

```

$Assumptions = {L > 0};

dofExpand[nodeIds_, n_] := Flatten[Transpose[Table[n nodeIds - (n - i), {i, 1, n}]]]
transformation[dir_] := Module[{y = {0, 1, 0}, γ},
  γ = FullSimplify[If[Abs[dir[[2]]] > 0.99,
    
$$\begin{pmatrix} 0 & \text{Sign[dir[[2]]}] & 0 \\ -\text{Sign[dir[[2]]}] & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

    {dir, Cross[Normalize[Cross[dir, y]], dir, Normalize[Cross[dir, y]]}]]];
  ArrayFlatten[
$$\begin{pmatrix} \gamma & 0 & 0 & 0 \\ 0 & \gamma & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & \gamma \end{pmatrix}$$
]
]

beamStiffness[E_, ν_, A_, Iy_, Iz_, L_] := Module[{G =  $\frac{E}{1 + 2\nu}$ , J = Iy + Iz},
  
$$\frac{E}{L} \begin{pmatrix} A & 0 & 0 & 0 & 0 & 0 & -A & 0 & 0 & 0 & 0 & 0 \\ 0 & 12 \frac{Iz}{L^2} & 0 & 0 & 0 & 6 \frac{Iz}{L} & 0 & -12 \frac{Iz}{L^2} & 0 & 0 & 0 & 6 \frac{Iz}{L} \\ 0 & 0 & 12 \frac{Iy}{L^2} & 0 & -6 \frac{Iy}{L} & 0 & 0 & 0 & -12 \frac{Iy}{L^2} & 0 & -6 \frac{Iy}{L} & 0 \\ 0 & 0 & 0 & \frac{GJ}{E} & 0 & 0 & 0 & 0 & 0 & -\frac{GJ}{E} & 0 & 0 \\ 0 & 0 & -6 \frac{Iy}{L} & 0 & 4 Iy & 0 & 0 & 0 & 6 \frac{Iy}{L} & 0 & 2 Iy & 0 \\ 0 & 6 \frac{Iz}{L} & 0 & 0 & 0 & 4 Iz & 0 & -6 \frac{Iz}{L} & 0 & 0 & 0 & 2 Iz \\ -A & 0 & 0 & 0 & 0 & 0 & A & 0 & 0 & 0 & 0 & 0 \\ 0 & -12 \frac{Iz}{L^2} & 0 & 0 & 0 & -6 \frac{Iz}{L} & 0 & 12 \frac{Iz}{L^2} & 0 & 0 & 0 & -6 \frac{Iz}{L} \\ 0 & 0 & -12 \frac{Iy}{L^2} & 0 & 6 \frac{Iy}{L} & 0 & 0 & 0 & 12 \frac{Iy}{L^2} & 0 & 6 \frac{Iy}{L} & 0 \\ 0 & 0 & 0 & -\frac{GJ}{E} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{E} & 0 & 0 \\ 0 & 0 & -6 \frac{Iy}{L} & 0 & 2 Iy & 0 & 0 & 0 & 6 \frac{Iy}{L} & 0 & 4 Iy & 0 \\ 0 & 6 \frac{Iz}{L} & 0 & 0 & 0 & 2 Iz & 0 & -6 \frac{Iz}{L} & 0 & 0 & 0 & 4 Iz \end{pmatrix}$$

]

beamMass[ρ_, A_, J_, L_] :=

```

$$\frac{\rho A L}{420} \begin{pmatrix} 140 & 0 & 0 & 0 & 0 & 0 & 70 & 0 & 0 & 0 & 0 & 0 \\ 0 & 156 & 0 & 0 & 0 & 22 L & 0 & 54 & 0 & 0 & 0 & -13 L \\ 0 & 0 & 156 & 0 & -22 L & 0 & 0 & 0 & 54 & 0 & 13 L & 0 \\ 0 & 0 & 0 & 140 \frac{J}{A} & 0 & 0 & 0 & 0 & 0 & 70 \frac{J}{A} & 0 & 0 \\ 0 & 0 & -22 L & 0 & 4 L^2 & 0 & 0 & 0 & -13 L & 0 & -3 L^2 & 0 \\ 0 & 22 L & 0 & 0 & 0 & 4 L^2 & 0 & 13 L & 0 & 0 & 0 & -3 L^2 \\ 70 & 0 & 0 & 0 & 0 & 0 & 140 & 0 & 0 & 0 & 0 & 0 \\ 0 & 54 & 0 & 0 & 0 & 13 L & 0 & 156 & 0 & 0 & 0 & -22 L \\ 0 & 0 & 54 & 0 & -13 L & 0 & 0 & 0 & 156 & 0 & 22 L & 0 \\ 0 & 0 & 0 & 70 \frac{J}{A} & 0 & 0 & 0 & 0 & 0 & 140 \frac{J}{A} & 0 & 0 \\ 0 & 0 & 13 L & 0 & -3 L^2 & 0 & 0 & 0 & 22 L & 0 & 4 L^2 & 0 \\ 0 & -13 L & 0 & 0 & 0 & -3 L^2 & 0 & -22 L & 0 & 0 & 0 & 4 L^2 \end{pmatrix};$$

```
makeBeam[{p_, q_}, nodeIds_, E_, v_, A_, Iy_, Iz_, rho_] := Module[
{
  L = FullSimplify[Norm[q - p]],
  J = Iy + Iz,
  Gamma = FullSimplify[transformation[Normalize[q - p]]]
},
<|
  "Gamma" -> Gamma,
  "gamma" -> Gamma[[1 ;; 3, 1 ;; 3]],
  "M" -> FullSimplify[beamMass[rho, A, J, L]],
  "K" -> FullSimplify[beamStiffness[E, v, A, Iy, Iz, L]],
  "endpoints" -> {p, q},
  "E" -> E,
  "v" -> v,
  "rho" -> rho,
  "A" -> A,
  "Iy" -> Iy,
  "Iz" -> Iz,
  "L" -> L,
  "nodeIds" -> nodeIds,
  "dofIds" -> dofExpand[nodeIds, 6],
  "displacement" -> ConstantArray[0.0, 12]
|>
]

psi[x_, L_] := {
  InterpolatingPolynomial[{{0, 1, 0}, {L, 0, 0}}, x],
  InterpolatingPolynomial[{{0, 1, 1}, {L, 0, 0}}, x],
  InterpolatingPolynomial[{{0, 0, 0}, {L, 1, 0}}, x],
  InterpolatingPolynomial[{{0, 0, 0}, {L, 0, 1}}, x]
}
```

