Econ 270 Section 16815 Exam II

University of Illinois at Chicago

2024-03-16

Name: ____

Instructions

Please read the following before starting the exam:

1. The exam is worth 200 points and consists of <?> multiple choice questions and <??> free response questions. Point values are included next to questions - please allocate your time appropriately
2. There is a blank page on the back of the last sheet that you may use if you run out of room writing any questions - just label appropriately so that I can follow your work
3. You must show all your work to receive partial credit for incorrect answers.
4. You may not communicate with classmates or other people about the exam.
5. Laptops, phones, and other electronics devices are not allowed during the exam. You may use a calculator.
6. Numeric answers may either be written as a decimal or as a reduced fraction
7. The exam is due at the end of class. No exams will be accepted after this time.

Question 1 (10 points)

For independent variables, $var(X - Y)$

- A $= var(X) - var(Y)$
- B $= var(X) + var(Y)$
- C $= var(X) * var(Y)$
- D Cannot be computed without additional information

Question 2 (10 points)

If $F(x)$ is the cumulative distribution function, the 90th percentile can be calculated as

- A $F(.9)$
- B $F(.1)$
- C x such that $F(x) = .9$
- D x such that $F(x) = .1$

Question 3 (10 points)

Which of the following is true regarding the normal distribution?

- A All large populations tend towards a normal distribution
- B If X is normal, then $Z = \frac{x-\mu}{\sigma} \sim N(\mu, \sigma)$
- C If X is normal, then $Z = \frac{x-\mu}{\sigma} \sim N(0, 1)$
- D None of the above

Question 4 (10 points)

The central limit theorem states that as the sample size grows large (with independent, identically distributed samples from a finite variance population):

- A The sampling variance approach zero
- B The sampling mean approaches the population mean
- C The sampling distribution approaches a normal distribution
- D The population distribution approaches a normal distribution

Question 5 (10 points)

If the sampling distribution has $\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$ then

- A $X \sim N(\mu, \frac{\sigma}{\sqrt{n}})$
- B $X \sim N(\mu, \sigma)$
- C The standard error is σ
- D $\bar{x}$ is unbiased

Question 6 (10 points)

If we calculate the mean of a sample and generate the 95% confidence interval, this confidence interval is a statement about

- A The individual sample we drew
- B The population distribution
- C The population mean
- D All of the above

Question 7 (10 points)

A p value of .01 means that

- A There is a 1 percent chance our conclusion is incorrect
- B There is a 1 percent chance we obtained our result by chance
- C There is a 1 percent chance that we would obtain a result at least as extreme as the one observed by chance given the null hypothesis
- D There is a 1% chance that the null hypothesis is true

Free Response

Question 8 (20 points)

You are given the following probability mass function:

$p(0) = .05, p(1) = .15, p(3) = .35, p(4) = .2, p(5) = .1, p(6) = .15$

Calculate:

a

$E[X]$

b

$E[X^2]$

c

σ_X

Question 9 (15 points)

You are given $E[X] = 5$, $\sigma_X = 2$. Let X_i be independent draws from X. Calculate the following:

a

$E[X_1 + X_2 + X_3]$

b

$var(X_1 + X_2 + X_3)$

c

$\sigma_{X_1+X_2+X_3}$

d

$\sigma_{\bar{X}}$, where $\bar{X} = \frac{X_1+X_2+X_3}{3}$

Question 10 (15 points)

A distribution has cumulative distribution function of $F(x) = 1 - e^{-2x}$ for $x \geq 0$. Calculate the 3rd quartile of X

Question 11 (20 points)

$X \sim N(5, 2)$. Calculate $P(3 \leq x \leq 9)$

Question 12 (30 points)

You randomly sample 25 individuals from a class and calculate their grades on an exam. You obtain the following statistics

$\bar{x} = 75, s = 15$

a

calculate the standard error of $\bar{X}$

b

Calculate a 99% confidence interval for $\bar{x}$

c

Using the normal approximation, what is the distribution of the sampling distribution of $\bar{x}$?

Question 13 (30 points)

Using the past 50 trading days, you observe that the stock market ended higher than it started in 70% of the days in your sample.

a

Calculate the standard error of $\hat{p}$

b

Construct a 95% confidence interval for $\hat{p}$

c

Suppose that the true proportion $p = .5$. Calculate the probability that $\hat{p} \geq .7$

Extra Credit

1

You are given $E[X] = 1$, $E[Y] = 2$, $E[X^2] = 10$, $E[Y^2] = 12$, $E[XY] = 5$. Calculate $var(2X - Y)$

2

X is a uniform distribution with CDF $F(x) = x, 0 \leq x \leq 1$. Calculate the probability that the maximum from a sample of two is at least 0.25, i.e. $P(max(X_1, X_2) \geq .25)$

3

Why is s^2 an unbiased estimator for σ^2?