1. An organization has members who possess IQs in the top 4% of the population. If IQs are normally distributed, with a mean of 100 and a standard deviation of 15, what is the minimum IQ required for admission into the organization?

Use Excel, and round your answer to the nearest integer: 126

2. The top 5% of applicants on a test will receive a scholarship. If the test scores are normally distributed with a mean of 600 and a standard distribution of 85, how low can an applicant score to still qualify for a scholarship?

Use Excel, and round your answer to the nearest integer. 740

-Here, the mean, $\mu$, is 600 and the standard deviation, $\sigma$, is 85. Let x be the score on the test. As the top 5% of the applicants will receive a scholarship, the area to the right of x is 5%=0.05. So the area to the left of x is 1−0.05=0.95. Use Excel to find x.

-Open Excel. Click on an empty cell. Type =NORM.INV(0.95,600,85) and press ENTER.

-The answer rounded to the nearest integer, is $x \approx 740$. Thus, an applicant can score a 740 and still be in the top 5% of applicants on a test in order to receive a scholarship.

3. The weights of oranges are normally distributed with a mean of 12.4 pounds and a standard deviation of 3 pounds. Find the minimum value that would be included in the top 5% of orange weights.

Use Excel, and round your answer to one decimal place. 17.3

-Here, the mean, $\mu$, is 12.4 and the standard deviation, $\sigma$, is 3. Let x be the minimum value that would be included in the top 5% of orange weights. The area to the right of x is 5%=0.05. So, the area to the left of x is 1−0.05=0.95. Use Excel to find x.

-1. Open Excel. Click on an empty cell. Type =NORM.INV(0.95,12.4,3) and press ENTER.

-The answer, rounded to one decimal place, is $x \approx 17.3$. Thus, the minimum value that would be included in the top 5% of orange weights is 17.3 pounds

4. Two thousand students took an exam. The scores on the exam have an approximate normal distribution with a mean of $\mu=81$ points and a standard deviation of $\sigma=4$ points. The middle 50% of the exam scores are between what two values?

Use Excel, and round your answers to the nearest integer. 78, 84

The probability to the left of x1 is 0.25. Use Excel to find x1.

1. Open Excel. Click on an empty cell. Type =NORM.INV(0.25,81,4) and press ENTER.

Rounding to the nearest integer, $x1 \approx 78$. The probability to the left of x2 is 0.25+0.50=0.75. Use Excel to find x2.

1. Open Excel. Click on an empty cell. Type =NORM.INV(0.75,81,4) and press ENTER.

Rounding to the nearest integer, $x2 \approx 84$. Thus, the middle 50% of the exam scores are between 78 and 84.

5. The number of walnuts in a mass-produced bag is modeled by a normal distribution with a mean of 44 and a standard deviation of 5. Find the number of walnuts in a bag that has more walnuts than 80% of the other bags.

Use Excel, and round your answer to the nearest integer. 48

Here, the mean, $\mu$, is 44 and the standard deviation, $\sigma$, is 5. Let x be the number of walnuts in the bag. The area to the left of x is 80%=0.80. Use Excel to find x.

1. Open Excel. Click on an empty cell. Type =NORM.INV(0.80,44,5) and press ENTER.

The answer, rounded to the nearest integer, is $x \approx 48$. Thus, there are 48 walnuts in a bag that has more walnuts than 80% of the other bags.

6. A firm's marketing manager believes that total sales for next year will follow the normal distribution, with a mean of $3.2 million and a standard deviation of $250,000. Determine the sales level that has only a 3% chance of being exceeded next year.

Use Excel, and round your answer to the nearest dollar.$3,670,198

Here, the mean, $\mu$, is 3.2 million =3,200,000 and the standard deviation, $\sigma$, is 250,000. Let x be sales for next year. To determine the sales level that has only a 3% chance of being exceeded next year, the area to the right of x is 0.03. So the area to the left of x is $1-0.03=0.97$. Use Excel to find x.

