Chain-Ladder vs. Bornhuetter-Ferguson: Workers' Compensation

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Executive Summary

The problem being studied in this paper is which reserving method is more reliable for the workers' compensation line of business; chain-ladder or Bornhuetter-Ferguson? This was studied by using a dataset with cumulative paid loss data and incurred loss data from U.S. property casualty insurers. We used a variety of loss development factor models and reconciled both types of loss data so as to find both the MAE for each individual company in the dataset as well as the total MAE for all companies in the dataset for each reserving method. Our findings based on our Mean Absolute Errors were that Bornhuetter-Ferguson method was the most reliable reserving method for this line of business in particular. However, Chain-Ladder should not be counted out as it was not too far behind and it is recommended to use both methods in tandem since whichever method performs best depends mostly on the data.

Section 1. Introduction

The methodologies that will be examined in this report are two mainstays of actuarial science. These methodologies are the chain-ladder method and the Bornhuetter-Ferguson method. The goal of this project is to analyze in-depth these two techniques for the purpose of gaining a better understanding of them and determine which one is more reliable for the workers' compensation business line using experimental data.

Previous research into these two procedures allows for a better perception as to what they entail. In Stochastic Claims Reserving in General Insurance by authors P.D. England and R.J. Verrall, the chain-ladder technique "consists of a way of obtaining forecasts of ultimate claims only". Regarding the Bornhuetter-Ferguson method, they state that it is favorable to the chain-ladder method in scenarios where "there is instability in the proportion of ultimate claims paid in the early development years". In their closing remarks they answer the question which model is best by saying there is no definitive answer since it all depends on the data. This means analyzing the dataset is critical in determining the best model. They also define the best estimate as "the expected value of the distribution of possible outcomes of the unpaid liabilities". Comparing paid data to incurred data, they state paid data is "a better option for methods because of the overestimation of aggregate case reserves." Because of this issue they propose modeling based on individual claims as opposed to the more traditional aggregate triangles, especially with the improvement in computing technology [1].

Additionally, in T. Mack's final remarks in Measuring the Variability of Chain Ladder Reserve Estimates, he highlights some of the disadvantages of the chain ladder methods such as "the fact that the estimators of the last two or three factors rely on very few observations and the fact that the known claims amount of the last accident year forms a very uncertain basis for the projection to ultimate." [2] Furthermore, in the textbook Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance by Robert L. Brown and W. Scott Lennox, when comparing the two methods they state that the "Bornhuetter-Ferguson method has the advantages of (a) being more stable than the chain-ladder method, and (b) allowing inclusion of data from other sources. Its major disadvantage is that it requires an assumption, perhaps from an outside data source, for the initial expected loss ratio, and such data may not be available" [3].

The data used for this paper is Loss Reserving Data pulled from NAIC Schedule P and comes from the Casualty Actuarial Society. Specifically, the data being analyzed comes from the workers' compensation line of business. Through the construction of loss triangles using this data, the two actuarial methodologies will be implemented to come up with estimated reserves and compared with the actual reserve provided in the data set as well as to each other to see which technique is the better fit. Three different modeling techniques will be utilized for the sake of attaining a reliable estimate: the average factor model, the five-year average factor model, and the volume-weighted average factor model. The objective of this analysis will be to determine the methodology that is the most appropriate for forecasting this dataset.

Section 2. Data Characteristics

This dataset contains observational and longitudinal loss reserving data for 132 U.S. property casualty insurers from the workers' compensation line of business. This dataset contains 13 variables, however we will only be using seven of them in addition to the variable containing the name of the company. For the purposes of constructing loss triangles, only data from years 1988 to 1997 are used. Although this dataset contains data for 132 insurance companies, after removing the companies with missing data, only fifty-seven were left. For a more thorough examination we observe the loss data for State Farm.

Table 2.1. *Summary Statistics for State Farm*

```
── Data Summary ────────────────────────
                           Values
Name                       data
Number of rows             55
Number of columns          8
_______________________
Column type frequency:
  character                1
  numeric                  7
________________________
Group variables            None

── Variable type: character ──────────────────────────────────────────────────────────
  skim_variable n_missing complete_rate min max empty n_unique whitespace
1 Company               0             1  18  18     0        1          0

── Variable type: numeric ────────────────────────────────────────────────────────────
  skim_variable    n_missing complete_rate    mean      sd     p0    p25    p50    p75   p100 hist
1 AccidentYear             0             1   1991    2.47   1988   1989   1991   1993   1997
2 DevelopmentYear          0             1   1994    2.47   1988   1992   1994   1996   1997
3 DevelopmentLag           0             1      4    2.47      1      2      4      6     10
4 IncurLoss                0             1 192632. 49844.  91892 153543 202911 245238 289785
5 CumPaidLoss              0             1 121520. 55610.  22190  84160 123492 161267 213904
6 EarnedPremNet            0             1 272427. 77849. 177104 201118 246010 340183 418755
7 PostedReserve97          0             1 542695       0 542695 542695 542695 542695 542695
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As can be noted from these summary statistics there are seven numeric variables that we will be utilizing for this project. This section aims to clearly define each one.

AccidentYear – This represents the year in which the accident that triggered the policy happened [5]. For this paper, we will be using accident years 1988 to 1997.

DevelopmentYear – This represents the number of years that has passed since the year in which the accident occurred, also known as the accident year. This means that this is the number of years that the claim has been in development. Even though this variable varies from 1988 to 2006, for the purposes of this project we have restricted the range from 1988 to 1997 so that only data relevant to creating loss triangles is included.

DevelopmentLag – This variable represents the number of years that have passed between the development year and the accident year plus 1. This is useful when constructing loss triangles to see which development year we are calculating for.

CumPaidLoss – This variable represents the cumulative paid loss and is purely objective. This is the amount that the insurer has paid thus far.

IncurLoss – This variable represents the incurred losses which is equal to cumulative paid loss plus case reserves. This data is subjective because although it consists of the objective data of the cumulative paid loss, it also includes the subjective data of the case reserves, which is the actuaries best estimate as to how much the insurance company needs to pay to ensure completion of their payments. This type of data comes under scrutiny for being over-estimated due to conservative estimates which in turn leads to negative incremental values when constructing the incurred loss triangle [1].

