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Ge 165 Final Project

Approximating a Receiver-Pair Green's Function Using a Third Receiver

Summary

Calculating the Green's Function between the locations of two receivers is an essential problem in seismology. This is typically done by cross-correlating the isotropic noise recorded by each pair of receivers. However, these cross-correlations require data over a very long time period in order to converge, and are very expensive to compute. In areas with a high number of receivers, it would therefore be useful to have a method which would reduce the number of cross-correlations that need to be calculated to compute a Green's Function for each receiver pair. It is shown here that, by convolving Green's Functions for different receiver pairs, one can approximate the Green's Functions for other receiver pairs.

Introduction

In seismology, the Earth can generally be assumed to function as a linear, time-invariant (LTI) system. The Green's Function essentially serves as the impulse response of the Earth between two receivers, and having a good approximation of this Green's Function is needed when taking seismic measurements in order to compensate for noise.

There are two primary approached through which Green's Function approximations have usually been calculated. The first is two measure differences in two receivers' measurements of an earthquake event. This is convenient because an Earthquake, as a brief, powerful event originating from a point source, closely resembles a theoretical impulse. However, this approach requires waiting for actual earthquakes to occur, which can take long and unpredictable amounts of time. The other approach, far more commonly used in modern seismology, is to perform a cross-correlation of the isotropic noise measured by two receivers. While this is convenient due to this noise continuously occurring, having these cross-correlations converge requires data collected over a very long period of time, and enormous time-series datasets which makes calculations very computationally intensive. In regions containing large numbers of receivers, where each receiver pair would require a separate calculation, these calculations would prove very costly. Therefore, it would be highly beneficial to have a method to reduce the number of cross-correlations that need to be calculated for a group of receivers. This would be particularly useful in Southern California, where there are many seismic receivers, and a continuous noise signal source, the pounding of waves from the south in Long Beach, which is then scattered to create effectively isotropic random noise.

The question, in its simplest form, therefore becomes: if receivers A, B, and C are in an area receiving the same isotropic noise signal, can the cross-correlation for receiver pair AC be derived from the AB and BC cross-correlations? Such a technique would greatly reduce the number of cross-correlations that must be calculated in an area containing many receivers.

Method and Data

Simulated data will be used for this experiment. While in the real world the data used is isotropic noise data, these cross-correlation properties would work for any signal. Therefore, a simulated sinc pulse will be used, to ease calculation and visualization for this experiment.

A software model will be developed, where a circle of identical point sources (in this experiment a total of 36 such sources) will be positioned equally spaced along a circle, with receivers near the center of the circle, to model the real-world environment in which data for these cross-correlations would be received. For the sake of simplicity, the software model will assume a non-dispersive medium with constant signal velocity and no energy loss.

Three receivers, called ‘A’, ‘B’, and ‘C’, will be placed near the center of the circle. At any one time, just a single source will be enabled, and all three receivers will record data, a sinc pulse, from the source (given the experimental design, the data recorded by each receiver should be the same, albeit with a time offset directly proportional to a given receiver’s distance from the source). The cross-correlations for receiver pairs AB, BC, and AC will then be calculated. The hypothesis is that convolving the AB and BC cross-correlations will yield the AC cross-correlation; therefore, the AB-BC convolution will be compared to the AC cross-correlation, both graphically and, since the cross-correlation of a sinc function is symmetric with a center peak, by comparing the location of the center peak for the actual AC cross-correlation to the AB-BC convolution. This will be repeated for each source along the circle.

