Formula Collection

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Integrals and Identities

Some integrals

Indefinite Integrals

$$\int \frac{x}{ax+b}dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b|$$

$$\int \frac{1}{x(ax+b)}dx = -\frac{1}{b}\ln\left|\frac{ax+b}{x}\right|$$

$$\int \frac{ax+b}{fx+g}dx = \frac{ax}{f} + \frac{bf-ag}{f^2}\ln|fx+g|$$

$$\int \frac{x}{(ax+b)(fx+g)}dx = \frac{1}{bf-ag}\left[\frac{b}{a}\ln|ax+b| - \frac{g}{f}\ln|fx+g|\right]$$

$$\text{Definition:}\quad \chi = \begin{cases} \frac{2}{\sqrt{4ac-b^2}}\arctan\frac{2ax+b}{\sqrt{4ac-b^2}} & \text{if } 4ac > b^2 \\ \frac{1}{a(p-q)}\ln\left|\frac{x-p}{x-q}\right| & \text{if } 4ac - b^2 \end{cases}$$

Where p and q are the roots of $ax^2+bx+c=0$.

$$\int \frac{1}{ax^2+bx+c}dx = \chi$$

$$\int \frac{x}{ax^2+bx+c}dx = \frac{1}{2a}\ln\left|ax^2+bx+c\right| - \frac{b}{2a}\chi$$

$$\int \frac{x^2}{ax^2+bx+c}dx = \frac{x}{a} - \frac{b}{2a^2}\ln\left|ax^2+bx+c\right| + \frac{b^2-2ac}{2a^2}\chi$$

$$\int \frac{1}{(ax^2+bx+c)^2}dx = \frac{2ax+b}{(4ac-b^2)(ax^2+bx+c)} + \frac{2a}{(4ac-b^2)}\chi$$

$$\int \frac{x}{(ax^2+bx+c)^2}dx = -\frac{bx+2c}{(4ac-b^2)(ax^2+bx+c)} - \frac{b}{(4ac-b^2)}\chi$$

$$\int \sqrt{ax+b}dx = \frac{2}{3a}(ax+b)^{3/2}$$

$$\int x\sqrt{ax+b}dx = \frac{2(3ax-2b)}{15a^2}(ax+b)^{3/2}$$

$$\int \frac{1}{\sqrt{ax+b}}dx = \frac{2\sqrt{ax+b}}{a}$$

$$\int \frac{x}{\sqrt{ax+b}}dx = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b}$$

$$\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\frac{x}{a}$$

$$\int x\sqrt{a^2-x^2}dx = -\frac{1}{3}(a^2-x^2)^{3/2}$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\frac{x}{a}$$

$$\int \frac{x}{\sqrt{a^2-x^2}}dx = -\sqrt{a^2-x^2}$$

$$\int \frac{1}{x\sqrt{a^2-x^2}}dx = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|$$

$$\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left|x+\sqrt{x^2-a^2}\right|$$

$$\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right|$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left|x+\sqrt{x^2+a^2}\right|$$

$$\int \frac{1}{\sin ax}dx = \frac{1}{a}\ln\left|\tan\frac{ax}{2}\right|$$

$$\int \frac{1}{\cos ax}dx = \frac{1}{a}\ln\left|\tan\left(\frac{ax}{2}+\frac{\pi}{4}\right)\right|$$

$$\int \tan ax\, dx = -\frac{1}{a} \ln |\cos ax|$$

$$\int \cot ax\, dx = \frac{1}{a} \ln |\sin ax|$$

$$\int x \sin x\, dx = \sin x - x \cos x$$

$$\int x \sin^2 x\, dx = \frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{\cos 2x}{8}$$

$$\int \sin^2 x\, dx = \frac{1}{2}(x - \sin x \cos x)$$

$$\int \ln x\, dx = x \ln |x| - x$$

$$\int x e^{ax}\, dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2 + b^2}(a \sin bx - b \cos bx)$$

Definite Integrals

$$\int_0^\infty x^{2n} \cdot e^{-ax^2}\, dx = \frac{(2n-1)!!}{2(2a)^n} \sqrt{\frac{\pi}{a}}$$

Where $a > 0$, $n! =$ negative integer.

$$\int_0^\infty x^{2n+1} \cdot e^{-ax^2}\, dx = \frac{n!}{2a^{n+1}}$$

$$\int_0^\infty x^k \cdot e^{-ax}\, dx = \Gamma(k+1) \cdot a^{-(k+1)}$$

$$I_k = \int_0^\infty \frac{x^k}{e^x - 1}\, dx = \Gamma(k+1) \cdot \zeta(k+1)$$

$$J_k = \int_0^\infty \frac{x^k}{e^x + 1}\, dx = (1 - 2^{-k}) \cdot I_k$$

$$\zeta(k) = \sum_{n=1}^\infty \frac{1}{n^k}$$

$$\int_0^{\pi/2} \sin^{2a+1} x \cdot \cos^{2b+1} x\, dx = \frac{\Gamma(a+1) \cdot \Gamma(b+1)}{2\Gamma(a+b+2)}$$

