Atomic Physics

Rydberg

$$\tilde{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right) Z^2$$

$$E_n = -hcR\frac{Z^2}{n^2}$$

$$hcR_\infty = \frac{m_e(e^2/4\pi\epsilon_0)^2}{2\hbar^2} = 13.606 \text{ eV}$$

$$R = R_\infty \cdot \frac{M_N}{m_e + M_N}$$

Alkaline-like System

$$n^* = n - \delta_l$$

$$E = -hcR_\infty \frac{Z_0^2}{n^{*2}}$$

$$\Delta E_{FS} = -\frac{Z_i^2 Z_0^2}{n^{*3} l(l+1)} \alpha^2 hcR_\infty$$

Hydrogen-like Atoms

$$V(r) = -\frac{Ze^2}{4\pi\epsilon_0 r} = -\frac{Ze_0^2}{r}$$

$$a_0 = \frac{\hbar^2 4\pi\epsilon_0}{m_e e^2}$$

In Bohrs model of the atom: $r_n = a_0 n^2/Z$

Radial Functions for Hydrogen-like Systems

$$R_{1,0} = \left(\frac{Z}{a_0}\right)^{3/2} 2e^{-Zr/a_0}$$

$$R_{2,0} = \left(\frac{Z}{2a_0}\right)^{3/2} 2\left(1 - \frac{Zr}{2a_0}\right) e^{-Zr/2a_0}$$

$$R_{2,0} = \left(\frac{Z}{2a_0}\right)^{3/2} \frac{2}{\sqrt{3}} \frac{Zr}{2a_0} e^{-Zr/2a_0}$$

Spherical Surface Functions

l	m	$\Upsilon_l^m(\theta, \varphi)$
0	0	$\Upsilon_0^0 = \frac{1}{\sqrt{4\pi}}$
1	0	$\Upsilon_1^0 = \sqrt{\frac{3}{4\pi}} \cos\theta$
1	± 1	$\Upsilon_1^{\pm 1} = \pm\sqrt{\frac{3}{8\pi}} \sin\theta e^{\pm i\varphi}$
2	0	$\Upsilon_2^0 = \sqrt{\frac{5}{16\pi}} \left(3\cos^2\theta - 1\right)$
2	± 1	$\Upsilon_2^{\pm 1} = \pm\sqrt{\frac{15}{8\pi}} \sin\theta\cos\theta e^{\pm i\varphi}$
2	± 2	$\Upsilon_2^{\pm 2} = \sqrt{\frac{15}{32\pi}} \sin^2\theta e^{\pm 2i\varphi}$

Hamilton Operator for Multi-electron Systems

$$\boldsymbol{H} = \sum_{i=1}^{N} \left(-\frac{\hbar^2}{2m}\nabla_i^2 - \frac{Ze^2/4\pi\epsilon_0}{r_i} + \sum_{j>i}^{N} \frac{e^2/4\pi\epsilon_0}{r_{ij}} \right)$$

$$\langle LM_L | l_1 | LM_L \rangle = \frac{\langle l_1 \cdot \boldsymbol{L} \rangle}{L(L+1)} \langle LM_L | \boldsymbol{L} | LM_L \rangle$$

LS coupling

$$\text{Terms:} \quad \begin{cases} L = |l_1 - l_2|, ..., l_1 + l_2 \\ S = |s_1 - s_2|, ..., s_1 + s_2 \end{cases}$$

$$\text{Levels:} \quad J = |L - S|, ..., L + S$$

Zeeman Effect

$$E_{ZE} = \begin{cases} g_J \mu_B B M_J & \text{(fine structure)} \\ g_F \mu_B B M_F & \text{(weak field, hfs)} \\ g_J \mu_B B M_J + A M_I M_J & \text{(strong field, } \mu_B B > A\text{)} \end{cases}$$

Connection between magnetic moment and momentum

$$g_S = 2$$

$$g_J = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$$

$$g_F = g_J \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)}$$

$$\boldsymbol{\mu}_I = g_I \mu_N \boldsymbol{I}$$

Doppler Width

$$\frac{\Delta\omega_D}{\omega_0} = 2\sqrt{\ln 2}\frac{u}{c} \approx 1.7\frac{u}{c}$$