$$\Psi[x_, L_] := \text{Transpose} \left[\begin{pmatrix} 1 - \frac{x}{L} & 0 & 0 \\ 0 & \psi[x, L][[1]] & 0 \\ 0 & 0 & \psi[x, L][[1]] \\ 0 & 0 & 0 \\ 0 & 0 & -\psi[x, L][[2]] \\ 0 & \psi[x, L][[2]] & 0 \\ \frac{x}{L} & 0 & 0 \\ 0 & \psi[x, L][[3]] & 0 \\ 0 & 0 & \psi[x, L][[3]] \\ 0 & 0 & 0 \\ 0 & 0 & -\psi[x, L][[4]] \\ 0 & \psi[x, L][[4]] & 0 \end{pmatrix} \right];$$

```

drawBeam[beam_, displacement_] := Module[
{
  p = beam["endpoints"][[1]],
  L = beam["L"],
  u
},
u = beam["Γ"].displacement + {0, 0, 0, 0, 0, 0, L, 0, 0, 0, 0, 0};
Line[
  Table[
    Transpose[beam["γ"]].Ψ[x, L].u + p
    ,
    {x, 0, L, 0.02 L}
  ]
]

drawBeamSimple[beam_, displacement_] :=
  Line[beam["endpoints"] + {displacement[[1 ;; 3]], displacement[[7 ;; 9]]}]

multiStoryFrame[xGrid_, yGrid_, zGrid_] :=
Module[{nodes, elems, fixed, free, dx, dy, dz, id},
  dx = 1;
  dy = Length[xGrid];
  dz = Length[xGrid] * Length[yGrid];
  nodes = Flatten[Table[{x, y, z}, {z, zGrid}, {y, yGrid}, {x, xGrid}], 2];

  elems = {};

  (* add all columns *)
  Do[
    id = (z - 1) * dz + (y - 1) * dy + x;
    AppendTo[elems, {id, id + dz}]
    ,
    {z, 1, Length[zGrid] - 1},
    {y, 1, Length[yGrid]},
    {x, 1, Length[xGrid]}
  ];

  (* beams in y-direction *)
  Do[
    id = (z - 1) * dz + (y - 1) * dy + x;

```

```

    AppendTo[elems, {id, id + dy}]
    ,
    {z, 2, Length[zGrid]},
    {y, 1, Length[yGrid] - 1},
    {x, 1, Length[xGrid]}
  ];

  (* beams in x-direction *)
  Do[
    id = (z - 1) * dz + (y - 1) * dy + x;
    AppendTo[elems, {id, id + dx}]
    ,
    {z, 2, Length[zGrid]},
    {y, 1, Length[yGrid]},
    {x, 1, Length[xGrid] - 1}
  ];

  fixed = {};
  Do[
    If[nodes[[i, 3]] == zGrid[[1]], AppendTo[fixed, i]]
    ,
    {i, 1, Length[nodes]}
  ];
  fixed = dofExpand[fixed, 6];
  free = Complement[Table[i, {i, 1, Length[nodes] * 6}], fixed];

  {nodes, elems, fixed, free}
]

assembleGlobalMatrices[beams_, ndof_] := Module[{
  Mglobal = ConstantArray[0.0, {ndof, ndof}],
  Kglobal = ConstantArray[0.0, {ndof, ndof}]
},
  Do[
    Mglobal[[b["dofIds"], b["dofIds"]]] += Transpose[b["r"]].b["M"].b["r"];
    Kglobal[[b["dofIds"], b["dofIds"]]] += Transpose[b["r"]].b["K"].b["r"];
    ,
    {b, beams}
  ];
  {Mglobal, Kglobal}
]

E = 200;
ν = 0.3;
A = 1430;
Iy = 1.26 * 106;
Iz = 1.26 * 106;
ρ =  $\frac{8050}{10^9}$ ;
L = 1000;

