Some integrals

Indefinite Integrals

$$\int \frac{x}{ax+b}dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b|$$

$$\int \frac{1}{x(ax+b)}dx = -\frac{1}{b}\ln\left|\frac{ax+b}{x}\right|$$

$$\int \frac{ax+b}{fx+g}dx = \frac{ax}{f} + \frac{bf-ag}{f^2}\ln|fx+g|$$

$$\int \frac{x}{(ax+b)(fx+g)}dx = \frac{1}{bf-ag}\left[\frac{b}{a}\ln|ax+b| - \frac{g}{f}\ln|fx+g|\right]$$

Definition: $\chi = \begin{cases} \frac{2}{\sqrt{4ac-b^2}}\arctan\frac{2ax+b}{\sqrt{4ac-b^2}} & \text{if } 4ac > b^2 \\ \frac{1}{a(p-q)}\ln\left|\frac{x-p}{x-q}\right| & \text{if } 4ac - b^2 \end{cases}$

Where p and q are the roots of $ax^2+bx+c=0$.

$$\int \frac{1}{ax^2+bx+c}dx = \chi$$

$$\int \frac{x}{ax^2+bx+c}dx = \frac{1}{2a}\ln\left|ax^2+bx+c\right| - \frac{b}{2a}\chi$$

$$\int \frac{x^2}{ax^2+bx+c}dx = \frac{x}{a} - \frac{b}{2a^2}\ln\left|ax^2+bx+c\right| + \frac{b^2-2ac}{2a^2}\chi$$

$$\int \frac{1}{(ax^2+bx+c)^2}dx = \frac{2ax+b}{(4ac-b^2)(ax^2+bx+c)} + \frac{2a}{(4ac-b^2)}\chi$$

$$\int \frac{x}{(ax^2+bx+c)^2}dx = -\frac{bx+2c}{(4ac-b^2)(ax^2+bx+c)} - \frac{b}{(4ac-b^2)}\chi$$

$$\int \sqrt{ax+b}dx = \frac{2}{3a}(ax+b)^{3/2}$$

$$\int x\sqrt{ax+b}dx = \frac{2(3ax-2b)}{15a^2}(ax+b)^{3/2}$$

$$\int \frac{1}{\sqrt{ax+b}}dx = \frac{2\sqrt{ax+b}}{a}$$

$$\int \frac{x}{\sqrt{ax+b}}dx = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b}$$

$$\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\frac{x}{a}$$

$$\int x\sqrt{a^2-x^2}dx = -\frac{1}{3}(a^2-x^2)^{3/2}$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\frac{x}{a}$$

$$\int \frac{x}{\sqrt{a^2-x^2}}dx = -\sqrt{a^2-x^2}$$

$$\int \frac{1}{x\sqrt{a^2-x^2}}dx = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|$$

$$\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left|x+\sqrt{x^2-a^2}\right|$$

$$\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right|$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left|x+\sqrt{x^2+a^2}\right|$$

$$\int \frac{1}{\sin ax}dx = \frac{1}{a}\ln\left|\tan\frac{ax}{2}\right|$$

$$\int \frac{1}{\cos ax}dx = \frac{1}{a}\ln\left|\tan\left(\frac{ax}{2}+\frac{\pi}{4}\right)\right|$$

$$\int \tan ax\, dx = -\frac{1}{a}\ln|\cos ax|$$

$$\int \cot ax\, dx = \frac{1}{a}\ln|\sin ax|$$

$$\int x \sin x \, dx = \sin x - x \cos x$$

$$\int x \sin^2 x \, dx = \frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{\cos 2x}{8}$$

$$\int \sin^2 x \, dx = \frac{1}{2}(x - \sin x \cos x)$$

$$\int \ln x \, dx = x \ln |x| - x$$

$$\int x e^{ax} \, dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2}(a \sin bx - b \cos bx)$$

Definite Integrals

$$\int_0^\infty x^{2n} \cdot e^{-ax^2} \, dx = \frac{(2n-1)!!}{2(2a)^n} \sqrt{\frac{\pi}{a}}$$

Where $a > 0$, $n! =$ negative integer.

$$\int_0^\infty x^{2n+1} \cdot e^{-ax^2} \, dx = \frac{n!}{2a^{n+1}}$$

$$\int_0^\infty x^k \cdot e^{-ax} \, dx = \Gamma(k+1) \cdot a^{-(k+1)}$$

$$I_k = \int_0^\infty \frac{x^k}{e^x - 1} \, dx = \Gamma(k+1) \cdot \zeta(k+1)$$

$$J_k = \int_0^\infty \frac{x^k}{e^x + 1} \, dx = (1 - 2^{-k}) \cdot I_k$$

$$\zeta(k) = \sum_{n=1}^\infty \frac{1}{n^k}$$

$$\int_0^{\pi/2} \sin^{2a+1} x \cdot \cos^{2b+1} x \, dx = \frac{\Gamma(a+1) \cdot \Gamma(b+1)}{2\Gamma(a+b+2)}$$

Stirling's approximation

$$N! \approx \sqrt{2\pi N} \left(\frac{N}{e}\right)^N \qquad N \gg 1$$

Error Function

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\xi^2} \, d\xi$$

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-\xi^2} \, d\xi$$

$$\operatorname{erf}(\infty) = 1$$