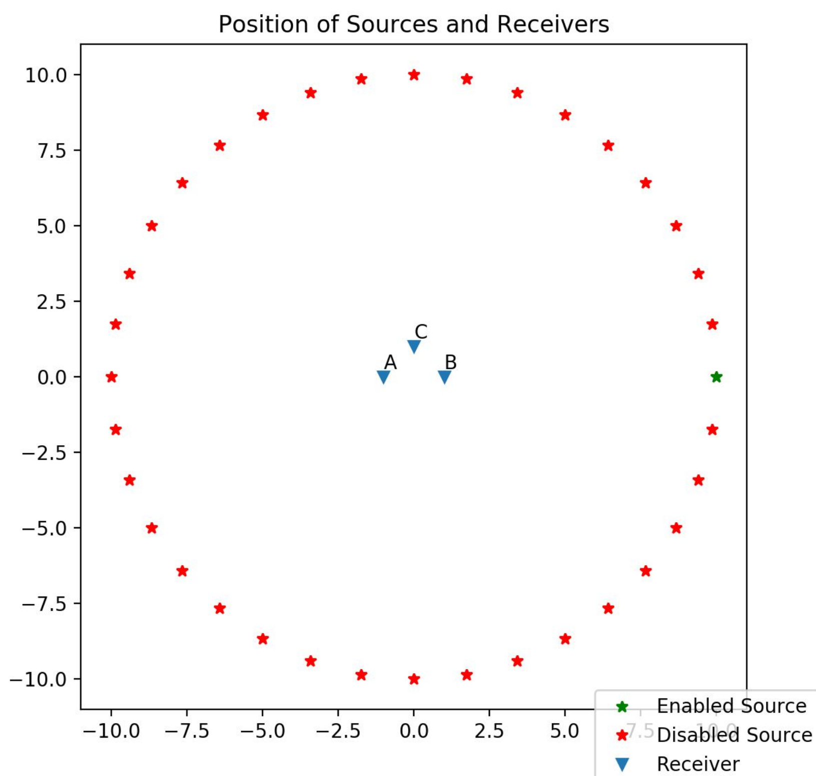


Fig. 1: arrangement of sources and receivers when only source at angle 0 is enabled.

Results

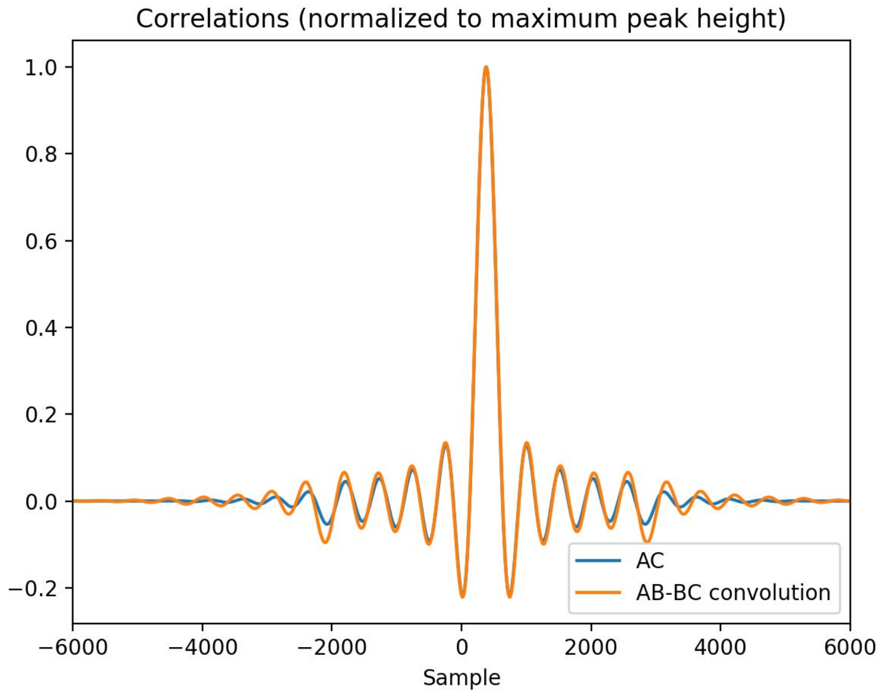


Fig. 2: AC cross-correlation compared to AB-BC convolution.