```

$$\mathbf{nodes} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 2.5 \\ -1 & 0 & 0 \\ 0 & 1.5 & 2.5 \\ 1 & 3 & 0 \\ 0 & 3 & 2.5 \\ -1 & 3 & 0 \end{pmatrix} \mathbf{L};$$

$$\text{elems} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \\ 2 & 4 \\ 4 & 6 \\ 5 & 6 \\ 6 & 7 \end{pmatrix};$$

```
ndof = Length[nodes] * 6;
fixed = {1, 2, 3, 13, 14, 15, 25, 26, 27, 37, 38, 39};
```

```
free = Complement[Table[i, {i, 1, ndof}], fixed];
```

```
ndof = Length[nodes] * 6;
beams = Table[makeBeam[nodes[[el]], el, E,  $\nu$ , A, Iy, Iz,  $\rho$ ], {el, elems}];
{Mglobal, Kglobal} = assembleGlobalMatrices[beams, ndof];
fglobal = -4.5 IdentityMatrix[ndof][[21]];
uglobal = ConstantArray[0.0, ndof];
uglobal[[free]] =
  LinearSolve[FullSimplify[Chop[Kglobal]][[free, free]], fglobal[[free]]];
```

```

{nodes, elems, fixed, free} =
  multiStoryFrame[{0, 1, 2, 3} L, {0, 1, 2} L, {0, 1, 2, 3} L];
ndof = Length[nodes] * 6;
beams = Table[makeBeam[nodes[[el]], el, E,  $\nu$ , A, Iy, Iz,  $\rho$ ], {el, elems}];
{Mglobal, Kglobal} = assembleGlobalMatrices[beams, ndof];
fglobal = Flatten[ $\left\{ \text{If}[\#[[1]] == 0, \frac{\#[[3]]}{I_x}, 0], 0, 0, 0, 0, 0 \right\} \& /@ \text{nodes}$ ];

```

```
uglobal = ConstantArray[0.0, ndof];
uglobal[[free]] = LinearSolve[Kglobal[[free, free]], fglobal[[free]]];
uglobal[[free]] =
  LinearSolve[FullSimplify[Chop[Kglobal]][[free, free]], fglobal[[free]]];
Partition[uglobal, 6] // MatrixForm
```

[illegible]

0.	0.	0.	0.	0.	0.
0.788283	-1.92687×10^{-13}	0.0128286	1.85361×10^{-16}	0.000703502	1.87789×10^{-16}
0.786097	-7.08329×10^{-14}	-0.00199893	1.3058×10^{-16}	0.00045684	8.13163×10^{-17}
0.784361	-1.44257×10^{-13}	0.00193357	3.12025×10^{-17}	0.000456051	1.16673×10^{-16}
0.783071	6.41688×10^{-14}	-0.0127632	-8.75898×10^{-17}	0.000698977	2.3217×10^{-16}
0.788283	-1.89843×10^{-13}	0.0128286	1.15087×10^{-16}	0.000703502	1.6374×10^{-16}
0.786097	-6.89994×10^{-14}	-0.00199893	8.382×10^{-17}	0.00045684	1.04206×10^{-16}
0.784361	-1.43062×10^{-13}	0.00193357	1.46092×10^{-17}	0.000456051	1.25059×10^{-16}
0.783071	6.33795×10^{-14}	-0.0127632	-5.87711×10^{-17}	0.000698977	1.83222×10^{-16}
0.788283	-1.88497×10^{-13}	0.0128286	1.82518×10^{-16}	0.000703502	1.25959×10^{-16}
0.786097	-6.57061×10^{-14}	-0.00199893	1.30875×10^{-16}	0.00045684	4.41676×10^{-17}
0.784361	-1.50357×10^{-13}	0.00193357	3.07991×10^{-17}	0.000456051	8.07738×10^{-17}
0.783071	6.22817×10^{-14}	-0.0127632	-8.75243×10^{-17}	0.000698977	1.7282×10^{-16}
1.728	-4.74444×10^{-13}	0.0196795	1.61805×10^{-16}	0.000550628	2.81892×10^{-16}
1.72242	-3.22644×10^{-13}	-0.00288802	8.65102×10^{-17}	0.000385679	2.56357×10^{-16}
1.71892	-4.75573×10^{-14}	0.00278401	-7.08154×10^{-17}	0.000384829	3.13729×10^{-16}
1.7175	2.57397×10^{-13}	-0.0195755	-1.02581×10^{-16}	0.000546674	3.44931×10^{-16}
1.728	-4.70819×10^{-13}	0.0196795	1.09045×10^{-16}	0.000550628	2.77301×10^{-16}
1.72242	-3.32499×10^{-13}	-0.00288802	6.00486×10^{-17}	0.000385679	2.66241×10^{-16}
1.71892	-4.6051×10^{-14}	0.00278401	-4.12528×10^{-17}	0.000384829	3.03888×10^{-16}
1.7175	2.555×10^{-13}	-0.0195755	-6.48704×10^{-17}	0.000546674	3.0823×10^{-16}
1.728	-4.67065×10^{-13}	0.0196795	1.62543×10^{-16}	0.000550628	1.89147×10^{-16}
1.72242	-3.27756×10^{-13}	-0.00288802	8.88302×10^{-17}	0.000385679	2.14811×10^{-16}
1.71892	-4.67754×10^{-14}	0.00278401	-7.15491×10^{-17}	0.000384829	2.74305×10^{-16}
1.7175	2.54209×10^{-13}	-0.0195755	-1.05155×10^{-16}	0.000546674	2.59844×10^{-16}
2.326	-6.52004×10^{-13}	0.0218191	6.98967×10^{-17}	0.000287891	3.71073×10^{-16}
2.3174	-3.12863×10^{-13}	-0.00319917	-9.4124×10^{-18}	0.000166855	3.67594×10^{-16}
2.31217	4.1632×10^{-14}	0.00307971	-3.46345×10^{-17}	0.000166442	3.57114×10^{-16}
2.3103	3.42609×10^{-13}	-0.0216996	-2.74033×10^{-17}	0.000284903	3.61895×10^{-16}
2.326	-6.49395×10^{-13}	0.0218191	4.39242×10^{-17}	0.000287891	3.60914×10^{-16}
2.3174	-3.13845×10^{-13}	-0.00319917	-3.09987×10^{-18}	0.000166855	3.5413×10^{-16}
2.31217	3.31562×10^{-14}	0.00307971	-1.73103×10^{-17}	0.000166442	3.49193×10^{-16}
2.3103	3.42255×10^{-13}	-0.0216996	-1.70324×10^{-17}	0.000284903	3.53247×10^{-16}
2.326	-6.48606×10^{-13}	0.0218191	7.18927×10^{-17}	0.000287891	3.03832×10^{-16}
2.3174	-3.15203×10^{-13}	-0.00319917	-1.83624×10^{-17}	0.000166855	3.26887×10^{-16}
2.31217	3.24515×10^{-14}	0.00307971	-3.58206×10^{-17}	0.000166442	3.21935×10^{-16}
2.3103	3.4194×10^{-13}	-0.0216996	-3.5404×10^{-17}	0.000284903	2.98497×10^{-16}

