth rate to find the value of the stock. This would assume the dividends increased each quarter, not each year.

- 31.** Here we have a stock with differential growth, but the dividend growth changes every year for the first four years. We can find the price of the stock in Year 3 since the dividend growth rate is constant after the third dividend. The price of the stock in Year 3 will be the dividend in Year 4, divided by the required return minus the constant dividend growth rate. So, the price in Year 3 will be:

$$P_3 = \$2.95(1.20)(1.15)(1.10)(1.05)/(.13 - .05)$$

$$P_3 = \$58.78$$

The price of the stock today will be the PV of the first three dividends, plus the PV of the stock price in Year 3, so:

$$P_0 = \$2.95(1.20)/(1.13) + \$2.95(1.20)(1.15)/1.13^2 + \$2.95(1.20)(1.15)(1.10)/1.13^3$$

$$+ \$58.78/1.13^3$$

$$P_0 = \$50.16$$

- 32.** Here we want to find the required return that makes the PV of the dividends equal to the current stock price. The equation for the stock price is:

$$P = \$2.95(1.20)/(1 + R) + \$2.95(1.20)(1.15)/(1 + R)^2 + \$2.95(1.20)(1.15)(1.10)/(1 + R)^3$$

$$+ [\$2.95(1.20)(1.15)(1.10)(1.05)/(R - .05)]/(1 + R)^3 = \$62.40$$

We need to find the roots of this equation. Using a spreadsheet, trial and error, or a calculator with a root solving function, we find that:

$$R = .1144, \text{ or } 11.44\%$$

CHAPTER 10

SOME LESSONS FROM CAPITAL MARKET HISTORY

Answers to Concepts Review and Critical Thinking Questions

1. They all wish they had! Since they didn't, it must have been the case that the stellar performance was not foreseeable, at least not by most.
2. As in the previous question, it's easy to see after the fact that the investment was terrible, but it probably wasn't so easy ahead of time.
3. No, stocks are riskier. Some investors are highly risk averse, and the extra possible return doesn't attract them relative to the extra risk.
4. Unlike gambling, the stock market is a positive sum game; everybody can win. Also, speculators provide liquidity to markets and thus help to promote efficiency.
5. T-bill rates were highest in the early eighties. This was during a period of high inflation and is consistent with the Fisher effect.
6. Before the fact, for most assets, the risk premium will be positive; investors demand compensation over and above the risk-free return to invest their money in the risky asset. After the fact, the observed risk premium can be negative if the asset's nominal return is unexpectedly low, the risk-free return is unexpectedly high, or if some combination of these two events occurs.
7. Yes, the stock prices are currently the same. Below is a diagram that depicts the stocks' price movements. Two years ago, each stock had the same price, P_0 . Over the first year, General Materials' stock price increased by 10 percent, or $(1.1) \times P_0$. Standard Fixtures' stock price declined by 10 percent, or $(.9) \times P_0$. Over the second year, General Materials' stock price decreased by 10 percent, or $(.9)(1.1) \times P_0$, while Standard Fixtures' stock price increased by 10 percent, or $(1.1)(.9) \times P_0$. Today, each of the stocks is worth 99 percent of its original value.

	<u>2 years ago</u>	<u>1 year ago</u>	<u>Today</u>
General Materials	P_0	$(1.1)P_0$	$(1.1)(.9)P_0 = (.99)P_0$
Standard Fixtures	P_0	$(.9)P_0$	$(.9)(1.1)P_0 = (.99)P_0$

8. The stock prices are not the same. The return quoted for each stock is the arithmetic return, not the geometric return. The geometric return tells you the wealth increase from the beginning of the period to the end of the period, assuming the asset had the same return each year. As such, it is a better measure of ending wealth. To see this, assuming each stock had a beginning price of \$100 per share, the ending price for each stock would be:

Lake Minerals ending price = $\$100(1.10)(1.10) = \121.00

Small Town Furniture ending price = $\$100(1.25)(.95) = \118.75

Whenever there is any variance in returns, the asset with the larger variance will always have the greater difference between the arithmetic and geometric return.

9. To calculate an arithmetic return, you sum the returns and divide by the number of returns. As such, arithmetic returns do not account for the effects of compounding. Geometric returns do account for the effects of compounding. As an investor, the more important return of an asset is the geometric return.
10. Risk premiums are about the same whether or not we account for inflation. The reason is that risk premiums are the difference between two returns, so inflation essentially nets out. Returns, risk premiums, and volatility would all be lower than we estimated because aftertax returns are smaller than pretax returns.

Solutions to Questions and Problems

NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The return of any asset is the increase in price, plus any dividends or cash flows, all divided by the initial price. The return of this stock is:

$$R = [(\$84 - 76) + 1.95]/\$76$$

$$R = .1309, \text{ or } 13.09\%$$

2. The dividend yield is the dividend divided by the price at the beginning of the period, so:

$$\text{Dividend yield} = \$1.95/\$76$$

$$\text{Dividend yield} = .0257, \text{ or } 2.57\%$$

And the capital gains yield is the increase in price divided by the initial price, so:

$$\text{Capital gains yield} = (\$84 - 76)/\$76$$

$$\text{Capital gains yield} = .1053, \text{ or } 10.53\%$$

3. Using the equation for total return, we find:

$$R = [(\$68 - 76) + 1.95]/\$76$$

$$R = -.0796, \text{ or } -7.96\%$$

And the dividend yield and capital gains yield are:

$$\text{Dividend yield} = \$1.95/\$76$$

$$\text{Dividend yield} = .0257, \text{ or } 2.57\%$$

$$\text{Capital gains yield} = (\$68 - 76)/\$76$$

$$\text{Capital gains yield} = -.1053, \text{ or } -10.53\%$$

Here's a question for you: Can the dividend yield ever be negative? No, that would mean you were paying the company for the privilege of owning the stock. It has happened on bonds.

4. The total dollar return is the change in price plus the coupon payment, so:

$$\text{Total dollar return} = \$1,052 - 1,010 + 49$$

$$\text{Total dollar return} = \$91$$

The total nominal percentage return of the bond is:

$$R = [(\$1,052 - 1,010) + 49]/\$1,010$$

$$R = .0901, \text{ or } 9.01\%$$

Notice here that we could have used the total dollar return of \$91 in the numerator of this equation.

Using the Fisher equation, the real return was:

$$(1 + R) = (1 + r)(1 + h)$$

$$r = (1.0901/1.030) - 1$$

$$r = .0583, \text{ or } 5.83\%$$

5. The nominal return is the stated return, which is 12.1 percent. Using the Fisher equation, the real return was:

$$(1 + R) = (1 + r)(1 + h)$$

$$r = (1.121)/(1.03) - 1$$

$$r = .0883, \text{ or } 8.83\%$$

6. Using the Fisher equation, the real returns for government and corporate bonds were:

$$(1 + R) = (1 + r)(1 + h)$$

$$r_G = 1.060/1.030 - 1$$

$$r_G = .0291, \text{ or } 2.91\%$$

$$r_C = 1.064/1.030 - 1$$

$$r_C = .0330, \text{ or } 3.30\%$$

7. The average return is the sum of the returns, divided by the number of returns. The average return for each stock was:

$$\bar{X} = \left[\sum_{i=1}^N x_i \right] / N = \frac{.12 + .24 - .27 + .14 + .19}{5} = .0840, \text{ or } 8.40\%$$

$$\bar{Y} = \left[\sum_{i=1}^N y_i \right] / N = \frac{.14 + .29 - .33 + .17 + .37}{5} = .1280, \text{ or } 12.80\%$$

We calculate the variance of each stock as:

$$\sigma_X^2 = \left[\sum_{i=1}^N (x_i - \bar{x})^2 \right] / (N - 1)$$

$$\sigma_X^2 = \frac{1}{5-1} \left[(.12 - .084)^2 + (.24 - .084)^2 + (-.27 - .084)^2 + (.14 - .084)^2 + (.19 - .084)^2 \right] = .04133$$

$$\sigma_Y^2 = \frac{1}{5-1} \left[(.14 - .128)^2 + (.29 - .128)^2 + (-.33 - .128)^2 + (.17 - .128)^2 + (.37 - .128)^2 \right] = .07412$$

The standard deviation is the square root of the variance, so the standard deviation of each stock is:

$$\sigma_X = .04133^{1/2}$$

$$\sigma_X = .2033, \text{ or } 20.33\%$$

$$\sigma_Y = .07412^{1/2}$$

$$\sigma_Y = .2722, \text{ or } 27.22\%$$

8. We will calculate the sum of the returns for each asset and the observed risk premium first. Doing so, we get:

<u>Year</u>	<u>Large co. stock return</u>	<u>T-bill return</u>	<u>Risk premium</u>
1973	-14.69%	7.29%	-21.98%
1974	-26.47	7.99	-34.46
1975	37.23	5.87	31.36
1976	23.93	5.07	18.86
1977	-7.16	5.45	-12.61
1978	<u>6.57</u>	<u>7.64</u>	<u>-1.07</u>
	19.41%	39.31%	-19.90%

- a. The average return for large company stocks over this period was:

$$\text{Large company stock average return} = 19.41\%/6$$

$$\text{Large company stock average return} = 3.24\%$$

And the average return for T-bills over this period was:

$$\text{T-bills average return} = 39.31\%/6$$

$$\text{T-bills average return} = 6.55\%$$

- b. Using the equation for variance, we find the variance for large company stocks over this period was:

$$\text{Variance} = 1/5[(-.1469 - .0324)^2 + (-.2647 - .0324)^2 + (.3723 - .0324)^2 + (.2393 - .0324)^2 + (-.0716 - .0324)^2 + (.0657 - .0324)^2]$$

$$\text{Variance} = .058136$$

And the standard deviation for large company stocks over this period was:

$$\text{Standard deviation} = (.058136)^{1/2}$$

$$\text{Standard deviation} = .2411, \text{ or } 24.11\%$$

Using the equation for variance, we find the variance for T-bills over this period was:

$$\text{Variance} = 1/5[(.0729 - .0655)^2 + (.0799 - .0655)^2 + (.0587 - .0655)^2 + (.0507 - .0655)^2 + (.0545 - .0655)^2 + (.0764 - .0655)^2]$$

$$\text{Variance} = .000153$$

And the standard deviation for T-bills over this period was:

$$\text{Standard deviation} = (.000153)^{1/2}$$

$$\text{Standard deviation} = .0124, \text{ or } 1.24\%$$

- c. The average observed risk premium over this period was:

$$\text{Average observed risk premium} = -19.90\%/6$$

$$\text{Average observed risk premium} = -3.32\%$$

The variance of the observed risk premium was:

$$\text{Variance} = 1/5[(-.2198 - (-.0332))^2 + (-.3446 - (-.0332))^2 + (.3136 - (-.0332))^2 + (.1886 - (-.0332))^2 + (-.1261 - (-.0332))^2 + (-.0107 - (-.0332))^2]$$

$$\text{Variance} = .062078$$

And the standard deviation of the observed risk premium was:

$$\text{Standard deviation} = (.062078)^{1/2}$$

$$\text{Standard deviation} = .2492, \text{ or } 24.92\%$$

9. a. To find the average return, we sum all the returns and divide by the number of returns, so:

$$\text{Arithmetic average return} = (.19 + .24 + .11 - .09 + .13)/5$$

$$\text{Arithmetic average return} = .1160, \text{ or } 11.60\%$$

- b. Using the equation to calculate variance, we find:

$$\text{Variance} = 1/4[(.19 - .116)^2 + (.24 - .116)^2 + (.11 - .116)^2 + (-.09 - .116)^2 + (.13 - .116)^2]$$

$$\text{Variance} = .01588$$

So, the standard deviation is:

$$\text{Standard deviation} = .01588^{1/2}$$

$$\text{Standard deviation} = .1260, \text{ or } 12.60\%$$

10. a. To calculate the average real return, we can use the average return of the asset and the average inflation rate in the Fisher equation. Doing so, we find:

$$(1 + R) = (1 + r)(1 + h)$$

$$\bar{r} = (1.116/1.036) - 1$$

$$\bar{r} = .0772, \text{ or } 7.72\%$$

- b. The average risk premium is the average return of the asset, minus the average real risk-free rate, so, the average risk premium for this asset would be:

$$\overline{RP} = \bar{R} - \bar{R}_f$$

$$\overline{RP} = .1160 - .0410$$

$$\overline{RP} = .0750, \text{ or } 7.50\%$$

11. We can find the average real risk-free rate using the Fisher equation. The average real risk-free rate was:

$$(1 + R) = (1 + r)(1 + h)$$

$$\bar{R}_f = (1.041/1.036) - 1$$

$$\bar{R}_f = .0048, \text{ or } .48\%$$

And to calculate the average real risk premium, we can subtract the average risk-free rate from the average real return. So, the average real risk premium was:

$$\overline{RP} = \bar{r} - \bar{R}_f$$

$$\overline{RP} = .0772 - .0048$$

$$\overline{RP} = .0724, \text{ or } 7.24\%$$

12. Applying the five-year holding-period return formula to calculate the total return of the stock over the five-year period, we find:

$$\text{5-year holding-period return} = [(1 + R_1)(1 + R_2)(1 + R_3)(1 + R_4)(1 + R_5)] - 1$$

$$\text{5-year holding-period return} = [(1 + .1279)(1 + .0921)(1 + .1468)(1 + .2183)(1 - .1034)] - 1$$

$$\text{5-year holding-period return} = .5430, \text{ or } 54.30\%$$

13. To find the return on the zero coupon bond, we first need to find the price of the bond today. Since one year has elapsed, the bond now has 20 years to maturity. Using semiannual compounding, the price today is:

$$P_1 = \$1,000/1.032^{40}$$

$$P_1 = \$283.67$$

There are no intermediate cash flows on a zero coupon bond, so the return is the capital gain, or:

$$R = (\$283.67 - 273.82)/\$273.82$$

$$R = .0360, \text{ or } 3.60\%$$

14. The return of any asset is the increase in price, plus any dividends or cash flows, all divided by the initial price. This preferred stock paid a dividend of \$3.10, so the return for the year was:

$$R = (\$97.18 - 94.82 + 3.10)/\$94.82$$

$$R = .0576, \text{ or } 5.76\%$$

15. The return of any asset is the increase in price, plus any dividends or cash flows, all divided by the initial price. This stock paid no dividend, so the return was:

$$R = (\$91.45 - 82.18)/\$82.18$$

$$R = .1128, \text{ or } 11.28\%$$

This is the return for three months, so the APR is:

$$\text{APR} = 4(11.28\%)$$

$$\text{APR} = 45.12\%$$

And the EAR is:

$$\text{EAR} = (1 + .1128)^4 - 1$$

$$\text{EAR} = .5335, \text{ or } 53.35\%$$

16. To find the real return each year, we will use the Fisher equation, which is:

$$1 + R = (1 + r)(1 + h)$$

Using this relationship for each year, we find:

	<u>T-bills</u>	<u>Inflation</u>	<u>Real Return</u>
1926	.0330	-.0112	.0447
1927	.0315	-.0226	.0554
1928	.0405	-.0116	.0527
1929	.0447	.0058	.0387
1930	.0227	-.0640	.0926
1931	.0115	-.0932	.1155
1932	.0088	-.1027	.1243

So, the average real return was:

$$\text{Average} = (.0447 + .0554 + .0527 + .0387 + .0926 + .1155 + .1243)/7$$

$$\text{Average} = .0748, \text{ or } 7.48\%$$

Notice the real return was higher than the nominal return during this period because of deflation, or negative inflation.

17. Looking at the long-term corporate bond return history in Table 10.2, we see that the mean return was 6.4 percent, with a standard deviation of 8.3 percent. The range of returns you would expect to see 68 percent of the time is the mean plus or minus 1 standard deviation, or:

$$R \in \mu \pm 1\sigma = 6.4\% \pm 8.3\% = -1.90\% \text{ to } 14.70\%$$

The range of returns you would expect to see 95 percent of the time is the mean plus or minus 2 standard deviations, or:

$$R \in \mu \pm 2\sigma = 6.4\% \pm 2(8.3\%) = -10.20\% \text{ to } 23.00\%$$

18. Looking at the large-company stock return history in Table 10.2, we see that the mean return was 12.1 percent, with a standard deviation of 19.8 percent. The range of returns you would expect to see 68 percent of the time is the mean plus or minus 1 standard deviation, or:

$$R \in \mu \pm 1\sigma = 12.1\% \pm 19.8\% = -7.70\% \text{ to } 31.90\%$$

The range of returns you would expect to see 95 percent of the time is the mean plus or minus 2 standard deviations, or:

$$R \in \mu \pm 2\sigma = 12.1\% \pm 2(19.8\%) = -27.50\% \text{ to } 51.70\%$$

Intermediate

19. Here we know the average stock return, and four of the five returns used to compute the average return. We can work the average return equation backward to find the missing return. The average return is calculated as:

$$5(.101) = .17 - .19 + .09 + .34 + R$$

$$R = .095, \text{ or } 9.5\%$$

The missing return has to be 9.5 percent. Now we can use the equation for the variance to find:

$$\text{Variance} = 1/4[(.17 - .101)^2 + (-.19 - .101)^2 + (.09 - .101)^2 + (.34 - .101)^2 + (.095 - .101)^2]$$

$$\text{Variance} = .03668$$

And the standard deviation is:

$$\text{Standard deviation} = .03668^{1/2}$$

$$\text{Standard deviation} = .1915, \text{ or } 19.15\%$$

20. The arithmetic average return is the sum of the known returns divided by the number of returns, so:

$$\text{Arithmetic average return} = (.23 + .11 + .37 - .03 + .22 - .17)/6$$

$$\text{Arithmetic average return} = .1217, \text{ or } 12.17\%$$

Using the equation for the geometric return, we find:

$$\text{Geometric average return} = [(1 + R_1) \times (1 + R_2) \times \dots \times (1 + R_T)]^{1/T} - 1$$

$$\text{Geometric average return} = [(1 + .23)(1 + .11)(1 + .37)(1 - .03)(1 + .22)(1 - .17)]^{1/6} - 1$$

$$\text{Geometric average return} = .1067, \text{ or } 10.67\%$$

Remember, the geometric average return will always be less than the arithmetic average return if the returns have any variation.

21. To calculate the arithmetic and geometric average returns, we must first calculate the return for each year. The return for each year is:

$$R_1 = (\$68.13 - \$64.12 + 1.15)/\$64.12 = .0805, \text{ or } 8.05\%$$

$$R_2 = (\$61.23 - \$68.13 + 1.25)/\$68.13 = -.0829, \text{ or } -8.29\%$$

$$R_3 = (\$74.27 - \$61.23 + 1.36)/\$61.23 = .2352, \text{ or } 23.52\%$$

$$R_4 = (\$77.38 - \$74.27 + 1.47)/\$74.27 = .0617, \text{ or } 6.17\%$$

$$R_5 = (\$86.19 - \$77.38 + 1.60)/\$77.38 = .1345, \text{ or } 13.45\%$$

The arithmetic average return was:

$$R_A = (.0805 - .0829 + .2352 + .0617 + .1345)/5$$

$$R_A = .0858, \text{ or } 8.58\%$$

And the geometric average return was:

$$R_G = [(1 + .0805)(1 - .0829)(1 + .2352)(1 + .0617)(1 + .1345)]^{1/5} - 1$$

$$R_G = .0807, \text{ or } 8.07\%$$

22. To find the real return we need to use the Fisher equation. Re-writing the Fisher equation to solve for the real return, we get:

$$r = [(1 + R)/(1 + h)] - 1$$

So, the real return each year was:

<u>Year</u>	<u>T-bill return</u>	<u>Inflation</u>	<u>Real return</u>
1973	.0729	.0871	-.0131
1974	.0799	.1234	-.0387
1975	.0587	.0694	-.0100
1976	.0507	.0486	.0020
1977	.0545	.0670	-.0117
1978	.0764	.0902	-.0127
1979	.1056	.1329	-.0241
<u>1980</u>	<u>.1210</u>	<u>.1252</u>	<u>-.0037</u>
	.6197	.7438	-.1120

- a. The average return for T-bills over this period was:

$$\text{Average return} = .6197/8$$

$$\text{Average return} = .0775, \text{ or } 7.75\%$$

And the average inflation rate was:

$$\text{Average inflation} = .7438/8$$

$$\text{Average inflation} = .0930, \text{ or } 9.30\%$$

- b. Using the equation for variance, we find the variance for T-bills over this period was:

$$\text{Variance} = 1/7[(.0729 - .0775)^2 + (.0799 - .0775)^2 + (.0587 - .0775)^2 + (.0507 - .0775)^2 + (.0545 - .0775)^2 + (.0764 - .0775)^2 + (.1056 - .0775)^2 + (.1210 - .0775)^2]$$

$$\text{Variance} = .000616$$

And the standard deviation for T-bills was:

$$\text{Standard deviation} = (.000616)^{1/2}$$

$$\text{Standard deviation} = .0248, \text{ or } 2.48\%$$

The variance of inflation over this period was:

$$\text{Variance} = 1/7[(.0871 - .0930)^2 + (.1234 - .0930)^2 + (.0694 - .0930)^2 + (.0486 - .0930)^2 + (.0670 - .0930)^2 + (.0902 - .0930)^2 + (.1329 - .0930)^2 + (.1252 - .0930)^2]$$

$$\text{Variance} = .000971$$

And the standard deviation of inflation was:

$$\text{Standard deviation} = (.000971)^{1/2}$$

$$\text{Standard deviation} = .0312, \text{ or } 3.12\%$$

- c. The average observed real return over this period was:

$$\text{Average observed real return} = -.1120/8$$

$$\text{Average observed real return} = -.0140, \text{ or } -1.40\%$$

- d. The statement that T-bills have no risk refers to the fact that there is only an extremely small chance of the government defaulting, so there is little default risk. Since T-bills are short term, there is also very limited interest rate risk. However, as this example shows, there is inflation risk, i.e. the purchasing power of the investment can actually decline over time even if the investor is earning a positive return.
23. To find the return on the coupon bond, we first need to find the price of the bond today. Since one year has elapsed, the bond now has six years to maturity, so the price today is:

$$P_1 = \$58(PVIFA_{5.4\%,6}) + \$1,000/1.054^6$$

$$P_1 = \$1,020.05$$

You received the coupon payments on the bond, so the nominal return was:

$$R = (\$1,020.05 - 1,027.50 + 58)/\$1,027.50$$

$$R = .0492, \text{ or } 4.92\%$$

And using the Fisher equation to find the real return, we get:

$$r = 1.0492/1.029 - 1$$

$$r = .0196, \text{ or } 1.96\%$$

24. Looking at the long-term government bond return history in Table 10.2, we see that the mean return was 6 percent, with a standard deviation of 9.9 percent. In the normal probability distribution, approximately 2/3 of the observations are within one standard deviation of the mean. This means that 1/3 of the observations are outside one standard deviation away from the mean. Or:

$$\Pr(R < -3.9 \text{ or } R > 15.9) \approx 1/3$$

But we are only interested in one tail here, that is, returns less than -3.9 percent, so:

$$\Pr(R < -3.9) \approx 1/6$$

You can use the z-statistic and the cumulative normal distribution table to find the answer as well. Doing so, we find:

$$z = (X - \mu)/\sigma$$

$$z = (-.039 - .06)/.099 = -1.00$$

Looking at the z-table, this gives a probability of 15.87%, or:

$$\Pr(R < -3.9) \approx .1587, \text{ or } 15.87\%$$

The range of returns you would expect to see 95 percent of the time is the mean plus or minus 2 standard deviations, or:

$$95\% \text{ level: } R \in \mu \pm 2\sigma = 6\% \pm 2(9.9\%) = -13.80\% \text{ to } 25.80\%$$

The range of returns you would expect to see 99 percent of the time is the mean plus or minus 3 standard deviations, or:

$$99\% \text{ level: } R \in \mu \pm 3\sigma = 6\% \pm 3(9.9\%) = -23.70\% \text{ to } 35.70\%$$

25. The mean return for small company stocks was 16.5 percent, with a standard deviation of 31.7 percent. Doubling your money is a 100% return, so if the return distribution is normal, we can use the z-statistic. So:

$$z = (X - \mu)/\sigma$$

$$z = (100\% - 16.5\%)/31.7\% = 2.634 \text{ standard deviations above the mean}$$

This corresponds to a probability of $\approx .422\%$, or about once every 200 years. Tripling your money would be:

$$z = (200\% - 16.5\%)/31.7\% = 5.789 \text{ standard deviations above the mean.}$$

This corresponds to a probability of (much) less than .5%. The actual answer is $\approx .00000035479\%$, or about once every 282 million years.

26. It is impossible to lose more than 100 percent of your investment. Therefore, return distributions are truncated on the lower tail at -100 percent.

Challenge

27. Using the z-statistic, we find:

$$z = (X - \mu)/\sigma$$

$$z = (0\% - 12.1\%)/19.8\% = -.611$$

$$\Pr(R \leq 0) \approx 27.06\%$$

28. For each of the questions asked here, we need to use the z-statistic, which is:

$$z = (X - \mu)/\sigma$$

$$a. \quad z_1 = (10\% - 6.4\%)/8.3\% = .4337$$

This z-statistic gives us the probability that the return is less than 10 percent, but we are looking for the probability the return is greater than 10 percent. Given that the total probability is 100 percent (or 1), the probability of a return greater than 10 percent is 1 minus the probability of a return less than 10 percent. Using the cumulative normal distribution table, we get:

$$\Pr(R \geq 10\%) = 1 - \Pr(R \leq 10\%) = 33.22\%$$

For a return less than 0 percent:

$$z_2 = (0\% - 6.4\%)/8.3\% = -.7711$$

$$\Pr(R < 10\%) = 1 - \Pr(R > 0\%) = 22.03\%$$

- b. The probability that T-bill returns will be greater than 10 percent is:

$$z_3 = (10\% - 3.4\%)/3.1\% = 2.1290$$

$$\Pr(R \geq 10\%) = 1 - \Pr(R \leq 10\%) = 1 - .9834 \approx 1.66\%$$

And the probability that T-bill returns will be less than 0 percent is:

$$z_4 = (0\% - 3.4\%)/3.1\% = -1.0968$$

$$\Pr(R \leq 0) \approx 13.64\%$$

- c. The probability that the return on long-term corporate bonds will be less than -4.18 percent is:

$$z_5 = (-4.18\% - 6.4\%)/8.3\% = -1.2747$$

$$\Pr(R \leq -4.18\%) \approx 10.12\%$$

And the probability that T-bill returns will be greater than 10.56 percent is:

$$z_6 = (10.56\% - 3.4\%)/3.1\% = 2.3097$$

$$\Pr(R \geq 10.56\%) = 1 - \Pr(R \leq 10.56\%) = 1 - .9895 \approx 1.05\%$$

CHAPTER 11

RISK AND RETURN: *THE CAPITAL ASSET PRICING MODEL (CAPM)*

Answers to Concepts Review and Critical Thinking Questions

1. Some of the risk in holding any asset is unique to the asset in question. By investing in a variety of assets, this unique portion of the total risk can be eliminated at little cost. On the other hand, there are some risks that affect all investments. This portion of the total risk of an asset cannot be costlessly eliminated. In other words, systematic risk can be controlled, but only by a costly reduction in expected returns.
2.
 - a. systematic
 - b. unsystematic
 - c. both; probably mostly systematic
 - d. unsystematic
 - e. unsystematic
 - f. systematic
3. No to both questions. The portfolio expected return is a weighted average of the asset's returns, so it must be less than the largest asset return and greater than the smallest asset return.
4. False. The variance of the individual assets is a measure of the total risk. The variance on a well-diversified portfolio is a function of systematic risk only.
5. Yes, the standard deviation can be less than that of every asset in the portfolio. However, β_p cannot be less than the smallest beta because β_p is a weighted average of the individual asset betas.
6. Yes. It is possible, in theory, to construct a zero beta portfolio of risky assets whose return would be equal to the risk-free rate. It is also possible to have a negative beta; the return would be less than the risk-free rate. A negative beta asset would carry a negative risk premium because of its value as a diversification instrument.
7. The covariance is a more appropriate measure of a security's risk in a well-diversified portfolio because the covariance reflects the effect of the security on the variance of the portfolio. Investors are concerned with the variance of their portfolios and not the variance of the individual securities. Since covariance measures the impact of an individual security on the variance of the portfolio, covariance is the appropriate measure of risk.