Stirling's approximation

$$N! \approx \sqrt{2\pi N}\left(\frac{N}{e}\right)^N \qquad N \gg 1$$

Error Function

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-\xi^2}\, d\xi$$

$$\mathrm{erfc}(x) = 1 - \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-\xi^2}\, d\xi$$

$$\mathrm{erf}(\infty) = 1$$

Power Series

Power Series

$$e^x = 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + ...$$

$$\sin(x) = \frac{1}{1!}x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - ...$$

$$\cos(x) = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - ...$$

$$\tan(x) = x + \frac{1}{3}x^3 + \frac{1}{15!}x^5 + ...|x| < \frac{\pi}{2}$$

$$\ln(1+x) = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + ...|x| < 1$$

$$(1+x)^a = 1 + \frac{a}{1!}x + \frac{a(a-1)}{2!}x^2 + ...$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x + \frac{1}{8}x^2 + \frac{1}{16}x^3 + ...$$

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + ...$$

$$\arctan(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - ...|x| < 1$$

$$\arcsin(x) = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + ...|x| < 1$$

$$\cosh(x) = 1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + ...$$

$$\sinh(x) = x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + ...$$

Trigonometric Functions

Trigonometric Functions

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$

$$\sin(3\alpha) = 3\sin\alpha - 4\sin^3\alpha$$

$$\cos(3\alpha) = 4\cos^3\alpha - 3\cos\alpha$$

$$\sin^2\frac{\alpha}{2} = \frac{1}{2}(1 - \cos\alpha)$$

$$\cos^2\frac{\alpha}{2} = \frac{1}{2}(1 + \cos\alpha)$$

$$\sin\alpha + \cos\beta = 2\sin\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$$

$$\sin\alpha - \cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\alpha-\beta)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$$

$$\cos\alpha - \cos\beta = -2\sin\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\alpha-\beta)$$

$$\sin\alpha = \frac{1}{2i}(e^{i\alpha} - e^{-i\alpha})$$

$$\cos\alpha = \frac{1}{2}(e^{i\alpha} + e^{-i\alpha})$$

Hyperbolic Functions

Hyperbolic Functions

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$

Vector Analysis

Vector Analysis

Vector Products

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

where $\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}}$ are unit vectors.

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b}(\boldsymbol{a} \cdot \boldsymbol{c}) - \boldsymbol{c}(\boldsymbol{a} \cdot \boldsymbol{b})$$

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) + \boldsymbol{b} \times (\boldsymbol{c} \times \boldsymbol{a}) + \boldsymbol{c} \times (\boldsymbol{a} \times \boldsymbol{b}) = 0$$

$$(\boldsymbol{a} \times \boldsymbol{b}) \cdot (\boldsymbol{c} \times \boldsymbol{d}) = (\boldsymbol{a} \cdot \boldsymbol{c})(\boldsymbol{b} \cdot \boldsymbol{d}) - (\boldsymbol{a} \cdot \boldsymbol{d})(\boldsymbol{b} \cdot \boldsymbol{c})$$

$$(\boldsymbol{a} \times \boldsymbol{b}) \times (\boldsymbol{c} \times \boldsymbol{d}) = \boldsymbol{c}((\boldsymbol{a} \times \boldsymbol{b}) \cdot \boldsymbol{d}) - \boldsymbol{d}((\boldsymbol{a} \times \boldsymbol{b}) \cdot \boldsymbol{c})$$

Gradient, Divergence, Curl and The Laplace Operator

$$\mathrm{grad} f = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = \left(\frac{\partial f}{\partial r}, \frac{1}{r}\frac{\partial f}{\partial \theta}, \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\right)$$

$$\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases}$$

$$\begin{aligned} \mathrm{div}\boldsymbol{a} = \nabla \cdot \boldsymbol{a} &= \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z} \\ &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 a_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta a_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial a_\varphi}{\partial \varphi} \end{aligned}$$

$$\begin{aligned}
\mathrm{rot}\boldsymbol{a} = \nabla \times \boldsymbol{a} &= \left(\frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}, \frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial x}, \frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}\right) \\
&= \left(\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta a_\varphi) - \frac{\partial a_\theta}{\partial\varphi}\right), \frac{1}{r\sin\theta}\frac{\partial a_r}{\partial\varphi} - \frac{1}{r}\frac{\partial}{\partial r}(r a_\varphi), \frac{1}{r}\frac{\partial}{\partial r}(r a_\theta) - \frac{1}{r}\frac{\partial a_r}{\partial\theta}\right)
\end{aligned}$$

