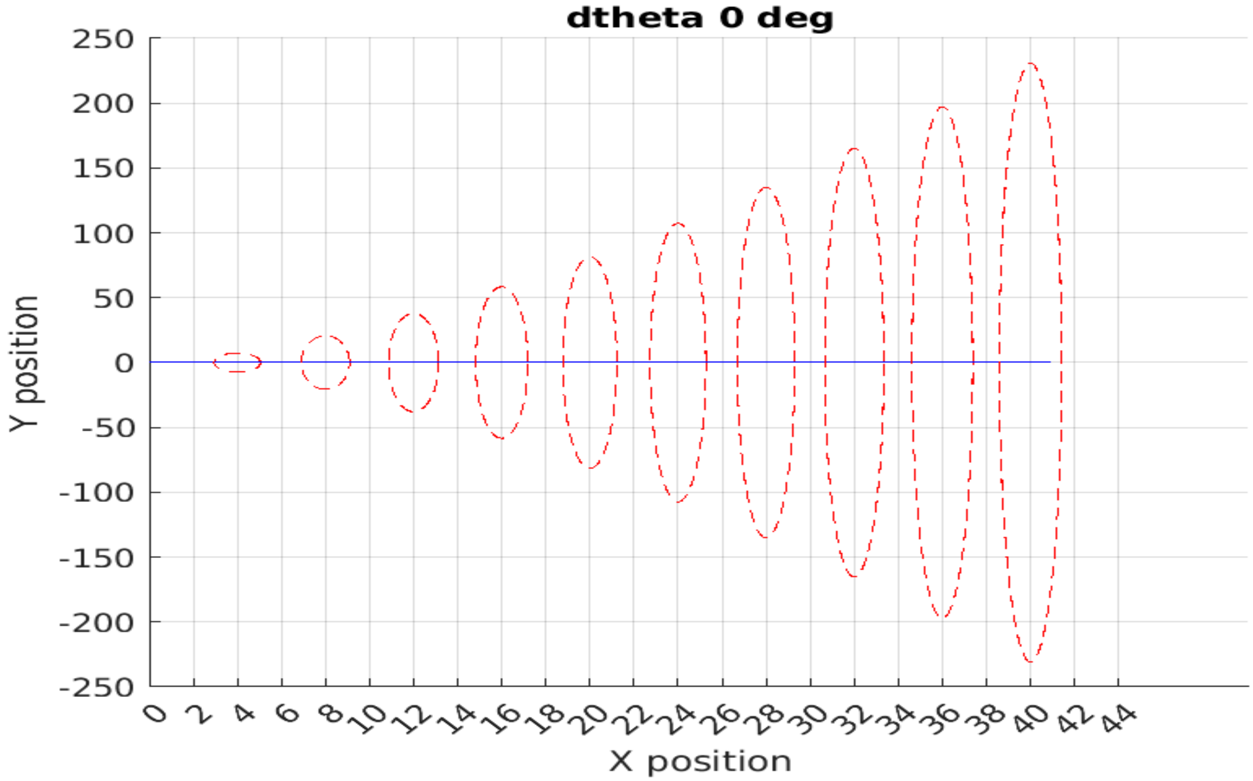
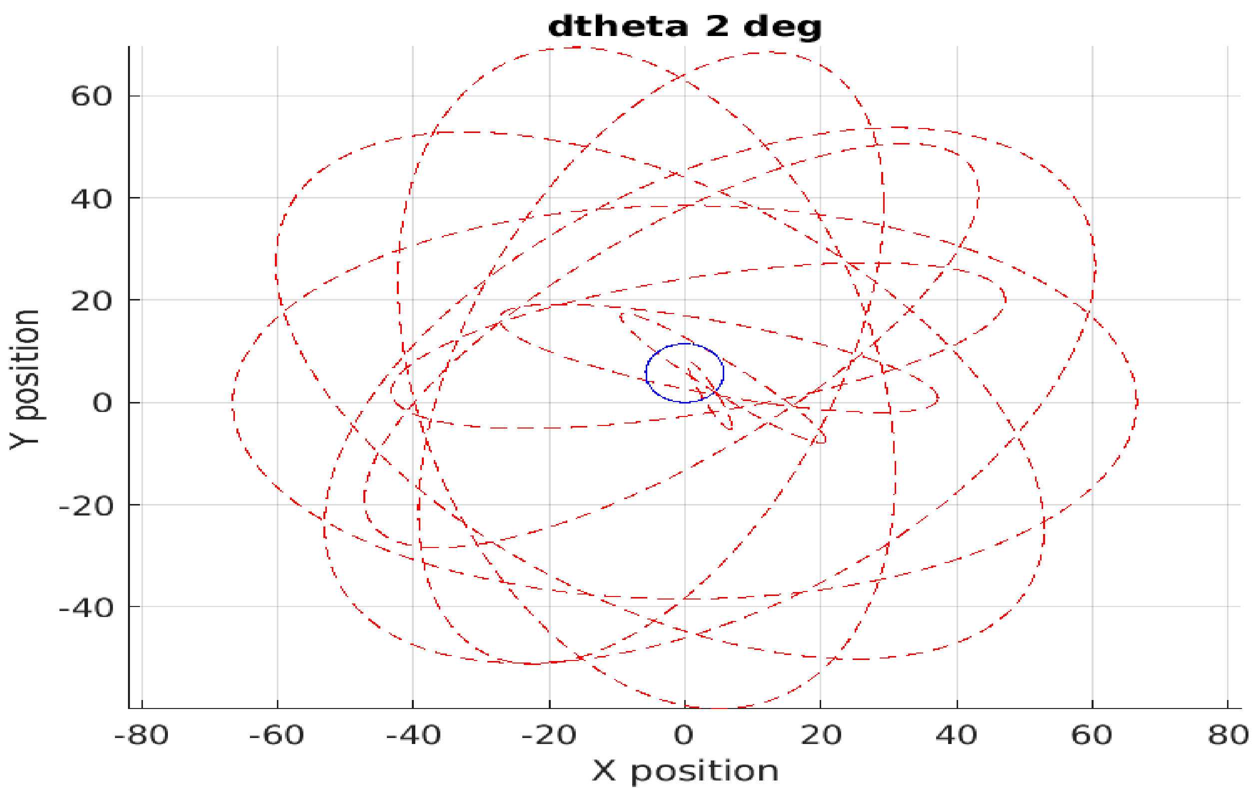
1. **Results**