Eigenvalues[Kglobal[[free, free]]];

{ Λ , Qfree} = **Eigensystem**[[Kglobal[[free, free]], Mglobal[[free, free]]];

ϕ = **ConstantArray**[0.0, {6, ndof}];

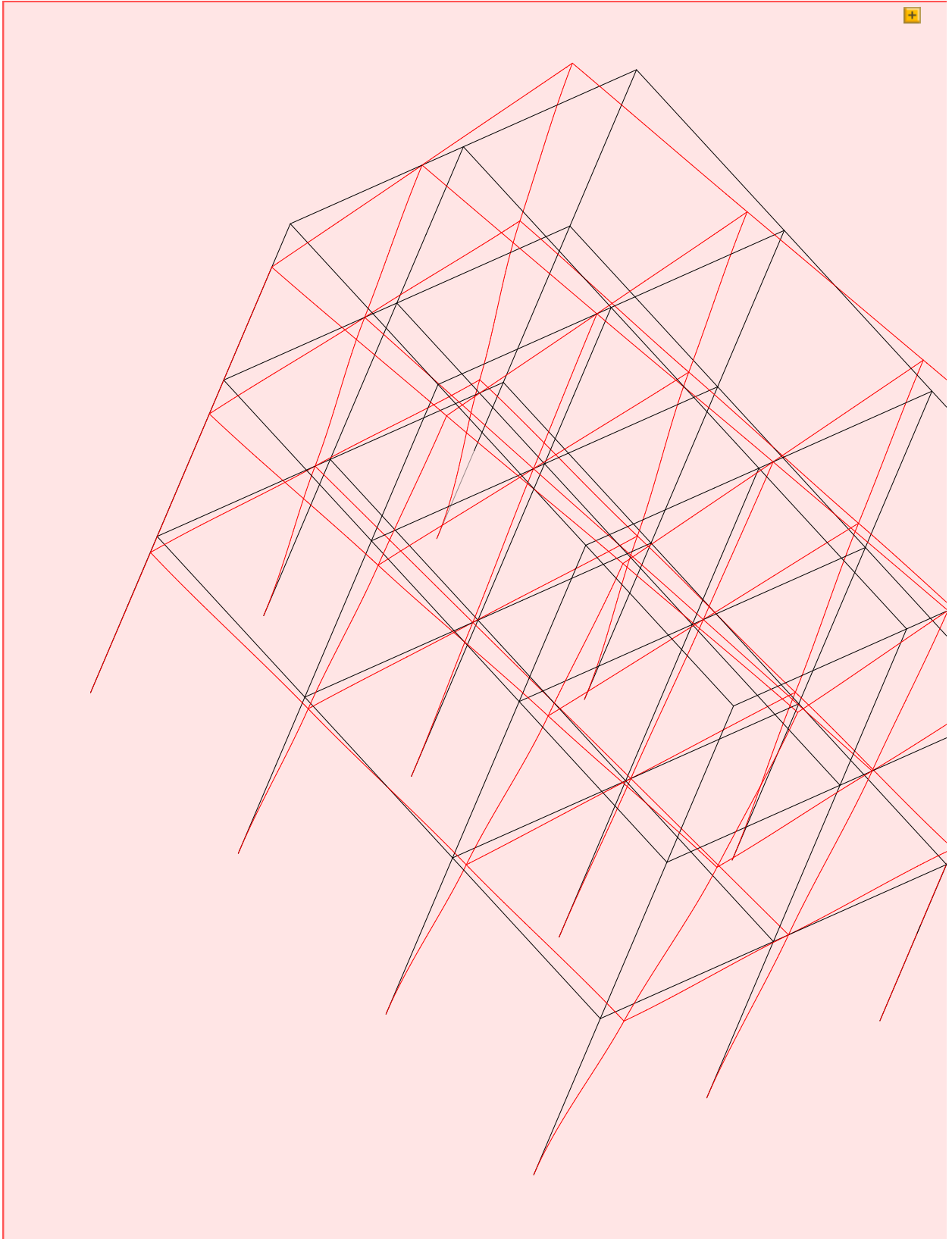
ϕ [[1 ;; 6, free]] = **Reverse**[Qfree[[-6 ;; -1, All]]];

```

mode = 3;

Graphics3D[{
  Opacity[0.4],
  Table[drawBeamSimple[b, 0 uglobal[[b["dofIds"]]]], {b, beams}],
  Opacity[1.0],
  Red,
  Table[drawBeam[b, 1000  $\phi$ [[mode, b["dofIds"]]]], {b, beams}]
},
(*PlotRange→{{-500, 3500}, {-500, 2500}, {0, 3500}},*)
ViewPoint → {5000, -2000, 2000},
ViewVertical → {0, 0, 1},
ImageSize → 800,
Boxed → False
]