8. If we assume that the market has not stayed constant during the past three years, then the lack in movement of Southern Co.'s stock price only indicates that the stock either has a standard deviation or a beta that is very near to zero. The large amount of movement in Texas Instruments' (TIs') stock price does not imply that the firm's beta is high. Total volatility (the price fluctuation) is a function of both systematic and unsystematic risks. The beta only reflects the systematic risk. Observing the standard deviation of price movements does not indicate whether the price changes were due to systematic factors or firm specific factors. Thus, if you observe large stock price movements like that of TI, you cannot claim that the beta of the stock is high. All you know is that the total risk of TI is high.
9. The wide fluctuations in the price of oil stocks do not indicate that these stocks are a poor investment. If an oil stock is purchased as part of a well-diversified portfolio, only its contribution to the risk of the entire portfolio matters. This contribution is measured by systematic risk or beta. Since price fluctuations in oil stocks reflect diversifiable plus non-diversifiable risks, observing the standard deviation of price movements is not an adequate measure of the appropriateness of adding oil stocks to a portfolio.
10. The statement is false. If a security has a negative beta, investors would want to hold the asset to reduce the variability of their portfolios. Those assets will have expected returns that are lower than the risk-free rate. To see this, examine the Capital Asset Pricing Model:

$$E(R_S) = R_f + \beta_S[E(R_M) - R_f]$$

If $\beta_S < 0$, then the $E(R_S) < R_f$

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The portfolio weight of an asset is total investment in that asset divided by the total portfolio value. First, we will find the portfolio value, which is:

$$\text{Total value} = 145(\$47) + 130(\$86)$$

$$\text{Total value} = \$17,995$$

The portfolio weight for each stock is:

$$X_A = 145(\$47)/\$17,995$$

$$X_A = .3787$$

$$X_B = 130(\$86)/\$17,995$$

$$X_B = .6213$$

2. The expected return of a portfolio is the sum of the weight of each asset times the expected return of each asset. The total value of the portfolio is:

$$\text{Total value} = \$3,100 + 4,600$$

$$\text{Total value} = \$7,700$$

So, the expected return of this portfolio is:

$$E(R_p) = (\$3,100/\$7,700)(.098) + (\$4,600/\$7,700)(.127)$$

$$E(R_p) = .1153, \text{ or } 11.53\%$$

3. The expected return of a portfolio is the sum of the weight of each asset times the expected return of each asset. So, the expected return of the portfolio is:

$$E(R_p) = .20(.105) + .45(.161) + .35(.124)$$

$$E(R_p) = .1369, \text{ or } 13.69\%$$

4. Here we are given the expected return of the portfolio and the expected return of each asset in the portfolio and are asked to find the weight of each asset. We can use the equation for the expected return of a portfolio to solve this problem. Since the total weight of a portfolio must equal 1 (100%), the weight of Stock Y must be one minus the weight of Stock X. Mathematically speaking, this means:

$$E(R_p) = .112 = .127X_x + .091(1 - X_x)$$

We can now solve this equation for the weight of Stock X as:

$$.112 = .127X_x + .091 - .091X_x$$

$$.021 = .036X_x$$

$$X_x = .5833$$

So, the dollar amount invested in Stock X is the weight of Stock X times the total portfolio value, or:

$$\text{Investment in X} = .5833(\$10,000)$$

$$\text{Investment in X} = \$5,833.33$$

And the dollar amount invested in Stock Y is:

$$\text{Investment in Y} = (1 - .5833)(\$10,000)$$

$$\text{Investment in Y} = \$4,166.67$$

5. The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each stock asset is:

$$E(R_A) = .25(.06) + .55(.07) + .20(.11)$$

$$E(R_A) = .0755, \text{ or } 7.55\%$$

$$E(R_B) = .25(-.20) + .55(.13) + .20(.33)$$

$$E(R_B) = .0875, \text{ or } 8.75\%$$

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance and standard deviation of each stock are:

$$\sigma_A^2 = .25(.06 - .0755)^2 + .55(.07 - .0755)^2 + .20(.11 - .0755)^2$$

$$\sigma_A^2 = .00031$$

$$\sigma_A = .00031^{1/2}$$

$$\sigma_A = .0177, \text{ or } 1.77\%$$

$$\sigma_B^2 = .25(-.20 - .0875)^2 + .55(.13 - .0875)^2 + .20(.33 - .0875)^2$$

$$\sigma_B^2 = .03342$$

$$\sigma_B = .03342^{1/2}$$

$$\sigma_B = .1828, \text{ or } 18.28\%$$

6. The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of the stock is:

$$E(R_A) = .15(-.148) + .30(.031) + .45(.162) + .10(.348)$$

$$E(R_A) = .0948, \text{ or } 9.48\%$$

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance and standard deviation are:

$$\sigma^2 = .15(-.148 - .0948)^2 + .30(.031 - .0948)^2 + .45(.162 - .0948)^2 + .10(.348 - .0948)^2$$

$$\sigma^2 = .01851$$

$$\sigma = .01851^{1/2}$$

$$\sigma = .1360, \text{ or } 13.60\%$$

7. The expected return of a portfolio is the sum of the weight of each asset times the expected return of each asset. So, the expected return of the portfolio is:

$$E(R_p) = .15(.09) + .60(.11) + .25(.14)$$

$$E(R_p) = .1145, \text{ or } 11.45\%$$

If we own this portfolio, we would expect to earn a return of 11.45 percent.

8. a. To find the expected return of the portfolio, we need to find the return of the portfolio in each state of the economy. This portfolio is a special case since all three assets have the same weight. To find the expected return in an equally weighted portfolio, we can sum the returns of each asset and divide by the number of assets, so the expected return of the portfolio in each state of the economy is:

$$\begin{aligned}\text{Boom: } R_p &= (.06 + .16 + .33)/3 \\ R_p &= .1833, \text{ or } 18.33\%\end{aligned}$$

$$\begin{aligned}\text{Bust: } R_p &= (.14 + .02 - .06)/3 \\ R_p &= .0333, \text{ or } 3.33\%\end{aligned}$$

To find the expected return of the portfolio, we multiply the return in each state of the economy by the probability of that state occurring, and then sum. Doing this, we find:

$$\begin{aligned}E(R_p) &= .75(.1833) + .25(.0333) \\ E(R_p) &= .1458, \text{ or } 14.58\%\end{aligned}$$

- b. This portfolio does not have an equal weight in each asset. We still need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

$$\begin{aligned}\text{Boom: } R_p &= .20(.06) + .20(.16) + .60(.33) \\ R_p &= .2420, \text{ or } 24.20\%\end{aligned}$$

$$\begin{aligned}\text{Bust: } R_p &= .20(.14) + .20(.02) + .60(-.06) \\ R_p &= -.0040, \text{ or } -.40\%\end{aligned}$$

And the expected return of the portfolio is:

$$\begin{aligned}E(R_p) &= .75(.2420) + .25(-.0040) \\ E(R_p) &= .1805, \text{ or } 18.05\%\end{aligned}$$

To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance of the portfolio is:

$$\begin{aligned}\sigma_p^2 &= .75(.2420 - .1805)^2 + .25(-.0040 - .1805)^2 \\ \sigma_p^2 &= .011347\end{aligned}$$

9. a. This portfolio does not have an equal weight in each asset. We first need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

$$\begin{aligned}\text{Boom: } R_p &= .30(.24) + .40(.45) + .30(.33) \\ R_p &= .3510, \text{ or } 35.10\%\end{aligned}$$

Good: $R_p = .30(.09) + .40(.10) + .30(.15)$
 $R_p = .1120$, or 11.20%

Poor: $R_p = .30(.03) + .40(-.10) + .30(-.05)$
 $R_p = -.0460$, or -4.60%

Bust: $R_p = .30(-.05) + .40(-.25) + .30(-.09)$
 $R_p = -.1420$, or -14.20%

And the expected return of the portfolio is:

$$E(R_p) = .20(.3510) + .35(.1120) + .40(-.0460) + .05(-.1420)$$

$$E(R_p) = .0839$$
, or 8.39%

- b. To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance and standard deviation of the portfolio are:

$$\sigma_p^2 = .20(.3510 - .0839)^2 + .35(.1120 - .0839)^2 + .40(-.0460 - .0839)^2 + .05(-.1420 - .0839)^2$$

$$\sigma_p^2 = .02385$$

$$\sigma_p = .02385^{1/2}$$

$$\sigma_p = .1544$$
, or 15.44%

10. The beta of a portfolio is the sum of the weight of each asset times the beta of each asset. So, the beta of the portfolio is:

$$\beta_p = .20(.75) + .30(1.90) + .15(1.38) + .35(1.16)$$

$$\beta_p = 1.33$$

11. The beta of a portfolio is the sum of the weight of each asset times the beta of each asset. If the portfolio is as risky as the market it must have the same beta as the market. Since the beta of the market is 1.0, we know the beta of our portfolio is 1.0. We also need to remember that the beta of the risk-free asset is zero. It has to be zero since the asset has no risk. Setting up the equation for the beta of our portfolio, we get:

$$\beta_p = 1.0 = \frac{1}{3}(0) + \frac{1}{3}(1.61) + \frac{1}{3}(\beta_X)$$

Solving for the beta of Stock X, we get:

$$\beta_X = 1.39$$

12. CAPM states the relationship between the risk of an asset and its expected return. CAPM is:

$$E(R_i) = R_f + [E(R_M) - R_f] \times \beta_i$$

Substituting the values we are given, we find:

$$E(R_i) = .038 + (.111 - .038)(1.15)$$

$$E(R_i) = .1220, \text{ or } 12.20\%$$

13. We are given the values for the CAPM except for the β of the stock. We need to substitute these values into the CAPM, and solve for the β of the stock. One important thing we need to realize is that we are given the market risk premium. The market risk premium is the expected return of the market minus the risk-free rate. We must be careful not to use this value as the expected return of the market. Using the CAPM, we find:

$$E(R_i) = .104 = .038 + .07\beta_i$$

$$\beta_i = .94$$

14. Here we need to find the expected return of the market using the CAPM. Substituting the values given, and solving for the expected return of the market, we find:

$$E(R_i) = .127 = .042 + [E(R_M) - .042](1.20)$$

$$E(R_M) = .1128, \text{ or } 11.28\%$$

15. Here we need to find the risk-free rate using the CAPM. Substituting the values given, and solving for the risk-free rate, we find:

$$E(R_i) = .109 = R_f + (.118 - R_f)(.90)$$

$$.109 = R_f + .1062 - .90R_f$$

$$R_f = .0280, \text{ or } 2.80\%$$

16. a. We have a special case where the portfolio is equally weighted, so we can sum the returns of each asset and divide by the number of assets. The expected return of the portfolio is:

$$E(R_p) = (.116 + .036)/2$$

$$E(R_p) = .0760, \text{ or } 7.60\%$$

- b. We need to find the portfolio weights that result in a portfolio with a β of .50. We know the β of the risk-free asset is zero. We also know the weight of the risk-free asset is one minus the weight of the stock since the portfolio weights must sum to one, or 100 percent. So:

$$\beta_p = .50 = X_S(1.08) + (1 - X_S)(0)$$

$$.50 = 1.08X_S + 0 - 0X_S$$

$$X_S = .50/1.08$$

$$X_S = .4630$$

And, the weight of the risk-free asset is:

$$X_{Rf} = 1 - .4630$$

$$X_{Rf} = .5370$$

- c. We need to find the portfolio weights that result in a portfolio with an expected return of 10 percent. We also know the weight of the risk-free asset is one minus the weight of the stock since the portfolio weights must sum to one, or 100 percent. So:

$$\begin{aligned} E(R_p) &= .105 = .116X_S + .036(1 - X_S) \\ .105 &= .116X_S + .036 - .036X_S \\ X_S &= .8625 \end{aligned}$$

So, the β of the portfolio will be:

$$\begin{aligned} \beta_p &= .8625(1.08) + (1 - .8625)(0) \\ \beta_p &= .932 \end{aligned}$$

- d. Solving for the β of the portfolio as we did in part b, we find:

$$\begin{aligned} \beta_p &= 2.16 = X_S(1.08) + (1 - X_S)(0) \\ X_S &= 2.16/1.08 \\ X_S &= 2 \end{aligned}$$

$$\begin{aligned} X_{Rf} &= 1 - 2 \\ X_{Rf} &= -1 \end{aligned}$$

The portfolio is invested 200% in the stock and -100% in the risk-free asset. This represents borrowing at the risk-free rate to buy more of the stock.

17. First, we need to find the β of the portfolio. The β of the risk-free asset is zero, and the weight of the risk-free asset is one minus the weight of the stock, so the β of the portfolio is:

$$\beta_p = X_w(1.2) + (1 - X_w)(0) = 1.2X_w$$

So, to find the β of the portfolio for any weight of the stock, we multiply the weight of the stock times its β .

Even though we are solving for the β and expected return of a portfolio of one stock and the risk-free asset for different portfolio weights, we are really solving for the SML. Any combination of this stock and the risk-free asset will fall on the SML. For that matter, a portfolio of any stock and the risk-free asset, or any portfolio of stocks, will fall on the SML. We know the slope of the SML line is the market risk premium, so using the CAPM and the information concerning this stock, the market risk premium is:

$$\begin{aligned} E(R_w) &= .123 = .04 + \text{MRP}(1.20) \\ \text{MRP} &= .083/1.2 \\ \text{MRP} &= .0692, \text{ or } 6.92\% \end{aligned}$$

So, now we know the CAPM equation for any stock is:

$$E(R_p) = .04 + .0692\beta_p$$

The slope of the SML is equal to the market risk premium, which is .0692. Using these equations to fill in the table, we get the following results:

X_w	$E(R_p)$	β_p
0%	.0400	0
25	.0608	.300
50	.0815	.600
75	.1023	.900
100	.1230	1.200
125	.1438	1.500
150	.1645	1.800

18. There are two ways to correctly answer this question. We will work through both. First, we can use the CAPM. Substituting in the value we are given for each stock, we find:

$$E(R_Y) = .045 + .071(1.15)$$

$$E(R_Y) = .1267, \text{ or } 12.67\%$$

It is given in the problem that the expected return of Stock Y is 11.8 percent, but according to the CAPM, the return of the stock based on its level of risk should be 12.67 percent. This means the stock return is too low, given its level of risk. Stock Y plots below the SML and is overvalued. In other words, its price must decrease to increase the expected return to 12.67 percent.

For Stock Z, we find:

$$E(R_Z) = .045 + .071(.85)$$

$$E(R_Z) = .1054, \text{ or } 10.54\%$$

The return given for Stock Z is 10.7 percent, but, according to the CAPM, the expected return of the stock should be 10.54 percent based on its level of risk. Stock Z plots above the SML and is undervalued. In other words, its price must increase to decrease the expected return to 10.54 percent.

We can also answer this question using the reward-to-risk ratio. All assets must have the same reward-to-risk ratio, that is, every asset must have the same ratio of the asset risk premium to its beta. This follows from the linearity of the SML in Figure 11.11. The reward-to-risk ratio is the risk premium of the asset divided by its β . This is also known as the Treynor ratio or Treynor index. We are given the market risk premium, and we know the β of the market is one, so the reward-to-risk ratio for the market is .071, or 7.1 percent. Calculating the reward-to-risk ratio for Stock Y, we find:

$$\text{Reward-to-risk ratio Y} = (.118 - .045)/1.15 = .0635$$

The reward-to-risk ratio for Stock Y is too low, which means the stock plots below the SML, and the stock is overvalued. Its price must decrease until its reward-to-risk ratio is equal to the market reward-to-risk ratio. For Stock Z, we find:

$$\text{Reward-to-risk ratio Z} = (.107 - .045)/.85 = .0729$$

The reward-to-risk ratio for Stock Z is too high, which means the stock plots above the SML, and the stock is undervalued. Its price must increase until its reward-to-risk ratio is equal to the market reward-to-risk ratio.

19. We need to set the reward-to-risk ratios of the two assets equal to each other (see the previous problem), which is:

$$(.118 - R_f)/1.15 = (.107 - R_f)/.85$$

We can cross multiply to get:

$$.85(.118 - R_f) = 1.15(.107 - R_f)$$

Solving for the risk-free rate, we find:

$$\begin{aligned} .1003 - .85R_f &= .1231 - 1.15R_f \\ R_f &= .0758, \text{ or } 7.58\% \end{aligned}$$

Intermediate

20. For a portfolio that is equally invested in large-company stocks and long-term bonds:

$$\begin{aligned} \text{Return} &= (12.1\% + 6.0\%)/2 \\ \text{Return} &= 9.05\% \end{aligned}$$

For a portfolio that is equally invested in small stocks and Treasury bills:

$$\begin{aligned} \text{Return} &= (16.5\% + 3.5\%)/2 \\ \text{Return} &= 10.00\% \end{aligned}$$

21. We know that the reward-to-risk ratios for all assets must be equal (See Question 19). This can be expressed as:

$$[E(R_A) - R_f]/\beta_A = [E(R_B) - R_f]/\beta_B$$

The numerator of each equation is the risk premium of the asset, so:

$$RP_A/\beta_A = RP_B/\beta_B$$

We can rearrange this equation to get:

$$\beta_B/\beta_A = RP_B/RP_A$$

If the reward-to-risk ratios are the same, the ratio of the betas of the assets is equal to the ratio of the risk premiums of the assets.

22. a. We need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

$$\begin{aligned} \text{Boom: } R_p &= .4(.25) + .4(.35) + .2(.40) \\ R_p &= .3200, \text{ or } 32.00\% \end{aligned}$$

$$\begin{aligned} \text{Normal: } R_p &= .4(.18) + .4(.13) + .2(.03) \\ R_p &= .1300, \text{ or } 13.00\% \end{aligned}$$

$$\begin{aligned}\text{Bust: } R_p &= .4(.03) + .4(-.18) + .2(-.45) \\ R_p &= -.1500, \text{ or } -15.00\%\end{aligned}$$

And the expected return of the portfolio is:

$$\begin{aligned}E(R_p) &= .25(.32) + .55(.130) + .20(-.150) \\ E(R_p) &= .1215, \text{ or } 12.15\%\end{aligned}$$

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, then add all of these up. The result is the variance. So, the variance and standard deviation of the portfolio are:

$$\begin{aligned}\sigma_p^2 &= .25(.32 - .1215)^2 + .55(.130 - .1215)^2 + .20(-.150 - .1215)^2 \\ \sigma_p^2 &= .02463\end{aligned}$$

$$\begin{aligned}\sigma_p &= .02463^{1/2} \\ \sigma_p &= .1569, \text{ or } 15.69\%\end{aligned}$$

- b. The risk premium is the return of a risky asset minus the risk-free rate. T-bills are often used as the risk-free rate, so:

$$\begin{aligned}RP_1 &= E(R_p) - R_f \\ RP_1 &= .1215 - .038 \\ RP_1 &= .0835, \text{ or } 8.35\%\end{aligned}$$

- c. The approximate expected real return is the expected nominal return minus the inflation rate, so:

$$\begin{aligned}\text{Approximate expected real return} &= .1215 - .035 \\ \text{Approximate expected real return} &= .0865, \text{ or } 8.65\%\end{aligned}$$

To find the exact real return, we will use the Fisher equation. Doing so, we get:

$$\begin{aligned}1 + E(R_i) &= (1 + h)[1 + e(r_i)] \\ 1.1215 &= (1.0350)[1 + e(r_i)] \\ e(r_i) &= (1.1215/1.035) - 1 \\ e(r_i) &= .0836, \text{ or } 8.36\%\end{aligned}$$

The approximate real risk-free rate is:

$$\begin{aligned}\text{Approximate expected real return} &= .038 - .035 \\ \text{Approximate expected real return} &= .003, \text{ or } .30\%\end{aligned}$$

And using the Fisher effect for the exact real risk-free rate, we find:

$$\begin{aligned}1 + E(R_i) &= (1 + h)[1 + e(r_i)] \\1.038 &= (1.0350)[1 + e(r_i)] \\e(r_i) &= (1.038/1.035) - 1 \\e(r_i) &= .0029, \text{ or } .29\%\end{aligned}$$

The approximate real risk premium is the approximate expected real return minus the risk-free rate, so:

$$\begin{aligned}\text{Approximate expected real risk premium} &= .0865 - .003 \\ \text{Approximate expected real risk premium} &= .0835, \text{ or } 8.35\%\end{aligned}$$

The exact real risk premium is the exact real return minus the risk-free rate, so:

$$\begin{aligned}\text{Exact expected real risk premium} &= .0836 - .0029 \\ \text{Exact expected real risk premium} &= .0807, \text{ or } 8.07\%\end{aligned}$$

23. We know the total portfolio value and the investment of two stocks in the portfolio, so we can find the weight of these two stocks. The weights of Stock A and Stock B are:

$$\begin{aligned}X_A &= \$190,000/\$1,000,000 \\ X_A &= .19\end{aligned}$$

$$\begin{aligned}X_B &= \$325,000/\$1,000,000 \\ X_B &= .325\end{aligned}$$

Since the portfolio is as risky as the market, the β of the portfolio must be equal to one. We also know the β of the risk-free asset is zero. We can use the equation for the β of a portfolio to find the weight of the third stock. Doing so, we find:

$$\beta_p = 1.0 = X_A(.83) + X_B(1.19) + X_C(1.45) + X_{Rf}(0)$$

Solving for the weight of Stock C, we find:

$$X_C = .31417241$$

So, the dollar investment in Stock C must be:

$$\begin{aligned}\text{Invest in Stock C} &= .31417241(\$1,000,000) \\ \text{Invest in Stock C} &= \$314,172.41\end{aligned}$$

We also know the total portfolio weight must be one, so the weight of the risk-free asset must be one minus the asset weight we know, or:

$$\begin{aligned}1 &= X_A + X_B + X_C + X_{Rf} \\ 1 &= .19 + .325 + .31417241 + X_{Rf} \\ X_{Rf} &= .17082759\end{aligned}$$

So, the dollar investment in the risk-free asset must be:

$$\text{Invest in risk-free asset} = .17082759(\$1,000,000)$$

$$\text{Invest in risk-free asset} = \$170,827.59$$

24. We are given the expected return of the assets in the portfolio. We also know the sum of the weights of each asset must be equal to one. Using this relationship, we can express the expected return of the portfolio as:

$$E(R_p) = .127 = X_X(.114) + X_Y(.0868)$$

$$.127 = X_X(.114) + (1 - X_X)(.0868)$$

$$.127 = .114X_X + .0868 - .0868X_X$$

$$.0402 = .0272X_X$$

$$X_X = 1.47794$$

And the weight of Stock Y is:

$$X_Y = 1 - 1.47794$$

$$X_Y = -.47794$$

The amount to invest in Stock Y is:

$$\text{Investment in Stock Y} = -.47794(\$100,000)$$

$$\text{Investment in Stock Y} = -\$47,794.12$$

A negative portfolio weight means that you short sell the stock. If you are unfamiliar with short selling, it means you borrow a stock today and sell it. You must then purchase the stock at a later date to repay the borrowed stock. If you short sell a stock, you make a profit if the stock decreases in value.

25. The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each stock is:

$$E(R_A) = .33(.099) + .33(.113) + .33(.059)$$

$$E(R_A) = .0903, \text{ or } 9.03\%$$

$$E(R_B) = .33(-.073) + .33(.128) + .33(.293)$$

$$E(R_B) = .1160, \text{ or } 11.60\%$$

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance and standard deviation of Stock A are:

$$\sigma_A^2 = .33(.099 - .0903)^2 + .33(.113 - .0903)^2 + .33(.059 - .0903)^2$$

$$\sigma_A^2 = .00052$$

$$\sigma_A = .00052^{1/2}$$

$$\sigma_A = .0229, \text{ or } 2.29\%$$

And the standard deviation of Stock B is:

$$\sigma_B^2 = .33(-.073 - .1160)^2 + .33(.128 - .1160)^2 + .33(.293 - .1160)^2$$

$$\sigma_B^2 = .02240$$

$$\sigma_B = .02240^{1/2}$$

$$\sigma_B = .1497, \text{ or } 14.97\%$$

To find the covariance, we multiply each possible state times the product of each asset's deviation from the mean in that state. The sum of these products is the covariance. So, the covariance is:

$$\text{Cov}(A,B) = .33(.099 - .0903)(-.073 - .1160) + .33(.113 - .0903)(.128 - .1160)$$

$$+ .33(.059 - .0903)(.293 - .1160)$$

$$\text{Cov}(A,B) = -.002304$$

And the correlation is:

$$\rho_{A,B} = \text{Cov}(A,B)/\sigma_A\sigma_B$$

$$\rho_{A,B} = -.002304/((.0229)(.1497))$$

$$\rho_{A,B} = -.6728$$

26. The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each stock is:

$$E(R_J) = .30(-.050) + .55(.118) + .15(.274)$$

$$E(R_J) = .0910, \text{ or } 9.10\%$$

$$E(R_K) = .30(.029) + .55(.074) + .15(.098)$$

$$E(R_K) = .0641, \text{ or } 6.41\%$$

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance and standard deviation of Stock J are:

$$\sigma_J^2 = .30(-.050 - .0910)^2 + .55(.118 - .0910)^2 + .15(.274 - .0910)^2$$

$$\sigma_J^2 = .01139$$

$$\sigma_J = .01139^{1/2}$$

$$\sigma_J = .1067, \text{ or } 10.67\%$$

And the standard deviation of Stock K is:

$$\sigma_K^2 = .30(.029 - .0641)^2 + .55(.074 - .0641)^2 + .15(.098 - .0641)^2$$

$$\sigma_K^2 = .00060$$

$$\sigma_K = .00060^{1/2}$$

$$\sigma_K = .0244, \text{ or } 2.44\%$$

To find the covariance, we multiply each possible state times the product of each asset's deviation from the mean in that state. The sum of these products is the covariance. So, the covariance is:

$$\text{Cov}(J,K) = .30(-.050 - .0910)(.029 - .0641) + .55(.118 - .0910)(.074 - .0641) \\ + .15(.274 - .0910)(.098 - .0641)$$

$$\text{Cov}(J,K) = .002562$$

And the correlation is:

$$\rho_{J,K} = \text{Cov}(J,K)/\sigma_J\sigma_K$$

$$\rho_{J,K} = .002562/((.1067)(.0244))$$

$$\rho_{J,K} = .9836$$

27. a. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:

$$E(R_P) = X_F E(R_F) + X_G E(R_G)$$

$$E(R_P) = .40(.10) + .60(.13)$$

$$E(R_P) = .1180, \text{ or } 11.80\%$$

- b. The variance of a portfolio of two assets can be expressed as:

$$\sigma_P^2 = X_F^2 \sigma_F^2 + X_G^2 \sigma_G^2 + 2X_F X_G \sigma_F \sigma_G \rho_{F,G}$$

$$\sigma_P^2 = .40^2(.53^2) + .60^2(.79^2) + 2(.40)(.60)(.53)(.79)(.35)$$

$$\sigma_P^2 = .33996$$

So, the standard deviation is:

$$\sigma_P = .33996^{1/2}$$

$$\sigma_P = .5831, \text{ or } 58.31\%$$

28. a. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:

$$E(R_P) = X_A E(R_A) + X_B E(R_B)$$

$$E(R_P) = .40(.11) + .60(.13)$$

$$E(R_P) = .1220, \text{ or } 12.20\%$$

The variance of a portfolio of two assets can be expressed as:

$$\sigma_P^2 = X_A^2 \sigma_A^2 + X_B^2 \sigma_B^2 + 2X_A X_B \sigma_A \sigma_B \rho_{A,B}$$

$$\sigma_P^2 = .40^2(.47^2) + .60^2(.81^2) + 2(.40)(.60)(.47)(.81)(.50)$$

$$\sigma_P^2 = .36291$$

So, the standard deviation is:

$$\sigma_P = .36291^{1/2}$$

$$\sigma_P = .6024, \text{ or } 60.24\%$$

$$\begin{aligned} b. \quad \sigma_P^2 &= X_A^2 \sigma_A^2 + X_B^2 \sigma_B^2 + 2X_A X_B \sigma_A \sigma_B \rho_{A,B} \\ \sigma_P^2 &= .40^2(.47^2) + .60^2(.81^2) + 2(.40)(.60)(.47)(.81)(-.50) \\ \sigma_P^2 &= .18017 \end{aligned}$$

So, the standard deviation is:

$$\sigma = .18017^{1/2}$$

$$\sigma = .4245, \text{ or } 42.45\%$$

- c. As Stock A and Stock B become less correlated, or more negatively correlated, the standard deviation of the portfolio decreases.