1. Open Excel. Click on an empty cell. Type =NORM.INV(0.97,3200000,250000) and press ENTER.

The answer, rounded to the nearest dollar, is $x \approx 3{,}670{,}198$. Thus, the sales level that has only a 3% chance of being exceeded next year is $3,670,198.

7. Suppose that the weight of navel oranges is normally distributed with a mean of $\mu=6$ ounces and a standard deviation of $\sigma=0.8$ ounces. Find the weight below that one can find the lightest 90% of all navel oranges.

Use Excel, and round your answer to two decimal places. 7.03

Here, the mean, $\mu$, is 6 and the standard deviation, $\sigma$, is 0.8. The area to the left of x is 90%=0.90. Use Excel to find x.

1. Open Excel. Click on an empty cell. Type =NORM.INV(0.90,6,0.8) and press ENTER.

The answer, rounded to two decimal places, is $x \approx 7.03$. Thus, navel oranges that weigh less than 7.03 ounces compose the lightest 90% of all navel oranges.

8. A tire company finds the lifespan for one brand of its tires is normally distributed with a mean of 47,500 miles and a standard deviation of 3,000 miles. What mileage would correspond to the the highest 3% of the tires?

Use Excel, and round your answer to the nearest integer. 53,142

Here, the mean, μ, is 47,500 and the standard deviation, σ, is 3,000. Let x be the minimum number of miles for a tire to be in the top 3%. The area to the right of x is 3%=0.03. So, the area to the left of x is 1−0.03=0.97. Use Excel to find x.

1. Open Excel. Click on an empty cell. Type =NORM.INV(0.97,47500,3000) and press ENTER.

The answer, rounded to the nearest integer, is x≈53,142. Thus, the approximate number of miles for the highest 3% of the tires is 53,142 miles.

1. The average credit card debt owed by Americans is $6375, with a standard deviation of $1200. Suppose a random sample of 36 Americans is selected. Identify each of the following:

    1. 6375
    2. 1200
    3. n=36
    4. 6375
    5. 200

2. The heights of all basketball players are normally distributed with a mean of 72 inches and a population standard deviation of 1.5 inches.  If a sample of 15 players are selected at random from the population, select the expected mean of the sampling distribution and the standard deviation of the sampling distribution below.

σx¯=0.387 inches

μx¯=72 inches

The standard deviation of the sampling distribution σx¯=σn√=1.51√5=0.387inches. Likewise, when the distribution is normal the mean of the sampling distribution is equal to the mean of the population μx¯=μ=72 inches.

1. After collecting the data, Peter finds that the standardized test scores of the students in a school are normally distributed with mean 85 points and standard deviation 3 points. Use the Empirical Rule to find the probability that a randomly selected student's score is greater than 76 points. Provide the final answer as a percent rounded to two decimal places.

99.85%

Notice that 76 points is 3 standard deviations less than the mean. Based on the Empirical Rule, approximately 99.7% of the scores are within 3 standard deviations of the mean. Since the normal distribution is symmetric, this implies that 0.15% of the scores are less than the score that is 3 standard deviations below the mean. Alternatively, 99.85% of the scores are greater than the score that is 3 standard deviations below the mean. Therefore, the probability that a randomly selected student's score is greater than 76 points is approximately 99.85%.

2. After collecting the data, Christopher finds that the total snowfall per year in Reamstown is normally distributed with mean 94 inches and standard deviation 14 inches. Which of the following gives the probability that in a randomly selected year, the snowfall was greater than 52 inches? Use the empirical rule

Provide the final answer as a percent rounded to two decimal places. 99.85%

3. The College Board conducted research studies to estimate the mean SAT score in 2016 and its standard deviation. The estimated mean was 1020 points out of 1600 possible points, and the estimated standard deviation was 192 points. Assume SAT scores follow a normal distribution. Using the Empirical Rule, about 95% of the scores lie between which two values?