EarnedPremNet – This value represents the difference between the direct premiums and the ceded premiums. According to Brown and Lennox, "earned premiums reflect the portion of the written premiums that were earned for the policy period as of the valuation date" [3]. This value is instrumental for the Bornhuetter-Ferguson method.

PostedReserve97 – This represents the actual reserve that we are trying to predict for, and so we will be using this value to compare our estimates to. We want our estimates to be as close to this value as possible. This is the actual amount that the insurer still needs to pay in 1997. This value includes net losses unpaid as well as unpaid loss adjustment expenses.

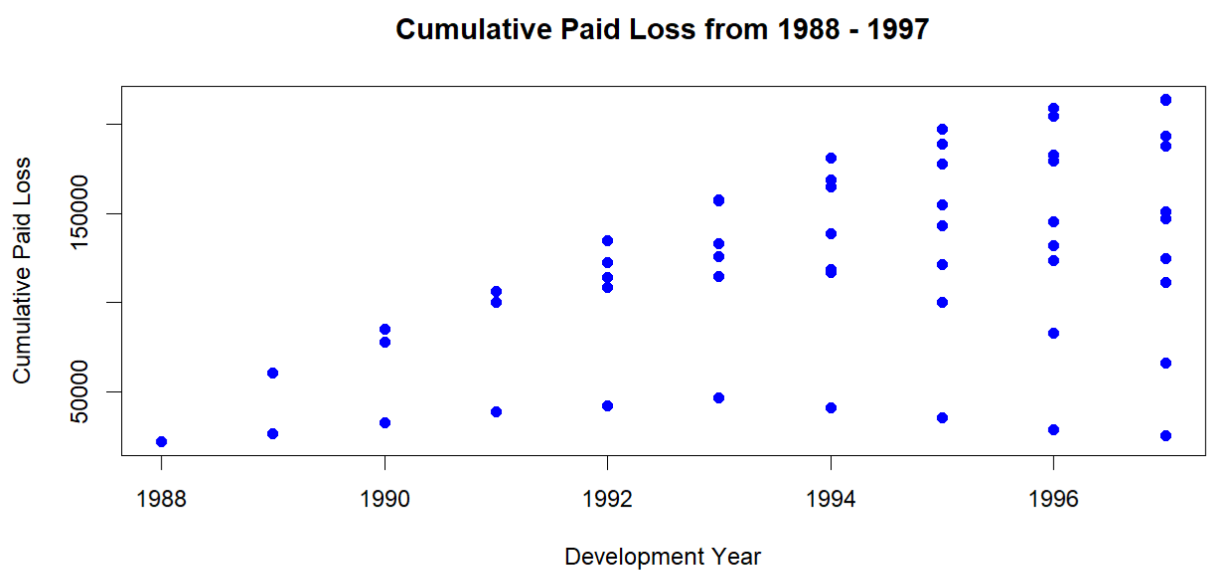

Fig. 2.1. Cumulative Paid Loss throughout Development Years 1988 – 1997. For each development year the cumulative paid loss increases.

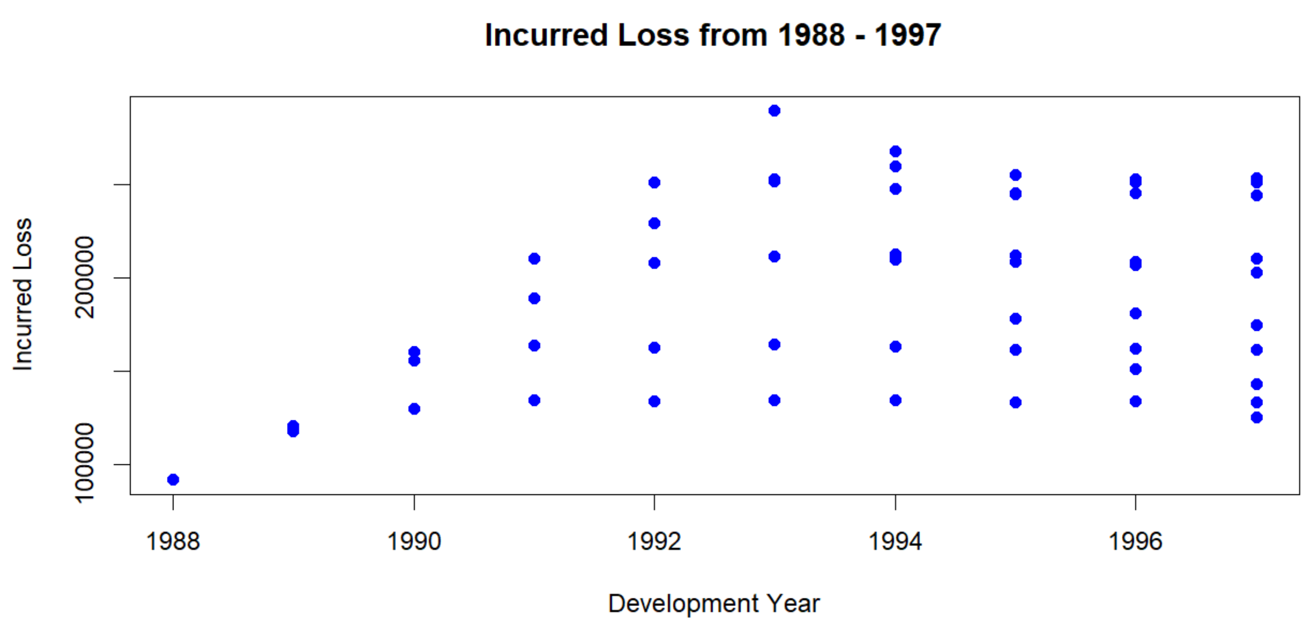

Fig. 2.2. Incurred Loss throughout Development Years 1988 – 1997. The incurred loss decreases after 1993.

Evidently, the difference between the two types of loss data is cumulative paid loss increases each development year while incurred loss does not since it decreases after 1993. This phenomenon of negative incremental values for incurred data is referred to in Stochastic Claims Reserving in General Insurance by P. D. England and R. J. Verrall. They state the cause of this is due to conservative over-estimation of case estimates. They also state that this can occur “due to timing of reinsurance or salvage recoveries, or premiums being included as negative loss amounts.” [1]

Section 3. Methodology

3.1 Chain-Ladder

Using the chain-ladder method and the data for State Farm Mutual Group, I will demonstrate how I calculated the reserves for each accident year from 1988 to 1997 first using the average factor model.