29. a. (i) Using the equation to calculate beta, we find:

$$\begin{aligned} \beta_A &= (\rho_{A,M})(\sigma_A)/\sigma_M \\ .85 &= (\rho_{A,M})(.31)/.20 \\ \rho_{A,M} &= .55 \end{aligned}$$

- (ii) Using the equation to calculate beta, we find:

$$\begin{aligned} \beta_B &= (\rho_{B,M})(\sigma_B)/\sigma_M \\ 1.40 &= (.50)(\sigma_B)/.20 \\ \sigma_B &= .56 \end{aligned}$$

- (iii) Using the equation to calculate beta, we find:

$$\begin{aligned} \beta_C &= (\rho_{C,M})(\sigma_C)/\sigma_M \\ \beta_C &= (.35)(.65)/.20 \\ \beta_C &= 1.14 \end{aligned}$$

- (iv) The market has a correlation of 1 with itself.

- (v) The beta of the market is 1.

- (vi) The risk-free asset has zero standard deviation.

- (vii) The risk-free asset has zero correlation with the market portfolio.

- (viii) The beta of the risk-free asset is 0.

- b. Using the CAPM to find the expected return of the stock, we find:

Firm A:

$$E(R_A) = R_f + \beta_A[E(R_M) - R_f]$$

$$E(R_A) = .05 + .85(.12 - .05)$$

$$E(R_A) = .1095, \text{ or } 10.95\%$$

According to the CAPM, the expected return on Firm A's stock should be 10.95 percent. However, the expected return on Firm A's stock given in the table is only 10 percent. Therefore, Firm A's stock is overpriced, and you should sell it.

Firm B:

$$E(R_B) = R_f + \beta_B[E(R_M) - R_f]$$

$$E(R_B) = .05 + 1.4(.12 - .05)$$

$$E(R_B) = .1480, \text{ or } 14.80\%$$

According to the CAPM, the expected return on Firm B's stock should be 14.80 percent. However, the expected return on Firm B's stock given in the table is 14 percent. Therefore, Firm B's stock is overpriced, and you should sell it.

Firm C:

$$E(R_C) = R_f + \beta_C[E(R_M) - R_f]$$

$$E(R_C) = .05 + 1.14(.12 - .05)$$

$$E(R_C) = .1296, \text{ or } 12.96\%$$

According to the CAPM, the expected return on Firm C's stock should be 12.96 percent. However, the expected return on Firm C's stock given in the table is 16 percent. Therefore, Firm C's stock is underpriced, and you should buy it.

30. Because a well-diversified portfolio has no unsystematic risk, this portfolio should lie on the Capital Market Line (CML). The slope of the CML equals:

$$\text{Slope}_{\text{CML}} = [E(R_M) - R_f]/\sigma_M$$

$$\text{Slope}_{\text{CML}} = (.115 - .041)/.19$$

$$\text{Slope}_{\text{CML}} = .38947$$

- a. The expected return on the portfolio equals:

$$E(R_P) = R_f + \text{Slope}_{\text{CML}}(\sigma_P)$$

$$E(R_P) = .041 + .38947(.09)$$

$$E(R_P) = .0761, \text{ or } 7.61\%$$

- b. The expected return on the portfolio equals:

$$E(R_P) = R_f + \text{Slope}_{\text{CML}}(\sigma_P)$$

$$.20 = .041 + .38947(\sigma_P)$$

$$\sigma_P = .4082, \text{ or } 40.82\%$$

31. First, we can calculate the standard deviation of the market portfolio using the Capital Market Line (CML). We know that the risk-free asset has a return of 4.1 percent and a standard deviation of zero and the portfolio has an expected return of 9 percent and a standard deviation of 16 percent. These two points must lie on the Capital Market Line. The slope of the Capital Market Line equals:

$$\text{Slope}_{\text{CML}} = \text{Rise/Run}$$

$$\text{Slope}_{\text{CML}} = \text{Increase in expected return/Increase in standard deviation}$$

$$\text{Slope}_{\text{CML}} = (.09 - .041)/(.16 - 0)$$

$$\text{Slope}_{\text{CML}} = .3063$$

According to the Capital Market Line:

$$E(R_i) = R_f + \text{Slope}_{\text{CML}}(\sigma_i)$$

Since we know the expected return on the market portfolio, the risk-free rate, and the slope of the Capital Market Line, we can solve for the standard deviation of the market portfolio which is:

$$E(R_M) = R_f + \text{Slope}_{\text{CML}}(\sigma_M)$$

$$.11 = .041 + (.3063)(\sigma_M)$$

$$\sigma_M = (.11 - .041)/.3063$$

$$\sigma_M = .2253, \text{ or } 22.53\%$$

Next, we can use the standard deviation of the market portfolio to solve for the beta of a security using the beta equation. Doing so, we find the beta of the security is:

$$\beta_i = (\rho_{iM})(\sigma_i)/\sigma_M$$

$$\beta_i = (.38)(.60)/.2253$$

$$\beta_i = 1.01$$

Now we can use the beta of the security in the CAPM to find its expected return, which is:

$$E(R_i) = R_f + \beta_i[E(R_M) - R_f]$$

$$E(R_i) = .041 + 1.01(.11 - .041)$$

$$E(R_i) = .1108, \text{ or } 11.08\%$$

32. First, we need to find the standard deviation of the market and the portfolio, which are:

$$\sigma_M = .0391^{1/2}$$

$$\sigma_M = .1977, \text{ or } 19.77\%$$

$$\sigma_Z = .3407^{1/2}$$

$$\sigma_Z = .5837, \text{ or } 58.37\%$$

Now we can use the equation for beta to find the beta of the portfolio, which is:

$$\beta_Z = (\rho_{Z,M})(\sigma_Z)/\sigma_M$$

$$\beta_Z = (.31)(.5837)/.1977$$

$$\beta_Z = .92$$

Now, we can use the CAPM to find the expected return of the portfolio, which is:

$$\begin{aligned} E(R_Z) &= R_f + \beta_Z[E(R_M) - R_f] \\ E(R_Z) &= .044 + .92(.109 - .044) \\ E(R_Z) &= .1035, \text{ or } 10.35\% \end{aligned}$$

Challenge

33. The amount of systematic risk is measured by the β of an asset. Since we know the market risk premium and the risk-free rate, if we know the expected return of the asset we can use the CAPM to solve for the β of the asset. The expected return of Stock I is:

$$E(R_I) = .15(.12) + .55(.17) + .30(.06) = .1295, \text{ or } 12.95\%$$

Using the CAPM to find the β of Stock I, we find:

$$\begin{aligned} .1295 &= .04 + .075\beta_I \\ \beta_I &= 1.19 \end{aligned}$$

The total risk of an asset is measured by its standard deviation, so we need to calculate the standard deviation of Stock I. Beginning with the calculation of the stock's variance, we find:

$$\begin{aligned} \sigma_I^2 &= .15(.12 - .1295)^2 + .55(.17 - .1295)^2 + .30(.06 - .1295)^2 \\ \sigma_I^2 &= .00236 \end{aligned}$$

$$\sigma_I = .00236^{1/2} = .0486, \text{ or } 4.86\%$$

Using the same procedure for Stock II, we find the expected return to be:

$$E(R_{II}) = .15(-.35) + .55(.12) + .30(.33) = .1125, \text{ or } 11.25\%$$

Using the CAPM to find the β of Stock II, we find:

$$\begin{aligned} .1125 &= .04 + .075\beta_{II} \\ \beta_{II} &= .97 \end{aligned}$$

And the standard deviation of Stock II is:

$$\begin{aligned} \sigma_{II}^2 &= .15(-.35 - .1125)^2 + .55(.12 - .1125)^2 + .30(.33 - .1125)^2 \\ \sigma_{II}^2 &= .04631 \end{aligned}$$

$$\sigma_{II} = .04631^{1/2} = .2152, \text{ or } 21.52\%$$

Although Stock II has more total risk than I, it has much less systematic risk, since its beta is much smaller than I's. Thus, I has more systematic risk, and II has more unsystematic and more total risk. Since unsystematic risk can be diversified away, I is actually the "riskier" stock despite the lack of volatility in its returns. Stock I will have a higher risk premium and a greater expected return.

34. Here we have the expected return and beta for two assets. We can express the returns of the two assets using CAPM. If the CAPM is true, then the security market line holds as well, which means all assets have the same risk premium. Setting the reward-to-risk ratios of the assets equal to each other and solving for the risk-free rate, we find:

$$\begin{aligned} (.1328 - R_f)/1.25 &= (.1012 - R_f)/.85 \\ .85(.1328 - R_f) &= 1.25(.1012 - R_f) \\ .11288 - .85R_f &= .12650 - 1.25R_f \\ .40R_f &= .01362 \\ R_f &= .0341, \text{ or } 3.41\% \end{aligned}$$

Using CAPM to find the expected return on the market with both stocks, we find:

$$\begin{aligned} .1328 &= .0341 + 1.25(R_M - .0341) & .1012 &= .0341 + .85(R_M - .0341) \\ R_M &= .1131, \text{ or } 11.31\% & R_M &= .1131, \text{ or } 11.31\% \end{aligned}$$

35. a. The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the expected return and standard deviation of each stock are:

Asset 1:

$$E(R_1) = .15(.20) + .35(.15) + .35(.10) + .15(.05) = .1250, \text{ or } 12.50\%$$

$$\sigma_1^2 = .15(.20 - .1250)^2 + .35(.15 - .1250)^2 + .35(.10 - .1250)^2 + .15(.05 - .1250)^2 = .00213$$

$$\sigma_1 = (.00213)^{1/2} = .0461, \text{ or } 4.61\%$$

Asset 2:

$$E(R_2) = .15(.20) + .35(.10) + .35(.15) + .15(.05) = .1250, \text{ or } 12.50\%$$

$$\sigma_2^2 = .15(.20 - .1250)^2 + .35(.10 - .1250)^2 + .35(.15 - .1250)^2 + .15(.05 - .1250)^2 = .00213$$

$$\sigma_2 = (.00213)^{1/2} = .0461, \text{ or } 4.61\%$$

Asset 3:

$$E(R_3) = .15(.05) + .35(.10) + .35(.15) + .15(.20) = .1250, \text{ or } 12.50\%$$

$$\sigma_3^2 = .15(.05 - .1250)^2 + .35(.10 - .1250)^2 + .35(.15 - .1250)^2 + .15(.20 - .1250)^2 = .00213$$

$$\sigma_3 = (.00213)^{1/2} = .0461, \text{ or } 4.61\%$$

- b. To find the covariance, we multiply each possible state times the product of each asset's deviation from the mean in that state. The sum of these products is the covariance. The correlation is the covariance divided by the product of the two standard deviations. So, the covariance and correlation between each possible set of assets are:

Asset 1 and Asset 2:

$$\begin{aligned}\text{Cov}(1,2) &= .15(.20 - .1250)(.20 - .1250) + .35(.15 - .1250)(.10 - .1250) \\ &\quad + .35(.10 - .1250)(.15 - .1250) + .15(.05 - .1250)(.05 - .1250) \\ \text{Cov}(1,2) &= .00125\end{aligned}$$

$$\begin{aligned}\rho_{1,2} &= \text{Cov}(1,2)/\sigma_1 \sigma_2 \\ \rho_{1,2} &= .00125/ (.0461)(.0461) \\ \rho_{1,2} &= .5882\end{aligned}$$

Asset 1 and Asset 3:

$$\begin{aligned}\text{Cov}(1,3) &= .15(.20 - .1250)(.05 - .1250) + .35(.15 - .1250)(.10 - .1250) \\ &\quad + .35(.10 - .1250)(.15 - .1250) + .15(.05 - .1250)(.20 - .1250) \\ \text{Cov}(1,3) &= -.002125\end{aligned}$$

$$\begin{aligned}\rho_{1,3} &= \text{Cov}(1,3)/\sigma_1 \sigma_3 \\ \rho_{1,3} &= -.002125/ (.0461)(.0461) \\ \rho_{1,3} &= -1\end{aligned}$$

Asset 2 and Asset 3:

$$\begin{aligned}\text{Cov}(2,3) &= .15(.20 - .1250)(.05 - .1250) + .35(.10 - .1250)(.10 - .1250) \\ &\quad + .35(.15 - .1250)(.15 - .1250) + .15(.05 - .1250)(.20 - .1250) \\ \text{Cov}(2,3) &= -.00125\end{aligned}$$

$$\begin{aligned}\rho_{2,3} &= \text{Cov}(2,3)/\sigma_2 \sigma_3 \\ \rho_{2,3} &= -.00125/ (.0461)(.0461) \\ \rho_{2,3} &= -.5882\end{aligned}$$

- c. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 1 and Asset 2:

$$\begin{aligned}E(R_p) &= X_1E(R_1) + X_2E(R_2) \\ E(R_p) &= .50(.1250) + .50(.1250) \\ E(R_p) &= .1250, \text{ or } 12.50\%\end{aligned}$$

The variance of a portfolio of two assets can be expressed as:

$$\begin{aligned}\sigma_p^2 &= X_1^2 \sigma_1^2 + X_2^2 \sigma_2^2 + 2X_1X_2\sigma_1\sigma_2\rho_{1,2} \\ \sigma_p^2 &= .50^2(.0461^2) + .50^2(.0461^2) + 2(.50)(.50)(.0461)(.0461)(.5882) \\ \sigma_p^2 &= .001688\end{aligned}$$

And the standard deviation of the portfolio is:

$$\begin{aligned}\sigma_p &= (.001688)^{1/2} \\ \sigma_p &= .0411, \text{ or } 4.11\%\end{aligned}$$

- d. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 1 and Asset 3:

$$\begin{aligned} E(R_p) &= X_1E(R_1) + X_3E(R_3) \\ E(R_p) &= .50(.1250) + .50(.1250) \\ E(R_p) &= .1250, \text{ or } 12.50\% \end{aligned}$$

The variance of a portfolio of two assets can be expressed as:

$$\begin{aligned} \sigma_p^2 &= X_1^2 \sigma_1^2 + X_3^2 \sigma_3^2 + 2X_1X_3\sigma_1\sigma_3\rho_{1,3} \\ \sigma_p^2 &= .50^2(.0461^2) + .50^2(.0461^2) + 2(.50)(.50)(.0461)(.0461)(-1) \\ \sigma_p^2 &= .000000 \end{aligned}$$

Since the variance is zero, the standard deviation is also zero.

- e. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 2 and Asset 3:

$$\begin{aligned} E(R_p) &= X_2E(R_2) + X_3E(R_3) \\ E(R_p) &= .50(.1250) + .50(.1250) \\ E(R_p) &= .1250, \text{ or } 12.50\% \end{aligned}$$

The variance of a portfolio of two assets can be expressed as:

$$\begin{aligned} \sigma_p^2 &= X_2^2 \sigma_2^2 + X_3^2 \sigma_3^2 + 2X_2X_3\sigma_2\sigma_3\rho_{2,3} \\ \sigma_p^2 &= .50^2(.0461^2) + .50^2(.0461^2) + 2(.50)(.50)(.0461)(.0461)(-.5882) \\ \sigma_p^2 &= .000438 \end{aligned}$$

And the standard deviation of the portfolio is:

$$\begin{aligned} \sigma_p &= (.000438)^{1/2} \\ \sigma_p &= .0209, \text{ or } 2.09\% \end{aligned}$$

- f. As long as the correlation between the returns on two securities is below 1, there is a benefit to diversification. A portfolio with negatively correlated securities can achieve greater risk reduction than a portfolio with positively correlated securities, holding the expected return on each stock constant. Applying proper weights on perfectly negatively correlated securities can reduce portfolio variance to 0.

36. a. The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each stock is:

$$E(R_A) = .15(-.13) + .60(.12) + .25(.34)$$

$$E(R_A) = .1375, \text{ or } 13.75\%$$

$$E(R_B) = .15(-.11) + .60(.10) + .25(.31)$$

$$E(R_B) = .1210, \text{ or } 12.10\%$$

- b. We can use the expected returns we calculated to find the slope of the Security Market Line. We know that the beta of Stock A is .25 greater than the beta of Stock B. Therefore, as beta increases by .25, the expected return on a security increases by .0165 ($= .1375 - .1210$). The slope of the security market line (SML) equals:

$$\text{Slope}_{\text{SML}} = \text{Rise/Run}$$

$$\text{Slope}_{\text{SML}} = \text{Increase in expected return/Increase in beta}$$

$$\text{Slope}_{\text{SML}} = (.1375 - .1210)/.25$$

$$\text{Slope}_{\text{SML}} = .0660, \text{ or } 6.60\%$$

Since the market's beta is 1 and the risk-free rate has a beta of zero, the slope of the Security Market Line equals the expected market risk premium. So, the expected market risk premium must be 6.6 percent.

We could also solve this problem using CAPM. The equations for the expected returns of the two stocks are:

$$.1375 = R_f + (\beta_B + .25)(\text{MRP})$$

$$.1210 = R_f + \beta_B(\text{MRP})$$

We can rewrite the CAPM equation for Stock A as:

$$.165 = R_f + \beta_B(\text{MRP}) + .25(\text{MRP})$$

Subtracting the CAPM equation for Stock B from this equation yields:

$$.0165 = .25\text{MRP}$$

$$\text{MRP} = .0660, \text{ or } 6.60\%$$

which is the same answer as our previous result.

37. a. A typical, risk-averse investor seeks high returns and low risks. For a risk-averse investor holding a well-diversified portfolio, beta is the appropriate measure of the risk of an individual security. To assess the two stocks, we need to find the expected return and beta of each of the two securities.

Stock A:

Since Stock A pays no dividends, the return on Stock A is: $(P_1 - P_0)/P_0$. So, the return for each state of the economy is:

$$R_{\text{Recession}} = (\$64 - \$75)/\$75 = -.147, \text{ or } -14.70\%$$

$$R_{\text{Normal}} = (\$87 - \$75)/\$75 = .160, \text{ or } 16.00\%$$

$$R_{\text{Expanding}} = (\$97 - \$75)/\$75 = .293, \text{ or } 29.30\%$$

The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of the stock is:

$$E(R_A) = .20(-.147) + .60(.160) + .20(.293) = .1253, \text{ or } 12.53\%$$

And the variance of the stock is:

$$\sigma_A^2 = .20(-.147 - .1253)^2 + .60(.160 - .1253)^2 + .20(.293 - .1253)^2$$

$$\sigma_A^2 = .0212$$

Which means the standard deviation is:

$$\sigma_A = .0212^{1/2}$$

$$\sigma_A = .1455, \text{ or } 14.55\%$$

Now we can calculate the stock's beta, which is:

$$\beta_A = (\rho_{A,M})(\sigma_A)/\sigma_M$$

$$\beta_A = (.70)(.1455)/.18$$

$$\beta_A = .566$$

For Stock B, we can directly calculate the beta from the information provided. So, the beta for Stock B is:

Stock B:

$$\beta_B = (\rho_{B,M})(\sigma_B)/\sigma_M$$

$$\beta_B = (.24)(.34)/.18$$

$$\beta_B = .453$$

The expected return on Stock B is higher than the expected return on Stock A. The risk of Stock B, as measured by its beta, is lower than the risk of Stock A. Thus, a typical risk-averse investor holding a well-diversified portfolio will prefer Stock B. Note, this situation implies that at least one of the stocks is mispriced since the higher risk (beta) stock has a lower return than the lower risk (beta) stock.

- b. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:

$$\begin{aligned} E(R_P) &= X_A E(R_A) + X_B E(R_B) \\ E(R_P) &= .70(.1253) + .30(.14) \\ E(R_P) &= .1297, \text{ or } 12.97\% \end{aligned}$$

To find the standard deviation of the portfolio, we first need to calculate the variance. The variance of the portfolio is:

$$\begin{aligned} \sigma_P^2 &= X_A^2 \sigma_A^2 + X_B^2 \sigma_B^2 + 2X_A X_B \sigma_A \sigma_B \rho_{A,B} \\ \sigma_P^2 &= (.70)^2 (.1455)^2 + (.30)^2 (.34)^2 + 2(.70)(.30)(.1455)(.34)(.36) \\ \sigma_P^2 &= .02825 \end{aligned}$$

And the standard deviation of the portfolio is:

$$\begin{aligned} \sigma_P &= .02825^{1/2} \\ \sigma_P &= .1681, \text{ or } 16.81\% \end{aligned}$$

- c. The beta of a portfolio is the weighted average of the betas of its individual securities. So the beta of the portfolio is:

$$\begin{aligned} \beta_P &= .70(.566) + .30(.453) \\ \beta_P &= .532 \end{aligned}$$

38. a. The variance of a portfolio of two assets equals:

$$\sigma_P^2 = X_A^2 \sigma_A^2 + X_B^2 \sigma_B^2 + 2X_A X_B \text{Cov}(A, B)$$

Since the weights of the assets must sum to one, we can write the variance of the portfolio as:

$$\sigma_P^2 = X_A^2 \sigma_A^2 + (1 - X_A)^2 \sigma_B^2 + 2X_A(1 - X_A)\text{Cov}(A, B)$$

To find the minimum for any function, we find the derivative and set the derivative equal to zero. Finding the derivative of the variance function with respect to the weight of Asset A, setting the derivative equal to zero, and solving for the weight of Asset A, we find:

$$X_A = [\sigma_B^2 - \text{Cov}(A, B)] / [\sigma_A^2 + \sigma_B^2 - 2\text{Cov}(A, B)]$$

Using this expression, we find the weight of Asset A must be:

$$\begin{aligned} X_A &= (.77^2 - .001) / [.41^2 + .77^2 - 2(.001)] \\ X_A &= .7798 \end{aligned}$$

This means the weight of Stock B is:

$$X_B = 1 - X_A$$

$$X_B = 1 - .7798$$

$$X_B = .2202$$

- b. Using the weights calculated in part a, the expected return of the portfolio is:

$$E(R_P) = X_A E(R_A) + X_B E(R_B)$$

$$E(R_P) = .7798(.10) + .2202(.16)$$

$$E(R_P) = .1132, \text{ or } 11.32\%$$

- c. Using the derivative from part a, with the new covariance, the weight of each stock in the minimum variance portfolio is:

$$X_A = [\sigma_B^2 - \text{Cov}(A,B)] / [\sigma_A^2 + \sigma_B^2 - 2\text{Cov}(A,B)]$$

$$X_A = (.77^2 + .05) / [.41^2 + .77^2 - 2(-.05)]$$

$$X_A = .7467$$

This implies the weight of Stock B is:

$$X_B = 1 - X_A$$

$$X_B = 1 - .7467$$

$$X_B = .2533$$

- d. The variance of the portfolio with the weights in part c is:

$$\sigma_P^2 = X_A^2 \sigma_A^2 + X_B^2 \sigma_B^2 + 2X_A X_B \text{Cov}(A,B)$$

$$\sigma_P^2 = (.7467)^2 (.41)^2 + (.2533)^2 (.77)^2 + 2(.7467)(.2533)(-.05)$$

$$\sigma_P^2 = .1129$$

And the standard deviation of the portfolio is:

$$\sigma_P = .1129^{1/2}$$

$$\sigma_P = .3359, \text{ or } 33.59\%$$

CHAPTER 12

AN ALTERNATIVE VIEW OF RISK AND RETURN: *THE ARBITRAGE PRICING THEORY*

Answers to Concept Questions

1. Systematic risk is risk that cannot be diversified away through formation of a portfolio. Generally, systematic risk factors are those factors that affect a large number of firms in the market, however, those factors will not necessarily affect all firms equally. Unsystematic risk is the type of risk that can be diversified away through portfolio formation. Unsystematic risk factors are specific to the firm or industry. Surprises in these factors will affect the returns of the firm in which you are interested, but they will have no effect on the returns of firms in a different industry and perhaps little effect on other firms in the same industry.
2. Any return can be explained with a large enough number of systematic risk factors. However, for a factor model to be useful as a practical matter, the number of factors that explain the returns on an asset must be relatively limited.
3. The market risk premium and inflation rates are probably good choices. The price of wheat, while a risk factor for Ultra Bread, is not a market risk factor and will not likely be priced as a risk factor common to all stocks. In this case, wheat would be a firm specific risk factor, not a market risk factor. A better model would employ macroeconomic risk factors such as interest rates, GDP, energy prices, and industrial production, among others.
4.
 - a. Real GNP was higher than anticipated. Since returns are positively related to the level of GNP, returns should rise based on this factor.
 - b. Inflation was exactly the amount anticipated. Since there was no surprise in this announcement, it will not affect Lewis-Striden returns.
 - c. Interest rates are lower than anticipated. Since returns are negatively related to interest rates, the lower than expected rate is good news. Returns should rise due to interest rates.
 - d. The president's death is bad news. Although the president was expected to retire, his retirement would not be effective for six months. During that period, he would still contribute to the firm. His untimely death means that those contributions will not be made. Since he was generally considered an asset to the firm, his death will cause returns to fall. However, since his departure was expected soon, the drop might not be very large.
 - e. The poor research results are also bad news. Since Lewis-Striden must continue to test the drug, it will not go into production as early as expected. The delay will affect expected future earnings, and thus it will dampen returns now.
 - f. The research breakthrough is positive news for Lewis Striden. Since it was unexpected, it will cause returns to rise.

- g. The competitor's announcement is also unexpected, but it is not a welcome surprise. This announcement will lower the returns on Lewis-Striden.

The systematic factors in the list are real GNP, inflation, and interest rates. The unsystematic risk factors are the president's ability to contribute to the firm, the research results, and the competitor.

5. The main difference is that the market model assumes that only one factor, usually a stock market aggregate, is enough to explain stock returns, while a k -factor model relies on k factors to explain returns.
6. The fact that APT does not give any guidance about the factors that influence stock returns is a commonly-cited criticism. However, in choosing factors, we should choose factors that have an economically valid reason for potentially affecting stock returns. For example, a smaller company has more risk than a larger company. Therefore, the size of a company can affect the returns of the company stock.
7. Assuming the market portfolio is properly scaled, it can be shown that the one-factor model is identical to the CAPM.
8. It is the weighted average of expected returns plus the weighted average of each security's beta times a factor F plus the weighted average of the unsystematic risks of the individual securities.
9. Choosing variables because they have been shown to be related to returns is data mining. The relation found between some attribute and returns can be accidental, thus overstated. For example, the occurrence of sunburns and ice cream consumption are related; however, sunburns do not necessarily cause ice cream consumption, or vice versa. For a factor to truly be related to asset returns, there should be sound economic reasoning for the relationship, not just a statistical one.
10. Using a benchmark composed of British stocks is wrong because the stocks included are not of the same style as those in a U.S. growth stock fund.