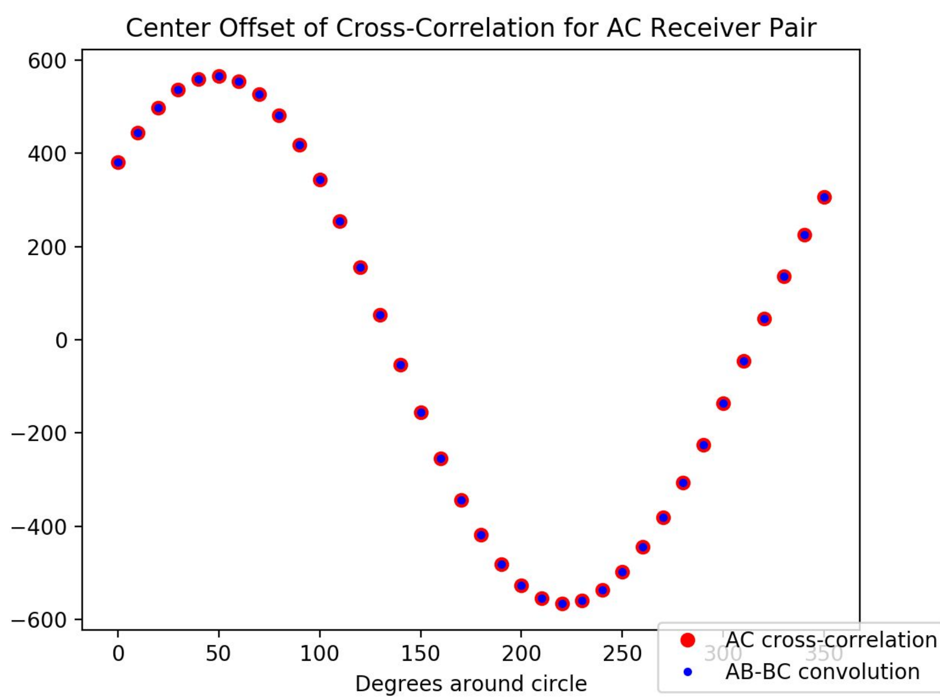


Fig. 3: Comparing center positions as enabled source moves around the circle.

The center for each AC cross-correlation was found to occur at the same sample as the center for each AB-BC convolution. While the AC cross-correlation and AB-BC convolution look very similar graphically, there is a small, but non-negligible, difference between the two.

Discussion

It is clear that the AB-BC convolution very closely approximates the AC cross-correlation (when normalizing to the height of the maximum peak). The center for each AC cross-correlation was found to occur at the same sample as the center for each AB-BC convolution. The AC cross-correlation and AB-BC convolution look very similar graphically, though there is a small, but non-negligible, difference between the two. Nevertheless, it appears that the convolution method is a viable technique for approximating Green's Functions for receiver pairs for which a noise cross-correlation has not been calculating.

In a real-world context, this approach can be used to greatly reduce the number of Green's Function approximations that need to be calculated in an area containing many receivers. For instance, if there are receivers A, B, C, D, etc., Green's Function approximations for AB and BC can be used to compute AC, as shown; then, Green's Function approximation CD also allows computation of AD and BD; DE would yield AE, BE, and CE; and so on, greatly reducing the number of Green's Function approximations that must be calculated as the number of receivers increases.

Conclusion and Future Work

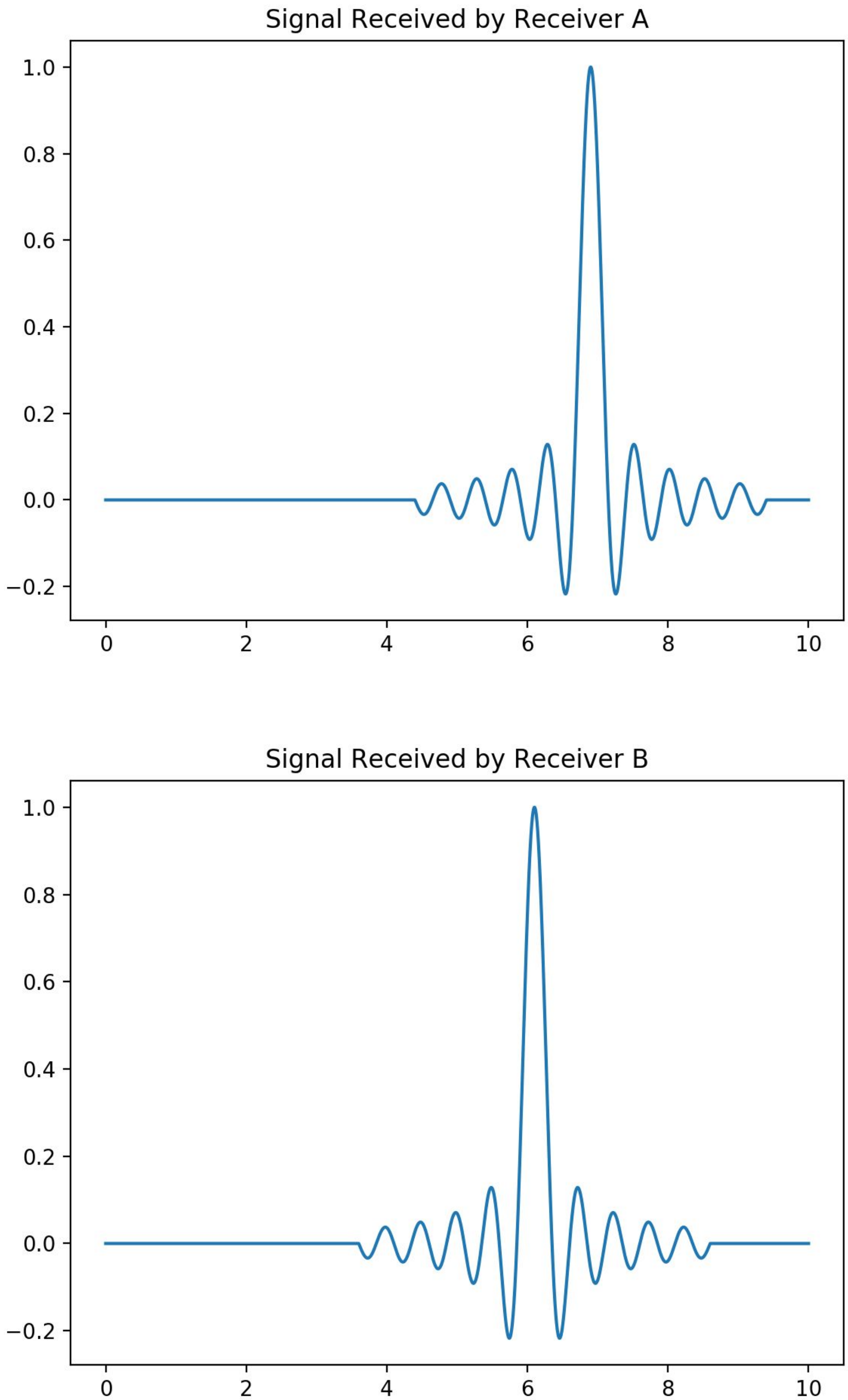
The simulation showed that, given receivers A, B, and C, convolving signal cross-correlations of AB and BC yields a close approximation of the cross-correlation of AC, allowing a large reduction in the number of Green's Function approximations for receiver pairs that must be calculated in a region with many receivers.