Table 3.1.1. *Cumulative Paid Loss for Accident Year 1988*

Accident Year	Development Year	Cumulative Paid Loss	1997 Posted Reserve
1988	1	22190	542695
1988	2	60834	542695
1988	3	85104	542695
1988	4	100151	542695
1988	5	108812	542695
1988	6	114967	542695
1988	7	118790	542695
1988	8	121558	542695
1988	9	123492	542695
1988	10	125049	542695

The columns displayed in table 3.1.1. are the only variables necessary for this calculation

The first step is constructing the loss triangle. This will be comprised of cumulative paid loss data with each row being a new accident year and each column being a new development year.

Table 3.1.2. *Cumulative Paid Loss Triangle*

AY	DY1	DY2	DY3	DY4	DY5	DY6	DY7	DY8	DY9	DY10
1988	22190	60834	85104	100151	108812	114967	118790	121558	123492	125049
1989	26542	77798	106407	122422	133359	138599	143029	145712	147358	NA
1990	32977	100494	134886	157758	168991	178065	182787	187760	NA	NA
1991	38604	114428	157103	181322	197411	208804	213396	NA	NA	NA
1992	42466	125820	164776	189045	204377	213904	NA	NA	NA	NA
1993	46447	116764	154897	179419	193676	NA	NA	NA	NA	NA
1994	41368	100344	132021	151081	NA	NA	NA	NA	NA	NA
1995	35719	83216	111268	NA	NA	NA	NA	NA	NA	NA
1996	28746	66033	NA	NA	NA	NA	NA	NA	NA	NA
1997	25265	NA	NA	NA	NA	NA	NA	NA	NA	NA

After constructing the loss triangle, the next step is to calculate the average factors. These average factors will be calculated using only the cumulative paid loss data. The average factor is calculated by dividing the values in one development year by the values in the previous development year for its corresponding accident year. Those quotients are then summed up and then divided again by the number of quotients summed up.

Table 3.1.3. *Average factors for each Development Year*

Development Year	Average Factors
1	2.690382
2	1.346355
3	1.157287
4	1.082720
5	1.050776
6	1.028432
7	1.023089
8	1.013603
9	1.012608
10	1.000000

After finding the average factors, we are now ready to fill in the loss triangle and estimate the ultimate losses for each accident year. We do this by multiplying the latest payment for each accident year by the corresponding average factor for the next development year. While the value in the 9th development year needs to only be multiplied by one average factor, the value in the 1st development year will be multiplied by all the average factors.

Table 3.1.4. *Full Triangle*

AY	DY1	DY2	DY3	DY4	DY5	DY6	DY7	DY8	DY9	DY10
1988	22190	60834.00	85104.00	100151.0	108812.0	114967.0	118790.0	121558.0	123492.0	125049.0
1989	26542	77798.00	106407.00	122422.0	133359.0	138599.0	143029.0	145712.0	147358.0	149215.9
1990	32977	100494.00	134886.00	157758.0	168991.0	178065.0	182787.0	187760.0	190314.1	192713.6
1991	38604	114428.00	157103.00	181322.0	197411.0	208804.0	213396.0	218323.1	221293.0	224083.0
1992	42466	125820.00	164776.00	189045.0	204377.0	213904.0	219985.6	225064.8	228126.4	231002.7
1993	46447	116764.00	154897.00	179419.0	193676.0	203510.1	209296.2	214128.6	217041.4	219777.9
1994	41368	100344.00	132021.00	151081.0	163578.4	171884.2	176771.2	180852.6	183312.8	185624.0
1995	35719	83216.00	111268.00	128769.0	139420.7	146499.9	150665.1	154143.8	156240.7	158210.6
1996	28746	66033.00	88903.87	102887.3	111398.1	117054.4	120382.5	123161.9	124837.3	126411.3
1997	25265	67972.51	91515.15	105909.3	114670.0	120492.5	123918.3	126779.4	128504.0	130124.2

This is the full loss triangle with the lower half being forecasted using the average factors.

Now that we have our full triangle we can move on to the next step, which is finding the reserve for each accident year. Since we are using the chain-ladder method, we will use the chain-ladder formula which is the estimated ultimate loss subtracted by the latest cumulative loss paid.

Formula 3.1.1. *Chain-Ladder Formula*

Loss Paid * Age-to-Ultimate Loss Factor – Loss Paid

LP * Fult - LP

Table 3.1.5. *Predicted Reserves for each Accident Year*

Ultimate Losses	Latest Cumulative Claims	Reserve	Reserve Sum
125049.0	125049	0.000	0.000
149215.9	147358	1857.905	1857.905
192713.6	187760	4953.633	6811.538
224083.0	213396	10687.043	17498.581
231002.7	213904	17098.671	34597.252
219777.9	193676	26101.913	60699.165
185624.0	151081	34542.995	95242.160
158210.6	111268	46942.578	142184.738
126411.3	66033	60378.306	202563.043
130124.2	25265	104859.240	307422.284

These reserves were calculated using the chain-ladder formula LP * Fult – LP.

Once we have the reserves all that's left to do is to sum them up and the result is the prediction for PostedReserve97.

Table 3.1.6. *Average Factor Model Prediction*

Actual	Prediction	Difference	Percentage
542695	307422.3	235272.7	56.64734

This average factor model used cumulative paid loss data for its prediction.

As is apparent, the prediction is not a very good one, with the difference between the actual reserve and the prediction being fairly large. To put it into perspective, the prediction is only about fifty-six percent of the posted reserve. We will see if our other two development factor models can perform better.

The process remains exactly the same for the other two development factors with only one difference, which is how the development factors are calculated. The five-year average factors are calculated slightly different from the average factors. They are calculated precisely the same way for the development years five to ten. However, once you get past the fifth latest development year the calculation changes. You divide the values in one development year by the values in the previous development year for the latest five corresponding accident years. These 5 quotients are then summed up and divided by 5.