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. Since we have the expected return of the stock, the revised expected return can be determined using the innovation, or surprise, in the risk factors. So, the revised expected return is:

$$R = .102 + 1.3(.032 - .035) - .47(.027 - .029)$$

$$R = .0990, \text{ or } 9.90\%$$

2. a. If m is the systematic risk portion of return, then:

$$m = \beta_{\text{GDP}}\Delta\text{GDP} + \beta_{\text{Inflation}}\Delta\text{Inflation} + \beta_r\Delta\text{Interest rates}$$

$$m = .0000734(\$19,843 - 19,571) - .90(.0270 - .0260) - .32(.0320 - .0340)$$

$$m = .0197, \text{ or } 1.97\%$$

- b. The unsystematic return is the return that occurs because of a firm specific factor such as the bad news about the company. So, the unsystematic return of the stock is $-.85$ percent. The total return is the expected return, plus the two components of unexpected return: the systematic risk portion of return and the unsystematic portion. So, the total return of the stock is:

$$\begin{aligned} R &= \bar{R} + m + \varepsilon \\ R &= .1090 + .0197 - .0085 \\ R &= .1202, \text{ or } 12.02\% \end{aligned}$$

3. a. If m is the systematic risk portion of return, then:

$$\begin{aligned} m &= \beta_{\text{GNP}} \Delta \% \text{GNP} + \beta_r \Delta \text{Interest rates} \\ m &= 1.47(.024 - .019) - .87(.036 - .032) \\ m &= .0039, \text{ or } .39\% \end{aligned}$$

- b. The unsystematic return is the return that occurs because of a firm specific factor such as the increase in market share. If ε is the unsystematic risk portion of the return, then:

$$\begin{aligned} \varepsilon &= .58(.15 - .11) \\ \varepsilon &= .0232, \text{ or } 2.32\% \end{aligned}$$

- c. The total return is the expected return, plus the two components of unexpected return: the systematic risk portion of return and the unsystematic portion. So, the total return of the stock is:

$$\begin{aligned} R &= \bar{R} + m + \varepsilon \\ R &= .105 + .0039 + .0232\% \\ R &= .1321, \text{ or } 13.21\% \end{aligned}$$

4. The beta for a particular risk factor in a portfolio is the weighted average of the betas of the assets. This is true whether the betas are from a single factor model or a multi-factor model. So, the betas of the portfolio are:

$$\begin{aligned} F_1 &= .20(1.55) + .20(.81) + .60(.73) \\ F_1 &= .91 \end{aligned}$$

$$\begin{aligned} F_2 &= .20(.80) + .20(1.25) + .60(-.14) \\ F_2 &= .33 \end{aligned}$$

$$\begin{aligned} F_3 &= .20(.05) + .20(-.20) + .60(1.24) \\ F_3 &= .71 \end{aligned}$$

So, the expression for the return of the portfolio is:

$$R_i = 3.2\% + .91F_1 + .33F_2 - .71F_3$$

Which means the return of the portfolio is:

$$\begin{aligned} R_i &= 3.2\% + .91(4.90\%) + .33(3.80\%) - .71(5.30\%) \\ R_i &= 5.11\% \end{aligned}$$

Intermediate

5. We can express the multifactor model for each portfolio as:

$$E(R_P) = R_F + \beta_1 F_1 + \beta_2 F_2$$

where F_1 and F_2 are the respective risk premiums for each factor. Expressing the return equation for each portfolio, we get:

$$16\% = 4\% + .85F_1 + 1.15F_2$$

$$12\% = 4\% + 1.45F_1 - .25F_2$$

We can solve the system of two equations with two unknowns. Multiplying each equation by the respective F_2 factor for the other equation, we get:

$$4.00\% = 1.0\% + .2125F_1 + .2875F_2$$

$$13.8\% = 4.6\% + 1.6675F_1 - .2875F_2$$

Summing the equations and solving F_1 for gives us:

$$17.8\% = 5.6\% + 1.88F_1$$

$$F_1 = 6.49\%$$

And now, using the equation for Portfolio A, we can solve for F_2 , which is:

$$16\% = 4\% + .85(6.49\%) + 1.15F_2$$

$$F_2 = 5.64\%$$

6. a. The market model is specified by:

$$R = \bar{R} + \beta(R_M - \bar{R}_M) + \varepsilon$$

so applying that to each stock:

Stock A:

$$R_A = \bar{R}_A + \beta_A(R_M - \bar{R}_M) + \varepsilon_A$$

$$R_A = 10.5\% + 1.2(R_M - 14.2\%) + \varepsilon_A$$

Stock B:

$$R_B = \bar{R}_B + \beta_B(R_M - \bar{R}_M) + \varepsilon_B$$

$$R_B = 13.0\% + .98(R_M - 14.2\%) + \varepsilon_B$$

Stock C:

$$R_C = \bar{R}_C + \beta_C(R_M - \bar{R}_M) + \varepsilon_C$$

$$R_C = 15.7\% + 1.37(R_M - 14.2\%) + \varepsilon_C$$

- b. Since we don't have the actual market return or unsystematic risk, we will get a formula with those values as unknowns:

$$\begin{aligned}
 R_P &= .30R_A + .45R_B + .25R_C \\
 R_P &= .30[10.5\% + 1.2(R_M - 14.2\%) + \varepsilon_A] + .45[13.0\% + .98(R_M - 14.2\%) + \varepsilon_B] \\
 &\quad + .25[15.7\% + 1.37(R_M - 14.2\%) + \varepsilon_C] \\
 R_P &= .30(10.5\%) + .45(13\%) + .25(15.7\%) + [.30(1.2) + .45(.98) + .25(1.37)](R_M - 14.2\%) \\
 &\quad + .30\varepsilon_A + .45\varepsilon_B + .25\varepsilon_C \\
 R_P &= 12.925\% + 1.1435(R_M - 14.2\%) + .30\varepsilon_A + .45\varepsilon_B + .25\varepsilon_C
 \end{aligned}$$

- c. Using the market model, if the return on the market is 15 percent and the systematic risk is zero, the return for each individual stock is:

$$\begin{aligned}
 R_A &= 10.5\% + 1.20(15\% - 14.2\%) \\
 R_A &= 11.46\%
 \end{aligned}$$

$$\begin{aligned}
 R_B &= 13\% + .98(15\% - 14.2\%) \\
 R_B &= 13.78\%
 \end{aligned}$$

$$\begin{aligned}
 R_C &= 15.70\% + 1.37(15\% - 14.2\%) \\
 R_C &= 16.80\%
 \end{aligned}$$

To calculate the return on the portfolio, we can use the equation from part b, so:

$$\begin{aligned}
 R_P &= 12.925\% + 1.1435(15\% - 14.2\%) \\
 R_P &= 13.84\%
 \end{aligned}$$

Alternatively, to find the portfolio return, we can use the return of each asset and its portfolio weight, or:

$$\begin{aligned}
 R_P &= X_1R_1 + X_2R_2 + X_3R_3 \\
 R_P &= .30(11.46\%) + .45(13.78\%) + .25(16.80\%) \\
 R_P &= 13.84\%
 \end{aligned}$$

7. a. Since the five stocks have the same expected returns and the same betas, the portfolio also has the same expected return and beta. However, the unsystematic risks might be different, so the expected return of the portfolio is:

$$\bar{R}_P = 11\% + .84F_1 + 1.69F_2 + (1/5)(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \varepsilon_5)$$

- b. Consider the expected return equation of a portfolio of five assets that we calculated in part a. Since we now have a very large number of stocks in the portfolio, as:

$$N \rightarrow \infty, \quad \frac{1}{N} \rightarrow 0$$

But, the ε s are infinite, so:

$$(1/N)(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \dots + \varepsilon_N) \rightarrow 0$$

Thus:

$$R_p = 11\% + .84F_1 + 1.69F_2$$

Challenge

8. To determine which investment an investor would prefer, you must compute the variance of portfolios created by many stocks from either market. Because you know that diversification is good, it is reasonable to assume that once an investor has chosen the market in which she will invest, she will buy many stocks in that market.

Known:

$$E_F = 0 \text{ and } \sigma = .10$$

$$E_\varepsilon = 0 \text{ and } S_{\varepsilon i} = .20 \text{ for all } i$$

If we assume the stocks in the portfolio are equally-weighted, the weight of each stock is $\frac{1}{N}$, that is:

$$X_i = \frac{1}{N} \text{ for all } i$$

If a portfolio is composed of N stocks each forming $1/N$ proportion of the portfolio, the return on the portfolio is $1/N$ times the sum of the returns on the N stocks. To find the variance of the respective portfolios in the two markets, we need to use the definition of variance from statistics:

$$\text{Var}(x) = E[x - E(x)]^2$$

In our case:

$$\text{Var}(R_p) = E[R_p - E(R_p)]^2$$

Note however, to use this, first we must find R_p and $E(R_p)$. So, using the assumption about equal weights and then substituting in the known equation for R_i :

$$\begin{aligned} R_p &= \frac{1}{N} \sum R_i \\ R_p &= \frac{1}{N} \sum (.10 + \beta F + \varepsilon_i) \\ R_p &= .10 + \beta F + \frac{1}{N} \sum \varepsilon_i \end{aligned}$$

Also, recall from statistics a property of expected value, that is:

$$\text{If: } \tilde{Z} = a\tilde{X} + \tilde{Y}$$

where a is a constant, and \tilde{Z} , \tilde{X} , and \tilde{Y} are random variables, then:

$$E(\tilde{Z}) = E(a)\tilde{X} + E(\tilde{Y})$$

and

$$E(a) = a$$

Now use the above to find $E(R_p)$:

$$\begin{aligned} E(R_p) &= E\left(.10 + \beta F + \frac{1}{N} \sum \varepsilon_i\right) \\ E(R_p) &= .10 + \beta E(F) + \frac{1}{N} \sum E(\varepsilon_i) \\ E(R_p) &= .10 + \beta(0) + \frac{1}{N} \sum 0 \\ E(R_p) &= .10 \end{aligned}$$

Next, substitute both of these results into the original equation for variance:

$$\begin{aligned} \text{Var}(R_p) &= E[R_p - E(R_p)]^2 \\ \text{Var}(R_p) &= E\left[.10 + \beta F + \frac{1}{N} \sum \varepsilon_i - .10\right]^2 \\ \text{Var}(R_p) &= E\left[\beta F + \frac{1}{N} \sum \varepsilon\right]^2 \\ \text{Var}(R_p) &= E\left[\beta^2 F^2 + 2\beta F \frac{1}{N} \sum \varepsilon + \frac{1}{N^2} \left(\sum \varepsilon\right)^2\right] \\ \text{Var}(R_p) &= \left[\beta^2 \sigma^2 + \frac{1}{N} \sigma^2 \varepsilon + \left(1 - \frac{1}{N}\right) \text{Cov}(\varepsilon_i, \varepsilon_j)\right]^2 \end{aligned}$$

Finally, since we can have as many stocks in each market as we want, in the limit, as $N \rightarrow \infty$,

$$\frac{1}{N} \rightarrow 0, \text{ so we get:}$$

$$\text{Var}(R_p) = \beta^2 \sigma^2 + \text{Cov}(\varepsilon_i, \varepsilon_j)$$

and, since:

$$\text{Cov}(\varepsilon_i, \varepsilon_j) = \sigma_i \sigma_j \rho(\varepsilon_i, \varepsilon_j)$$

and the problem states that $\sigma_1 = \sigma_2 = .10$, so:

$$\begin{aligned}\text{Var}(R_p) &= \beta^2 \sigma^2 + \sigma_1 \sigma_2 \rho(\varepsilon_i, \varepsilon_j) \\ \text{Var}(R_p) &= \beta^2 (.01) + .04 \rho(\varepsilon_i, \varepsilon_j)\end{aligned}$$

So now, to summarize what we have so far:

$$\begin{aligned}R_{1i} &= .10 + 1.5F + \varepsilon_{1i} \\ R_{2i} &= .10 + .5F + \varepsilon_{2i} \\ E(R_{1p}) &= E(R_{2p}) = .10 \\ \text{Var}(R_{1p}) &= .0225 + .04\rho(\varepsilon_{1i}, \varepsilon_{1j}) \\ \text{Var}(R_{2p}) &= .0025 + .04\rho(\varepsilon_{2i}, \varepsilon_{2j})\end{aligned}$$

Finally we can begin answering parts *a*, *b*, and *c* for various values of the correlations:

- a.* Substitute $\rho(\varepsilon_{1i}, \varepsilon_{1j}) = \rho(\varepsilon_{2i}, \varepsilon_{2j}) = 0$ into the respective variance formulas:

$$\begin{aligned}\text{Var}(R_{1p}) &= .0225 \\ \text{Var}(R_{2p}) &= .0025\end{aligned}$$

Since $\text{Var}(R_{1p}) > \text{Var}(R_{2p})$, and expected returns are equal, a risk averse investor will prefer to invest in the second market.

- b.* If we assume $\rho(\varepsilon_{1i}, \varepsilon_{1j}) = .9$, and $\rho(\varepsilon_{2i}, \varepsilon_{2j}) = 0$, the variance of each portfolio is:

$$\begin{aligned}\text{Var}(R_{1p}) &= .0225 + .04\rho(\varepsilon_{1i}, \varepsilon_{1j}) \\ \text{Var}(R_{1p}) &= .0225 + .04(.9) \\ \text{Var}(R_{1p}) &= .0585\end{aligned}$$

$$\begin{aligned}\text{Var}(R_{2p}) &= .0025 + .04\rho(\varepsilon_{2i}, \varepsilon_{2j}) \\ \text{Var}(R_{2p}) &= .0025 + .04(0) \\ \text{Var}(R_{2p}) &= .0025\end{aligned}$$

Since $\text{Var}(R_{1p}) > \text{Var}(R_{2p})$, and expected returns are equal, a risk averse investor will prefer to invest in the second market.

- c.* If we assume $\rho(\varepsilon_{1i}, \varepsilon_{1j}) = 0$, and $\rho(\varepsilon_{2i}, \varepsilon_{2j}) = .5$, the variance of each portfolio is:

$$\begin{aligned}\text{Var}(R_{1p}) &= .0225 + .04\rho(\varepsilon_{1i}, \varepsilon_{1j}) \\ \text{Var}(R_{1p}) &= .0225 + .04(0) \\ \text{Var}(R_{1p}) &= .0225\end{aligned}$$

$$\begin{aligned}\text{Var}(R_{2p}) &= .0025 + .04\rho(\varepsilon_{2i}, \varepsilon_{2j}) \\ \text{Var}(R_{2p}) &= .0025 + .04(.5) \\ \text{Var}(R_{2p}) &= .0225\end{aligned}$$

Since $\text{Var}(R_{1p}) = \text{Var}(R_{2p})$, and expected returns are equal, a risk averse investor will be indifferent between the two markets.

- d. Since the expected returns are equal, indifference implies that the variances of the portfolios in the two markets are also equal. So, set the variance equations equal, and solve for the correlation of one market in terms of the other:

$$\begin{aligned}\text{Var}(R_{1P}) &= \text{Var}(R_{2P}) \\ .0225 + .04\rho(\varepsilon_{1i}, \varepsilon_{1j}) &= .0025 + .04\rho(\varepsilon_{2i}, \varepsilon_{2j}) \\ \rho(\varepsilon_{2i}, \varepsilon_{2j}) &= \rho(\varepsilon_{1i}, \varepsilon_{1j}) + .5\end{aligned}$$

Therefore, for any set of correlations that have this relationship (as found in part c), a risk adverse investor will be indifferent between the two markets.

9. a. In order to find standard deviation, σ , you must first find the variance, since $\sigma = \sqrt{\text{Var}}$. Recall from statistics a property of variance:

$$\text{If: } \tilde{Z} = a\tilde{X} + \tilde{Y}$$

where a is a constant, and \tilde{Z} , \tilde{X} , and \tilde{Y} are random variables, then:

$$\text{Var}(\tilde{Z}) = a^2 \text{Var}(\tilde{X}) + \text{Var}(\tilde{Y})$$

and:

$$\text{Var}(a) = 0$$

The problem states that return-generation can be described by:

$$R_{i,t} = \alpha_i + \beta_i(R_M) + \varepsilon_{i,t}$$

Realize that $R_{i,t}$, R_M , and $\varepsilon_{i,t}$ are random variables, and α_i and β_i are constants. Then, applying the above properties to this model, we get:

$$\text{Var}(R_i) = \beta_i^2 \text{Var}(R_M) + \text{Var}(\varepsilon_i)$$

and now we can find the standard deviation for each asset:

$$\begin{aligned}\sigma_A^2 &= .70^2(.0121) + .01 = .015929 \\ \sigma_A &= \sqrt{.015929} = .1262, \text{ or } 12.62\%\end{aligned}$$

$$\begin{aligned}\sigma_B^2 &= 1.2^2(.0121) + .0144 = .031824 \\ \sigma_B &= \sqrt{.031824} = .1784, \text{ or } 17.84\%\end{aligned}$$

$$\begin{aligned}\sigma_C^2 &= 1.5^2(.0121) + .0225 = .049725 \\ \sigma_C &= \sqrt{.049725} = .2230, \text{ or } 22.30\%\end{aligned}$$

- b. From the above formula for variance, note that as $N \rightarrow \infty$, $\frac{\text{Var}(\varepsilon_i)}{N} \rightarrow 0$, so you get:

$$\text{Var}(R_i) = \beta_i^2 \text{Var}(R_M)$$

So, the variances for the assets are:

$$\begin{aligned}\sigma_A^2 &= .7^2(.0121) = .005929 \\ \sigma_B^2 &= 1.2^2(.0121) = .017424 \\ \sigma_C^2 &= 1.5^2(.0121) = .027225\end{aligned}$$

- c. We can use the model:

$$R_i = R_F + \beta_i(\bar{R}_M - R_F)$$

which is the CAPM (or APT Model when there is one factor and that factor is the Market). So, the expected return of each asset is:

$$\begin{aligned}\bar{R}_A &= 3.3\% + .70(10.6\% - 3.3\%) = 8.41\% \\ \bar{R}_B &= 3.3\% + 1.2(10.6\% - 3.3\%) = 12.06\% \\ \bar{R}_C &= 3.3\% + 1.5(10.6\% - 3.3\%) = 14.25\%\end{aligned}$$

We can compare these results for expected asset returns as per CAPM or APT with the expected returns given in the table. This shows that Assets A and B are accurately priced, but Asset C is overpriced (the model shows the return should be higher). Thus, rational investors will not hold Asset C.

- d. If short selling is allowed, rational investors will sell short Asset C, causing the price of Asset C to decrease until no arbitrage opportunity exists. In other words, the price of Asset C should decrease until the return becomes 14.25 percent.

10. a. Let:

X_1 = the proportion of Security 1 in the portfolio, and
 X_2 = the proportion of Security 2 in the portfolio

and note that since the weights must sum to 1.0:

$$X_1 = 1 - X_2$$

Recall from Chapter 11 that the beta for a portfolio (or in this case the beta for a factor) is the weighted average of the security betas, so

$$\begin{aligned}\beta_{P1} &= X_1\beta_{11} + X_2\beta_{21} \\ \beta_{P1} &= X_1\beta_{11} + (1 - X_1)\beta_{21}\end{aligned}$$

Now, apply the condition given in the hint that the return of the portfolio does not depend on F_1 . This means that the portfolio beta for that factor will be 0, so:

$$\begin{aligned}\beta_{P1} &= 0 = X_1\beta_{11} + (1 - X_1)\beta_{21} \\ \beta_{P1} &= 0 = X_1(1.0) + (1 - X_1)(.5)\end{aligned}$$

and solving for X_1 and X_2 :

$$\begin{aligned}X_1 &= -1 \\ X_2 &= 2\end{aligned}$$

Thus, sell short Security 1 and buy Security 2.

To find the expected return on that portfolio, use

$$R_p = X_1R_1 + X_2R_2$$

so applying the above:

$$\begin{aligned}E(R_p) &= -1(20\%) + 2(20\%) \\ E(R_p) &= 20\%\end{aligned}$$

$$\begin{aligned}\beta_{P1} &= -1(1) + 2(.5) \\ \beta_{P1} &= 0\end{aligned}$$

- b. Following the same logic as in part a, we have

$$\begin{aligned}\beta_{P2} &= 0 = X_3\beta_{31} + (1 - X_3)\beta_{41} \\ \beta_{P2} &= 0 = X_3(1) + (1 - X_3)(1.5)\end{aligned}$$

and

$$\begin{aligned}X_3 &= 3 \\ X_4 &= -2\end{aligned}$$

Thus, sell short Security 4 and buy Security 3. Then,

$$\begin{aligned}E(R_{P2}) &= 3(10\%) + (-2)(10\%) \\ E(R_{P2}) &= 10\%\end{aligned}$$

$$\begin{aligned}\beta_{P2} &= 3(.5) - 2(.75) \\ \beta_{P2} &= 0\end{aligned}$$

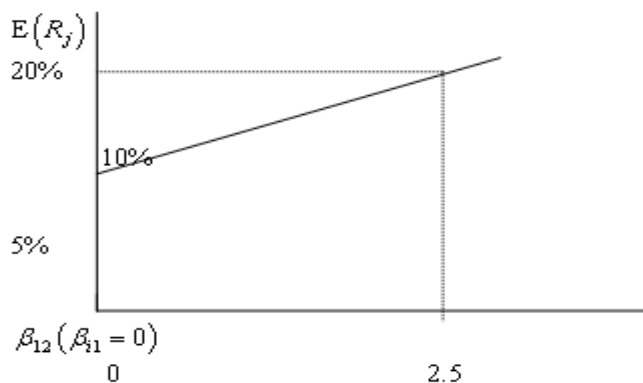
Note that since both β_{P1} and β_{P2} are 0, this is a risk-free portfolio!

- c. The portfolio in part b provides a risk-free return of 10%, which is higher than the 5% return provided by the risk-free security. To take advantage of this opportunity, borrow at the risk-free rate of 5% and invest the funds in a portfolio built by selling short Security 4 and buying Security 3 with weights (3,-2) as in part b.

- d. First assume that the risk-free security will not change. The price of Security 4 (that everyone is trying to sell short) will decrease, and the price of Security 3 (that everyone is trying to buy) will increase. Hence the return of Security 4 will increase and the return of Security 3 will decrease.

The alternative is that the prices of Securities 3 and 4 will remain the same, and the price of the risk-free security drops until its return is 10%.

Finally, a combined movement of all security prices is also possible. The prices of Security 4 and the risk-free security will decrease and the price of Security 3 will increase until the opportunity disappears.



CHAPTER 13

RISK, COST OF CAPITAL, AND CAPITAL BUDGETING

Answers to Concepts Review and Critical Thinking Questions

1. No. The cost of capital depends on the risk of the project, not the source of the money.
2. Interest expense is tax-deductible. There is no difference between pretax and aftertax equity costs.
3. You are assuming that the new project's risk is the same as the risk of the firm as a whole, and that the firm is financed entirely with equity.
4. Two primary advantages of the SML approach are that the model explicitly incorporates the relevant risk of the stock and the method is more widely applicable than is the DCF model, since the SML doesn't make any assumptions about the firm's dividends. The primary disadvantages of the SML method are (1) three parameters (the risk-free rate, the expected return on the market, and beta) must be estimated, and (2) the method essentially uses historical information to estimate these parameters. The risk-free rate is usually estimated to be the yield on very short maturity T-bills and is, hence, observable; the market risk premium is usually estimated from historical risk premiums and, hence, is not observable. The stock beta, which is unobservable, is usually estimated either by determining some average historical beta from the firm and the market's return data, or by using beta estimates provided by analysts and investment firms.
5. The appropriate aftertax cost of debt to the company is the interest rate it would have to pay if it were to issue new debt today. Hence, if the YTM on outstanding bonds of the company is observed, the company has an accurate estimate of its cost of debt. If the debt is privately-placed, the firm could still estimate its cost of debt by (1) looking at the cost of debt for similar firms in similar risk classes, (2) looking at the average debt cost for firms with the same credit rating (assuming the firm's private debt is rated), or (3) consulting analysts and investment bankers. Even if the debt is publicly traded, an additional complication arises when the firm has more than one issue outstanding; these issues rarely have the same yield because no two issues are ever completely homogeneous.
6.
 - a. This only considers the dividend yield component of the required return on equity.
 - b. This is the current yield only, not the promised yield to maturity. In addition, it is based on the book value of the liability, and it ignores taxes.
 - c. Equity is inherently riskier than debt (except, perhaps, in the unusual case where a firm's assets have a negative beta). For this reason, the cost of equity exceeds the cost of debt. If taxes are considered in this case, it can be seen that at reasonable tax rates, the cost of equity does exceed the cost of debt.

7. $R_{\text{Sup}} = .12 + .75(.08) = .1800$, or 18.00%

Both should proceed. The appropriate discount rate does not depend on which company is investing; it depends on the risk of the project. Since Superior is in the business, it is closer to a pure play. Therefore, its cost of capital should be used. With an 18 percent cost of capital, the project has an NPV of \$1 million regardless of who takes it.

8. If the different operating divisions were in much different risk classes, then separate cost of capital figures should be used for the different divisions; the use of a single, overall cost of capital would be inappropriate. If the single hurdle rate were used, riskier divisions would tend to receive more funds for investment projects, since their return would exceed the hurdle rate despite the fact that they may actually plot below the SML and, hence, be unprofitable projects on a risk-adjusted basis. The typical problem encountered in estimating the cost of capital for a division is that it rarely has its own securities traded on the market, so it is difficult to observe the market's valuation of the risk of the division. Two typical ways around this are to use a pure play proxy for the division, or to use subjective adjustments of the overall firm hurdle rate based on the perceived risk of the division.
9. The discount rate for the projects should be lower than the rate implied by the security market line. The security market line is used to calculate the cost of equity. The appropriate discount rate for the projects is the firm's weighted average cost of capital. Since the firm's cost of debt is generally less than the firm's cost of equity, the rate implied by the security market line will be too high.
10. Beta measures the responsiveness of a security's returns to movements in the market. Beta is determined by the cyclical nature of a firm's revenues. This cyclical nature is magnified by the firm's operating and financial leverage. The following three factors will impact the firm's beta. (1) Revenues. The cyclical nature of a firm's sales is an important factor in determining beta. In general, stock prices will rise when the economy expands and will fall when the economy contracts. As we said above, beta measures the responsiveness of a security's returns to movements in the market. Therefore, firms whose revenues are more responsive to movements in the economy will generally have higher betas than firms with less-cyclical revenues. (2) Operating leverage. Operating leverage is the percentage change in earnings before interest and taxes (EBIT) for a percentage change in sales. A firm with high operating leverage will have greater fluctuations in EBIT for a change in sales than a firm with low operating leverage. In this way, operating leverage magnifies the cyclical nature of a firm's revenues, leading to a higher beta. (3) Financial leverage. Financial leverage arises from the use of debt in the firm's capital structure. A levered firm must make fixed interest payments regardless of its revenues. The effect of financial leverage on beta is analogous to the effect of operating leverage on beta. Fixed interest payments cause the percentage change in net income to be greater than the percentage change in EBIT, magnifying the cyclical nature of a firm's revenues. Thus, returns on highly-levered stocks should be more responsive to movements in the market than the returns on stocks of firms with little or no debt in their capital structures.

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. With the information given, we can find the cost of equity using the CAPM. The cost of equity is:

$$R_S = .027 + .95(.10 - .027)$$

$$R_S = .0964, \text{ or } 9.64\%$$

2. The pretax cost of debt is the YTM of the company's bonds, so:

$$P_0 = \$950 = \$30(\text{PVIFA}_{R\%,34}) + \$1,000(\text{PVIF}_{R\%,34})$$

$$R = 3.245\%$$

$$R_B = 2 \times 3.245\%$$

$$R_B = 6.49\%$$

And the aftertax cost of debt is:

$$\text{Aftertax cost of debt} = .0649(1 - .21)$$

$$\text{Aftertax cost of debt} = .0513, \text{ or } 5.13\%$$

3. a. The pretax cost of debt is the YTM of the company's bonds, so:

$$P_0 = \$1,060 = \$29.50(\text{PVIFA}_{R\%,54}) + \$1,000(\text{PVIF}_{R\%,54})$$

$$R = 2.736\%$$

$$R_B = 2 \times 2.736\%$$

$$R_B = 5.47\%$$

- b. The aftertax cost of debt is:

$$\text{Aftertax cost of debt} = .0547(1 - .22)$$

$$\text{Aftertax cost of debt} = .0427, \text{ or } 4.27\%$$

- c. The aftertax rate is more relevant because that is the actual cost to the company.