Future work can investigate the causes of the small, but non-negligible, discrepancies between the Green's Function approximations found by AC cross-correlation and AB-BC convolution. Additionally, further testing using either real data, or signals more similar to isotropic noise and a more complex medium than the simple one assumed for this model, can confirm the effectiveness of this approach in practice.

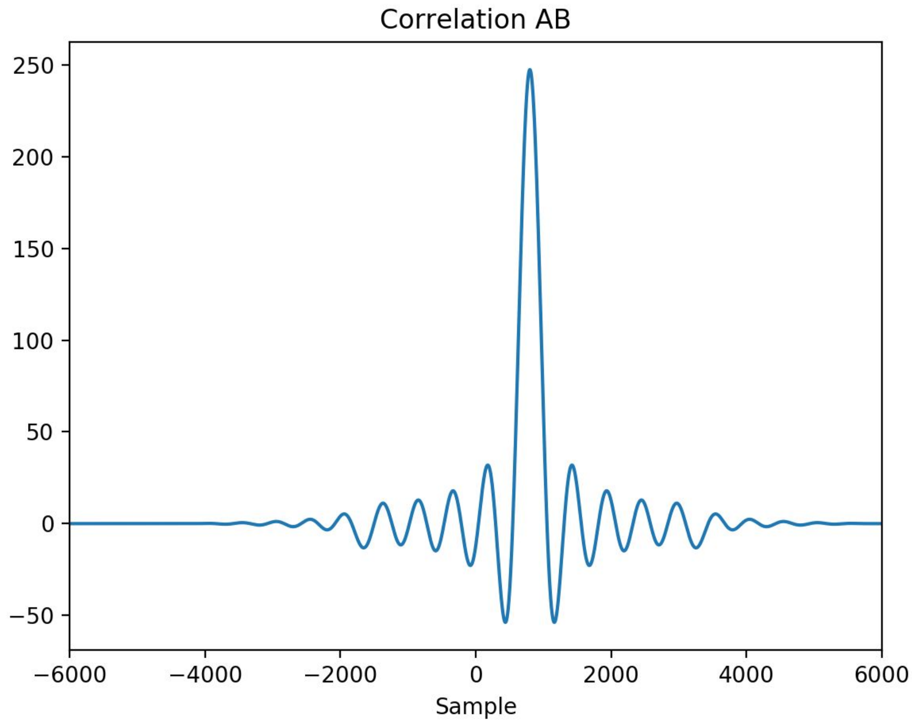
Supplementary Material

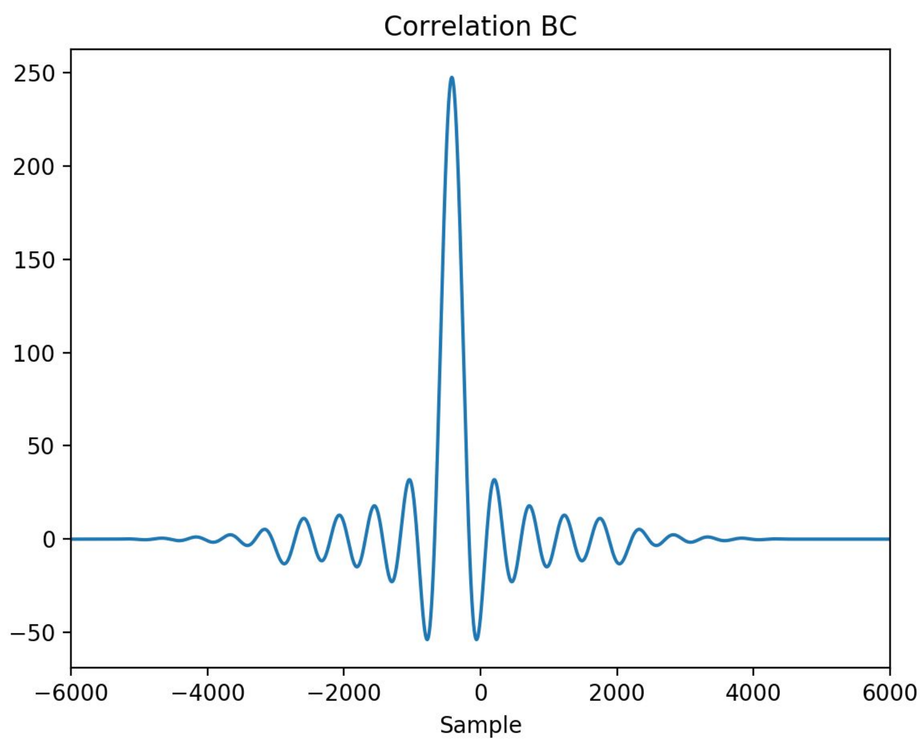