Table 3.1.7. *Five-Year Average Factors*

Development Year	Average Factors	5-Year Average Factors
1	2.690382	2.505853
2	1.346355	1.332385
3	1.157287	1.154739
4	1.082720	1.081968
5	1.050776	1.050776
6	1.028432	1.028432
7	1.023089	1.023089
8	1.013603	1.013603
9	1.012608	1.012608

Ostensibly, the five-year average factors for development years 5 – 9 are identical to the average factors. However, they are different for development years 1 – 4. Utilizing the chain-ladder formula in concert with the latest cumulative paid loss and these five-year average factors results in the following forecast.

Table 3.1.8. *Five-Year Average Factor Model Predicted Reserves*

Ultimate Losses	Latest Cumulative Claims	Reserve	Reserve Sum
125049.0	125049	0.000	0.000
149215.9	147358	1857.905	1857.905
192713.6	187760	4953.633	6811.538
224083.0	213396	10687.043	17498.581
231002.7	213904	17098.671	34597.252
219777.9	193676	26101.913	60699.165
185495.1	151081	34414.079	95113.244
157752.6	111268	46484.589	141597.833
124737.5	66033	58704.452	200302.285
119594.4	25265	94329.351	294631.635

Table 3.1.9. *Predicted Reserves for AFM and Five-Year AFM*

	Actual	Prediction	Difference	Percentage
Average Factor Model	542695	307422.3	235272.7	56.64734
5-Year Average Factor Model	542695	294631.6	248063.4	54.29046

As shown from table 3.1.9 the forecast using this five-year average factor model performs even worse than the average factor model as the prediction is further off than the actual posted reserve by about two percent.

The next development factor model is the volume-weighted average factor model and is the most unique out of the three models. Although the average factor model and the five-year average factor model are calculated similarly and differ only after the fifth latest development year, the volume-weighted average factor model is calculated differently for all the development years except the last one. For the other two models we divided the values in one development year by the values in the previous development year for its corresponding accident year and then summed up those quotients and divided that sum by the number of quotients. For this model we first find the sum of the values for the development year and then divide that sum by the sum of the values for the corresponding accident years. This results in the following factors.

Table 3.1.10. *Volume-Weighted Average Factors*

Development Year	Average Factors	5-Year Average Factors	Volume-Weighted Average Factors
1	2.690382	2.505853	2.684358
2	1.346355	1.332385	1.342138
3	1.157287	1.154739	1.156122
4	1.082720	1.081968	1.082257
5	1.050776	1.050776	1.050912
6	1.028432	1.028432	1.027430
7	1.023089	1.023089	1.023445
8	1.013603	1.013603	1.013395
9	1.012608	1.012608	1.012608

As illustrated, the volume-weighted average factors are different for each development year except for the last one due to the difference in calculations. Combining these factors with the chain-ladder formula and our cumulative paid loss triangle gives us the following results.

Table 3.1.11. *Predicted Reserve for each Development Factor Model*

	Actual	Prediction	Difference	Percentage
Average Factor Model	542695	307422.3	235272.7	56.64734
5-Year Average Factor Model	542695	294631.6	248063.4	54.29046
Volume-Weighted Average Factor Model	542695	304881.9	237813.1	56.17924

Purportedly, the prediction using the volume-weighted average factor model is more accurate than that of the five-year average factor model but is less accurate than that of the average factor model by a slight margin, about half of a percent. Thus, the average factor model is the "best model" using the chain-ladder methodology and cumulative loss paid date compared to the other two models.

We will repeat this process but with incurred loss data instead of cumulative paid loss data.

Table 3.1.12. *Loss Triangle Using Incurred Data*

AY	DY1	DY2	DY3	DY4	DY5	DY6	DY7	DY8	DY9	DY10
1988	91892	120466	129785	134401	134051	134264	134169	133158	133800	133513
1989	117540	160490	163802	162824	164650	163326	161246	161836	161673	NA
1990	155671	210607	208274	211660	209823	208629	208662	210204	NA	NA
1991	189021	251182	253292	247771	244882	245666	244669	NA	NA	NA
1992	229435	289785	268235	255528	252878	253878	NA	NA	NA	NA
1993	251804	259975	245594	251196	251129	NA	NA	NA	NA	NA
1994	212555	212160	206811	202911	NA	NA	NA	NA	NA	NA
1995	178437	181175	174496	NA	NA	NA	NA	NA	NA	NA
1996	151415	143042	NA	NA	NA	NA	NA	NA	NA	NA
1997	125429	NA	NA	NA	NA	NA	NA	NA	NA	NA

Repeating the same calculations for each development factor model results in the following development factors.

Table 3.1.13. *Link Ratios using Incurred Loss Data*

Development Year	Average Factors	5-Year Average Factors	Volume-Weighted Average Factors
1	1.1790878	1.0507350	1.1591563
2	0.9879448	0.9633282	0.9789120
3	0.9972337	0.9902080	0.9935614
4	0.9962723	0.9960476	0.9952770
5	0.9990026	0.9990026	0.9994823
6	0.9956642	0.9956642	0.9958252
7	1.0011712	1.0011712	1.0022239
8	1.0019071	1.0019071	1.0016238
9	0.9978550	0.9978550	0.9978550

As can be seen, most of the average factors for incurred loss are less than one, as opposed to those for cumulative paid loss where every single factor was above one. On the authority of Robert L. Brown and W. Scott Lennox in their text Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance, they maintain that "it is possible for incurred loss development factors to be less than one if the claim file estimates are conservative or because of salvage or subrogation" whereas "incurred loss development factors greater than one indicate that claim file estimates were inadequate, due to gross IBNR." [3]

These in turn result in the following predictions.

Table 3.1.14. *Incurred Data Predictions*

	Actual	Prediction	Difference	Percentage
Average Factor Model	542695	10149.765	532545.2	1.8702522
5-Year Average Factor Model	542695	-15478.312	558173.3	-2.8521199
Volume-Weighted Average Factor Model	542695	4449.545	538245.5	0.8198979

Seemingly, the forecast using the incurred loss triangle is extremely inaccurate. However, the reason as to why this is the case is because unlike with the cumulative paid loss data, there is an extra step we must take. We must add these estimated incurred loss reserves to the case reserves.