4. The book value of debt is the total par value of all outstanding debt, so:

$$BV_B = \$25,000,000 + 60,000,000$$

$$BV_B = \$85,000,000$$

To find the market value of debt, we find the price of the bonds and multiply by the number of bonds. Alternatively, we can multiply the price quote of the bond times the par value of the bonds. Doing so, we find:

$$B = 1.06(\$25,000,000) + .68(\$60,000,000)$$

$$B = \$67,300,000$$

The YTM of the zero coupon bonds is:

$$P_Z = \$680 = \$1,000(PVIF_{R\%,18})$$

$$R = 2.166\%$$

$$YTM = 2 \times 2.166\%$$

$$YTM = 4.33\%$$

So, the aftertax cost of the zero coupon bonds is:

$$\text{Aftertax cost of debt} = .0433(1 - .22)$$

$$\text{Aftertax cost of debt} = .0338, \text{ or } 3.38\%$$

The aftertax cost of debt for the company is the weighted average of the aftertax cost of debt for all outstanding bond issues. We need to use the market value weights of the bonds. The total aftertax cost of debt for the company is:

$$\text{Aftertax cost of debt} = .0427[1.06(\$25)/\$67.3] + .0338[.68(\$60)/\$67.3]$$

$$\text{Aftertax cost of debt} = .0373, \text{ or } 3.73\%$$

5. Using the equation to calculate the WACC, we find:

$$R_{WACC} = .70(.109) + .30(.057)(1 - .23)$$

$$R_{WACC} = .0895, \text{ or } 8.95\%$$

6. Here we need to use the debt-equity ratio to calculate the WACC. Doing so, we find:

$$R_{WACC} = .118(1/1.40) + .065(.40/1.40)(1 - .21)$$

$$R_{WACC} = .0990, \text{ or } 9.90\%$$

7. Here we have the WACC and need to find the debt-equity ratio of the company. Setting up the WACC equation, we find:

$$R_{WACC} = .0910 = .11(S/V) + .064(B/V)(1 - .21)$$

Rearranging the equation, we find:

$$.0910(V/S) = .11 + .064(.79)(B/S)$$

Now we must realize that the V/S is just the equity multiplier, which is equal to:

$$V/S = 1 + B/S$$

$$.0910(B/S + 1) = .11 + .05056(B/S)$$

Now we can solve for B/S as:

$$.04044(B/S) = .019$$

$$B/S = .4698$$

8. a. The book value of equity is the book value per share times the number of shares, and the book value of debt is the face value of the company's debt, so:

$$\text{Equity} = 7,600,000(\$4) = \$30,400,000$$

$$\text{Debt} = \$80,000,000 + 65,000,000 = \$145,000,000$$

So, the total book value of the company is:

$$\text{Book value} = \$30,400,000 + 145,000,000 = \$175,400,000$$

And the book value weights of equity and debt are:

$$\text{Equity/Value} = \$30,400,000/\$175,400,000 = .1733$$

$$\text{Debt/Value} = 1 - \text{Equity/Value} = .8267$$

- b. The market value of equity is the share price times the number of shares, so:

$$S = 7,600,000(\$67) = \$509,200,000$$

Using the relationship that the total market value of debt is the price quote times the par value of the bond, we find the market value of debt is:

$$B = 1.095(\$80,000,000) + 1.124(\$65,000,000) = \$160,660,000$$

This makes the total market value of the company:

$$V = \$509,200,000 + 160,660,000 = \$669,860,000$$

And the market value weights of equity and debt are:

$$S/V = \$509,200,000/\$669,860,000 = .7602$$

$$B/V = 1 - S/V = .2398$$

- c. The market value weights are more relevant.

9. First, we will find the cost of equity for the company. The information provided allows us to solve for the cost of equity using the CAPM, so:

$$R_S = .029 + 1.10(.07)$$

$$R_S = .1060, \text{ or } 10.60\%$$

Next, we need to find the YTM on both bond issues. Doing so, we find:

$$P_1 = \$1,095 = \$34(PVIFA_{R\%,18}) + \$1,000(PVIF_{R\%,18})$$

$$R = 2.725\%$$

$$YTM = 2.725\% \times 2 = 5.45\%$$

$$P_2 = \$1,124 = \$35.50(PVIFA_{R\%,50}) + \$1,000(PVIF_{R\%,50})$$

$$R = 3.062\%$$

$$YTM = 3.062\% \times 2 = 6.12\%$$

To find the weighted average aftertax cost of debt, we need the weight of each bond as a percentage of the total debt. We find:

$$X_{B1} = 1.095(\$80,000,000)/\$160,660,000 = .545$$

$$X_{B2} = 1.124(\$65,000,000)/\$160,660,000 = .455$$

Now we can multiply the weighted average cost of debt times one minus the tax rate to find the weighted average aftertax cost of debt. This gives us:

$$R_B = (1 - .23)[(.545)(.0545) + (.455)(.0612)]$$

$$R_B = .0443, \text{ or } 4.43\%$$

Using these costs and the weight of debt we calculated earlier, the WACC is:

$$R_{WACC} = .7602(.1060) + .2398(.0443)$$

$$R_{WACC} = .0912, \text{ or } 9.12\%$$

10. a. Using the equation to calculate WACC, we find:

$$R_{WACC} = .101 = (1/1.38)(.12) + (.38/1.38)(1 - .25)R_B$$

$$R_B = .0680, \text{ or } 6.80\%$$

- b. Using the equation to calculate WACC, we find:

$$R_{WACC} = .101 = (1/1.38)R_S + (.38/1.38)(.064)$$

$$R_S = .1151, \text{ or } 11.51\%$$

11. We will begin by finding the market value of each type of financing. We find:

$$B = 17,000(\$2,000)(1.05) = \$35,700,000$$

$$S = 425,000(\$67) = \$28,475,000$$

And the total market value of the firm is:

$$V = \$35,700,000 + 28,475,000$$

$$V = \$64,175,000$$

Now, we can find the cost of equity using the CAPM. The cost of equity is:

$$R_S = .035 + .88(.07)$$

$$R_S = .0966, \text{ or } 9.66\%$$

The cost of debt is the YTM of the bonds, so:

$$P_0 = \$2,100 = \$49(\text{PVIFA}_{R\%,40}) + \$1,000(\text{PVIF}_{R\%,40})$$

$$R = 2.259\%$$

$$\text{YTM} = 2.259\% \times 2 = 4.52\%$$

And the aftertax cost of debt is:

$$R_B = (1 - .21)(.0452)$$

$$R_B = .0357, \text{ or } 3.57\%$$

Now we have all of the components to calculate the WACC. The WACC is:

$$R_{\text{WACC}} = .0357(\$35,700,000/\$64,175,000) + .0966(\$28,475,000/\$64,175,000)$$

$$R_{\text{WACC}} = .0627, \text{ or } 6.27\%$$

Notice that we didn't include the $(1 - T_C)$ term in the WACC equation. We used the aftertax cost of debt in the equation, so the term is not needed here.

12. a. We will begin by finding the market value of each type of financing. We find:

$$B = 175,000(\$1,000)(1.06) = \$185,500,000$$

$$S = 6,400,000(\$53) = \$339,200,000$$

And the total market value of the firm is:

$$V = \$185,500,000 + \$339,200,000$$

$$V = \$524,700,000$$

So, the market value weights of the company's financing are:

$$B/V = \$185,500,000/\$524,700,000 = .3535$$

$$S/V = \$339,200,000/\$524,700,000 = .6465$$

- b. For projects equally as risky as the firm itself, the WACC should be used as the discount rate.

First we can find the cost of equity using the CAPM. The cost of equity is:

$$R_S = .031 + 1.15(.068)$$

$$R_S = .1092, \text{ or } 10.92\%$$

The cost of debt is the YTM of the bonds, so:

$$P_0 = \$1,060 = \$31(\text{PVIFA}_{R\%,50}) + \$1,000(\text{PVIF}_{R\%,50})$$

$$R = 2.872\%$$

$$\text{YTM} = 2.872\% \times 2 = 5.74\%$$

And the aftertax cost of debt is:

$$R_B = (1 - .22)(.0574)$$

$$R_B = .0448, \text{ or } 4.48\%$$

Now we can calculate the WACC as:

$$R_{WACC} = .6465(.1092) + .3535(.0448)$$

$$R_{WACC} = .0864, \text{ or } 8.64\%$$

13. a. Projects Y and Z.

- b. Using the CAPM to consider the projects, we need to calculate the expected return of each project given its level of risk. This expected return should then be compared to the expected return of the project. If the return calculated using the CAPM is lower than the project expected return, we should accept the project; if not, we reject the project. After considering risk via the CAPM:

$$E[W] = .035 + .75(.12 - .035) = .0988 > .094, \text{ so reject W}$$

$$E[X] = .035 + .90(.12 - .035) = .1115 < .112, \text{ so accept X}$$

$$E[Y] = .035 + 1.15(.12 - .035) = .1328 < .141, \text{ so accept Y}$$

$$E[Z] = .035 + 1.45(.12 - .035) = .1583 > .155, \text{ so reject Z}$$

- c. Project X would be incorrectly rejected; Project Z would be incorrectly accepted.

14. a. He should look at the weighted average flotation cost, not just the debt cost. If the company uses only debt to fund this project, it will have to issue equity later to maintain its capital structure.

- b. The weighted average flotation cost is the weighted average of the flotation costs for debt and equity, so:

$$f_A = .02(.65/1.65) + .06(1/1.65)$$

$$f_A = .0442, \text{ or } 4.42\%$$

- c. The total cost of the equipment including flotation costs is:

$$\text{Amount raised}(1 - .0442) = \$43,000,000$$

$$\text{Amount raised} = \$43,000,000/(1 - .0442)$$

$$\text{Amount raised} = \$44,990,488$$

Even if the specific funds are actually being raised completely from debt, the flotation costs, and hence true investment cost, should be valued as if the firm's target capital structure is used.

15. We first need to find the weighted average flotation cost. Doing so, we find:

$$f_A = .70(.07) + .05(.04) + .25(.03)$$

$$f_A = .059, \text{ or } 5.9\%$$

And the total cost of the equipment including flotation costs is:

$$\begin{aligned}\text{Amount raised}(1 - .059) &= \$75,000,000 \\ \text{Amount raised} &= \$75,000,000/(1 - .059) \\ \text{Amount raised} &= \$79,660,117\end{aligned}$$

Intermediate

16. Using the debt-equity ratio to calculate the WACC, we find:

$$\begin{aligned}R_{WACC} &= (.65/1.65)(.043) + (1/1.65)(.11) \\ R_{WACC} &= .0836, \text{ or } 8.36\%\end{aligned}$$

Since the project is riskier than the company, we need to adjust the project discount rate for the additional risk. Using the subjective risk factor given, we find:

$$\begin{aligned}\text{Project discount rate} &= 8.36\% + 2\% \\ \text{Project discount rate} &= 10.36\%\end{aligned}$$

We should accept the project if the NPV is positive. The NPV is the PV of the cash outflows plus the PV of the cash inflows. Since we are seeking the break-even initial cost, we need to find the PV of future inflows. The cash inflows are a growing perpetuity. If you remember, the equation for the PV of a growing perpetuity is the same as the dividend growth equation, so:

$$\begin{aligned}\text{PV of future CF} &= \$1,850,000/ (.1036 - .03) \\ \text{PV of future CF} &= \$25,133,800\end{aligned}$$

The project should only be undertaken if its cost is less than \$25,133,800 since a cost less than this amount will result in a positive NPV.

17. We will begin by finding the market value of each type of financing. We will use B1 to represent the coupon bond, and B2 to represent the zero coupon bond. The market value of the firm's financing is:

$$\begin{aligned}B_{B1} &= 40,000(\$1,000)(1.065) = \$42,600,000 \\ B_{B2} &= 40,000(\$10,000)(.218) = \$87,200,000 \\ P &= 135,000(\$87) = \$11,745,000 \\ S &= 1,900,000(\$73) = \$138,700,000\end{aligned}$$

And the total market value of the firm is:

$$\begin{aligned}V &= \$42,600,000 + \$87,200,000 + \$11,745,000 + \$138,700,000 \\ V &= \$280,245,000\end{aligned}$$

Now, we can find the cost of equity using the CAPM. The cost of equity is:

$$\begin{aligned}R_S &= .036 + 1.15(.07) \\ R_S &= .1165, \text{ or } 11.65\%\end{aligned}$$

The cost of debt is the YTM of the bonds, so:

$$P_0 = \$1,065 = \$24.50(PVIFA_{R\%,30}) + \$1,000(PVIF_{R\%,30})$$

$$R = 2.154\%$$

$$YTM = 2.154\% \times 2 = 4.31\%$$

And the aftertax cost of debt is:

$$R_{B1} = (1 - .23)(.0431)$$

$$R_{B1} = .0332, \text{ or } 3.32\%$$

And the aftertax cost of the zero coupon bonds is:

$$P_0 = \$2,180 = \$10,000(PVIF_{R\%,60})$$

$$R = 2.571\%$$

$$YTM = 2.571\% \times 2 = 5.14\%$$

$$R_{B2} = (1 - .23)(.0514)$$

$$R_{B2} = .0396, \text{ or } 3.96\%$$

Even though the zero coupon bonds make no payments, the calculation for the YTM (or price) still assumes semiannual compounding, consistent with a coupon bond. Also remember that, even though the company does not make interest payments, the accrued interest is still tax deductible for the company.

To find the required return on preferred stock, we can use the preferred stock pricing equation, which is the level perpetuity equation, so the required return on the company's preferred stock is:

$$R_P = D_1/P_0$$

$$R_P = \$3.50/\$87$$

$$R_P = .0402, \text{ or } 4.02\%$$

Notice that the required return on the preferred stock is lower than the required return on the bonds. This result is inconsistent with the risk levels of the two instruments but is a common occurrence. There is a practical reason for this: Assume Company A owns stock in Company B. The tax code allows Company A to exclude at least 70 percent of the dividends received from Company B, meaning Company A does not pay taxes on this amount. In practice, much of the outstanding preferred stock is owned by other companies, who are willing to take the lower return since much of the return is effectively tax exempt for the investing company.

Now we have all of the components to calculate the WACC. The WACC is:

$$WACC = .0332(\$42,600,000/\$280,245,000) + .0396(\$87,200,000/\$280,245,000) \\ + .1165(\$138,700,000/\$280,245,000) + .0402(\$11,745,000/\$280,245,000)$$

$$WACC = .0767, \text{ or } 7.67\%$$

- 18.** The total cost of the equipment including flotation costs was:

$$\text{Total costs} = \$30,000,000 + 1,900,000$$

$$\text{Total costs} = \$31,900,000$$

Using the equation to calculate the total cost including flotation costs, we get:

Amount raised $(1 - f_A)$ = Amount needed after flotation costs

$$\$31,900,000(1 - f_A) = \$30,000,000$$

$$f_A = .0596, \text{ or } 5.96\%$$

Now, we know the weighted average flotation cost. The equation to calculate the percentage flotation costs is:

$$f_A = .0596 = .07(S/V) + .03(B/V)$$

We can solve this equation to find the debt-equity ratio as follows:

$$.0596(V/S) = .07 + .03(B/S)$$

We must recognize that the V/S term is the equity multiplier, which is $(1 + B/S)$, so:

$$.0596(B/S + 1) = .07 + .03(B/S)$$

$$B/S = .3531$$

19. a. Using the dividend discount model, the cost of equity is:

$$R_S = [(.75)(1.045)/\$84] + .045$$

$$R_S = .0543, \text{ or } 5.43\%$$

- b. Using the CAPM, the cost of equity is:

$$R_S = .037 + 1.15(.11 - .037)$$

$$R_S = .1210, \text{ or } 12.10\%$$

- c. When using the dividend discount model or the CAPM, you must remember that both are estimates for the cost of equity. Additionally, and perhaps more importantly, each method of estimating the cost of equity depends upon different assumptions.

20. We are given the total cash flow for the current year. To value the company, we need to calculate the cash flows until the growth rate levels off at a constant perpetual rate. So, the cash flows each year will be:

$$\text{Year 1: } \$7,100,000(1 + .10) = \$7,810,000$$

$$\text{Year 2: } \$7,810,000(1 + .10) = \$8,591,000$$

$$\text{Year 3: } \$8,591,000(1 + .10) = \$9,450,100$$

$$\text{Year 4: } \$9,450,100(1 + .10) = \$10,395,110$$

$$\text{Year 5: } \$10,395,110(1 + .10) = \$11,434,621$$

$$\text{Year 6: } \$11,434,621(1 + .04) = \$11,892,006$$

We can calculate the terminal value in Year 5 since the cash flows begin a perpetual growth rate. Since we are valuing Arras, we need to use the cost of capital for that company since this rate is based on the risk of Arras. The cost of capital for Schultz is irrelevant in this case. So, the terminal value is:

$$\begin{aligned} TV_5 &= CF_6 / (R_{WACC} - g) \\ TV_5 &= \$11,892,006 / (.09 - .04) \\ TV_5 &= \$237,840,117 \end{aligned}$$

Now we can discount the cash flows for the first 5 years as well as the terminal value back to today. Again, using the cost of capital for Arras, we find the value of the company today is:

$$\begin{aligned} V_0 &= \$7,810,000/1.09 + \$8,591,000/1.09^2 + \$9,450,100/1.09^3 + \$10,395,110/1.09^4 \\ &\quad + (\$11,434,621 + 237,840,117)/1.09^5 \\ V_0 &= \$191,068,855 \end{aligned}$$

The market value of the equity is the market value of the company minus the market value of the debt, or:

$$\begin{aligned} S &= \$191,068,855 - 22,000,000 \\ S &= \$169,068,855 \end{aligned}$$

To find the maximum offer price, we divide the market value of equity by the shares outstanding, or:

$$\begin{aligned} \text{Share price} &= \$169,068,855 / 2,500,000 \\ \text{Share price} &= \$67.63 \end{aligned}$$

21. a. To begin the valuation of Joe's, we will begin by calculating the R_{WACC} for Happy Times. Since both companies are in the same industry, it is likely that the R_{WACC} for both companies will be the same. The weights of debt and equity are:

$$\begin{aligned} X_B &= \$115,000,000 / (\$115,000,000 + 360,000,000) = .2421, \text{ or } 24.21\% \\ X_S &= \$360,000,000 / (\$115,000,000 + 360,000,000) = .7579, \text{ or } 75.79\% \end{aligned}$$

The R_{WACC} for Happy Times is:

$$\begin{aligned} R_{WACC} &= .2421(.06)(1 - .21) + .7579(.11) \\ R_{WACC} &= .0948, \text{ or } 9.48\% \end{aligned}$$

Next, we need to calculate the cash flows for each year. The EBIT will grow at 10 percent per year for 5 years. Net working capital, capital spending, and depreciation are 9 percent, 15 percent, and 8 percent of EBIT, respectively. So, the cash flows for each year over the next 5 years will be:

	<i>Year 1</i>	<i>Year 2</i>	<i>Year 3</i>	<i>Year 4</i>	<i>Year 5</i>
EBIT	\$17,300,000	\$19,030,000	\$20,933,000	\$23,026,300	\$25,328,930
Taxes*	3,633,000	3,996,300	4,395,930	4,835,523	5,319,075
Net income	13,667,000	15,033,700	16,537,070	18,190,777	20,009,855
Depreciation	1,384,000	1,522,400	1,674,640	1,842,104	2,026,314
OCF	\$15,051,000	\$16,556,100	\$18,211,710	\$20,032,881	\$22,036,169
– Capital spending	2,595,000	2,854,500	3,139,950	3,453,945	3,799,340
– Change in NWC	1,557,000	1,712,700	1,883,970	2,072,367	2,279,604
CFA*	\$10,899,000	\$11,988,900	\$13,187,790	\$14,506,569	\$15,957,226

After Year 5 the cash flows will grow at 3 percent in perpetuity. We can find the terminal value of the company in Year 5, using the cash flow in Year 6, as:

$$TV_5 = CF_6 / (R_{WACC} - g)$$

$$TV_5 = \$15,957,226(1 + .03) / (.0948 - .03)$$

$$TV_5 = \$253,468,159$$

Now we can discount the cash flows and terminal value to today. Doing so, we find:

$$V_0 = \$10,899,000/1.0948 + \$11,988,900/1.0948^2 + \$13,187,790/1.0948^3$$

$$+ \$14,506,569/1.0948^4 + (\$15,957,226 + 253,468,159)/1.0948^5$$

$$V_0 = \$211,369,780$$

The market value of the equity is the market value of the company minus the market value of the debt, or:

$$S = \$211,369,780 - 45,000,000$$

$$S = \$166,369,780$$

To find the maximum offer price, we divide the market value of equity by the shares outstanding, or:

$$\text{Share price} = \$166,369,780/1,950,000$$

$$\text{Share price} = \$85.32$$

- b. To calculate the terminal value using the EV/EBITDA multiple we need to calculate the Year 5 EBITDA, which is EBIT plus depreciation, or:

$$\text{EBITDA} = \$25,328,930 + 2,026,314$$

$$\text{EBITDA} = \$27,355,244$$

We can now calculate the terminal value of the company using the Year 5 EBITDA, which will be:

$$TV_5 = \$27,355,244(9)$$

$$TV_5 = \$246,197,200$$

Note, this is the terminal value in Year 5 since we used the Year 5 EBITDA. We need to calculate the present value of the cash flows for the first 4 years, plus the present value of the Year 5 terminal value. We do not need to include the Year 5 cash flow since it is included in the Year 5 terminal value. So, the value of the company today is:

$$\begin{aligned} V_0 &= \$10,899,000/1.0948 + \$11,988,900/1.0948^2 + \$13,187,790/1.0948^3 \\ &\quad + \$14,506,569/1.0948^4 + (\$15,957,226 + 246,197,200)/1.0948^5 \\ V_0 &= \$196,604,093 \end{aligned}$$

The market value of the equity is the market value of the company minus the market value of the debt, or:

$$S = \$196,604,093 - 45,000,000$$

$$S = \$151,604,093$$

To find the maximum offer price, we divide the market value of equity by the shares outstanding, or:

$$\text{Share price} = \$151,604,093/1,950,000$$

$$\text{Share price} = \$77.75$$

Challenge

22. We can use the debt-equity ratio to calculate the weights of equity and debt. For the debt of the company, there is a weight for long-term debt and a weight for accounts payable. We can use the target ratio given for accounts payable to calculate the weight of accounts payable and the weight of long-term debt. The weight of each will be:

$$\text{Accounts payable weight} = .20/1.20 = .17$$

$$\text{Long-term debt weight} = 1/1.20 = .83$$

Since the accounts payable has the same cost as the overall WACC, we can write the equation for the WACC as:

$$R_{WACC} = (1/1.40)(.13) + (.40/1.40)[(.20/1.2)R_{WACC} + (1/1.2)(.06)(1 - .21)]$$

Solving for WACC, we find:

$$R_{WACC} = .0929 + .2857[(.20/1.2)R_{WACC} + .0395]$$

$$R_{WACC} = .0929 + (.0476)R_{WACC} + .0113$$

$$(.9524)R_{WACC} = .1041$$

$$R_{WACC} = .1094, \text{ or } 10.94\%$$

We will use the same equation to calculate the weighted average flotation cost, except we will use the flotation cost for each form of financing. Doing so, we get:

$$\text{Flotation costs} = (1/1.40)(.07) + (.40/1.40)[(.20/1.2)(0) + (1/1.2)(.02)] = .0548, \text{ or } 5.48\%$$

The total amount we need to raise to fund the new equipment will be:

$$\text{Amount raised cost} = \$60,000,000/(1 - .0548)$$

$$\text{Amount raised} = \$63,476,071$$

Since the cash flows go to perpetuity, we can calculate the present value using the equation for the PV of a perpetuity. The NPV is:

$$\text{NPV} = -\$63,476,071 + (\$9,600,000/.1094)$$

$$\text{NPV} = \$24,315,425$$

23. We can use the debt-equity ratio to calculate the weights of equity and debt. The weight of debt in the capital structure is:

$$X_B = .65/1.65 = .3939, \text{ or } 39.39\%$$

And the weight of equity is:

$$X_S = 1 - .3939 = .6061, \text{ or } 60.61\%$$

Now we can calculate the weighted average flotation costs for the various percentages of internally raised equity. To find the portion of equity flotation costs, we can multiply the equity costs by the percentage of equity raised externally, which is one minus the percentage raised internally. So, if the company raises all equity externally, the flotation costs are:

$$f_A = (.6061)(.08)(1 - 0) + (.3939)(.035)$$

$$f_A = .0623, \text{ or } 6.23\%$$

The initial cash outflow for the project needs to be adjusted for the flotation costs. To account for the flotation costs:

$$\text{Amount raised}(1 - .0623) = \$137,000,000$$

$$\text{Amount raised} = \$137,000,000/(1 - .0623)$$

$$\text{Amount raised} = \$146,097,916$$

If the company uses 60 percent internally generated equity, the flotation cost is:

$$f_A = (.6061)(.08)(1 - .60) + (.3939)(.035)$$

$$f_A = .0332, \text{ or } 3.32\%$$

And the initial cash flow will be:

$$\text{Amount raised}(1 - .0332) = \$137,000,000$$

$$\text{Amount raised} = \$137,000,000/(1 - .0332)$$

$$\text{Amount raised} = \$141,701,928$$

If the company uses 100 percent internally generated equity, the flotation cost is:

$$f_A = (.6061)(.08)(1 - 1) + (.3939)(.035)$$

$$f_A = .0138, \text{ or } 1.38\%$$

And the initial cash flow will be:

$$\text{Amount raised}(1 - .0138) = \$137,000,000$$

$$\text{Amount raised} = \$137,000,000 / (1 - .0138)$$

$$\text{Amount raised} = \$138,915,348$$

24. The \$7.3 million cost of the land 3 years ago is a sunk cost and irrelevant; the \$7.5 million appraised value of the land is an opportunity cost and is relevant. The \$7.9 million land value in 5 years is a relevant cash flow as well. The fact that the company is keeping the land rather than selling it is unimportant. The land is an opportunity cost in 5 years and is a relevant cash flow for this project. The market value capitalization weights are:

$$B = 130,000(\$2,000)(1.04) = \$270,400,000$$

$$S = 9,900,000(\$68) = \$673,200,000$$

$$P = 400,000(\$87) = \$34,800,000$$

The total market value of the company is:

$$V = \$270,400,000 + \$673,200,000 + \$34,800,000 = \$978,400,000$$

The weight of each form of financing in the company's capital structure is:

$$X_B = \$270,400,000 / \$978,400,000 = .2764$$

$$X_S = \$673,200,000 / \$978,400,000 = .6881$$

$$X_P = \$34,800,000 / \$978,400,000 = .0356$$

Next we need to find the cost of funds. We have the information available to calculate the cost of equity using the CAPM, so:

$$R_S = .031 + 1.20(.07)$$

$$R_S = .1150, \text{ or } 11.50\%$$

The cost of debt is the YTM of the company's outstanding bonds, so:

$$P_0 = \$2,080 = \$61(PVIFA_{R\%,50}) + \$2,000(PVIF_{R\%,50})$$

$$R = 2.898\%$$

$$\text{YTM} = 2.898\% \times 2 = 5.80\%$$

And the aftertax cost of debt is:

$$R_B = (1 - .25)(.0580)$$

$$R_B = .0435, \text{ or } 4.35\%$$

The cost of preferred stock is:

$$R_P = \$4.20/\$87$$

$$R_P = .0483, \text{ or } 4.83\%$$

- a. The weighted average flotation cost is the sum of the weight of each source of funds in the capital structure of the company times the flotation costs, so:

$$f_A = .6881(.065) + .2764(.03) + .0356(.045)$$

$$f_A = .0546, \text{ or } 5.46\%$$

The initial cash outflow for the project needs to be adjusted for the flotation costs. To account for the flotation costs:

$$\text{Amount raised}(1 - .0546) = \$55,000,000$$

$$\text{Amount raised} = \$55,000,000/(1 - .0546)$$

$$\text{Amount raised} = \$58,177,399$$

So the cash flow at time zero will be:

$$CF_0 = -\$7,500,000 - 58,177,399 - 2,500,000$$

$$CF_0 = -\$68,177,399$$

There is an important caveat to this solution. This solution assumes that the increase in net working capital does not require the company to raise outside funds; therefore the flotation costs are not included. However, this is an assumption and the company could need to raise outside funds for the NWC. If this is true, the initial cash outlay includes these flotation costs, so:

Total cost of NWC including flotation costs:

$$\$2,500,000/(1 - .0546) = \$2,644,427$$

This would make the total initial cash flow:

$$CF_0 = -\$7,500,000 - 58,177,399 - 2,644,427$$

$$CF_0 = -\$68,321,827$$

- b. To find the required return on this project, we first need to calculate the WACC for the company. The company's WACC is:

$$R_{WACC} = .6881(.1150) + .2764(.0435) + .0356(.0483)$$

$$R_{WACC} = .0929, \text{ or } 9.29\%$$

The company wants to use the subjective approach to this project because it is located overseas. The adjustment factor is 2 percent, so the required return on this project is:

$$\text{Project required return} = 9.29\% + 2\%$$

$$\text{Project required return} = 11.29\%$$

- c. The annual depreciation for the equipment will be:

$$\$55,000,000/8 = \$6,875,000$$

So, the book value of the equipment at the end of five years will be:

$$\begin{aligned} BV_5 &= \$55,000,000 - 5(\$6,875,000) \\ BV_5 &= \$20,625,000 \end{aligned}$$

So, the aftertax salvage value will be:

$$\begin{aligned} \text{Aftertax salvage value} &= \$8,900,000 + .25(\$20,625,000 - 8,900,000) \\ \text{Aftertax salvage value} &= \$11,831,250 \end{aligned}$$

- d. Using the tax shield approach, the OCF for this project is:

$$\begin{aligned} \text{OCF} &= [(P - v)Q - FC](1 - T_c) + T_c D \\ \text{OCF} &= [(\$11,600 - 9,750)(18,500) - 8,100,000](1 - .25) + .25(\$55,000,000/8) \\ \text{OCF} &= \$21,312,500 \end{aligned}$$

- e. The accounting break-even sales figure for this project is:

$$\begin{aligned} Q_A &= (FC + D)/(P - v) \\ Q_A &= (\$8,100,000 + 6,875,000)/(\$11,600 - 9,750) \\ Q_A &= 8,095 \text{ units} \end{aligned}$$

- f. We have calculated all cash flows of the project. We need to make sure that in Year 5 we add back the aftertax salvage value and the recovery of the initial NWC. The cash flows for the project are:

<u>Year</u>	<u>Flow Cash</u>
0	−\$68,177,399
1	21,312,500
2	21,312,500
3	21,312,500
4	21,312,500
5	43,543,750

Using the required return of 11.29 percent, the NPV of the project is:

$$\begin{aligned} \text{NPV} &= -\$68,177,399 + \$21,312,500(\text{PVIFA}_{11.29\%,4}) + \$43,543,750/1.1129^5 \\ \text{NPV} &= \$23,053,458 \end{aligned}$$

And the IRR is:

$$\begin{aligned} \text{NPV} = 0 &= -\$68,177,399 + \$21,312,500(\text{PVIFA}_{\text{IRR}\%,4}) + \$43,543,750/(1 + \text{IRR})^5 \\ \text{IRR} &= 22.67\% \end{aligned}$$

If the initial NWC is assumed to be financed from outside sources, the cash flows are:

<i>Year</i>	<i>Flow Cash</i>
0	−\$68,321,827
1	21,312,500
2	21,312,500
3	21,312,500
4	21,312,500
5	43,543,750

With this assumption, and the required return of 11.29 percent, the NPV of the project is:

$$\text{NPV} = -\$68,321,827 + \$21,312,500(\text{PVIFA}_{11.29\%,4}) + \$43,543,750/1.1129^5$$

$$\text{NPV} = \$22,909,030.76$$

And the IRR is:

$$\text{IRR} = 0 = -\$68,321,827 + \$21,312,500(\text{PVIFA}_{\text{IRR}\%,4}) + \$43,543,750/(1 + \text{IRR})^5$$

$$\text{IRR} = 22.58\%$$

CHAPTER 14

CORPORATE FINANCING DECISIONS AND EFFICIENT CAPITAL MARKETS

Answers to Concepts Review and Critical Thinking Questions

1. To create value, firms should accept financing proposals with positive net present values. Firms can create valuable financing opportunities in three ways: 1) Fool investors. A firm can issue a complex security to receive more than the fair market value. Financial managers attempt to package securities to receive the greatest value. 2) Reduce costs or increase subsidies. A firm can package securities to reduce taxes. Such a security will increase the value of the firm. In addition, financing techniques involve many costs, such as accountants, lawyers, and investment bankers. Packaging securities in a way to reduce these costs will also increase the value of the firm. 3) Create a new security. A previously unsatisfied investor may pay extra for a specialized security catering to his or her needs. Corporations gain from developing unique securities by issuing these securities at premium prices.
2. The three forms of the efficient markets hypothesis are: 1) Weak form. Market prices reflect information contained in historical prices. Investors are unable to earn abnormal returns using historical prices to predict future price movements. 2) Semistrong form. In addition to historical data, market prices reflect all publicly-available information. Investors with insider, or private information, are able to earn abnormal returns. 3) Strong form. Market prices reflect all information, public or private. Investors are unable to earn abnormal returns using insider information or publicly-available information.
3.
 - a. False. Market efficiency implies that prices reflect all available information, but it does not imply certain knowledge. Many pieces of information that are available and reflected in prices are fairly uncertain. Efficiency of markets does not eliminate that uncertainty and therefore does not imply perfect forecasting ability.
 - b. True. Market efficiency exists when prices reflect all available information. To be efficient in the weak form, the market must incorporate all historical data into prices. Under the semistrong form of the hypothesis, the market incorporates all publicly-available information in addition to the historical data. In strong form efficient markets, prices reflect all publicly and privately available information.
 - c. False. Market efficiency implies that market participants are rational. Rational people will immediately act upon new information and will bid prices up or down to reflect that information.
 - d. False. In efficient markets, prices reflect all available information. Thus, prices will fluctuate whenever new information becomes available.

- e. True. Competition among investors results in the rapid transmission of new market information. In efficient markets, prices immediately reflect new information as investors bid the stock price up or down.
4. On average, the only return that is earned is the required return—investors buy assets with returns in excess of the required return (positive NPV), bidding up the price and thus causing the return to fall to the required return (zero NPV); investors sell assets with returns less than the required return (negative NPV), driving the price lower and thus causing the return to rise to the required return (zero NPV).
5. The market is not weak form efficient.
6. Yes, historical information is also public information; weak form efficiency is a subset of semistrong form efficiency.
7. Ignoring trading costs, on average, such investors merely earn what the market offers; the trades all have zero NPV. If trading costs exist, then these investors lose by the amount of the costs.
8. Unlike gambling, the stock market is a positive sum game; everybody can win. Also, speculators provide liquidity to markets and thus help to promote efficiency.
9. The EMH only says, within the bounds of increasingly strong assumptions about the information processing of investors, that assets are fairly priced. An implication of this is that, on average, the typical market participant cannot earn excessive profits from a particular trading strategy. However, that does not mean that a few particular investors cannot outperform the market over a particular investment horizon. Certain investors who do well for a period of time get a lot of attention from the financial press, but the scores of investors who do not do well over the same period of time generally get considerably less attention from the financial press.
10.
 - a. If the market is not weak form efficient, then this information could be acted on and a profit earned from following the price trend. Under (2), (3), and (4), this information is fully impounded in the current price and no abnormal profit opportunity exists.
 - b. Under (2), if the market is not semistrong form efficient, then this information could be used to buy the stock “cheap” before the rest of the market discovers the financial statement anomaly. Since (2) is stronger than (1), both imply that a profit opportunity exists; under (3) and (4), this information is fully impounded in the current price and no profit opportunity exists.
 - c. Under (3), if the market is not strong form efficient, then this information could be used as a profitable trading strategy by noting the buying activity of the insiders as a signal that the stock is underpriced or that good news is imminent. Since (1) and (2) are weaker than (3), all three imply that a profit opportunity exists. Note that this assumes the individual who sees the insider trading is the only one who sees the trading. If the information about the trades made by company management is public information, it will be discounted in the stock price and no profit opportunity exists. Under (4), this information does not signal any profit opportunity for traders; any pertinent information the manager-insiders may have is fully reflected in the current share price.
11. A technical analyst would argue that the market is not efficient. Since a technical analyst examines past prices, the market cannot be weak form efficient for technical analysis to work. If the market is not weak form efficient, it cannot be efficient under stronger assumptions about the information available.

12. Investor sentiment captures the mood of the investing public. If investors are bearish in general, it may be that the market is headed down in the future since investors are less likely to invest. If the sentiment is bullish, it would be taken as a positive signal to the market. To use investor sentiment in technical analysis, you would probably want to construct a ratio such as a bulls/bears ratio. To use the ratio, compare the historical ratio to the market to determine if a certain level on the ratio indicates a market upturn or downturn. Of course, there is a group of investors called contrarians who view the market signals as reversed. That is, if the number of bearish investors reaches a certain level, the market will head up. For a contrarian, these signals are reversed.
13. Taken at face value, this fact suggests that markets have become more efficient. The increasing ease with which information is available over the Internet lends strength to this conclusion. On the other hand, during this particular period, large-capitalization growth stocks were the top performers. Value-weighted indexes such as the S&P 500 are naturally concentrated in such stocks, thus making them especially hard to beat during this period. So, it may be that the dismal record compiled by the pros is a matter of bad luck or benchmark error.
14. It is likely the market has a better estimate of the stock price, assuming it is semistrong form efficient. However, semistrong form efficiency only states that you cannot easily profit from publicly available information. If financial statements are unavailable, the market can still price stocks based upon the available public information, limited though it may be. Therefore, it may have been as difficult to examine the limited public information and make an extra return.
15.
 - a. Aerotech's stock price should rise immediately after the announcement of the positive news.
 - b. Only scenario (ii) indicates market efficiency. In that case, the price of the stock rises immediately to the level that reflects the new information, eliminating all possibility of abnormal returns. In the other two scenarios, there are periods of time during which an investor could trade on the information and earn abnormal returns.
16. False. The stock price would have adjusted before the founder's death only if investors had perfect forecasting ability. The 12.5 percent increase in the stock price after the founder's death indicates that either the market did not anticipate the death or that the market had anticipated it imperfectly. However, the market reacted immediately to the new information, implying efficiency. It is interesting that the stock price rose after the announcement of the founder's death. This price behavior indicates that the market felt he was a liability to the firm.
17. The announcement should not deter investors from buying UPC's stock. If the market is semistrong form efficient, the stock price will have already reflected the present value of the payments that UPC must make. The expected return after the announcement should still be equal to the expected return before the announcement. UPC's current stockholders bear the burden of the loss, since the stock price falls on the announcement. After the announcement, the expected return moves back to its original level.
18. The market is often considered to be relatively efficient up to the semistrong form. If so, no systematic profit can be made by trading on publicly-available information. Although illegal, the lead engineer of the device can profit from purchasing the firm's stock *before* the news release on the implementation of the new technology. The price should immediately and fully adjust to the new information in the article. Thus, no abnormal return can be expected from purchasing after the publication of the article.

19. Under the semistrong form of market efficiency, the stock price should stay the same. The accounting system changes are publicly available information. Investors would identify no changes in either the firm's current or its future cash flows. Thus, the stock price will not change after the announcement of increased earnings.
20. Because the number of subscribers has increased dramatically, the time it takes for information in the newsletter to be reflected in prices has shortened. With shorter adjustment periods, it becomes impossible to earn abnormal returns with the information provided by Durkin. If Durkin is using only publicly-available information in its newsletter, its ability to pick stocks is inconsistent with the efficient markets hypothesis. Under the semi-strong form of market efficiency, all publicly-available information should be reflected in stock prices. The use of private information for trading purposes is illegal.
21. You should not agree with your broker. The performance ratings of the small manufacturing firms were published and became public information. Prices should adjust immediately to the information, thus preventing future abnormal returns.
22. Stock prices should immediately and fully rise to reflect the announcement. Thus, one cannot expect abnormal returns following the announcement.
23.
 - a. No. Earnings information is in the public domain and reflected in the current stock price.
 - b. Possibly. If the rumors were publicly disseminated, the prices would have already adjusted for the possibility of a merger. If the rumor is information that you received from an insider, you could earn excess returns, although trading on that information is illegal.
 - c. No. The information is already public, and thus, already reflected in the stock price.
24. Serial correlation occurs when the current value of a variable is related to the future value of the variable. If the market is efficient, the information about the serial correlation in the macroeconomic variable and its relationship to net earnings should already be reflected in the stock price. In other words, although there is serial correlation in the variable, there will not be serial correlation in stock returns. Therefore, knowledge of the correlation in the macroeconomic variable will not lead to abnormal returns for investors.
25. The statement is false because every investor has a different risk preference. Although the expected return from every well-diversified portfolio is the same after adjusting for risk, investors still need to choose funds that are consistent with their particular risk level.
26. The share price will decrease immediately to reflect the new information. At the time of the announcement, the price of the stock should immediately decrease to reflect the negative information.

27. In an efficient market, the cumulative abnormal return (CAR) for Prospectors would rise substantially at the announcement of a new discovery. The CAR falls slightly on any day when no discovery is announced. There is a small positive probability that there will be a discovery on any given day. If there is no discovery on a particular day, the price should fall slightly because the good event did not occur. The substantial price increases on the rare days of discovery should balance the small declines on the other days, leaving CARs that are horizontal over time.
28. Behavioral finance attempts to explain both the 1987 stock market crash and the internet bubble by changes in investor sentiment and psychology. These changes can lead to non-random price behavior.

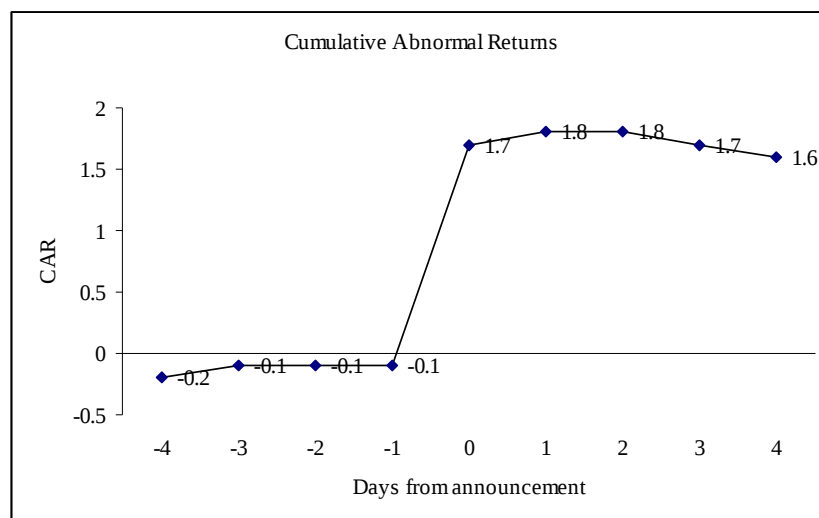
Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. To find the cumulative abnormal returns, we chart the abnormal returns for each of the three airlines for the days preceding and following the announcement. The abnormal return is calculated by subtracting the market return from a stock's return on a particular day, $R_i - R_M$. Group the returns by the number of days before or after the announcement for each respective airline. Calculate the cumulative average abnormal return by adding each abnormal return to the previous day's abnormal return.

Days from announcement	<u>Abnormal returns ($R_i - R_M$)</u>				Average abnormal return	Cumulative average residual
	Delta	United	American	Sum		
-4	-.2	-.2	-.2	-.6	-.2	-.2
-3	.2	-.1	.2	.3	.1	-.1
-2	.2	-.2	0	0	0	-.1
-1	.2	.2	-.4	0	0	-.1
0	3.3	.2	1.9	5.4	1.8	1.7
1	.2	.1	0	.3	.1	1.8
2	-.1	0	.1	0	0	1.8
3	-.2	.1	-.2	-.3	-.1	1.7
4	-.1	-.1	-.1	-.3	-.1	1.6



The market reacts favorably to the announcements. Moreover, the market reacts only on the day of the announcement. Before and after the event, the cumulative abnormal returns are relatively flat. This behavior is consistent with market efficiency.

2. The diagram does not support the efficient markets hypothesis. The CAR should remain relatively flat following the announcements. The diagram reveals that the CAR rose in the first month, only to drift down to lower levels during later months. Such movement violates the semistrong form of the efficient markets hypothesis because an investor could earn abnormal profits while the stock price gradually decreases.
3.
 - a. Supports. The CAR remained constant after the event at Time 0. This result is consistent with market efficiency, because prices adjust immediately to reflect the new information. Drops in CAR prior to an event can easily occur in an efficient capital market. For example, consider a sample of forced removals of the CEO. Since any CEO is more likely to be fired following bad rather than good stock performance, CARs are likely to be negative prior to removal. Because the firing of the CEO is announced at Time 0, one cannot use this information to trade profitably *before* the announcement. Thus, price drops prior to an event are neither consistent nor inconsistent with the efficient markets hypothesis.
 - b. Rejects. Because the CAR increases after the event date, one can profit by buying after the event. This possibility is inconsistent with the efficient markets hypothesis.
 - c. Supports. The CAR does not fluctuate after the announcement at Time 0. While the CAR was rising before the event, insider information would be needed for profitable trading. Thus, the graph is consistent with the semistrong form of efficient markets.

- d.* Supports. The diagram indicates that the information announced at Time 0 was of no value. Similar to part *a*, such movement is neither consistent nor inconsistent with the efficient markets hypothesis (EMH). Movements at the event date are neither consistent nor inconsistent with the efficient markets hypothesis.
- 4. Once the verdict is reached, the diagram shows that the CAR continues to decline after the court decision, allowing investors to earn abnormal returns. The CAR should remain constant, on average, even if an appeal is in progress, because no new information about the company is being revealed. Thus, the diagram is inconsistent with the efficient markets hypothesis (EMH).

CHAPTER 15

LONG-TERM FINANCING

Answers to Concepts Review and Critical Thinking Questions

1. The indenture is a legal contract and can run into 100 pages or more. Bond features which would be included are: the basic terms of the bond, the total amount of the bonds issued, description of the property used as security, repayment arrangements, call provisions, convertibility provisions, and details of protective covenants.
2. The differences between preferred stock and debt are:
 - a. The dividends on preferred stock cannot be deducted as interest expense when determining taxable corporate income. From the individual investor's point of view, preferred dividends are ordinary income for tax purposes. For corporate investors, 70% of the amount they receive as dividends from preferred stock are exempt from income taxes.
 - b. In case of liquidation (at bankruptcy), preferred stock is junior to debt and senior to common stock.
 - c. There is no legal obligation for firms to pay out preferred dividends as opposed to the obligated payment of interest on bonds. Therefore, firms cannot be forced into default if a preferred stock dividend is not paid in a given year. Preferred dividends can be cumulative or non-cumulative, and they can also be deferred indefinitely (of course, indefinitely deferring the dividends might have an undesirable effect on the market value of the stock).
3. Some firms can benefit from issuing preferred stock. The reasons can be:
 - a. Public utilities can pass the tax disadvantage of issuing preferred stock on to their customers, so there is a substantial amount of straight preferred stock issued by utilities.
 - b. Firms reporting losses to the IRS already don't have positive income for any tax deductions, so they are unaffected by the tax disadvantage of dividends versus interest payments. They may be willing to issue preferred stock.
 - c. Firms that issue preferred stock can avoid the threat of bankruptcy that exists with debt financing because preferred dividends are not a legal obligation like interest payments on corporate debt.
4. The return on non-convertible preferred stock is lower than the return on corporate bonds for two reasons: 1) Corporate investors receive 70 percent tax deductibility on dividends if they hold the stock. Therefore, they are willing to pay more for the stock; that lowers its return. 2) Issuing corporations are willing and able to offer higher returns on debt since the interest on the debt reduces their tax liabilities. Preferred dividends are paid out of net income, hence they provide no tax shield.

Corporate investors are the primary holders of preferred stock since, unlike individual investors, they can deduct 70 percent of the dividend when computing their tax liabilities. Therefore, they are willing to accept the lower return that the stock generates.

5. The following table summarizes the main differences between debt and equity:

	<u>Debt</u>	<u>Equity</u>
Repayment is an obligation of the firm	Yes	No
Grants ownership of the firm	No	Yes
Provides a tax shield	Yes	No
Liquidation will result if not paid	Yes	No

Companies often issue hybrid securities because of the potential tax shield and the bankruptcy advantage. If the IRS accepts the security as debt, the firm can use it as a tax shield. If the security maintains the bankruptcy and ownership advantages of equity, the firm has the best of both worlds.

6. There are two benefits. First, the company can take advantage of interest rate declines by calling in an issue and replacing it with a lower coupon issue. Second, a company might wish to eliminate a covenant for some reason. Calling the issue does this. The cost to the company is a higher coupon. A put provision is desirable from an investor's standpoint, so it helps the company by reducing the coupon rate on the bond. The cost to the company is that it may have to buy back the bond at an unattractive price.
7. It is the grant of authority by a shareholder to someone else to vote his or her shares.
8. Preferred stock is similar to both debt and common equity. Preferred shareholders receive a stated dividend only, and if the corporation is liquidated, preferred stockholders get a stated value. However, unpaid preferred dividends are not debts of a company and preferred dividends are not a tax deductible business expense.
9. A company has to issue more debt to replace the old debt that comes due if the company wants to maintain its capital structure. There is also the possibility that the market value of a company continues to increase (we hope). This means that to maintain a specific capital structure on a market value basis, the company has to issue new debt, since the market value of existing debt generally does not increase as the value of the company increases (at least by not as much).
10. Internal financing comes from internally generated cash flows and does not require issuing securities. In contrast, external financing requires the firm to issue new securities.
11. The three basic factors that affect the decision to issue external equity are: 1) The general economic environment, specifically, business cycles. 2) The level of stock prices, and 3) The availability of positive NPV projects.
12. When a company has dual class stock, the difference in the share classes is the voting rights. Dual share classes allow minority shareholders to retain control of the company even though they do not own a majority of the total shares outstanding. Often, dual share companies were started by a family, and then taken public, but the founders want to retain control of the company.
13. The statement is true. In an efficient market, the callable bonds will be sold at a lower price than the non-callable bonds, other things being equal. This is because the holder of callable bonds effectively sold a call option to the bond issuer. Since the issuer holds the right to call the bonds, the price of the bonds will reflect the disadvantage to the bondholders and the advantage to the bond issuer (i.e., the bondholder has the obligation to surrender his/her bonds when the call option is exercised by the bond issuer.)

14. As interest rates fall, the call option on the callable bonds is more likely to be exercised by the bond issuer. Since the non-callable bonds do not have such a drawback, the value of the bond will go up to reflect the decrease in the market rate of interest. Thus, the price of non-callable bonds will move higher than that of the callable bonds.
15. Sinking funds provide additional security to bonds. If a firm is experiencing financial difficulty, it is likely to have trouble making its sinking fund payments. Thus, the sinking fund provides an early warning system to the bondholders about the quality of the bonds. A drawback to sinking funds is that they give the firm an option that the bondholders may find distasteful. If bond prices are low, the firm may satisfy its sinking fund obligation by buying bonds in the open market. If bond prices are high though, the firm may satisfy its obligation by purchasing bonds at face value (or another fixed price, depending on the specific terms). Those bonds being repurchased are chosen through a lottery.

Solutions to Questions and Problems

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Basic

1. If the company uses straight voting, the board of directors is elected one at a time. You will need to own one-half of the shares, plus one share, in order to guarantee enough votes to win the election. So, the number of shares needed to guarantee election under straight voting will be:

$$\begin{aligned}\text{Shares needed} &= (850,000 \text{ shares}/2) + 1 \\ \text{Shares needed} &= 425,001\end{aligned}$$

And the total cost to you will be the shares needed times the price per share, or:

$$\begin{aligned}\text{Total cost} &= 425,001 \times \$61 \\ \text{Total cost} &= \$25,925,061\end{aligned}$$

If the company uses cumulative voting, the board of directors are all elected at once. You will need $1/(N + 1)$ percent of the stock (plus one share) to guarantee election, where N is the number of seats up for election. So, the percentage of the company's stock you need is:

$$\begin{aligned}\text{Percent of stock needed} &= 1/(N + 1) \\ \text{Percent of stock needed} &= 1/(7 + 1) \\ \text{Percent of stock needed} &= .1250, \text{ or } 12.50\%\end{aligned}$$

So, the number of shares you need to purchase is:

$$\begin{aligned}\text{Number of shares to purchase} &= (850,000 \times .1250) + 1 \\ \text{Number of shares to purchase} &= 106,251\end{aligned}$$

And the total cost to you will be the shares needed times the price per share, or:

$$\text{Total cost} = 106,251 \times \$61$$

$$\text{Total cost} = \$6,481,311$$

2. If the company uses cumulative voting, the directors are all elected at once. You will need $1/(N + 1)$ percent of the stock (plus one share) to guarantee election, where N is the number of seats up for election. So, the percentage of the company's stock you need is:

$$\text{Percent of stock needed} = 1/(N + 1)$$

$$\text{Percent of stock needed} = 1/(3 + 1)$$

$$\text{Percent of stock needed} = .25, \text{ or } 25\%$$

So, the number of shares you need is:

$$\text{Number of shares to purchase} = (19,600 \times .25) + 1$$

$$\text{Number of shares to purchase} = 4,901$$

So, the number of additional shares you need to purchase is:

$$\text{New shares to purchase} = 4,901 - 500$$

$$\text{New shares to purchase} = 4,401$$

3. If the company uses cumulative voting, the directors are all elected at once. You will need $1/(N + 1)$ percent of the stock (plus one share) to guarantee election, where N is the number of seats up for election. So, the percentage of the company's stock you need is:

$$\text{Percent of stock needed} = 1/(N + 1)$$

$$\text{Percent of stock needed} = 1/(4 + 1)$$

$$\text{Percent of stock needed} = .20, \text{ or } 20\%$$

So, the number of shares you need to purchase is:

$$\text{Number of shares to purchase} = (14,300,000 \times .20) + 1$$

$$\text{Number of shares to purchase} = 2,860,001$$

And the total cost will be the shares needed times the price per share, or:

$$\text{Total cost} = 2,860,001 \times \$15.25$$

$$\text{Total cost} = \$43,615,015$$

4. Under cumulative voting, she will need $1/(N + 1)$ percent of the stock (plus one share) to guarantee election, where N is the number of seats up for election. So, the percentage of the company's stock she needs is:

$$\text{Percent of stock needed} = 1/(N + 1)$$

$$\text{Percent of stock needed} = 1/(12 + 1)$$

$$\text{Percent of stock needed} = .0769, \text{ or } 7.69\%$$

Her nominee is guaranteed election. If the elections are staggered, the percentage of the company's stock needed is:

$$\text{Percent of stock needed} = 1/(N + 1)$$

$$\text{Percent of stock needed} = 1/(4 + 1)$$

$$\text{Percent of stock needed} = .2000, \text{ or } 20.00\%$$

Her nominee is no longer guaranteed election.

