Econ 270 Lecture 9

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Confidence Intervals and Hypothesis Tests Summary

- We often want to estimate a parameter from a sample, i.e. we estimate μ from $\bar{x}$
- We can use the statistical properties of samples to construct a standard error and confidence interval
- We can also formally test whether a parameter is equal to some value
    - In the case of joint tests, we won't have a standard error or confidence interval, but can still conduct a hypothesis test

Confidence Intervals

- We estimate the unknown parameter μ using $\bar{x}$
- The standard error $\sigma_{\bar{x}}$ is the standard deviation of the sampling distribution
  - The abstract process that generates the sample mean
- The 95% confidence interval is constructed such that 95% of all confidence intervals will contain the true mean
  - $\bar{x} \pm z_{\alpha/2}\sigma_{\bar{x}}$

Standard error formulas

- Always of the form $\sigma_{\bar{x}} = \frac{s}{\sqrt{n}}$. This is directly used in a single mean
- Single proportion: $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
- Difference in means: $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
- Difference in proportions: $\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_1}}$

Critical Value

- For proportion and differences in proportions, we get $z_{\alpha/2}$ from a standard normal table
  - 95% corresponds to .975
- For means and differences in means, we instead use a t-distribution with $n-1$ (or $(n_1-1)+(n_2-1)$) degrees of freedom
  - If n is large, we can just use a standard normal table

Hypothesis Testing

- We always set up a null hypothesis to start
  - $H_0 : \mu = 0;\ \mu_1 = \mu_2;\ p_1 = p_2;\ p_1 = .5;\ \mu_1 = \mu_2 = \mu_3$
- The alternative hypothesis is always the negation of the null hypothesis for a two-tailed test
- We calculate a p-value: the probability of observing a result at least as extreme as what we observed if the null hypotheis were true
- Compare to α. Either reject or fail to reject
  - Results in either a type I or type II error (or a correct decision)

Hypothesis Testing Steps

- In hypothesis testing, we first calculate a test statistic
- For univariate tests: $\frac{\hat{\theta}-\mu_0}{\sigma_{\hat{\theta}}}$
  - i.e. the standardized point estimate. μ_0 is the null hypothesis, $\sigma_{\hat{\theta}}$ is the standard error
- Chi-square: $\sum \frac{(observed-expected)^2}{expected}$
- F: $\frac{MSG}{MSE} = \frac{\frac{1}{k-1}\sum n_i(\bar{x}_i-\bar{x})^2}{\frac{1}{n-k}\sum(n_i-1)s_i^2}$

P-values

- Proportions use a standard normal table. Means use a t-table
- chi-square and F use a table with only 1 tail

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A basic Z Table

```
      x F(x)          x F(x)
1: -3.0 0.00        0.0 0.50
2: -2.5 0.01        0.5 0.69
3: -2.0 0.02        1.0 0.84
4: -1.5 0.07        1.5 0.93
5: -1.0 0.16        2.0 0.98
6: -0.5 0.31        2.5 0.99
7:  0.0 0.50        3.0 1.00
```

Candy!

	Bag1	Bag2	Bag3	Tot
SK	7	4	7	18
MP	2	6	6	14
WB	3	2	4	9
SO	4	5	1	10
Tot	16	17	18	51