To get a better understanding as to why we must add these estimates to the case reserves, we consult the literature. Section 2 of the Faculty and Institute of Actuaries Claims Reserving Manual v.1 states that "incurred claims (paid plus case reserves) may be used as an alternative to paid claims only, and can give a further perspective on the estimating of ultimate liability. However, whereas methods based on paid data require only a stable settlement pattern, those based on incurred data also require a stable reporting pattern and consistency in the setting of case reserves." [4] Brown and Lennox define case reserves as "the aggregate of the individual claim file estimates (split by line of business and accident year)." [3]

With the intention of understanding why we add case reserves to the incurred loss reserves we must first understand what the difference is between the cumulative losses paid triangle and the incurred losses triangle. The values in the cumulative losses paid triangle is purely objective and solely represents the losses that have been paid by the insurer. Therefore, the predicted value $307,422.3 represents the amount that the insurer is predicting will need to be paid out taking into account previous payments. The predicted value $10,149.765

from the incurred loss triangle represents something completely different. The incurred loss triangle is subjective. It's the same as the cumulative losses paid triangle, but the insurers are adding additional values to this data. This is so that when the lower half of the triangle is filled out, the amount left that the insurer would need to pay is 0. Therefore, the $10,149.765 represents a minor adjustment to the estimates that has already been made that is added to this data based on the average factor model. In other words, the estimate for the reserves is already in the values in the triangle. In order to extract that estimate, we must subtract the values in the triangle by the cumulative losses paid and we get the pure estimate the insurers made that needs to be paid. We then add that estimate to the minor adjustment and we get the full predicted estimate.

Table 3.1.15. *Case Reserves*

	Cumulative Paid Loss + Case Reserves	Cumulative Paid Loss	Case Reserves	Case Reserves Sum
1988	133513	125049	8464	8464
1989	161673	147358	14315	22779
1990	210204	187760	22444	45223
1991	244669	213396	31273	76496
1992	253878	213904	39974	116470
1993	251129	193676	57453	173923
1994	202911	151081	51830	225753
1995	174496	111268	63228	288981
1996	143042	66033	77009	365990
1997	125429	25265	100164	466154

The case reserves are the estimates made by actuaries with the assumption that if the lower half of the triangle was full, or at the end of the 10 years, the amount left that would need to be paid is 0. Therefore, the sum of the case reserves is more along the lines of what we are looking for compared to the sum of the incurred loss reserves. The $10,149.765 in incurred loss reserves represents the amount left that needs to be paid after taking into account the case reserves. So adding these two estimates together, the full estimate is $476,303.7653.

Table 3.1.16. *Final Incurred Loss Prediction*

	Incurred Loss Reserve	Case Reserve	Total Reserve
Average Factor Model	10149.765	466154	476303.8
5-Year Average Factor Model	-15478.312	466154	450675.7
Volume-Weighted Average Factor Model	4449.545	466154	470603.5

Therefore, the estimates for the incurred loss reserve can be interpreted as the forecasted amount left that needed to be paid if the lower half of the triangle was filled using that model. The estimates for the case reserve can be interpreted as the subjective amount the actuaries estimated needed to be paid.

Table 3.1.17. *Cumulative and Incurred Loss Predictions*

	Actual	Prediction	Difference	Percentage
Cumulative Average Factor Model	542695	307422.3	235272.72	56.64734
Cumulative 5-Year Average Factor Model	542695	294631.6	248063.36	54.29046
Cumulative Volume-Weighted Average Factor Model	542695	304881.9	237813.09	56.17924
Incurred Average Factor Model	542695	476303.8	66391.23	87.76638
Incurred 5-Year Average Factor Model	542695	450675.7	92019.31	83.04401
Incurred Volume-Weighted Average Factor Model	542695	470603.5	72091.46	86.71603

As evidenced, the combination of loss data and development factor model that produces a reserve closest to the reported one is the incurred loss data together with the average factor model.

3.2 Bornhuetter-Ferguson

The Bornhuetter-Ferguson method improves on the shortcomings of the chain-ladder method by combining it with the expected loss ratio with the aim of stabilizing immature data [3].

Formula 3.2.1. *The Bornhuetter–Ferguson Formula*

Earned Premium * Estimated Loss Ratio * (1 – 1 / Age-to-Ultimate Loss Factor)

EP * ELR * (1 – 1 / Fult)

Formula 3.2.2. *The Estimated Loss Ratio Formula*

Cumulative Paid Loss of the 1st Accident Year / Net Earned Premium

EP is the net earned premium corresponding to each accident year. ELR is the estimated loss ratio which is estimated by dividing the full cumulative paid loss at the end of the 10th development year for the 1st accident year divided by the earned premium net for that accident year. That specific ELR is then used for all the accident years. Fortunately, the age-to-age loss development factors remain the same for both methods since it only concerns the loss triangle and the formula for each development factor model. Therefore, the only difference in calculation between the methods will be calculating the reserves. The age-to-ultimate loss development factors for each accident year is simply the cumulative product of the link ratios for each previous accident year.

Table 3.2.1. *Calculation of the Age-to-Ultimate Loss Development Factors*

	Average Factors	Fult (Age-to-Ultimate Factors)
1988	1.000000	1.000000
1989	1.012608	1.012608
1990	1.013603	1.026383
1991	1.023089	1.050081
1992	1.028432	1.079936
1993	1.050776	1.134771
1994	1.082720	1.228639
1995	1.157287	1.421887
1996	1.346355	1.914366
1997	2.690382	5.150376

Table 3.2.2. *Cumulative Paid Loss Calculated Reserves for the Bornhuetter-Ferguson Method*

	EP	ELR	1-1/Fult	Reserve	Sum of the Reserves
1988	177104	0.7060767	0.00000000	0.000	0.000
1989	201118	0.7060767	0.01245112	1768.118	1768.118
1990	246010	0.7060767	0.02570463	4464.944	6233.062
1991	286019	0.7060767	0.04769233	9631.531	15864.592
1992	340183	0.7060767	0.07401936	17779.102	33643.694
1993	418755	0.7060767	0.11876495	35115.605	68759.299
1994	366031	0.7060767	0.18609121	48094.520	116853.818
1995	338186	0.7060767	0.29670948	70849.843	187703.662
1996	286631	0.7060767	0.47763375	96665.170	284368.831
1997	245378	0.7060767	0.80583940	139616.252	423985.083

This shows the process for the average factor model for the Bornhuetter-Ferguson method. The reserve is calculated by taking the product of the first three columns. The final prediction is the sum of all the reserves.

After repeating this process for the other two development factor models with their respective link ratios we receive the following results.