```



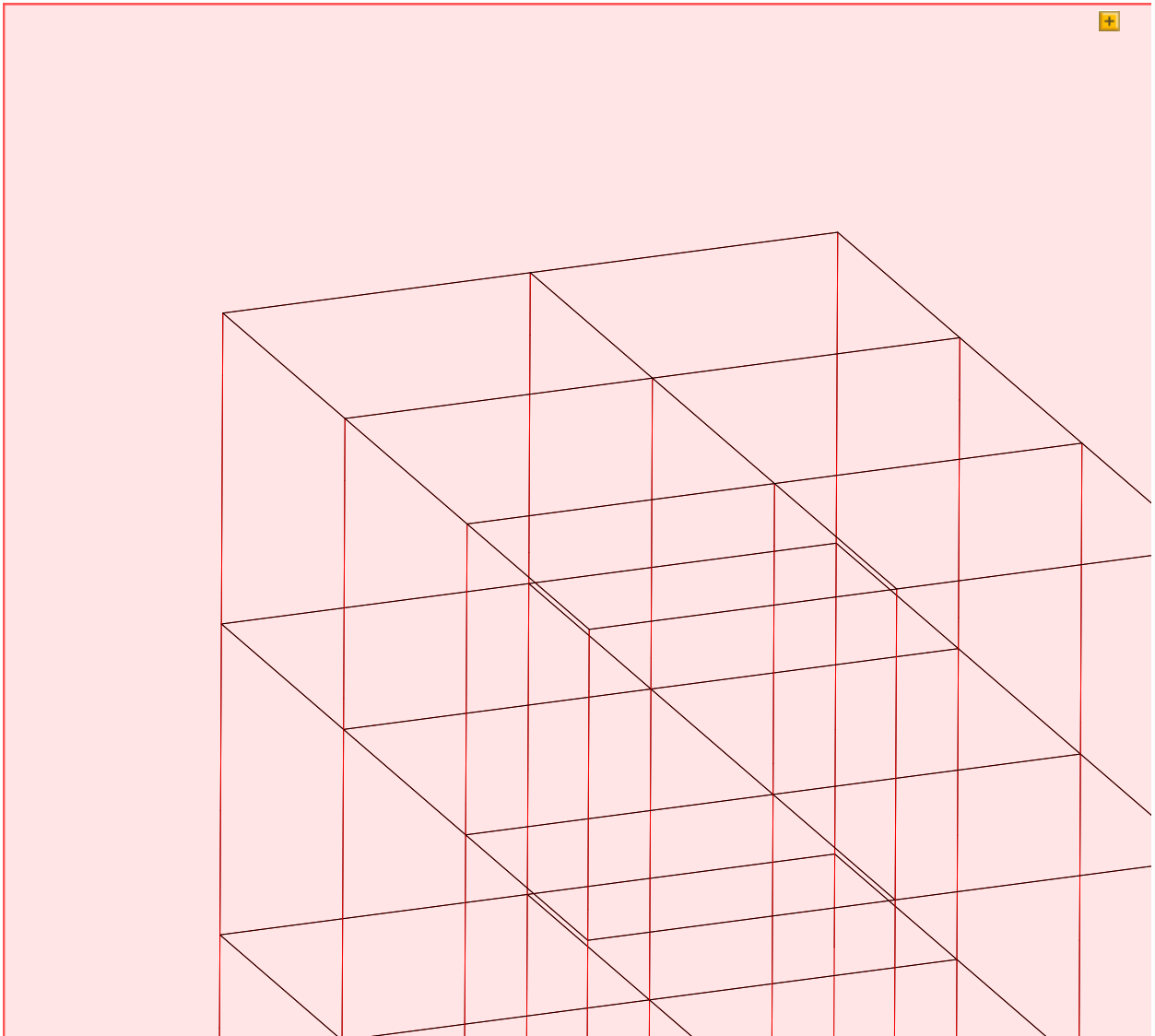

```

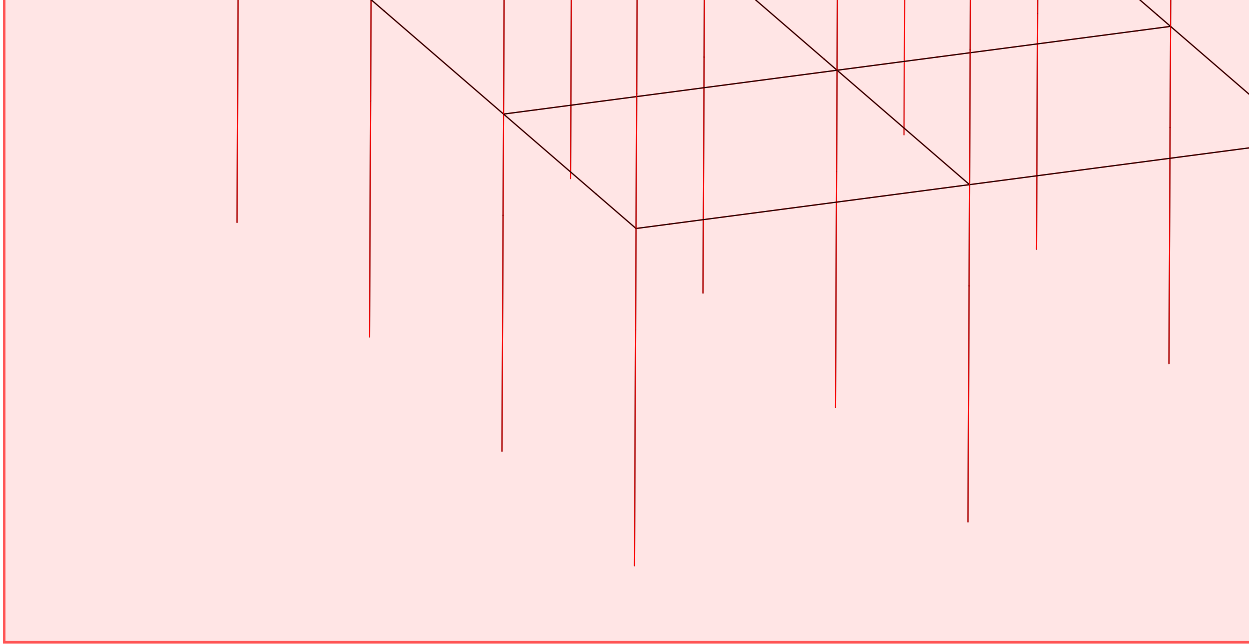
drawBeamSimple[beams[[1]], 0 uglobal[[beams[[1]]["dofIds"]]]]
Line[{{0., 0., 0.}, {0., 0., 1000.}}]

mode = 3;

Graphics3D[{
  Opacity[0.4],
  Table[drawBeamSimple[b, 0 uglobal[[b["dofIds"]]]], {b, beams}],
  Opacity[1.0],
  Red,
  Table[drawBeamSimple[b, 0  $\phi$ [[mode, b["dofIds"]]]], {b, beams}
],
PlotRange → {{-500, 3500}, {-500, 2500}, {0, 3500}},
ViewPoint → {5000, -2000, 2000},
ViewVertical → {0, 0, 1},
ImageSize → 800,
Boxed → False
]