636 to 1404

4. After collecting the data, Kenneth finds that the body weights of the forty students in a class are normally distributed with mean 140 pounds and standard deviation 9 pounds. Use the Empirical Rule to find the probability that a randomly selected student has a body weight of greater than 113 pounds. Provide the final answer as a percent rounded to two decimal places.

99.85%

5. Mrs. Miller's science test scores are normally distributed with a mean score of 77 ($\mu$) and a standard deviation of 3 ($\sigma$). Using the Empirical Rule, about 68% of the scores lie between which two values?

74-80

6. Brenda has collected data to find that the finishing times for cyclists in a race has a normal distribution. What is the probability that a randomly selected race participant had a finishing time of greater than 154 minutes if the mean is 143 minutes and the standard deviation is 11 minutes? Use the empirical rule.

16%

Notice that 154 minutes is one standard deviation greater than the mean. Based on the Empirical Rule, 68% of the finishing times are within one standard deviation of the mean, so 100%−68%=32% of the finishing times are outside of one standard deviation from the mean in either direction. Since the normal distribution is symmetric, half of the 32% will be less than the mean and half will be greater, so 16% of the finishing times are greater than one standard deviation more than the mean.

7. Suppose $X \sim N(20,2)$, and $x=26$. Find and interpret the z-score of the standardized normal random variable.

3, 20, 30

8. Isabella averages 17 points per basketball game with a standard deviation of 4 points. Suppose Isabella's points per basketball game are normally distributed. Let X= the number of points per basketball game. Then $X \sim N(17,4)$.

3, 17, 3

9. Suppose X~N(6.5,1.5), and x=3.5. Find and interpret the z-score of the standardized normal random variable.

-2, 6.5, 2

| When $z$ is **positive**: | When $z$ is **negative**: | When $z$ is **zero**: |
|---|---|---|
| • the $x$-value is greater than $\mu$<br>• the $x$-value is above (to the right of) $\mu$ | • the $x$-value is less than $\mu$<br>• the $x$-value is below or (to the left of) $\mu$ | • the $x$-value is equal to $\mu$ |

For example, let's say the mean of a normal distribution is 5 and the standard deviation is 3. Then the value 8 is one standard deviation above the mean, because 5+1·3=8. The value 11 is two standard deviations above the mean, because 5+2·3=11. The value 14 is three standard deviations above the mean, because 5+3·3=14. (If the value is below the mean, then we would subtract the value of standard deviation from the mean.)

10. Suppose X~N(5.5,2), and x=7.5. Find and interpret the z-score of the standardized normal random variable.
This means that x=7.5 is one standard deviation (1σ) above or to the right of the mean, μ=5.5.

11. Jerome averages 16 points a game with a standard deviation of 4 points. Suppose Jerome's points per game are normally distributed. Let X = the number of points per game. Then X~N(16,4).

"Suppose Jerome scores 10 points in the game on Monday. The z-score when x=10 is z=−1.5. This z-score tells you that x=10 is 1.5 standard deviations to the left of the mean, 16."

12. Josslyn was told that her score on an aptitude test was 3 standard deviations above the mean. If test scores were approximately normal with μ=79 and σ=9, what was Josslyn's score? Do not include units in your answer. For example, if you found that the score was 79 points, you would enter 79.

106

13. Marc's points per game of bowling are normally distributed with a standard deviation of 13 points. If Marc scores 231 points, and the z-score of this value is 4, then what is his mean points in a game? Do not include the units in your answer. For example, if you found that the mean is 150 points, you would enter 150.

179
We can think of this conceptually as well. We know that the z-score is 4, which tells us that x=231 is four standard deviations to the right of the mean, and each standard deviation is 13. So four standard deviations is (4)(13)=52 points. So, now we know that 231 is 52 units to the right of the mean. (In other words, the mean is 52 units to the left of x=231.) So the mean is 231−52=179.

14. Floretta's points per basketball game are normally distributed with a standard deviation of 4 points. If Floretta scores 10 points, and the z-score of this value is −4, then what is her mean points in a game? Do not include the units in your answer. For example, if you found that the mean is 33 points, you would enter 33.