Table 3.2.3. *Bornhuetter-Ferguson Method with Cumulative Paid Loss Data Predictions*

	Actual	Prediction	Difference	Percentage
Average Factor Model	542695	423985.1	118709.9	78.12585
5-Year Average Factor Model	542695	418970.9	123724.1	77.20190
Volume-Weighted Average Factor Model	542695	422200.0	120495.0	77.79692

According to these results, the development factor model that produces a reserve closest to the reported one is again the average factor model.

A question some may ask is whether or not the Bornhuetter-Ferguson method has the ability to use incurred loss data. The answer to that question is yes it can, and the reason why is because the Bornhuetter-Ferguson technique "is essentially a blend of the development and expected claims techniques" and "combines the two techniques by splitting ultimate claims into two components: actual reported (or paid) claims and expected unreported (or unpaid) claims. As experience matures, more weight is given to the actual claims and the expected claims become gradually less important." [5] Since incurred loss data consists of paid claims plus expected claims, in theory this type of data should work well with this method.

Table 3.2.4. *Bornhuetter-Ferguson Method with Incurred Loss Data Predictions*

	Actual	Prediction	Difference	Percentage
Average Factor Model	542695	478052.4	64642.58	88.08860
5-Year Average Factor Model	542695	443223.6	99471.39	81.67085
Volume-Weighted Average Factor Model	542695	470536.5	72158.54	86.70367

Table 3.2.5. *All Combinations of Reserving Methods, LDF Models, and Types of Loss Data*

	Actual	Prediction	Difference	Percentage
BF Incurred AFM	542695	478052.4	64642.58	88.08860
CL Incurred AFM	542695	476303.8	66391.23	87.76638
CL Incurred Volume-Weighted AFM	542695	470603.5	72091.46	86.71603
BF Incurred Volume-Weighted AFM	542695	470536.5	72158.54	86.70367
CL Incurred 5-Year AFM	542695	450675.7	92019.31	83.04401
BF Incurred 5-Year AFM	542695	443223.6	99471.39	81.67085
BF Cumulative AFM	542695	423985.1	118709.92	78.12585
BF Cumulative Volume-Weighted AFM	542695	422200.0	120495.01	77.79692
BF Cumulative 5-Year AFM	542695	418970.9	123724.14	77.20190
CL Cumulative AFM	542695	307422.3	235272.72	56.64734
CL Cumulative Volume-Weighted AFM	542695	304881.9	237813.09	56.17924
CL Cumulative 5-Year AFM	542695	294631.6	248063.36	54.29046

As displayed, out of the twelve combinations of loss data and development factor models, the combination that produces a reserve closest to the reported one is the Bornhuetter-Ferguson method with incurred loss data. What's interesting to note is that every method combined with incurred loss data beat every method combined with cumulative loss data for State Farm. This makes sense since incurred loss data includes the actuaries' best subjective estimate for the reserves so we would hope this would be more accurate than the purely objective data of the cumulative losses paid triangle.

3.3 Mean Absolute Error

Now that we have demonstrated how to calculate the predicted sum of all reserves for every combination of the Bornhuetter-Ferguson method and the chain-ladder method with cumulative paid loss and incurred loss data, we will now set out to answer the question as to which method produces the most reliable reserves for the workers' compensation business line. We will do this by using all the companies in our dataset with complete data for the ten accident years we need to construct our loss triangles, which comes out to fifty-seven companies.

Using general actuarial theory and statistical theory, the measure that will be used for the purpose of evaluating the goodness of a reserving method will be the Mean Absolute Error. According to Cort J. Willmott and Kenji Matsuura, "MAE is the most natural measure of average error magnitude, and that … it is an unambiguous measure of average error magnitude. It seems to us that all dimensioned evaluations and inter-comparisons of average model-performance error should be based on MAE." [6]

Formula 3.3.1. *Mean Absolute Error Formula [6]*

$$\mathrm{MAE} = \left[n^{-1} \sum_{i=1}^{n} |e_i| \right]$$

When calculating our value for the MAE, we will be taking the absolute value of the difference between the actual value, which is the posted reserve for the year 1997, and the predicted value, which is the sum of all reserves predicted by one of our development factor models.

According to Brown and Lennox, “Generally it is a good idea to do both a paid-loss-development and an incurred-loss-development analysis. … In practice, the reserve values derived from the expected ultimate values of these two bases will differ, and the reconciliation of the difference will assist in defining the loss reserve estimate.” [3]

With respect to this consideration, when calculating MAE, we will reconcile the difference between the estimates for incurred loss data and cumulative paid loss data by taking the average of the two predicted values for each of the three development factor models resulting in a reconciled predicted value. After subtracting these three predictions from the actual posted reserve, we will then sum the three differences up and then divide them by the number of observations, which is three. We will then compare the Bornhuetter-Ferguson MAE to the chain-ladder MAE for each individual company and choose the method that has the lower MAE.

To demonstrate this procedure, we will continue to use the data for State Farm.

Table 3.3.1. *Reconciliated Reserves*

	Cumulative Reserve	Incurred Reserve	Reconciliated Reserve
Chain-Ladder AFM	307422.3	476303.8	391863.0
Chain-Ladder 5-Year AFM	294631.6	450675.7	372653.7
Chain-Ladder Volume-Weighted AFM	304881.9	470603.5	387742.7
Bornhuetter-Ferguson AFM	423985.1	478052.4	451018.8
Bornhuetter-Ferguson 5-Year AFM	418970.9	443223.6	431097.2
Bornhuetter-Ferguson Volume-Weighted AFM	422200.0	470536.5	446368.2

The average of the two estimated reserves is equal to the reconciliated reserve.

Once we find the predicted sum of all reserves using cumulative paid loss data and incurred loss data for each development factor model, we then reconcile the two estimates by taking their average. After we have the reconciliated reserves for each combination of development factor model and reserving method, we can subtract each one by the actual reserve and take the absolute value of that difference for the sake of finding the absolute error.

