Resolution Theorem Proving

step 1) Convert all to CNF (conjunctive normal form)
2) negate all
3) resolve to prove counterargument/contradiction

if contradiction, conclusion is true
else False

example:

cats like fish
cats eat what they like
Fluff is a cat
does fluff eat fish?

1) to CNF

a) $cat(x) \rightarrow likes(x, y)$
b) $\{cat(x) \wedge likes(x,y)\} \rightarrow eats(x, y)$
c) $cat(fluff)$

2) to NEGATION

note: $x \rightarrow y = \neg x \vee y$

a) equivalent to $\neg cat(x) \vee likes(x,y)$
negate $(\neg cat(x) \vee likes(x,y)) = \neg(\neg cat(x) \vee likes(x,y))$
by DeMorgans $= \underline{cat(x) \wedge \neg likes(x,y)}$

b) equivalent to $(\neg(cat(x) \wedge likes(x,y))) \vee eats(x,y)$
by DeMorgans $(\neg cat(x) \vee \neg likes(x,y)) \vee eats(x,y)$
negate $((\neg cat(x) \vee \neg likes) \vee eats(x,y)) = \neg((\neg cat(x) \vee \neg likes(x,y)) \vee eats(x,y))$
by DeMorgans $= \neg(\neg cat(x) \vee \neg likes(x,y)) \wedge \neg eats(x,y)$
" " $= \underline{cat(x) \wedge likes(x,y) \wedge \neg eats(x,y)}$

c) $\underline{\neg cat(Fluffy)}$

3) to RESOLUTION
???

2 additions:

1) Unit Preference
- prefer unit clauses (clause w/ one literal)
- prefer shorter clauses, zero length clause is contradiction

2) Set of Support
- choose resolutions involving negated goal or clauses derived from negated goal

First Order Resolution:

$$\frac{\forall x \{P(x) \rightarrow Q(x)\} \quad P(A)}{Q(A)}$$

$\Downarrow$

$$\frac{\forall x \{\neg P(x) \vee Q(x)\} \quad P(A)}{Q(A)} \Rightarrow \frac{\neg P(A) \vee Q(A) \quad P(A)}{Q(A)}$$

uppercase = const.
lowercase = variable

syllogism:

socrates is a man
men are mortal
---
socrates is mortal

Resolution Rule

$$\frac{x \vee y \quad \neg y \vee z}{x \vee z}$$

example II:

$P \vee Q$
$P \rightarrow R$
$Q \rightarrow R$

Step	Formula	Derivation
1	$P \vee Q$	given
2	$\neg P \vee R$	given
3	$\neg Q \vee R$	given
4	$\neg R$	Negated Conclusion
5	$Q \vee R$	1, 2
6	$\neg P$	2, 4
7	$\neg Q$	3, 4
8	R	5, 7
9	-	4, 8

False $\vee$ R
$\neg$R $\vee$ false
---
false $\vee$ false

Note: $(P \wedge \neg P) \rightarrow Z$ is valid

Unification:

finding a substitution that makes two expressions match exactly

ω_1 & ω_2 are unifiable iff there is a sub for which

$\omega_1 s = \omega_2 s$

Aside: Substitutions

given: $P(x, F(y), B)$: an atomic sentence

Substitution Instances	Substitution $\{v_1/t_1, \ldots, v_n/t_n\}$	Comment
$P(z, F(w), B)$	$\{x/z, y/w\}$	Alphabetic Variant
$P(x, F(A), B)$	$\{y/A\}$	
$P(G(z), F(A), B)$	$\{x/G(z), y/A\}$	
$P(C, F(A), B)$	$\{x/C, y/A\}$	Ground Instance