The source code for the software model, along with the simulations and experiments run using this model, can be found at https://github.com/samuellush/signal_simulation.

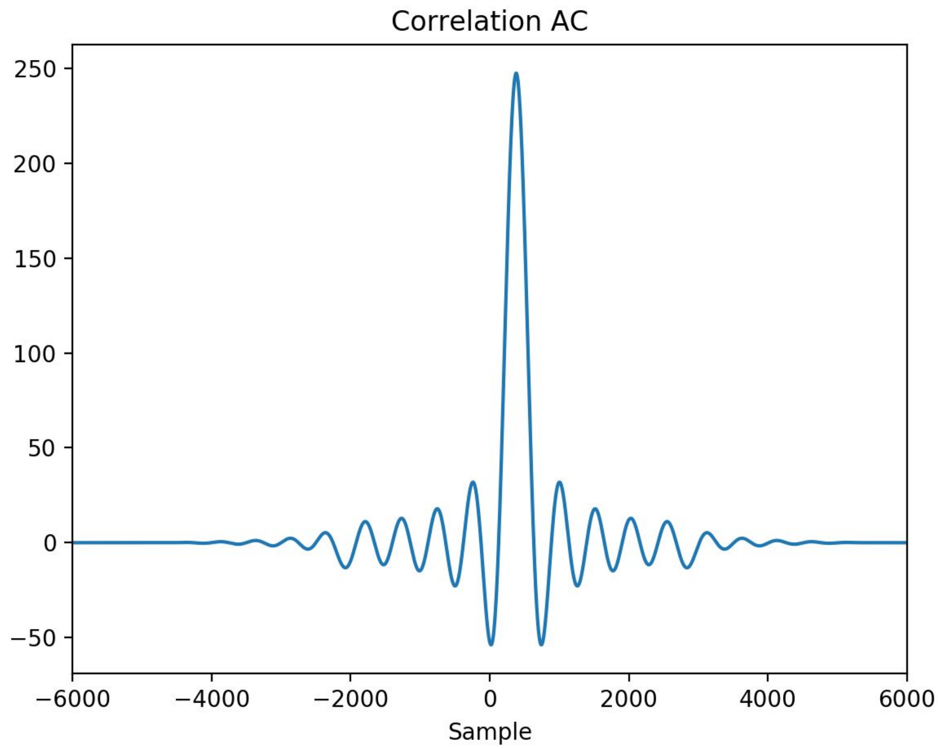
Additional helpful graphs are attached.



Figs. 4a & 4b: Signals received by A and B when only the source at angle 0 is enabled; note the time delay (x-axis units are seconds).


Correlation AB
250
200
150
100
50
0
−50
−6000
−4000
−2000
0
2000
4000
6000
Sample


Correlation BC
250
200
150
100
50
0
−50
−6000
−4000
−2000
0
2000
4000
6000
Sample



Figs. 5a, 5b, and 5c: AB, BC, & AC cross-correlations when only source at angle 0 is enabled.