5. We first need to find the market value of equity, which is:

$$\text{Market value of equity} = \$43(1,600,000)$$

$$\text{Market value of equity} = \$68,800,000$$

So, the debt-equity ratio is:

$$D/E = \text{Market value of debt} / \text{Market value of equity}$$

$$D/E = \$55,000,000 / \$68,800,000$$

$$D/E = .80 \text{ times}$$

6. To find the price of the company's bonds, we need to find the present value of the bond's cash flows. So, the price of the bond is:

$$P = \$30(\text{PVIFA}_{2.80\%, 26}) + \$1,000(\text{PVIF}_{2.80\%, 26})$$

$$P = \$1,036.59$$

So, the total market value of the company's debt is:

$$\text{Market value of debt} = 325,000(\$1,036.59)$$

$$\text{Market value of debt} = \$336,891,994$$

And the market value of equity is:

$$\text{Market value of equity} = \$71(10,000,000)$$

$$\text{Market value of equity} = \$710,000,000$$

So, the debt-equity ratio is:

$$D/E = \text{Market value of debt} / \text{Market value of equity}$$

$$D/E = \$336,891,994 / \$710,000,000$$

$$D/E = .47 \text{ times}$$

7. a. The debt-equity ratio based on the book value of debt and equity is:

$$D/E = \text{Book value of debt} / \text{Book value of equity}$$

$$D/E = \$185,000,000 / \$210,000,000$$

$$D/E = .881 \text{ times}$$

- b. The debt-equity ratio based on the market value of debt and equity is:

$$D/E = \text{Market value of debt} / \text{Market value of equity}$$

$$D/E = \$195,000,000 / \$590,000,000$$

$$D/E = .331 \text{ times}$$

- c. Market values are preferred over book, or accounting, values since market values represent the current value, while book value represents historical cost.

Intermediate

8. a. The price of the bond today is the present value of the expected price in one year. So, the price of the bond in one year if interest rates increase will be:

$$P_1 = \$35(PVIFA_{4.5\%,58}) + \$1,000(PVIF_{4.5\%,58})$$

$$P_1 = \$795.08$$

If interest rates fall, the price if the bond in one year will be:

$$P_1 = \$35(PVIFA_{2.5\%,58}) + \$1,000(PVIF_{2.5\%,58})$$

$$P_1 = \$1,304.48$$

Now we can find the price of the bond today, which will be:

$$P_0 = [.50(\$795.08) + .50(\$1,304.48)] / 1.03^2 + \$35 / 1.03 + \$35 / 1.03^2$$

$$P_0 = \$1,056.49$$

For students who have studied term structure, the assumption of risk-neutrality implies that the forward rate is equal to the expected future spot rate.

- b. If the bond is callable, then the bond value will be less than the amount computed in part a. If the bond price rises above the call price, the company will call it. Therefore, bondholders will not pay as much for a callable bond.
9. The price of the bond today is the present value of the expected price in one year. The bond will be called whenever the price of the bond is greater than the call price of \$1,150. First, we need to find the expected price in one year. If interest rates increase next year, the price of the bond will be the present value of the perpetual interest payments, so:

$$P_1 = (\$90 / .11)$$

$$P_1 = \$818.18$$

This is lower than the call price, so the bond will not be called. If the interest rates fall next year, the price of the bond will be:

$$P_1 = (\$90 / .07)$$

$$P_1 = \$1,285.71$$

This is greater than the call price, so the bond will be called. The present value of the expected value of the bond price in one year, plus the coupon payment made in one year, is:

$$P_0 = [.60(\$818.18) + .40(\$1,150)]/1.09 + \$90/1.09$$

$$P_0 = \$954.96$$

10. If interest rates rise, the price of the bonds will fall. If the price of the bonds is low, the company will not call them. The firm would be foolish to pay the call price for something worth less than the call price. In this case, the bondholders will receive the coupon payment, C , plus the present value of the remaining payments. So, if interest rates rise, the price of the bonds in one year will be:

$$P_1 = C + C/.10$$

If interest rates fall, the assumption is that the bonds will be called. In this case, the bondholders will receive the call price, plus the coupon payment, C . So, the price of the bonds if interest rates fall will be:

$$P_1 = \$1,175 + C$$

The selling price today of the bonds is the PV of the expected payoffs to the bondholders. To find the coupon rate, we can set the desired issue price equal to the present value of the expected value of end of year payoffs, and solve for C . Doing so, we find:

$$P_0 = \$1,000 = [.60(C + C/.10) + .40(\$1,175 + C)]/1.09$$

$$C = \$88.57$$

So the coupon rate necessary to sell the bonds at par value will be:

$$\text{Coupon rate} = \$88.57/\$1,000$$

$$\text{Coupon rate} = .0886, \text{ or } 8.86\%$$

11. a. The price of the bond today is the present value of the expected price in one year. So, the price of the bond in one year if interest rates increase will be:

$$P_1 = \$65 + \$65/.08$$

$$P_1 = \$877.50$$

If interest rates fall, the price of the bond in one year will be:

$$P_1 = \$65 + \$65/.05$$

$$P_1 = \$1,365.00$$

Now we can find the price of the bond today, which will be:

$$P_0 = [.35(\$877.50) + .65(\$1,365.00)]/1.065$$

$$P_0 = \$1,121.48$$

- b. If interest rates rise, the price of the bonds will fall. If the price of the bonds is low, the company will not call them. The firm would be foolish to pay the call price for something worth less than the call price. In this case, the bondholders will receive the coupon payment, C , plus the present value of the remaining payments. So, if interest rates rise, the price of the bonds in one year will be:

$$P_1 = C + C/.08$$

If interest rates fall, the assumption is that the bonds will be called. In this case, the bondholders will receive the call price, plus the coupon payment, C . The call premium is not fixed, but it is the same as the coupon rate, so the price of the bonds if interest rates fall will be:

$$P_1 = (\$1,000 + C) + C$$

$$P_1 = \$1,000 + 2C$$

The selling price today of the bonds is the PV of the expected payoffs to the bondholders. To find the coupon rate, we can set the desired issue price equal to the present value of the expected value of end of year payoffs, and solve for C . Doing so, we find:

$$P_0 = \$1,000 = [.35(C + C/.08) + .65(\$1,000 + 2C)]/1.065$$

$$C = \$68.88$$

So the coupon rate necessary to sell the bonds at par value will be:

$$\text{Coupon rate} = \$68.88/\$1,000$$

$$\text{Coupon rate} = .0689, \text{ or } 6.89\%$$

- c. To the company, the value of the call provision will be given by the difference between the value of an outstanding, non-callable bond and the call provision. So, the value of a non-callable bond with the same coupon rate would be:

$$\text{Non-callable bond value} = \$68.88/.05$$

$$\text{Non-callable bond value} = \$1,377.59$$

So, the value of the call provision to the company is:

$$\text{Value} = .65(\$1,377.59 - 1,068.88)/1.065$$

$$\text{Value} = \$188.42$$

Challenge

12. In general, this is not likely to happen, although it can (and did). The reason that this bond has a negative YTM is that it is a callable U.S. Treasury bond. Market participants know this. Given the high coupon rate of the bond, it is extremely likely to be called, which means the bondholder will not receive all the cash flows promised. A better measure of the return on a callable bond is the yield to call (YTC). The YTC calculation is basically the same as the YTM calculation, but the number of periods is the number of periods until the call date. If the YTC were to be calculated on this bond, it would be positive.

CHAPTER 16

CAPITAL STRUCTURE: BASIC CONCEPTS

Answers to Concepts Review and Critical Thinking Questions

1. Assumptions of the Modigliani-Miller theory in a world without taxes: 1) Individuals can borrow at the same interest rate at which the firm borrows. Since investors can purchase securities on margin, an individual's effective interest rate is probably no higher than that for a firm. Therefore, this assumption is reasonable when applying MM's theory to the real world. If a firm were able to borrow at a rate lower than individuals could borrow, the firm's value would increase through corporate leverage. As MM Proposition I states, this is not the case in a world with no taxes. 2) There are no taxes. In the real world, firms do pay taxes. In the presence of corporate taxes, the value of a firm is positively related to its debt level. Since interest payments are deductible, increasing debt reduces taxes and raises the value of the firm. 3) There are no costs of financial distress. In the real world, costs of financial distress can be substantial. Since stockholders eventually bear these costs, there are incentives for a firm to lower the amount of debt in its capital structure. This topic will be discussed in more detail in later chapters.
2. False. A reduction in leverage will decrease both the risk of the stock and its expected return. Modigliani and Miller state that, in the absence of taxes, these two effects exactly cancel each other out and leave the price of the stock and the overall value of the firm unchanged.
3. False. Modigliani-Miller Proposition II (No Taxes) states that the required return on a firm's equity is positively related to the firm's debt-equity ratio $[R_S = R_0 + (B/S)(R_0 - R_B)]$. Therefore, any increase in the amount of debt in a firm's capital structure will increase the required return on the firm's equity.
4. Interest payments are tax deductible, where payments to shareholders (dividends) are not tax deductible.
5. Business risk is the equity risk arising from the nature of the firm's operating activity, and is directly related to the systematic risk of the firm's assets. Financial risk is the equity risk that is due entirely to the firm's chosen capital structure. As financial leverage, or the use of debt financing, increases, so does financial risk and, hence, the overall risk of the equity. Thus, Firm B could have a higher cost of equity if it uses greater leverage.
6. No, it doesn't follow. While it is true that the equity and debt costs are rising, the key thing to remember is that the cost of debt is still less than the cost of equity. Since we are using more and more debt, the WACC does not necessarily rise.

7. Because many relevant factors such as bankruptcy costs, tax asymmetries, and agency costs cannot easily be identified or quantified, it is practically impossible to determine the precise debt-equity ratio that maximizes the value of the firm. However, if the firm's cost of new debt suddenly becomes much more expensive, it's probably true that the firm is too highly leveraged.
8. It's called leverage (or "gearing" in the UK) because it magnifies gains or losses.
9. Homemade leverage refers to the use of borrowing on the personal level as opposed to the corporate level.
10. The basic goal is to minimize the value of non-marketed claims.

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. a. A table outlining the income statement for the three possible states of the economy is shown below. The EPS is the net income divided by the 5,000 shares outstanding. The last row shows the percentage change in EPS the company will experience in a recession or an expansion economy.

	<u>Recession</u>	<u>Normal</u>	<u>Expansion</u>
EBIT	\$11,400	\$19,000	\$23,750
Interest	<u>0</u>	<u>0</u>	<u>0</u>
NI	<u>\$11,400</u>	<u>\$19,000</u>	<u>\$23,750</u>
EPS	\$ 2.28	\$ 3.80	\$ 4.75
%ΔEPS	-40	—	+25

- b. If the company undergoes the proposed recapitalization, it will repurchase:

Share price = Market value/Shares outstanding

Share price = \$245,000/5,000

Share price = \$49

Shares repurchased = Debt issued/Share price

Shares repurchased = \$58,800/\$49

Shares repurchased = 1,200

The interest payment each year under all three scenarios will be:

Interest payment = \$58,800(.08)

Interest payment = \$4,704

The last row shows the percentage change in EPS the company will experience in a recession or an expansion economy under the proposed recapitalization.

	<u>Recession</u>	<u>Normal</u>	<u>Expansion</u>
EBIT	\$11,400	\$19,000	\$23,750
Interest	<u>4,704</u>	<u>4,704</u>	<u>4,704</u>
NI	<u>\$ 6,696</u>	<u>\$14,296</u>	<u>\$19,046</u>
EPS	\$1.76	\$ 3.76	\$ 5.01
%ΔEPS	-53.16	—	+33.23%

2. a. A table outlining the income statement with taxes for the three possible states of the economy is shown below. The share price is \$49, and there are 5,000 shares outstanding. The last row shows the percentage change in EPS the company will experience in a recession or an expansion economy.

	<u>Recession</u>	<u>Normal</u>	<u>Expansion</u>
EBIT	\$11,400	\$19,000	\$23,750
Interest	0	0	0
Taxes	<u>2,394</u>	<u>3,990</u>	<u>4,988</u>
NI	<u>\$ 9,006</u>	<u>\$15,010</u>	<u>\$18,763</u>
EPS	\$1.80	\$3.00	\$3.75
%ΔEPS	-40	—	+25

- b. A table outlining the income statement with taxes for the three possible states of the economy and assuming the company undertakes the proposed capitalization is shown below. The interest payment and shares repurchased are the same as in part b of Problem 1.

	<u>Recession</u>	<u>Normal</u>	<u>Expansion</u>
EBIT	\$11,400	\$19,000	\$23,750
Interest	4,704	4,704	4,704
Taxes	<u>1,406</u>	<u>3,002</u>	<u>4,000</u>
NI	<u>\$5,290</u>	<u>\$11,294</u>	<u>\$15,046</u>
EPS	\$1.39	\$2.97	\$3.96
%ΔEPS	-53.16	—	+33.23

Notice that the percentage change in EPS is the same both with and without taxes.

3. a. Since the company has a market-to-book ratio of 1.0, the total equity of the firm is equal to the market value of equity. Using the equation for ROE:

$$\text{ROE} = \text{NI}/\$245,000$$

The ROE for each state of the economy under the current capital structure and no taxes is:

	<u>Recession</u>	<u>Normal</u>	<u>Expansion</u>
ROE	4.65%	7.76%	9.69%
%ΔROE	-40	—	+25

The second row shows the percentage change in ROE from the normal economy.

- b. If the company undertakes the proposed recapitalization, the new equity value will be:

$$\text{Equity} = \$245,000 - 58,800$$

$$\text{Equity} = \$186,200$$

So, the ROE for each state of the economy is:

$$\text{ROE} = \text{NI}/\$186,200$$

	<u>Recession</u>	<u>Normal</u>	<u>Expansion</u>
ROE	3.60%	7.68%	10.23%
%ΔROE	-53.16	—	+33.23

- c. If there are corporate taxes and the company maintains its current capital structure, the ROE is:

ROE	3.68%	6.13%	7.66%
%ΔROE	-40	—	+25

If the company undertakes the proposed recapitalization, and there are corporate taxes, the ROE for each state of the economy is:

ROE	2.84%	6.07%	8.08%
%ΔROE	-53.16	—	+33.23

Notice that the percentage change in ROE is the same as the percentage change in EPS. The percentage change in ROE is also the same with or without taxes.

4. a. Under Plan I, the unlevered company, net income is the same as EBIT with no corporate tax. The EPS under this capitalization will be:

$$\text{EPS} = \$1,250,000/365,000 \text{ shares}$$

$$\text{EPS} = \$3.42$$

Under Plan II, the levered company, EBIT will be reduced by the interest payment. The interest payment is the amount of debt times the interest rate, so:

$$\text{NI} = \$1,250,000 - .10(\$4,560,000)$$

$$\text{NI} = \$794,000$$

And the EPS will be:

$$\begin{aligned}\text{EPS} &= \$794,000/245,000 \text{ shares} \\ \text{EPS} &= \$3.24\end{aligned}$$

Plan I has the higher EPS when EBIT is \$1,250,000.

- b. Under Plan I, the net income is \$1,750,000 and the EPS is:

$$\begin{aligned}\text{EPS} &= \$1,750,000/365,000 \text{ shares} \\ \text{EPS} &= \$4.79\end{aligned}$$

Under Plan II, the net income is:

$$\begin{aligned}\text{NI} &= \$1,750,000 - .10(\$4,560,000) \\ \text{NI} &= \$1,294,000\end{aligned}$$

And the EPS is:

$$\begin{aligned}\text{EPS} &= \$1,294,000/245,000 \text{ shares} \\ \text{EPS} &= \$5.28\end{aligned}$$

Plan II has the higher EPS when EBIT is \$1,750,000.

- c. To find the break-even EBIT for two different capital structures, we set the equations for EPS equal to each other and solve for EBIT. The break-even EBIT is:

$$\begin{aligned}\text{EBIT}/365,000 &= [\text{EBIT} - .10(\$4,560,000)]/245,000 \\ \text{EBIT} &= \$1,387,000\end{aligned}$$

5. We can find the price per share by dividing the amount of debt used to repurchase shares by the number of shares repurchased. Doing so, we find the share price is:

$$\begin{aligned}\text{Share price} &= \$4,560,000/(365,000 - 245,000) \\ \text{Share price} &= \$38.00 \text{ per share}\end{aligned}$$

The value of the company under the all-equity plan is:

$$\begin{aligned}V &= \$38(365,000 \text{ shares}) \\ V &= \$13,870,000\end{aligned}$$

And the value of the company under the levered plan is:

$$\begin{aligned}V &= \$38(245,000 \text{ shares}) + \$4,560,000 \text{ debt} \\ V &= \$13,870,000\end{aligned}$$

6. a. The income statement for each capitalization plan is:

	<i>I</i>	<i>II</i>	<i>All-equity</i>
EBIT	\$14,800	\$14,800	\$14,800
Interest	<u>3,744</u>	<u>6,656</u>	<u>0</u>
NI	<u>\$11,056</u>	<u>\$8,144</u>	<u>\$14,800</u>
EPS	\$ 3.16	\$ 2.91	\$ 3.36

The all-equity plan has the highest EPS; Plan II has the lowest EPS.

- b. The break-even level of EBIT occurs when the capitalization plans result in the same EPS. The EPS is calculated as:

$$\text{EPS} = (\text{EBIT} - R_B B) / \text{Shares outstanding}$$

This equation calculates the interest payment ($R_B B$) and subtracts it from the EBIT, which results in the net income. Dividing by the shares outstanding gives us the EPS. For the all-equity capital structure, the interest paid is zero. To find the break-even EBIT for two different capital structures, we set the equations equal to each other and solve for EBIT. The break-even EBIT between the all-equity capital structure and Plan I is:

$$\begin{aligned} \text{EBIT}/4,400 &= [\text{EBIT} - .10(\$37,440)]/3,500 \\ \text{EBIT} &= \$18,304 \end{aligned}$$

And the break-even EBIT between the all-equity capital structure and Plan II is:

$$\begin{aligned} \text{EBIT}/4,400 &= [\text{EBIT} - .10(\$66,560)]/2,800 \\ \text{EBIT} &= \$18,304 \end{aligned}$$

The break-even levels of EBIT are the same because of M&M Proposition I.

- c. Setting the equations for EPS from Plan I and Plan II equal to each other and solving for EBIT, we get:

$$\begin{aligned} [\text{EBIT} - .10(\$37,440)]/3,500 &= [\text{EBIT} - .10(\$66,560)]/2,800 \\ \text{EBIT} &= \$18,304 \end{aligned}$$

This break-even level of EBIT is the same as in part *b* again because of M&M Proposition I.

- d. The income statement for each capitalization plan with corporate income taxes is:

	<i>I</i>	<i>II</i>	<i>All-equity</i>
EBIT	\$14,800	\$14,800	\$14,800
Interest	3,744	6,656	0
Taxes	<u>2,322</u>	<u>1,710</u>	<u>3,108</u>
NI	<u>\$8,734</u>	<u>\$6,434</u>	<u>\$11,692</u>
EPS	\$ 2.50	\$ 2.30	\$ 2.66

The all-equity plan has the highest EPS; Plan I has the lowest EPS.

We can calculate the EPS as:

$$\text{EPS} = [(\text{EBIT} - R_B D)(1 - T_C)] / \text{Shares outstanding}$$

This is similar to the equation we used before, except that now we need to account for taxes. Again, the interest expense term is zero in the all-equity capital structure. So, the break-even EBIT between the all-equity plan and Plan I is:

$$\begin{aligned} \text{EBIT}(1 - .21)/4,400 &= [\text{EBIT} - .10(\$37,440)](1 - .21)/3,500 \\ \text{EBIT} &= \$18,304 \end{aligned}$$

The break-even EBIT between the all-equity plan and Plan II is:

$$\begin{aligned} \text{EBIT}(1 - .21)/4,400 &= [\text{EBIT} - .10(\$66,560)](1 - .21)/2,800 \\ \text{EBIT} &= \$18,304 \end{aligned}$$

And the break-even between Plan I and Plan II is:

$$\begin{aligned} [\text{EBIT} - .10(\$37,440)](1 - .21)/3,500 &= [\text{EBIT} - .10(\$66,560)](1 - .21)/2,800 \\ \text{EBIT} &= \$18,304 \end{aligned}$$

The break-even levels of EBIT do not change because the addition of taxes reduces the income of all three plans by the same percentage; therefore, they do not change relative to one another.

7. To find the value per share of the stock under each capitalization plan, we can calculate the price as the value of shares repurchased divided by the number of shares repurchased. The dollar value of the shares repurchased is the increase in the value of the debt used to repurchase shares, or:

$$\text{Dollar value of repurchase} = \$66,560 - 37,440$$

$$\text{Dollar value of repurchase} = \$29,120$$

The number of shares repurchased is the decrease in shares outstanding, or:

$$\text{Number of shares repurchased} = 3,500 - 2,800$$

$$\text{Number of shares repurchased} = 700$$

So, under Plan I, the value per share is:

$$P = \$29,120/700 \text{ shares}$$

$$P = \$41.60 \text{ per share}$$

And under Plan II, the number of shares repurchased from the all equity plan by the \$37,440 in debt is:

$$\text{Number of shares repurchased} = 4,400 - 3,500$$

$$\text{Number of shares repurchased} = 900$$

So the share price is:

$$P = \$37,440/900 \text{ shares}$$

$$P = \$41.60 \text{ per share}$$

This shows that when there are no corporate taxes, the stockholder does not care about the capital structure decision of the firm. This is M&M Proposition I without taxes.

8. a. The earnings per share are:

$$\text{EPS} = \$43,600/5,000 \text{ shares}$$

$$\text{EPS} = \$8.72$$

So, the cash flow for the shareholder is:

$$\text{Cash flow} = \$8.72(100 \text{ shares})$$

$$\text{Cash flow} = \$872$$

- b. To determine the cash flow to the shareholder, we need to determine the EPS of the firm under the proposed capital structure. The market value of the firm is:

$$V = \$49(5,000)$$

$$V = \$245,000$$

Under the proposed capital structure, the firm will raise new debt in the amount of:

$$B = .35(\$245,000)$$

$$B = \$85,750$$

This means the number of shares repurchased will be:

$$\text{Shares repurchased} = \$85,750 / \$49$$

$$\text{Shares repurchased} = 1,750$$

Under the new capital structure, the company will have to make an interest payment on the new debt. The net income with the interest payment will be:

$$\text{NI} = \$43,600 - .07(\$85,750)$$

$$\text{NI} = \$37,598$$

This means the EPS under the new capital structure will be:

$$\text{EPS} = \$37,598 / (5,000 - 1,750 \text{ shares})$$

$$\text{EPS} = \$11.57$$

Since all earnings are paid as dividends, the shareholder will receive:

$$\text{Shareholder cash flow} = \$11.57(100 \text{ shares})$$

$$\text{Shareholder cash flow} = \$1,156.85$$

- c. To replicate the proposed capital structure, the shareholder should sell 35 percent of their shares, or 35 shares, and lend the proceeds at 7 percent. The shareholder will have an interest cash flow of:

$$\text{Interest cash flow} = 35(\$49)(.07)$$

$$\text{Interest cash flow} = \$120.05$$

The shareholder will receive dividend payments on the remaining 65 shares, so the dividends received will be:

$$\text{Dividends received} = \$11.57(65 \text{ shares})$$

$$\text{Dividends received} = \$751.95$$

The total cash flow for the shareholder under these assumptions will be:

$$\text{Total cash flow} = \$120.05 + 751.95$$

$$\text{Total cash flow} = \$872$$

This is the same cash flow we calculated in part a.

- d. The capital structure is irrelevant because shareholders can create their own leverage or unlever the stock to create the payoff they desire, regardless of the capital structure the firm actually chooses.
9. a. The rate of return earned will be the dividend yield. The company has debt, so it must make an interest payment. The net income for the company is:

$$\text{NI} = \$61,000 - .08(\$270,000)$$

$$\text{NI} = \$39,400$$

The investor will receive dividends in proportion to the percentage of the company's shares he owns. The total dividends received by the shareholder will be:

$$\text{Dividends received} = \$39,400(\$30,000/\$270,000)$$

$$\text{Dividends received} = \$4,378$$

So the return the shareholder expects is:

$$R = \$4,378/\$30,000$$

$$R = .1459, \text{ or } 14.59\%$$

- b. To generate exactly the same cash flows in the other company, the shareholder needs to match the capital structure of ABC. The shareholder should sell all shares in XYZ. This will net \$30,000. The shareholder should then borrow \$30,000. This will create an interest cash flow of:

$$\text{Interest cash flow} = .08(-\$30,000)$$

$$\text{Interest cash flow} = -\$2,400$$

The investor should then use the proceeds of the stock sale and the loan to buy shares in ABC. The investor will receive dividends in proportion to the percentage of the company's share he owns. The total dividends received by the shareholder will be:

$$\text{Dividends received} = \$61,000(\$60,000/\$540,000)$$

$$\text{Dividends received} = \$6,778$$

The total cash flow for the shareholder will be:

$$\text{Total cash flow} = \$6,778 - 2,400$$

$$\text{Total cash flow} = \$4,378$$

The shareholder's return in this case will be:

$$R = \$4,378/\$30,000$$

$$R = .1459, \text{ or } 14.59\%$$

- c. ABC is an all equity company, so:

$$R_S = R_A = \$61,000/\$540,000$$

$$R_S = .1130, \text{ or } 11.30\%$$

To find the cost of equity for XYZ, we need to use M&M Proposition II, so:

$$R_S = R_A + (R_A - R_B)(B/S)(1 - T_C)$$

$$R_S = .1130 + (.1130 - .08)(1)(1)$$

$$R_S = .1459, \text{ or } 14.59\%$$

- d. To find the WACC for each company, we need to use the WACC equation:

$$\text{WACC} = (S/V)R_S + (B/V)R_B(1 - T_C)$$

So, for ABC, the WACC is:

$$\begin{aligned}\text{WACC} &= (1)(.1130) + (0)(.08) \\ \text{WACC} &= .1130, \text{ or } 11.30\%\end{aligned}$$

And for XYZ, the WACC is:

$$\begin{aligned}\text{WACC} &= (1/2)(.1459) + (1/2)(.08) \\ \text{WACC} &= .1130, \text{ or } 11.30\%\end{aligned}$$

When there are no corporate taxes, the cost of capital for the firm is unaffected by the capital structure; this is M&M Proposition I without taxes.

10. With no taxes, the value of an unlevered firm is the EBIT divided by the unlevered cost of equity, so:

$$\begin{aligned}V &= \text{EBIT}/\text{WACC} \\ \$37,000,000 &= \text{EBIT}/.086 \\ \text{EBIT} &= .086(\$37,000,000) \\ \text{EBIT} &= \$3,182,000\end{aligned}$$

11. If there are corporate taxes, the value of an unlevered firm is:

$$V_U = \text{EBIT}(1 - T_C)/R_U$$

Using this relationship, we can find EBIT as:

$$\begin{aligned}\$37,000,000 &= \text{EBIT}(1 - .23)/.086 \\ \text{EBIT} &= \$4,132,467.53\end{aligned}$$

The WACC remains at 8.6 percent. Due to taxes, EBIT for an all-equity firm would have to be higher for the firm to still be worth \$37 million.