Table 3.3.2. *Absolute Errors*

	Actual Reserve	Predicted Reserve	Absolute Error
Chain-Ladder AFM	542695	391863.0	150831.98
Chain-Ladder 5-Year AFM	542695	372653.7	170041.34
Chain-Ladder Volume-Weighted AFM	542695	387742.7	154952.27
Bornhuetter-Ferguson AFM	542695	451018.8	91676.25
Bornhuetter-Ferguson 5-Year AFM	542695	431097.2	111597.76
Bornhuetter-Ferguson Volume-Weighted AFM	542695	446368.2	96326.78

Formula 3.3.2. *Absolute Error Formula*

Absolute Error = |Actual Reserve – Predicted Reserve|

Once we have the absolute error, we can then find the Mean Absolute Error of each reserving method by taking the average of the three absolute errors corresponding to the three development factor models.

Table 3.3.3. *MAE of each Reserving Method*

	AFM	5-Year AFM	Volume-Weighted AFM	MAE
Chain-Ladder	150831.98	170041.3	154952.27	158608.53
Bornhuetter-Ferguson	91676.25	111597.8	96326.78	99866.93

As revealed by this table, the MAE for chain-ladder is much higher than it is for Bornhuetter-Ferguson. Therefore, Bornhuetter-Ferguson is the most reliable reserving method for State Farm in particular. We will now apply this process to fifty-seven companies in our dataset and see which reserving method is the most reliable for the workers' compensation business line.

Section 4. Results and Discussion

Table 4.1. *Number of Companies with the Lower MAE*

	Chain-Ladder	Bornhuetter-Ferguson
Companies	21	36

As presented, the Bornhuetter-Ferguson method was more reliable for 36 companies compared to 21 companies for the chain-ladder method. This shows that although Bornhuetter-Ferguson method is most likely to be the more reliable method, we should not disregard the chain-ladder method because there's a good chance that it might be more reliable than the Bornhuetter-Ferguson method.

We achieved the results in table 4.1 by first finding the MAE of each reserving method for each individual company and then comparing the two estimates to see which one had the lowest Mean Absolute Error. Now, we will find the MAE of each reserving method for all fifty-seven companies and compare the two values to each other to see which method has the lowest Mean Absolute Error.

Table 4.2. *Total MAE for each Reserving Method*

	Chain-Ladder	Bornhuetter-Ferguson
Mean Absolute Error	30190.75	27315.93

Our findings in table 4.2 agrees with the results found in table 4.1 that Bornhuetter-Ferguson is the most reliable reserving method for this workers' compensation line of business since the MAE for the Bornhuetter-Ferguson method is lower. However, this shows that the results were closer than table 4.1 portrayed since the MAE for chain-ladder is only 3,000 larger than Bornhuetter-Ferguson.

Section 5. Summary and Concluding Remarks

In conclusion, the objective of this analysis was to determine the methodology that is the most appropriate for forecasting this dataset and we have found that methodology to be the Bornhuetter-Ferguson method. Out of the 57 companies tested, 36 had a lower MAE for the Bornhuetter-Ferguson method compared to that of the chain-ladder method. Also, the Bornhuetter-Ferguson method had a lower MAE (27,315.93) across all 57 companies compared to the MAE for the chain-ladder method (30,190.75). One possible reason why the Bornhuetter-Ferguson method performed better is because the Bornhuetter-Ferguson method is more stable than the chain-ladder method [3]. Another explanation could be its use of both the development and expected claims technique, where the expected claims technique is used more in the early years and the development method is used more in the latter years [5].

Even though the Bornhuetter-Ferguson method was proven to be the more reliable reserving method for the workers' compensation business line, the chain-ladder method should still be used in conjunction with the Bornhuetter-Ferguson method since we saw that there was still a significant amount of companies that the chain-ladder method had a lower MAE for and the overall MAE for the chain-ladder method was not too far off from the MAE for the Bornhuetter-Ferguson method. It is possible that improvements could be made to the chain-ladder method by increasing its stability through deleting the highest and lowest loss development factors in each column or by using a harmonic or geometric mean to deal with outliers [3]. The chain-ladder method works best "for high-frequency, low-severity lines with stable and relatively time reporting of claims, especially where the claims are evenly spread through the accident year" [5]. This is because "when there is not an even spread of claims throughout the year, the development technique can distort the projected ultimate claims for an accident year" [5]. This is where the Bornhuetter-Ferguson method improves on the chain-ladder method since it "combines the expected loss ratio and chain-ladder methods" as a means to stabilize "long-tail lines or immature data" and in doing so "assumes that past experience is not fully representative of the future" [3]. While the chain-ladder method assumes that there is no prior information about the row parameters, the Bornhuetter-Ferguson method assumes that there is perfect prior information about the row parameters. Thus, it is preferable to use these two techniques in tandem with each other [1].

We also observed and got a better understanding of cumulative paid loss data and incurred loss data. Cumulative paid loss data is purely objective data and is simply the amount that the insurer has paid. Incurred loss data adds an extra amount to the cumulative paid loss data necessary in an effort to fully pay out all the claims. This is called the case reserves which is a subjective estimate set by actuaries. Common criticism against incurred loss data is that it "may include over-estimation of case reserves leading to negative incremental values" which may cause problems [1]. Because of these potential issues, it is recommended that both types of data be used so that their differences be reconciliated [3]. Additionally, one advantage that incurred loss data has over cumulative paid data is that because of its subjectivity, incurred loss data will adjust to reflect current events while cumulative paid data will have to wait until it has been affected [3].

In closing, although the Bornhuetter-Ferguson method was proven to be the most reliable reserving method for the workers' compensation business line, the chain-ladder method should not be counted out as there is a good chance that it may provide a better estimate. Depending on the type of data, one method may be preferable to another, thus there is no truly best method. Because of this, all combinations of loss data, loss development factor models, and reserving methods should be tested to achieve the best estimate.