`Confidence:0.6827`





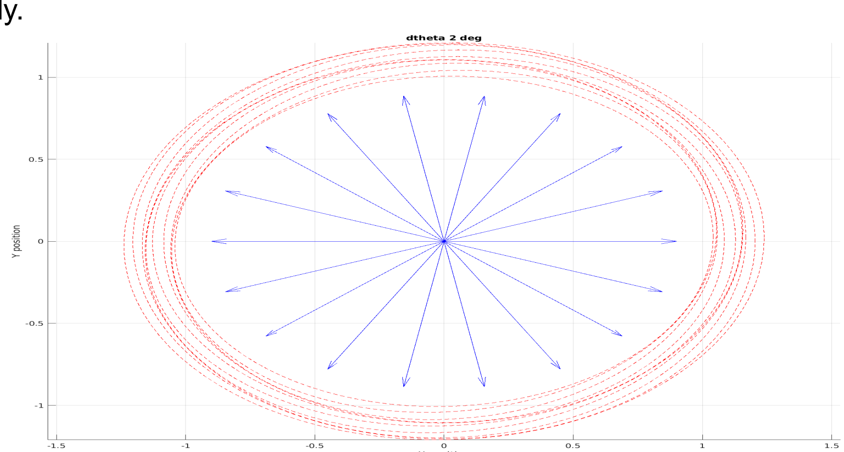
2. Discuss the change of the pose covariance over time for (a) and (b).

In case (a), the movement is along a straight line. Here, the covariance ellipses grow in size gradually over time, particularly in the direction perpendicular to movement. This is because we **don't have a sensor measurement in the perpendicular direction(i.e. Both ds_left and ds_right measurements are in the movement direction)** and hence, the uncertainty is large. For example, small deviations in wheel rotations affect the perpendicular direction more significantly when there is no change in heading.

In case (b), the robot is moving in a circular path. Here, we observe the uncertainty grow as the time progresses as well. Although the main axis of the ellipse not perpendicular to the direction movement, we still observe that error grows much faster in nearly orthogonal direction(i.e. Direction of no measurement).

3. Discuss why the covariance ellipse does not remain perpendicular to the direction of movement in (b).

Ans: I think it is because of the turn introduced in case (b).
Observation: The level of non-perpendicularity gets worse as time progresses. Initially, it is nearly perpendicular, but it gets worse as the time passes. Let's make **ds=0** and observe the effect of turn only.



The introduction of turn makes uncertainty grow as the time passes. This will be superimposed to the uncertainty because of **ds!=0** when **dtheta=0**. Super-imposing an ellipse of the perpendicular main axis and circle may introduce the coupling in the covariance matrix in the direction of motion and perpendicular direction of the motion.

And also from the following equation:

$$p' = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \Delta s \cos(\theta + \Delta\theta/2) \\ \Delta s \sin(\theta + \Delta\theta/2) \\ \Delta\theta \end{bmatrix}$$

$$= \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{\Delta s_r + \Delta s_l}{2} \cos\left(\theta + \frac{\Delta s_r - \Delta s_l}{2b}\right) \\ \frac{\Delta s_r + \Delta s_l}{2} \sin\left(\theta + \frac{\Delta s_r - \Delta s_l}{2b}\right) \\ \frac{\Delta s_r - \Delta s_l}{b} \end{bmatrix}$$

>> When dtheta not zero, **ds_left and ds_right** have a distribution at different averages(i.e. Mean value). So, the gaussian distribution of one is the shift of the other one(if we think in 1D). And also P' is no longer a linear combination of **ds_left and ds_right**. So, it is somehow evident that we are losing symmetry. And hence, orthogonal decomposition along the direction of motion is impossible because of this coupling.