```





uglobal

```
{0., 0., 0., 0.000757374, -2.5418 × 10-6, 0.0013384, 8.6417 × 10-17, 0.00262786,
-0.0122854, -0.00258862, -2.08541 × 10-20, 6.76508 × 10-19, 0., 0., 0., 0.000757374,
2.5418 × 10-6, -0.0013384, -2.77004 × 10-16, -2.27106 × 10-12, -4.46491,
-2.21706 × 10-16, -2.4112 × 10-20, -2.18994 × 10-20, 0., 0., 0., -0.000757374,
-2.5418 × 10-6, -0.0013384, -1.47441 × 10-18, -0.00262786, -0.0122854, 0.00258862,
-7.21386 × 10-21, -1.92672 × 10-19, 0., 0., 0., -0.000757374, 2.5418 × 10-6, 0.0013384}
```

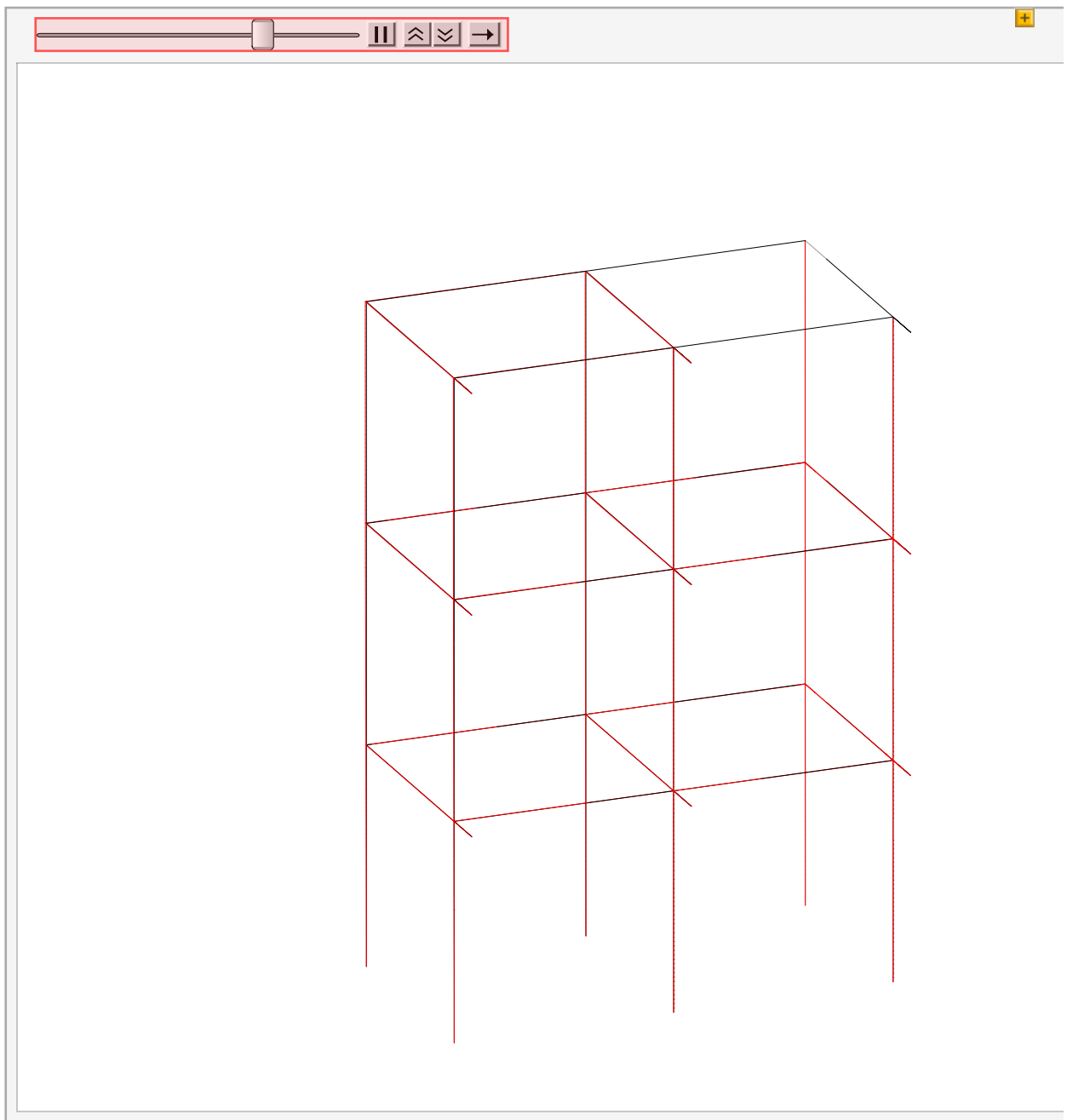
```
frames = ParallelTable[
  pos = nodes + α Partition[uglobal, 6][[All, 1 ;; 3]];
  Graphics3D[{
    Opacity[0.4],
    Table[drawBeamSimple[b, 0 uglocal[[b["dofIds"]]]], {b, beams}},
    Opacity[1.0],
    Red,
    Table[drawBeam[b, α ϕ[[4, b["dofIds"]]]], {b, beams}]
  ],
  PlotRange → {{-1200, 1200}, {-1000, 4000}, {0, 3000}},
  ViewPoint → {5000, -2000, 2000},
  ImageSize → 800,
  Boxed → False
];
```

```

frames = ParallelTable[
  pos = nodes +  $\alpha$  Partition[uglobal, 6][[All, 1 ;; 3]];
  Graphics3D[{
    Table[
      drawBeam[b,  $\alpha$  uglobal[[b["dofIds"]]]], {b, beams}
    ],
    Red,
    Arrowheads[0.03],
    Arrow[{pos[[4]] + {0, 0, 500}, pos[[4]]}]
  },
  PlotRange  $\rightarrow$  {{-1200, 1200}, {-1000, 4000}, {0, 3000}},
  ViewPoint  $\rightarrow$  {5000, -2000, 2000},
  ImageSize  $\rightarrow$  800,
  Boxed  $\rightarrow$  False
]
,
{ $\alpha$ , 0, 100, 2}
];