Appendix

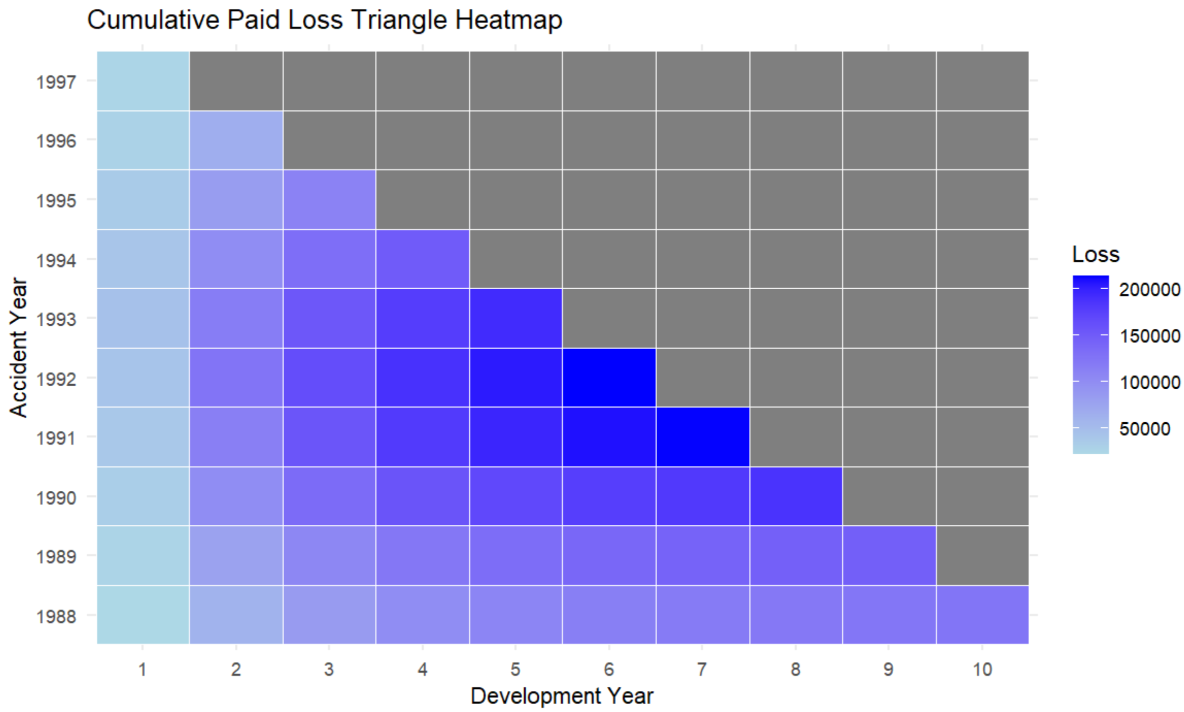

Fig. A.1. Heatmap of the Loss Triangle Constructed with Cumulative Paid Loss Data. Each row gets darker as the development year increases signifying the cumulated paid loss.

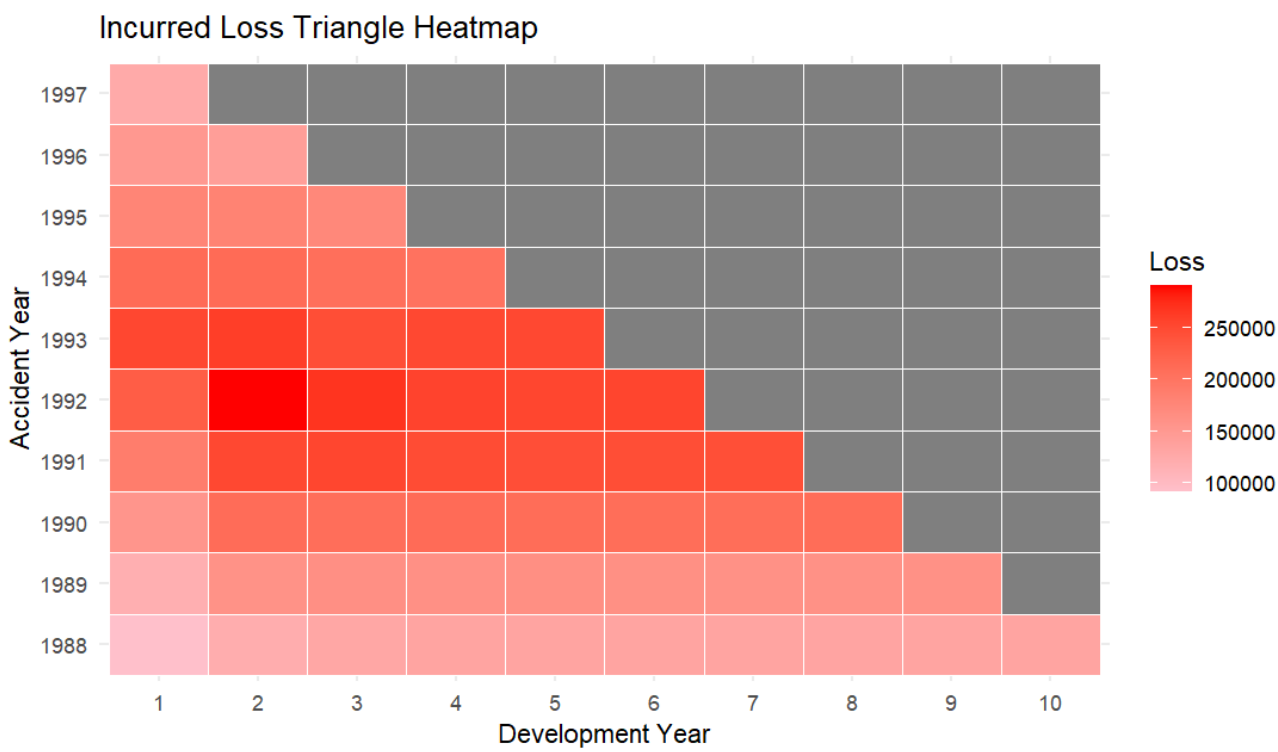

Fig. A.2. Heatmap of the Loss Triangle Constructed with Incurred Data. Here you can clearly see how the incurred loss data differs from the cumulative paid loss data since the incurred loss decreases as the development year increases for some accident years.

Table A.1. *Comparing Reserving Methods using MSE*

	MSE	Companies
Chain-Ladder Cumulative Paid Loss	16584784011	19
Bornhuetter-Ferguson Cumulative Paid Loss	11816992828	38

Here the chain-ladder and Bornhuetter-Ferguson methods are compared with each other using cumulative paid loss data only. As you can see, the Bornhuetter-Ferguson cumulative paid loss data is better than the chain-ladder method when only using cumulative paid loss data. Twice as many companies have a lower MSE for Bornhuetter-Ferguson than chain-ladder and Bornhuetter-Ferguson also has an overall lower MSE. The Bornhuetter-Ferguson method performing more accurately than the chain-ladder method makes sense theoretically because “it assumes that past experience is not fully representative of the future” [3].

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