

Atomic Physics

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Rydberg

$$\tilde{\nu} = \frac{1}{\lambda} = R \left(\frac{1}{n^2} - \frac{1}{m^2} \right) Z^2$$

$$E_n = -hcR \frac{Z^2}{n^2}$$

$$hcR_\infty = \frac{m_e (e^2/4\pi\epsilon_0)^2}{2\hbar^2} = 13.606 \text{ eV}$$

$$R = R_\infty \cdot \frac{M_N}{m_e + M_N}$$

Alkaline-like System

$$n^* = n - \delta_l$$

$$E = -hcR_\infty \frac{Z_0^2}{n^{*2}}$$

$$\Delta E_{FS} = -\frac{Z_i^2 Z_0^2}{n^{*3} l(l+1)} \alpha^2 hcR_\infty$$

Hydrogen-like Atoms

$$V(r) = -\frac{Ze^2}{4\pi\epsilon_0 r} = -\frac{Ze_0^2}{r}$$

$$a_0 = \frac{\hbar^2 4\pi\epsilon_0}{m_e e^2}$$

In Bohrs model of the atom: $r_n = a_0 n^2 / Z$

Radial Functions for Hydrogen-like Systems

$$R_{1,0} = \left(\frac{Z}{a_0} \right)^{3/2} 2e^{-Zr/a_0}$$

$$R_{2,0} = \left(\frac{Z}{2a_0} \right)^{3/2} 2 \left(1 - \frac{Zr}{2a_0} \right) e^{-Zr/2a_0}$$

$$R_{2,0} = \left(\frac{Z}{2a_0} \right)^{3/2} \frac{2}{\sqrt{3}} \frac{Zr}{2a_0} e^{-Zr/2a_0}$$

Spherical Surface Functions

l	m	$\Upsilon_l^m(\theta, \varphi)$
0	0	$\Upsilon_0^0 = \frac{1}{\sqrt{4\pi}}$
1	0	$\Upsilon_1^0 = \sqrt{\frac{3}{4\pi}} \cos \theta$
1	± 1	$\Upsilon_1^{\pm 1} = \pm \sqrt{\frac{3}{8\pi}} \sin \theta e^{\pm i\varphi}$
2	0	$\Upsilon_2^0 = \sqrt{\frac{5}{16\pi}} (3 \cos^2 \theta - 1)$
2	± 1	$\Upsilon_2^{\pm 1} = \pm \sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{\pm i\varphi}$
2	± 2	$\Upsilon_2^{\pm 2} = \sqrt{\frac{15}{32\pi}} \sin^2 \theta e^{\pm 2i\varphi}$

Hamilton Operator for Multi-electron Systems

$$H = \sum_{i=1}^N \left(-\frac{\hbar^2}{2m} \nabla_i^2 - \frac{Ze^2/4\pi\epsilon_0}{r_i} + \sum_{j>i}^N \frac{e^2/4\pi\epsilon_0}{r_{ij}} \right)$$

$$\langle LM_L | l_1 | LM_L \rangle = \frac{\langle l_1 \cdot \mathbf{L} \rangle}{L(L+1)} \langle LM_L | \mathbf{L} | LM_L \rangle$$

LS coupling

$$\text{Terms: } \begin{cases} L = |l_1 - l_2|, \dots, l_1 + l_2 \\ S = |s_1 - s_2|, \dots, s_1 + s_2 \end{cases}$$

$$\text{Levels: } J = |L - S|, \dots, L + S$$

Zeeman Effect

$$E_{ZE} = \begin{cases} g_J \mu_B B M_J & (\text{fine structure}) \\ g_F \mu_B B M_F & (\text{weak field, hfs}) \\ g_J \mu_B B M_J + A M_I M_J & (\text{strong field, } \mu_B B > A) \end{cases}$$

Connection between magnetic moment and Integrals momentum

$$g_S = 2$$

$$g_J = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$$

$$g_F = g_J \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)}$$

$$\mu_I = g_I \mu_N \mathbf{I}$$

Doppler Width

$$\frac{\Delta\omega_D}{\omega_0} = 2\sqrt{\ln 2} \frac{u}{c} \approx 1.7 \frac{u}{c}$$