```

```
ListAnimate[frames]
```



```
Export["swingset_mode2.gif",  
  Join[frames[[1 ;; -2]], Reverse[frames[[2 ;; -1]]], Antialiasing -> True]  
swingset_mode2.gif
```

```

(*scales = {1, 1, 1, 0.01, 0.01, 0.01};
{ $\Lambda$ , Qfree}=Eigensystem[{Kglobal[[free, free]], Mglobal[[free, free]]}];
 $\phi$  = ConstantArray[0.0, {6, ndof}];
 $\phi$ [[1;;6,free]] = Reverse[Qfree[[-6;;-1, All]]];
mode = 1;
Manipulate[
  Graphics3D[{
    Opacity[0.5],
    Table[
      drawBeam[b, 0 $\phi$ [[mode,b["dofIds"]]]], {b, beams}
    ],
    Opacity[1.0],
    Table[
      drawBeam[b,  $\alpha$  scales[[mode]] $\phi$ [[mode,b["dofIds"]]]], {b, beams}
    ]
  },
  PlotRange→{{-1200, 1200}, {-1000, 4000}, {0, 3000}},
  Boxed→False,
  ViewPoint→{5000, -2000, 2000},
  ImageSize→800
],
{ $\alpha$ , 0, 1000}
]*)

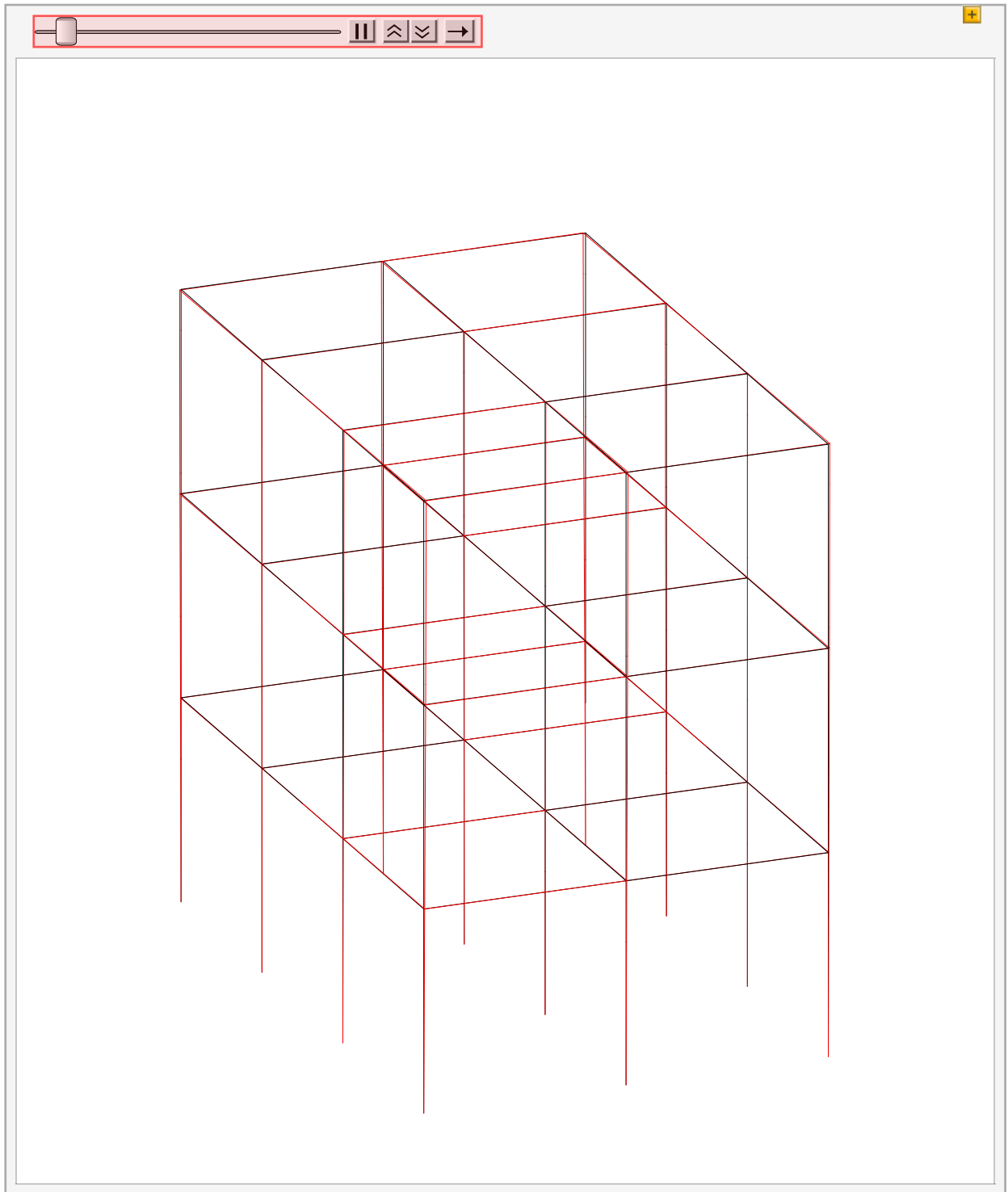
```

```

frames = Table[
  Graphics3D[{
    Opacity[0.4],
    Table[drawBeamSimple[b, 0 uglobal[[b["dofIds"]]]], {b, beams}],
    Opacity[1.0],
    Red,
    Table[drawBeamSimple[b,  $\alpha$   $\phi$ [[mode, b["dofIds"]]]], {b, beams}
  ],
  PlotRange → {{-500, 3500}, {-500, 2500}, {0, 3500}},
  ViewPoint → {5000, -2000, 2000},
  ViewVertical → {0, 0, 1},
  ImageSize → 600,
  Boxed → False
],
{ $\alpha$ , 0, 1000, 10}
];

```

```
ListAnimate[frames]
```



```
$Aborted
```

```

Graphics3D[{
  Opacity[0.2],
  Table[drawBeamSimple[b, 0 uglobal[[b["dofIds"]]]], {b, beams}],
  Opacity[1.0],
  Red,
  Table[drawBeam[b, 100 uglobal[[b["dofIds"]]]], {b, beams}]
},
PlotRange → {{-500, 3500}, {-500, 2500}, {0, 3500}},
ViewPoint → {5000, -2000, 2000},
ViewVertical → {0, 0, 1},
ImageSize → 600,
Boxed → False
]

```