26
We can think of this conceptually as well. We know that the z-score is −4, which tells us that x=10 is four standard deviations to the left of the mean, and each standard deviation is 4. So four standard deviations is (−4)(4)=−16 points. So, now we know that 10 is 16 units to the left of the mean. (In other words, the mean is 16 units to the right of x=10.) So the mean is 10+16=26.

15. Jamie was told that her score on an aptitude test was 3 standard deviations below the mean. If test scores were approximately normal with $\mu=94$ and $\sigma=6$, what was Jamie's score? Do not include units in your answer. For example, if you found that the score was 94 points, you would enter 94.

76
We can think of this conceptually as well. We know that the z-score is −3, which tells us that x is three standard deviations to the left of the mean, 94. So we can think of the distance between 94 and the x-value as (3)(6)=18. So Jamie's score is 94−18=76.

16. A normal distribution is observed from the number of points per game for a certain basketball player. If the mean is 16 points and the standard deviation is 2 points, what is the probability that in a randomly selected game, the player scored between 12 and 20 points? Use the empirical rule

Provide the final answer as a percent. 95%

17. A random sample of vehicle mileage expectancies has a sample mean of $\bar{x}$=169,200 miles and sample standard deviation of s=19,400 miles. Use the Empirical Rule to estimate the percentage of vehicle mileage expectancies that are more than 188,600 miles.

16%

Since the mean is 169,200, a mileage of 188,600 is 188,600−169,200=19,400 miles more than the mean, which is one standard deviation.

By the Empirical Rule, we know that about 68% of the data lies within one standard deviation of the mean. Therefore, 100%−68%=32% of the data lie more than one standard deviation away from the mean.

Since the distribution of vehicle expectancies is symmetric, about half of the remaining 32% will lie to either extreme. So about 16% of vehicle expectancies are more than 188,600 miles.

18. A random sample of lobster tail lengths has a sample mean of $\bar{x}$=4.7 inches and sample standard deviation of s=0.4 inches. Use the Empirical Rule to determine the approximate percentage of lobster tail lengths that lie between 4.3 and 5.1 inches.

Round your answer to the nearest whole number (percent). 68%

19. A random sample of SAT scores has a sample mean of $\bar{x}$=1060 and sample standard deviation of s=195. Use the Empirical Rule to estimate the approximate percentage of SAT scores that are less than 865.

Round your answer to the nearest whole number (percent). 16%

20. The number of pages per book on a bookshelf is normally distributed with mean 248 pages and standard deviation 21 pages. Using the empirical rule, what is the probability that a randomly selected book has less than 206 pages?

2.5%

The Empirical Rule states:
For a normal distribution, nearly all of the data will fall within 3 standard deviations of the mean.
The empirical rule can be broken down into three parts:

68% of data falls within one standard deviation from the mean (between $-1\sigma$ and $1\sigma$).
95% fall within two standard deviations from the mean (between $-2\sigma$ and $2\sigma$).
99.7% fall within three standard deviations from the mean (between $-3\sigma$ and $3\sigma$).
The empirical rule is also known as the 68–95–99.7 Rule.

21. Mr. Karly's math test scores are normally distributed with a mean score of 87 ($\mu$) and a standard deviation of 4 ($\sigma$). Using the Empirical Rule, about 99.7% of the data values lie between which two values?

75-99%

22. In 2014, the CDC estimated that the mean height for adult women in the U.S. was 64 inches with a standard deviation of 4 inches. Suppose X, height in inches of adult women, follows a normal distribution. Which of the following gives the probability that a randomly selected woman has a height of greater than 68 inches?

16%

23. A normal distribution is observed from the number of points per game for a certain basketball player. The mean for this distribution is 20 points and the standard deviation is 3

points. Use the empirical rule for normal distributions to estimate the probability that in a randomly selected game the player scored less than 26 points.

Provide the final answer as a percent rounded to one decimal place.

