

1. For part a, the order of the group is 61 meaning that there can be a total number of 60 generators in Z , where generators are less than 61 and are co-prime to 61. Because 61 is a prime number, we can see cycle for some value (0 to 60) where the result of the discrete logarithm will result in a cycle. Two examples of the generators can be 2 & 6. For part b, using the Lagrange theorem, I found all possible subgroups in the group 61, and found a subgroup with order 5. The answer is : 1, 9, 20, 34, 58.
2. In order to find the answer, we first need to find the different variables. $\phi(n)$ where $n=p*q$ is a multiplicative function, meaning that $\phi(n) = \phi(p) * \phi(q)$, where $\phi(p) = p-1$ and $\phi(q) = q-1$. This function works because both p and q are prime numbers. Then I used a built-in function of python, `pow()`, to solve for m . The answer is :
302562423231164711433775790368517349565995667849578087779000841593443114033480772
433054688939009443567978654937428707236407863980260309231777601473087110374524033
894222588248814856928431210421802802119552503278962630326720552941119001410709815
23217773015658586930923177732713966172972763510074059270940820416
3. For part a, the $\gcd(a,n)$ is 2, and because d is divisible by b , there may be two possible solutions. For part b, the $\gcd(a,n)$ is also 2, but because d is not divisible by b , there are no solutions for this linear congruence. For part c, the $\gcd(a,n)=1$, meaning that there is a unique solution to the equation. The inverse of a is: 87208981786492864261728145663, and the answer is:
327252728639173874206458501252
4. The functions in `lfsr.py` was used for this question. For the first equation, the result of the first period is equal to 2^L-1 , which is 31. Therefore, it can be considered as a maximum period. For the second equation, the first period was equal to three, which was not equal to the maximum period value.
5. For this question, the `FindPeriod` and `BM` function was used to calculate the period of the bit sequence and the shortest LFSR. For all three of the examples(X_1, X_2, X_3), the first period was 100 out of 100 length bit sequence, and the highest degree polynomials were 31. Furthermore, all the polynomials were the same with $x^{31}+x^{28}+1$. This means that they are unpredictable because it has the biggest period possible, considering the fact that the bit sequence were 100 bits long.