Most Probable Speed

$$u = 2230\sqrt{\frac{T}{300M}} \quad \text{m/s}$$

Dopplershift

$$\delta = kv = \frac{\omega v}{c}$$

Natural Width

$$\Delta\omega_N = \Gamma = A_{21} = 1\frac{1}{\tau}$$

$$\Delta f_N = \frac{\Delta\omega_N}{2\pi}$$

Hyper Fine Structure

$$H = -\boldsymbol{\mu}_I \cdot \boldsymbol{B}_e = A\boldsymbol{I} \cdot \boldsymbol{J}$$

For S-electrons in Hydrogen-like Systems

$$A = \frac{2}{3}\mu_0 g_S \mu_B g_I \mu_N \frac{Z^3}{\pi a_0^3 n^3}$$

Boltzman Distribution

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-\Delta E/(kT)}$$

Integrals

$$\int_0^\infty x^n e^{-\alpha x}\, dx = \frac{n!}{\alpha^{n+1}}$$

$$\int_0 \infty x^{2n+1} e^{-\alpha x^2}\, dx = \frac{n!}{2\alpha^{n+1}}$$

$$\int_0^\infty e^{-\alpha x^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

$$\int_0^\infty x^{2n} e^{-\alpha x^2}\, dx = \frac{(2n-1)!!}{2(2\alpha)^n}\sqrt{\frac{\pi}{\alpha}}$$

Operators

$$\boldsymbol{p} = -i\hbar\nabla$$

$$\boldsymbol{L} = -i\hbar\boldsymbol{r} \times \nabla$$

$$\boldsymbol{H} = -\frac{\hbar^2}{2m}\nabla^2 + V \qquad \text{(standard)}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{r^2}\left(\frac{\partial^2}{\partial\theta^2} + \frac{1}{\tan\theta}\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right)$$

Dirac Notation

$$< \boldsymbol{H} >=< \psi|\boldsymbol{H}|\psi >= \int_{\mathbb{R}} \psi^* \boldsymbol{H}\psi\, dv$$

Commutators

$$[A, B] = AB - BA$$
$$[A, B] = -[B, A]$$
$$[A, B + C] = [A, B] + [A, C]$$
$$[AB, C] = A[B, C] + [A, C]B$$

Schrödinger Equation

$$\boldsymbol{H}\psi = E\psi \qquad \text{(time independent)}$$

$$\boldsymbol{H}\psi = i\hbar\frac{\partial}{\partial t}\psi \qquad \text{(time dependent)}$$

Configuration	$\prod n_i l_i^{\omega_i}$
Terms	L and S(^{2S+1}L)
Levels	J
States (ZE-sublevels)	M_J
Hyperfinivåer	F

1	$\Delta J = 0, \pm 1$	$(J = 0 \not\leftrightarrow J' = 0)$	level
2	$\Delta M_J = 0, \pm 1$	$(M_J = 0 \not\leftrightarrow M_{j'} = 0$ if $\Delta J = 0)$	state
3	break parity		configuration
4	$\Delta l = \pm 1$		
5	$\Delta L = 0, \pm 1$	$(L = 0 \not\leftrightarrow L' = 0)$	term
6	$\Delta S = 0$		term

$1, 2$ are replaced for similar formulas for F and M_F if F is a good quantum number. $5, 6$ only hold if L and S are good quantum numbers.

	Fine Structure - LS	Hyper Fine Structure - IJ
interaction	$\beta \boldsymbol{L} \cdot \boldsymbol{S}$	$A \boldsymbol{I} \cdot \boldsymbol{J}$
moment	$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$	$\boldsymbol{F} = \boldsymbol{I} + \boldsymbol{J}$
eigen-states	$\lvert LSJM_J \rangle$	$\lvert IJFM_F \rangle$
energy	$\beta/2(J(J+1) - L(L+1) - S(S+1))$	$A/2(F(F+1) - I(I+1) - J(J+1))$
interval	$E_J - E_{J-1} = \beta J$	$E_F - E_{F-1} = AF$
	(if $E_{S-O} \ll E_{re}$)	(if $A \gg \Delta E_{quadrupole}$)