Most Probable Speed

$$u = 2230 \sqrt{\frac{T}{300M}} \quad \text{m/s}$$

Dopplershift

$$\delta = kv = \frac{\omega v}{c}$$

Natural Width

$$\Delta\omega_N = \Gamma = A_{21} = 1 \frac{1}{\tau}$$

$$\Delta f_N = \frac{\Delta\omega_N}{2\pi}$$

Hyper Fine Structure

$$H = -\mu_I \cdot \mathbf{B}_e = A \mathbf{I} \cdot \mathbf{J}$$

For S-electrons in Hydrogen-like Systems

$$A = \frac{2}{3} \mu_0 g_S \mu_B g_I \mu_N \frac{Z^3}{\pi a_0^3 n^3}$$

Boltzman Distribution

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-\Delta E/(kT)}$$

$$\int_0^\infty x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}}$$

$$\int_0^\infty x^{2n+1} e^{-\alpha x^2} dx = \frac{n!}{2\alpha^{n+1}}$$

$$\int_0^\infty e^{-\alpha x^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{\alpha}}$$

$$\int_0^\infty x^{2n} e^{-\alpha x^2} dx = \frac{(2n-1)!!}{2(2\alpha)^n} \sqrt{\frac{\pi}{\alpha}}$$

Operators

$$\mathbf{p} = -i\hbar \nabla$$

$$\mathbf{L} = -i\hbar \mathbf{r} \times \nabla$$

$$\mathbf{H} = -\frac{\hbar^2}{2m} \nabla^2 + V \quad (\text{standard})$$

$$\nabla^2 = \frac{1}{r} \frac{\partial^2}{\partial r^2} r + \frac{1}{r^2} \left(\frac{\partial^2}{\partial \theta^2} + \frac{1}{\tan \theta} \frac{\partial}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right)$$

Dirac Notation

$$\langle \mathbf{H} \rangle = \langle \psi | \mathbf{H} | \psi \rangle = \int_{\mathbb{R}} \psi^* \mathbf{H} \psi dv$$

Commutators

$$[A, B] = AB - BA$$

$$[A, B] = -[B, A]$$

$$[A, B + C] = [A, B] + [A, C]$$

$$[AB, C] = A[B, C] + [A, C]B$$

Schrödinger Equation

$$\mathbf{H}\psi = E\psi \quad (\text{time independent})$$

$$\mathbf{H}\psi = i\hbar \frac{\partial}{\partial t} \psi \quad (\text{time dependent})$$

Configuration	$\prod n_i l_i^{\omega_i}$
Terms	L and $S(^{2S+1}L)$
Levels	J
States (ZE-sublevels)	M_J
Hyperfinitivåer	F

1	$\Delta J = 0, \pm 1$	$(J = 0 \leftrightarrow J' = 0)$	level
2	$\Delta M_J = 0, \pm 1$	$(M_J = 0 \leftrightarrow M_{J'} = 0 \text{ if } \Delta J = 0)$	state
3	break parity		configuration
4	$\Delta l = \pm 1$		
5	$\Delta L = 0, \pm 1$	$(L = 0 \leftrightarrow L' = 0)$	term
6	$\Delta S = 0$		term

1, 2 are replaced for similar formulas for F and M_F if F is a good quantum number. 5, 6 only hold if L and S are good quantum numbers.

	Fine Structure - LS	Hyper Fine Structure - IJ
interaction	$\beta \mathbf{L} \cdot \mathbf{S}$	$A \mathbf{I} \cdot \mathbf{J}$
moment	$\mathbf{J} = \mathbf{L} + \mathbf{S}$	$\mathbf{F} = \mathbf{I} + \mathbf{J}$
eigen-states	$ LSJM_J\rangle$	$ IJFM_F\rangle$
energy	$\beta/2(J(J+1) - L(L+1) - S(S+1))$	$A/2(F(F+1) - I(I+1) - J(J+1))$
interval	$E_J - E_{J-1} = \beta J$ (if $E_{S-O} \ll E_{re}$)	$E_F - E_{F-1} = AF$ (if $A \gg \Delta E_{quadrupole}$)