12. a. With the information provided, we can use the equation for calculating WACC to find the cost of equity. The equation for WACC is:

$$\text{WACC} = (S/V)R_S + (B/V)R_B(1 - T_C)$$

The company has a debt-equity ratio of 1.5, which implies the weight of debt is 1.5/2.5, and the weight of equity is 1/2.5, so

$$\begin{aligned}\text{WACC} &= .098 = (1/2.5)R_S + (1.5/2.5)(.06)(1 - .21) \\ R_S &= .1739, \text{ or } 17.39\%\end{aligned}$$

- b. To find the unlevered cost of equity, we need to use M&M Proposition II with taxes, so:

$$R_S = R_0 + (R_0 - R_B)(B/S)(1 - T_C)$$

$$.1739 = R_0 + (R_0 - .06)(1.5)(1 - .21)$$

$$R_0 = .1121, \text{ or } 11.21\%$$

- c. To find the cost of equity under different capital structures, we can again use M&M Proposition II with taxes. With a debt-equity ratio of 2, the cost of equity is:

$$R_S = R_0 + (R_0 - R_B)(B/S)(1 - T_C)$$

$$R_S = .1121 + (.1121 - .06)(2)(1 - .21)$$

$$R_S = .1945, \text{ or } 19.45\%$$

With a debt-equity ratio of 1.0, the cost of equity is:

$$R_S = .1121 + (.1121 - .06)(1)(1 - .21)$$

$$R_S = .1533, \text{ or } 15.33\%$$

And with a debt-equity ratio of 0, the cost of equity is:

$$R_S = .1121 + (.1121 - .06)(0)(1 - .21)$$

$$R_S = R_0 = .1121, \text{ or } 11.21\%$$

13. a. For an all-equity financed company:

$$WACC = R_0 = R_S = .091, \text{ or } 9.1\%$$

- b. To find the cost of equity for the company with leverage, we need to use M&M Proposition II with taxes, so:

$$R_S = R_0 + (R_0 - R_B)(B/S)(1 - T_C)$$

$$R_S = .091 + (.091 - .058)(.25/.75)(1 - .22)$$

$$R_S = .0996, \text{ or } 9.96\%$$

- c. Using M&M Proposition II with taxes again, we get:

$$R_S = R_0 + (R_0 - R_B)(B/S)(1 - T_C)$$

$$R_S = .091 + (.091 - .058)(.50/.50)(1 - .22)$$

$$R_S = .1167, \text{ or } 11.67\%$$

- d. The WACC with 25 percent debt is:

$$WACC = (S/V)R_S + (B/V)R_B(1 - T_C)$$

$$WACC = .75(.0996) + .25(.058)(1 - .22)$$

$$WACC = .0860, \text{ or } 8.60\%$$

And the WACC with 50 percent debt is:

$$WACC = (S/V)R_S + (B/V)R_D(1 - T_C)$$

$$WACC = .50(.1167) + .50(.058)(1 - .22)$$

$$WACC = .0810, \text{ or } 8.10\%$$

14. a. The value of the unlevered firm is:

$$\begin{aligned} V &= \text{EBIT}(1 - T_C)/R_0 \\ V &= \$163,000(1 - .23)/.15 \\ V &= \$836,733.33 \end{aligned}$$

- b. The value of the levered firm is:

$$\begin{aligned} V &= V_U + T_C B \\ V &= \$836,733.33 + .23(\$185,000) \\ V &= \$879,283.33 \end{aligned}$$

15. We can find the cost of equity using M&M Proposition II with taxes. First, we need to find the market value of equity, which is:

$$\begin{aligned} V &= B + S \\ \$879,283.33 &= \$185,000 + S \\ S &= \$694,283.33 \end{aligned}$$

Now we can find the cost of equity, which is:

$$\begin{aligned} R_S &= R_0 + (R_0 - R_B)(B/S)(1 - T_C) \\ R_S &= .15 + (.15 - .08)(\$185,000/\$694,283.33)(1 - .23) \\ R_S &= .1644, \text{ or } 16.44\% \end{aligned}$$

Using this cost of equity, the WACC for the firm after recapitalization is:

$$\begin{aligned} \text{WACC} &= (S/V)R_S + (B/V)R_B(1 - T_C) \\ \text{WACC} &= (\$694,283.33/\$879,283.33)(.1644) + (\$185,000/\$879,283.33)(.08)(1 - .23) \\ \text{WACC} &= .1427, \text{ or } 14.27\% \end{aligned}$$

When there are corporate taxes, the overall cost of capital for the firm declines the more highly leveraged is the firm's capital structure. This is M&M Proposition I with taxes.

16. Since Unlevered is an all-equity firm, its value is equal to the market value of its outstanding shares. Unlevered has 3.8 million shares of common stock outstanding, worth \$71 per share. Therefore, the value of Unlevered:

$$\begin{aligned} V_U &= 3,800,000(\$71) \\ V_U &= \$269,800,000 \end{aligned}$$

Modigliani-Miller Proposition I states that, in the absence of taxes, the value of a levered firm equals the value of an otherwise identical unlevered firm. Since Levered is identical to Unlevered in every way except its capital structure and neither firm pays taxes, the value of the two firms should be equal. Therefore, the market value of Levered, Inc., should also be \$269.8 million. Since Levered has 1.9 million outstanding shares, worth \$98 per share, the market value of Levered's equity is:

$$\begin{aligned} S_L &= 1,900,000(\$98) \\ S_L &= \$186,200,000 \end{aligned}$$

The market value of Levered's debt is \$65 million. The value of a levered firm equals the market value of its debt plus the market value of its equity. Therefore, the current market value of Levered is:

$$\begin{aligned} V_L &= B + S \\ V_L &= \$65,000,000 + 186,200,000 \\ V_L &= \$251,200,000 \end{aligned}$$

The market value of Levered's equity needs to be \$269.8 million, \$18.6 million higher than its current market value of \$251.2 million, for MM Proposition I to hold. Since Levered's market value is less than Unlevered's market value, Levered is relatively underpriced and an investor should buy shares of the firm's stock.

Intermediate

17. To find the value of the levered firm, we first need to find the value of an unlevered firm. So, the value of the unlevered firm is:

$$\begin{aligned} V_U &= \text{EBIT}(1 - T_C)/R_0 \\ V_U &= (\$63,000)(1 - .21)/.12 \\ V_U &= \$414,750 \end{aligned}$$

Now we can find the value of the levered firm as:

$$\begin{aligned} V_L &= V_U + T_C B \\ V_L &= \$414,750 + .21(\$115,000) \\ V_L &= \$438,900 \end{aligned}$$

Applying M&M Proposition I with taxes, the firm has increased its value by issuing debt. As long as M&M Proposition I holds, that is, there are no bankruptcy costs and so forth, then the company should continue to increase its debt-equity ratio to maximize the value of the firm.

18. a. With no debt, we are finding the value of an unlevered firm, so:

$$\begin{aligned} V &= \text{EBIT}(1 - T_C)/R_0 \\ V &= \$31,480(1 - .21)/.14 \\ V &= \$177,637.14 \end{aligned}$$

- b. The general expression for the value of a leveraged firm is:

$$V_L = V_U + T_C B$$

If debt is 50 percent of V_U , then $D = (.50)V_U$, and we have:

$$\begin{aligned} V_L &= V_U + T_C[(.50)V_U] \\ V_L &= \$177,637.14 + .21(.50)(\$177,637.14) \\ V_L &= \$196,289.04 \end{aligned}$$

And if debt is 100 percent of V_U , then $D = (1.0)V_U$, and we have:

$$V_L = V_U + T_c[(1.0)V_U]$$

$$V_L = \$177,637.14 + .21(1.0)(\$177,637.14)$$

$$V_L = \$214,940.94$$

- c. According to M&M Proposition I with taxes:

$$V_L = V_U + T_c B$$

With debt being 50 percent of the value of the levered firm, D must equal $(.50)V_L$, so:

$$V_L = V_U + T_c[(.50)V_L]$$

$$V_L = \$177,637.14 + .21(.50)(V_L)$$

$$V_L = \$198,477.25$$

If the debt is 100 percent of the levered value, D must equal V_L , so:

$$V_L = V_U + T_c[(1.0)(V_L)]$$

$$V_L = \$177,637.14 + .21(1.0)(V_L)$$

$$V_L = \$224,857.14$$

19. According to M&M Proposition I with taxes, the increase in the value of the company will be the present value of the interest tax shield. Since the loan will be repaid in equal installments, we need to find the loan interest and the interest tax shield each year. The loan schedule will be:

Year	Loan Balance	Interest	Tax Shield
0	\$3,100,000		
1	\$1,550,000	\$248,000	\$59,520
2		124,000	29,760

So, the increase in the value of the company is:

$$\text{Value increase} = \$59,520/1.08 + \$29,760/(1.08)^2$$

$$\text{Value increase} = \$80,625.51$$

20. a. Since Alpha Corporation is an all-equity firm, its value is equal to the market value of its outstanding shares. Alpha has 18,000 shares of common stock outstanding, worth \$35 per share, so the value of Alpha Corporation is:

$$V_{\text{Alpha}} = 18,000(\$35)$$

$$V_{\text{Alpha}} = \$630,000$$

- b. Modigliani-Miller Proposition I states that in the absence of taxes, the value of a levered firm equals the value of an otherwise identical unlevered firm. Since Beta Corporation is identical to Alpha Corporation in every way except its capital structure and neither firm pays taxes, the value of the two firms should be equal. So, the value of Beta Corporation is \$630,000 as well.

- c. The value of a levered firm equals the market value of its debt plus the market value of its equity. So, the value of Beta's equity is:

$$\begin{aligned}V_L &= B + S \\ \$630,000 &= \$85,000 + S \\ S &= \$545,000\end{aligned}$$

- d. The investor would need to invest 20 percent of the total market value of Alpha's equity, which is:

$$\begin{aligned}\text{Amount to invest in Alpha} &= .20(\$630,000) \\ \text{Amount to invest in Alpha} &= \$126,000\end{aligned}$$

Beta has less equity outstanding, so to purchase 20 percent of Beta's equity, the investor would need:

$$\begin{aligned}\text{Amount to invest in Beta} &= .20(\$545,000) \\ \text{Amount to invest in Beta} &= \$109,000\end{aligned}$$

- e. Alpha has no interest payments, so the dollar return to an investor who owns 20 percent of the company's equity would be:

$$\begin{aligned}\text{Dollar return on Alpha investment} &= .20(\$93,000) \\ \text{Dollar return on Alpha investment} &= \$18,600\end{aligned}$$

Beta Corporation has an interest payment due on its debt in the amount of:

$$\begin{aligned}\text{Interest on Beta's debt} &= .09(\$85,000) \\ \text{Interest on Beta's debt} &= \$7,650\end{aligned}$$

So, the investor who owns 20 percent of the company would receive 20 percent of EBIT minus the interest expense, or:

$$\begin{aligned}\text{Dollar return on Beta investment} &= .20(\$93,000 - 7,650) \\ \text{Dollar return on Beta investment} &= \$17,070\end{aligned}$$

- f. From part d, we know the initial cost of purchasing 20 percent of Alpha Corporation's equity is \$126,000, but the cost to an investor of purchasing 20 percent of Beta Corporation's equity is only \$109,000. In order to purchase \$126,000 worth of Alpha's equity using only \$109,000 of his own money, the investor must borrow \$17,000 to cover the difference. The investor will receive the same dollar return from the Alpha investment, but will pay interest on the amount borrowed, so the net dollar return to the investment is:

$$\begin{aligned}\text{Net dollar return} &= \$18,600 - .09(\$17,000) \\ \text{Net dollar return} &= \$17,070\end{aligned}$$

Notice that this amount exactly matches the dollar return to an investor who purchases 20 percent of Beta's equity.

- g. The equity of Beta Corporation is riskier. Beta must pay off its debt holders before its equity holders receive any of the firm's earnings. If the firm does not do particularly well, all of the firm's earnings may be needed to repay its debt holders, and equity holders will receive nothing.
21. a. A firm's debt-equity ratio is the market value of the firm's debt divided by the market value of a firm's equity. So, the debt-equity ratio of the company is:

$$\begin{aligned}\text{Debt-equity ratio} &= \text{MV of debt} / \text{MV of equity} \\ \text{Debt-equity ratio} &= \$7,300,000 / \$18,500,000 \\ \text{Debt-equity ratio} &= .39\end{aligned}$$

- b. We first need to calculate the cost of equity. To do this, we can use the CAPM, which gives us:

$$\begin{aligned}R_S &= R_F + \beta[E(R_M) - R_F] \\ R_S &= .04 + 1.15(.11 - .04) \\ R_S &= .1205, \text{ or } 12.05\%\end{aligned}$$

We need to remember that an assumption of the Modigliani-Miller theorem is that the company debt is risk-free, so we can use the Treasury bill rate as the cost of debt for the company. In the absence of taxes, a firm's weighted average cost of capital is equal to:

$$\begin{aligned}R_{WACC} &= [B/(B + S)]R_B + [S/(B + S)]R_S \\ R_{WACC} &= (\$7,300,000 / \$25,800,000)(.04) + (\$18,500,000 / \$25,800,000)(.1205) \\ R_{WACC} &= .0977, \text{ or } 9.77\%\end{aligned}$$

- c. According to Modigliani-Miller Proposition II with no taxes:

$$\begin{aligned}R_S &= R_0 + (B/S)(R_0 - R_B) \\ .1205 &= R_0 + (.39)(R_0 - .04) \\ R_0 &= .0977, \text{ or } 9.77\%\end{aligned}$$

This is consistent with Modigliani-Miller's proposition that, in the absence of taxes, the cost of capital for an all-equity firm is equal to the weighted average cost of capital of an otherwise identical levered firm.

22. a. To purchase 5 percent of Knight's equity, the investor would need:

$$\begin{aligned}\text{Knight investment} &= .05(\$3,550,000) \\ \text{Knight investment} &= \$177,500\end{aligned}$$

And to purchase 5 percent of Veblen without borrowing would require:

$$\begin{aligned}\text{Veblen investment} &= .05(\$4,400,000) \\ \text{Veblen investment} &= \$220,000\end{aligned}$$

In order to compare dollar returns, the initial net cost of both positions should be the same. Therefore, the investor will need to borrow the difference between the two amounts, or:

$$\begin{aligned}\text{Amount to borrow} &= \$220,000 - 177,500 \\ \text{Amount to borrow} &= \$42,500\end{aligned}$$

An investor who owns 5 percent of Knight's equity will be entitled to 5 percent of the firm's earnings available to common stockholders at the end of each year. While Knight's expected operating income is \$610,000, it must pay \$96,000 to debt holders before distributing any of its earnings to stockholders. So, the amount available to this shareholder will be:

$$\text{Cash flow from Knight to shareholder} = .05(\$610,000 - 96,000)$$

$$\text{Cash flow from Knight to shareholder} = \$25,700$$

Veblen will distribute all of its earnings to shareholders, so the shareholder will receive:

$$\text{Cash flow from Veblen to shareholder} = .05(\$610,000)$$

$$\text{Cash flow from Veblen to shareholder} = \$30,500$$

However, to have the same initial cost, the investor has borrowed \$42,500 to invest in Veblen, so interest must be paid on the borrowings. The net cash flow from the investment in Veblen will be:

$$\text{Net cash flow from Veblen investment} = \$30,500 - .06(\$42,500)$$

$$\text{Net cash flow from Veblen investment} = \$27,950$$

For the same initial cost, the investment in Veblen produces a higher dollar return.

- b. Both of the two strategies have the same initial cost. Since the dollar return to the investment in Veblen is higher, all investors will choose to invest in Veblen over Knight. The process of investors purchasing Veblen's equity rather than Knight's will cause the market value of Veblen's equity to rise and/or the market value of Knight's equity to fall. Any differences in the dollar returns to the two strategies will be eliminated, and the process will cease when the total market values of the two firms are equal.
23. a. Before the announcement of the stock repurchase plan, the market value of the outstanding debt is \$2,700,000. Using the debt-equity ratio, we can find that the value of the outstanding equity must be:

$$\text{Debt-equity ratio} = B/S$$

$$.35 = \$2,700,000/S$$

$$S = \$7,714,286$$

The value of a levered firm is equal to the sum of the market value of the firm's debt and the market value of the firm's equity, so:

$$V_L = B + S$$

$$V_L = \$2,700,000 + 7,714,286$$

$$V_L = \$10,414,286$$

According to MM Proposition I without taxes, changes in a firm's capital structure have no effect on the overall value of the firm. Therefore, the value of the firm will not change after the announcement of the stock repurchase plan

- b. The expected return on a firm's equity is the ratio of annual earnings to the market value of the firm's equity, or return on equity. Before the restructuring, the company was expected to pay interest in the amount of:

$$\text{Interest payment} = .064(\$2,700,000)$$

$$\text{Interest payment} = \$172,800$$

The return on equity, which is equal to R_S , will be:

$$\text{ROE} = R_S = (\$940,000 - 172,800)/\$7,714,286$$

$$R_S = .0995, \text{ or } 9.95\%$$

- c. According to Modigliani-Miller Proposition II with no taxes:

$$R_S = R_0 + (B/S)(R_0 - R_B)$$

$$.0995 = R_0 + (.35)(R_0 - .064)$$

$$R_0 = .0903, \text{ or } 9.03\%$$

This problem can also be solved in the following way:

$$R_0 = \text{Earnings before interest}/V_U$$

According to Modigliani-Miller Proposition I, in a world with no taxes, the value of a levered firm equals the value of an otherwise-identical unlevered firm. Since the value of the company as a levered firm is \$10,414,286 (= \$2,700,000 + 7,714,286) and since the firm pays no taxes, the value of the company as an unlevered firm is also \$10,414,286. So:

$$R_0 = \$940,000/\$10,414,286$$

$$R_0 = .0903, \text{ or } 9.03\%$$

- d. In part c, we calculated the cost of an all-equity firm. We can use Modigliani-Miller Proposition II with no taxes again to find the cost of equity for the firm with the new leverage ratio. The cost of equity under the stock repurchase plan will be:

$$R_S = R_0 + (B/S)(R_0 - R_B)$$

$$R_S = .0903 + (.50)(.0903 - .064)$$

$$R_S = .1034, \text{ or } 10.34\%$$

24. a. The expected return on a firm's equity is the ratio of annual aftertax earnings to the market value of the firm's equity. The amount the firm must pay each year in taxes will be:

$$\text{Taxes} = .21(\$1,450,000)$$

$$\text{Taxes} = \$304,500$$

So, the return on the unlevered equity will be:

$$R_0 = (\$1,450,000 - 304,500)/\$6,100,000$$

$$R_0 = .1878, \text{ or } 18.78\%$$

Notice that perpetual aftertax annual earnings of \$1,145,500, discounted at 18.78 percent, yields the market value of the firm's equity.

- b. The company's market value balance sheet before the announcement of the debt issue is:

		Debt	0
Assets	<u>\$6,100,000</u>	Equity	<u>\$6,100,000</u>
Total assets	<u>\$6,100,000</u>	Total D&E	<u>\$6,100,000</u>

The price per share is the total market value of the stock divided by the shares outstanding, or:

$$\text{Price per share} = \$6,100,000 / 280,000$$

$$\text{Price per share} = \$21.79$$

- c. Modigliani-Miller Proposition I states that in a world with corporate taxes:

$$V_L = V_U + T_C B$$

When the company announces the debt issue, the value of the firm will increase by the present value of the tax shield on the debt. The present value of the tax shield is:

$$\text{PV(Tax Shield)} = T_C B$$

$$\text{PV(Tax Shield)} = .21(\$1,600,000)$$

$$\text{PV(Tax Shield)} = \$336,000$$

Therefore, the value of the company will increase by \$336,000 as a result of the debt issue. The value after the repurchase announcement is:

$$V_L = V_U + T_C B$$

$$V_L = \$6,100,000 + .21(\$1,600,000)$$

$$V_L = \$6,436,000$$

Since the firm has not yet issued any debt, its equity is also worth \$6,436,000.

The company's market value balance sheet after the announcement of the debt issue is:

Old assets	\$6,100,000	Debt	0
PV(tax shield)	<u>336,000</u>	Equity	<u>\$6,436,000</u>
Total assets	<u>\$6,436,000</u>	Total D&E	<u>\$6,436,000</u>

- d. The share price immediately after the announcement of the debt issue will be:

$$\text{New share price} = \$6,436,000 / 280,000$$

$$\text{New share price} = \$22.99$$

- e. The number of shares repurchased will be the amount of the debt issue divided by the new share price, or:

$$\text{Shares repurchased} = \$1,600,000 / \$22.99$$

$$\text{Shares repurchased} = 69,608.45$$

The number of shares outstanding will be the current number of shares minus the number of shares repurchased, or:

$$\text{New shares outstanding} = 280,000 - 69,608.45$$

$$\text{New shares outstanding} = 210,391.55$$

- f. The share price will remain the same after restructuring takes place. The total market value of the outstanding equity in the company will be:

$$\text{Market value of equity} = \$22.99(210,391.55)$$

$$\text{Market value of equity} = \$4,836,000$$

The market-value balance sheet after the restructuring is:

Old assets	\$6,100,000	Debt	\$1,600,000
PV(tax shield)	<u>336,000</u>	Equity	<u>4,836,000</u>
Total assets	<u>\$6,436,000</u>	Total D&E	<u>\$6,436,000</u>

- g. According to Modigliani-Miller Proposition II with corporate taxes:

$$R_S = R_0 + (B/S)(R_0 - R_B)(1 - T_C)$$

$$R_S = .1878 + (\$1,600,000/\$4,836,000)(.1878 - .06)(1 - .21)$$

$$R_S = .2212, \text{ or } 22.12\%$$

25. a. In a world with corporate taxes, a firm's weighted average cost of capital is equal to:

$$R_{WACC} = [B/(B + S)](1 - T_C)R_B + [S/(B + S)]R_S$$

We do not have the company's debt-to-value ratio or the equity-to-value ratio, but we can calculate either from the debt-to-equity ratio. With the given debt-equity ratio, we know the company has 2.5 dollars of debt for every dollar of equity. Since we only need the ratio of debt-to-value and equity-to-value, we can say:

$$B/(B + S) = 2.3/(2.3 + 1) = .6970$$

$$S/(B + S) = 1/(2.3 + 1) = .3030$$

We can now use the weighted average cost of capital equation to find the cost of equity, which is:

$$.10 = (.6970)(1 - .24)(.06) + (.3030)(R_S)$$

$$R_S = .2251, \text{ or } 22.51\%$$

- b. We can use Modigliani-Miller Proposition II with corporate taxes to find the unlevered cost of equity. Doing so, we find:

$$R_S = R_0 + (B/S)(R_0 - R_B)(1 - T_C)$$

$$.2251 = R_0 + (2.3)(R_0 - .06)(1 - .24)$$

$$R_0 = .1201, \text{ or } 12.01\%$$

- c. We first need to find the debt-to-value ratio and the equity-to-value ratio. We can then use the cost of levered equity equation with taxes, and finally the weighted average cost of capital equation. So:

If debt-equity = .75

$$B/(B + S) = .75/ (.75 + 1) = .4286$$

$$S/(B + S) = 1/ (.75 + 1) = .5714$$

The cost of levered equity will be:

$$R_S = R_0 + (B/S)(R_0 - R_B)(1 - T_C)$$

$$R_S = .1201 + (.75)(.1201 - .06)(1 - .24)$$

$$R_S = .1543, \text{ or } 15.43\%$$

And the weighted average cost of capital will be:

$$R_{WACC} = [B/(B + S)](1 - T_C)R_B + [S/(B + S)]R_S$$

$$R_{WACC} = (.4286)(1 - .24)(.06) + (.5714)(.1543)$$

$$R_{WACC} = .1077, \text{ or } 10.77\%$$

If debt-equity = 1.30

$$B/(B + S) = 1.30/(1.30 + 1) = .5652$$

$$S/(B + S) = 1/(1.30 + 1) = .4348$$

The cost of levered equity will be:

$$R_S = R_0 + (B/S)(R_0 - R_B)(1 - T_C)$$

$$R_S = .1201 + (1.30)(.1201 - .06)(1 - .24)$$

$$R_S = .1795, \text{ or } 17.95\%$$

And the weighted average cost of capital will be:

$$R_{WACC} = [B/(B + S)](1 - T_C)R_B + [S/(B + S)]R_S$$

$$R_{WACC} = (.5652)(1 - .24)(.06) + (.4348)(.1795)$$

$$R_{WACC} = .1038, \text{ or } 10.38\%$$

Challenge

26. M&M Proposition II states:

$$R_S = R_0 + (R_0 - R_B)(B/S)(1 - T_C)$$

And the equation for WACC is:

$$WACC = (S/V)R_S + (B/V)R_B(1 - T_C)$$

Substituting the M&M Proposition II equation into the equation for WACC, we get:

$$WACC = (S/V)[R_0 + (R_0 - R_B)(B/S)(1 - T_C)] + (B/V)R_B(1 - T_C)$$

Rearranging and reducing the equation, we get:

$$WACC = R_0[(S/V) + (S/V)(B/S)(1 - T_C)] + R_B(1 - T_C)[(B/V) - (S/V)(B/S)]$$

$$WACC = R_0[(S/V) + (B/V)(1 - T_C)]$$

$$WACC = R_0[(S+B)/V - T_C(B/V)]$$

$$WACC = R_0[1 - T_C(B/V)]$$

27. The return on equity is net income divided by equity. Net income can be expressed as:

$$NI = (EBIT - R_B B)(1 - T_C)$$

So, ROE is:

$$R_S = (EBIT - R_B B)(1 - T_C)/S$$

Now we can rearrange and substitute as follows to arrive at M&M Proposition II with taxes:

$$R_S = [EBIT(1 - T_C)/S] - [R_B(B/S)(1 - T_C)]$$

$$R_S = R_0 V_U/S - [R_B(B/S)(1 - T_C)]$$

$$R_S = R_0(V_L - T_C B)/S - [R_B(B/S)(1 - T_C)]$$

$$R_S = R_0(S + B - T_C B)/S - [R_B(B/S)(1 - T_C)]$$

$$R_S = R_0 + (R_0 - R_B)(B/S)(1 - T_C)$$

28. M&M Proposition II, with no taxes is:

$$R_S = R_A + (R_A - R_f)(B/S)$$

Note that we use the risk-free rate as the return on debt. This is an important assumption of M&M Proposition II. The CAPM to calculate the cost of equity is expressed as:

$$R_S = \beta_S(R_M - R_f) + R_f$$

We can rewrite the CAPM to express the return on an unlevered company as:

$$R_0 = \beta_A(R_M - R_f) + R_f$$

We can now substitute the CAPM for an unlevered company into M&M Proposition II. Doing so and rearranging the terms, we get:

$$R_S = \beta_A(R_M - R_f) + R_f + [\beta_A(R_M - R_f) + R_f - R_f](B/S)$$

$$R_S = \beta_A(R_M - R_f) + R_f + [\beta_A(R_M - R_f)](B/S)$$

$$R_S = (1 + B/S)\beta_A(R_M - R_f) + R_f$$

Now we set this equation equal to the CAPM equation to calculate the cost of equity and reduce:

$$\beta_S(R_M - R_f) + R_f = (1 + B/S)\beta_A(R_M - R_f) + R_f$$

$$\beta_S(R_M - R_f) = (1 + B/S)\beta_A(R_M - R_f)$$

$$\beta_S = \beta_A(1 + B/S)$$

29. Using the equation we derived in Problem 28:

$$\beta_S = \beta_A(1 + B/S)$$

The equity beta for the respective asset betas is:

<u>Debt-equity ratio</u>	<u>Equity beta</u>
0	$1(1 + 0) = 1$
1	$1(1 + 1) = 2$
5	$1(1 + 5) = 6$
20	$1(1 + 20) = 21$

The equity risk to the shareholder is composed of both business and financial risks. Even if the assets of the firm are not very risky, the risk to the shareholder can still be large if the financial leverage is high. These higher levels of risk will be reflected in the shareholder's required rate of return R_S , which will increase with higher debt-equity ratios.

- 30.** We first need to set the cost of capital equation equal to the cost of capital for an all-equity firm, so:

$$\frac{B}{B+S} R_B + \frac{S}{B+S} R_S = R_0$$

Multiplying both sides by $(B + S)/S$ yields:

$$\frac{B}{S} R_B + R_S = \frac{B+S}{S} R_0$$

We can rewrite the right-hand side as:

$$\frac{B}{S} R_B + R_S = \frac{B}{S} R_0 + R_0$$

Moving $(B/S)R_B$ to the right-hand side and rearranging gives us:

$$R_S = R_0 + \frac{B}{S} (R_0 - R_B)$$

