Disturbance Calculations

Time independent disturbance:

$$\left.\begin{array}{l}\left(H^0+H'\right)\boldsymbol{\psi}'_m = E'_m\boldsymbol{\psi}'_m \\ H^0\boldsymbol{\psi}_n = E^0_n\boldsymbol{\psi}_n\end{array}\right\} \Longrightarrow$$

$$E'_m = E^0_m + \langle m|H'|m\rangle + \sum_{n\neq m}\frac{|\langle m|H'|n\rangle|^2}{E^0_m - E^0_n}$$

$$\boldsymbol{\psi}'_m = \boldsymbol{\psi}_m + \sum_{n\neq m}\frac{\int \boldsymbol{\psi}^*_n H'\boldsymbol{\psi}_m\, d^3r}{E^0_m - E^0_n}\boldsymbol{\psi}_n$$

Time dependent disturbance:

$$\left.\begin{array}{l}H = H^0 + H' \\ H^0 \text{ Time independent} \\ H^0\boldsymbol{\psi}_n = E^0_n\boldsymbol{\psi}_n \\ H\boldsymbol{\psi}' = i\hbar\frac{\partial}{\partial t}\boldsymbol{\psi}'\end{array}\right\} \Longrightarrow$$

$$\boldsymbol{\psi}'_m = \sum_n a_{mn}(t)\boldsymbol{\psi}_n$$

$$\dot{a}_{mn} = -\frac{i}{\hbar}e^{-i(E_m-E_n)t/\hbar}\cdot H'_{nm}$$

"Golden Rule"

The transition probability per unit of time $w_{f\leftarrow i}$ for a transition from the state $\boldsymbol{\psi}_i$ to a group of states $F = \{\boldsymbol{\psi}_f\}$ with energy $\sin E^0_i$ for a system characterized by the state density $\rho(E)$ is given by:

$$w_{f\leftarrow i} = \frac{2\pi}{\hbar}|\langle f|H'|i\rangle|^2_{E^0_i\approx E^0_f}\cdot\rho(E^0_f)$$

Dispersion (Born Approximation)

$$\frac{d\sigma}{d\Omega} = |f(\xi,\eta)|^2$$

$$f(\xi,\eta) = \frac{m}{2\pi\hbar^2}\int e^{i(\boldsymbol{k}_i-\boldsymbol{k}_f)\cdot\boldsymbol{r}}\cdot v(\boldsymbol{r})\, d^3r$$

For spherical symmetrical potential:

$$f(\xi,\eta) = \frac{2m}{\hbar^2 K}\int_0^\infty \sin(Kr)\cdot r\cdot v(r)dr, \qquad |K| = 2k\cdot\sin\left(\frac{\xi}{2}\right)$$

Spherical box-potential:

$$V(r) = \begin{cases}-V_0 & r\leq a \\ 0 & r > a\end{cases}$$

$$f(\xi,\eta) = -\frac{2mV_0}{\hbar^2}\cdot\frac{\sin(Ka) - Ka\cos(Ka)}{K^3}$$

Screened Coulomb Potential:

$$v(r) = -\frac{A}{r}\cdot e^{-\alpha r}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{2mA}{\hbar^2\left(\alpha^2 + 4k^2\sin^2(\xi/2)\right)}\right)^2$$

$$\sigma = \left(\frac{Am}{\hbar^2}\right)^2\frac{16\pi}{\alpha^2\left(\alpha^2+4k^2\right)}$$

$$\text{When } \alpha\to 0, \quad \frac{d\sigma}{d\Omega}\to\left(\frac{Am}{\hbar^2}\right)^2\frac{1}{4\left(k\sin(\xi/2)\right)^4}$$

Periodic Potential

$$V(x) = \left.\begin{array}{ll}0 & n(a+b) < x < n(a+b)+a \\ V_0 & n(a+b)+a < x < (n+1)(a+b)\end{array}\right\}$$

Continuity Requirements:

$$\cos k_1a\cdot\cos k_2b - \frac{k_1^2+k_2^2}{2k_1k_2}\sin k_1a\cdot\sin k_2b = \cos(k(a+b)), \quad V_0 < E$$

$$\cos k_1a\cdot\cosh\kappa b - \frac{k_1^2+\kappa^2}{2k_1\kappa}\sin k_1a\cdot\sinh\kappa b = \cos(k(a+b)), \quad V_0 < E$$

Phase and group speed:

$$v_f = \frac{\omega}{k}, \quad v_g = \frac{d\omega}{dk} = \frac{dE}{dp}$$

Effective mass:

$$m^* = \left(\frac{1}{\hbar^2}\frac{d^2E}{dk^2}\right)^{-1}$$