97.5%

24. A normal distribution is observed from the number of points per game for a certain basketball player. If the mean is 15 points and the standard deviation is 3 points, what is the probability that in a randomly selected game, the player scored greater than 24 points? Use the empirical rule

15%

25. The College Board conducted research studies to estimate the mean SAT score in 2016 and its standard deviation. The estimated mean was 1020 points out of 1600 possible points, and the estimated standard deviation was 192 points. Assume SAT scores follow a normal distribution. Using the Empirical Rule, about 95% of the scores lie between which two values?

636-1404

26. The typing speeds for the students in a typing class is normally distributed with mean 44 words per minute and standard deviation 6 words per minute. What is the probability that a randomly selected student has a typing speed of less than 38 words per minute? Use the empirical rule

Provide the final answer as a percent. If necessary round the percent to the nearest whole number.

16%

Notice that 38 words per minute is one standard deviation less than the mean. Based on the Empirical Rule, 68% of the typing speeds are within one standard deviation of the mean. Since the normal distribution is symmetric, this implies that 16% of the typing speeds are less than one standard deviation less than the mean.

27. Nick has collected data to find that the body weights of the forty students in a class has a normal distribution. What is the probability that a randomly selected student has a body weight of greater than 169 pounds if the mean is 142 pounds and the standard deviation is 9 pounds? Use the empirical rule.

.15%

Notice that 169 pounds is three standard deviations greater than the mean. Based on the Empirical Rule, 99.7% of the body weights are within three standard deviations of the mean. Since the normal distribution is symmetric, this implies that 0.15% of the body weights are greater than three standard deviation more than the mean.

The pink section represents the 68% of the data that lies within one standard deviation of the mean. This section is split in half, with 34% to the left of the mean and 34% to the right of the mean.

The smaller red sections (marked with 13.5%), along with the inner pink sections, represent the 95% of the data that lies within two standard deviations of the mean. (If we add the four sections together, we will get 95%: 13.5%+34%+34%+13.5%=95%.)

The small blue sections (marked with 2.35%), along with the four inner sections, represent the 99.7% of the data that lies within three standard deviations of the mean. (If we add the sections together, we will get 99.7%: 2.35%+13.5%+34%+34%+13.5%+2.35%=99.7%.)

*Remember: 50% of the data lie above the mean, and 50% of the data lie below the mean in normal distributions. So, if we have a normally distributed data set, with a mean of 10.5, then we can say that 50% of the data is less than 10.5 and 50% of the data is greater than 10.5.

28. The times to complete an obstacle course is normally distributed with mean 73 seconds and standard deviation 9 seconds. What is the probability using the Empirical Rule that a randomly selected finishing time is less than 100 seconds?

99.85%

29. After collecting the data, Douglas finds that the finishing times for cyclists in a race is normally distributed with mean 149 minutes and standard deviation 16 minutes. What is the probability that a randomly selected race participant had a finishing time of less than 165 minutes? Use the empirical rule

84%

30. Charles has collected data to find that the total snowfall per year in Reamstown has a normal distribution. Using the Empirical Rule, what is the probability that in a randomly selected year, the snowfall was less than 87 inches if the mean is 72 inches and the standard deviation is 15 inches?

84%

31. Christopher has collected data to find that the total snowfall per year in Laytonville has a normal distribution. What is the probability that in a randomly selected year, the snowfall was greater than 53 inches if the mean is 92 inches and the standard deviation is 13 inches? Use the empirical rule

Notice that 53 inches is three standard deviations less than the mean. Based on the Empirical Rule, 99.7% of the yearly snowfalls are within three standard deviations of the mean. Since the normal distribution is symmetric, this implies that 0.15% of the yearly snowfalls are less than three standard deviations below the mean. Alternatively, 99.85% of the yearly snowfalls are greater than three standard deviations below the mean.

32. The times to complete an obstacle course is normally distributed with mean 87 seconds and standard deviation 7 seconds. What is the probability that a randomly selected finishing time is greater than 80 seconds? Use the empirical rule

Alternatively, 84% of the finishing times are greater than one standard deviation below the mean.

33.