$$\begin{aligned}
\Delta &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \\
&= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}
\end{aligned}$$

$$\Delta f(r) = \frac{1}{r}\frac{d^2}{dr^2}(rf), \quad r \neq 0$$

$$\begin{gathered}
\nabla \times (\nabla\mathcal{U}) = 0 \\
\nabla \cdot (\nabla\mathcal{U}) = \nabla\mathcal{U} \\
\nabla \cdot (\nabla \times \boldsymbol{A}) = 0 \\
\nabla \times (\nabla \times \boldsymbol{A}) = \nabla(\nabla \cdot \boldsymbol{A}) - \Delta\boldsymbol{A} \\
\nabla \cdot (\mathcal{U}V) = \mathcal{U}\Delta V + 2\nabla\mathcal{U} \cdot \nabla V + V\Delta\mathcal{U} \\
\nabla \cdot (\mathcal{U}\boldsymbol{V}) = \mathcal{U}\Delta\boldsymbol{V} + 2(\nabla\mathcal{U} \cdot \nabla)\boldsymbol{V} + \boldsymbol{V}\Delta\mathcal{U} \\
\nabla \times (\mathcal{U}\boldsymbol{V}) = \mathcal{U}\nabla \times \boldsymbol{V} + (\nabla\mathcal{U}) \times \boldsymbol{V} \\
\nabla \cdot (\boldsymbol{A} \times \boldsymbol{B}) = \boldsymbol{B} \cdot (\nabla \times \boldsymbol{A}) - \boldsymbol{A} \cdot (\nabla \times \boldsymbol{B}) \\
\nabla(\boldsymbol{A} \cdot \boldsymbol{B}) = \boldsymbol{A} \times (\nabla \times \boldsymbol{B}) + \boldsymbol{B} \times (\nabla \times \boldsymbol{A}) + (\boldsymbol{B} \cdot \nabla)\boldsymbol{A} + (\boldsymbol{A} \cdot \nabla)\boldsymbol{B} \\
\nabla \times (\boldsymbol{A} \times \boldsymbol{B}) = (\boldsymbol{B} \cdot \nabla)\boldsymbol{A} - (\boldsymbol{A} \cdot \nabla)\boldsymbol{B} + \boldsymbol{A}(\nabla \cdot \boldsymbol{B}) - \boldsymbol{B}(\nabla \cdot \boldsymbol{A})
\end{gathered}$$

Gauss sats

$$\oint_{S(V)} \boldsymbol{a} \cdot d\boldsymbol{S} = \int_V (\nabla \cdot \boldsymbol{a}) dV$$

Where dV in polar coordinates are $r^2 \sin\theta \, dr \, d\theta \, d\varphi$

Stokes sats

$$\oint_{C(S)} \boldsymbol{a} \cdot d\boldsymbol{l} = \int_S (\nabla \times \boldsymbol{a}) \cdot d\boldsymbol{S}$$

Where S is an arbitrary surface with border $C(S)$

Greens sats

$$\oint_{S(V)} (\Psi\nabla\varphi - \varphi\nabla\Psi) \cdot d\boldsymbol{S} = \int_V (\Psi\Delta\varphi - \varphi\Delta\Psi) dV$$

Electromagnetic field theory

Statics

Coulombs law

The force F on a point charge q_1 at the point $\mathbf{r_1}$ caused by a point charge q_2 at the point $\mathbf{r_2}$

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 |r_1 - r_2|^2} \frac{(r_1 - r_2)}{|r_1 - r_2|}$$

Electric Field Strength

From a point charge q in $\boldsymbol{r'}$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{q}{4\pi\epsilon_0 R^2} \boldsymbol{e}_R$$

From charge distribution

$$\boldsymbol{E}(\boldsymbol{r}) = \int \frac{1}{4\pi\epsilon_0 R^2} \boldsymbol{e}_R dq(\boldsymbol{r'}),$$

$$dq(\boldsymbol{r'}) = \begin{cases} \rho_{tot}(\boldsymbol{r'})dv' = \rho(\boldsymbol{r'}) + \rho_p(\boldsymbol{r'}))dv' \\ \rho_{tot,s}(\boldsymbol{r'})dS' = \rho_s(\boldsymbol{r'}) + \rho_{p,s}(\boldsymbol{r'}))dS' \\ \rho_l(\boldsymbol{r'})dl' \end{cases}$$

From point dipole $\boldsymbol{p} = p\boldsymbol{e}_z$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{p}{4\pi\epsilon_0 r^3}(2\cos(\theta)\boldsymbol{e}_r + \sin(\theta)\boldsymbol{e}_\theta)$$

From line charge ρ_l

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{\rho_l}{2\pi\epsilon_0 r_c} \boldsymbol{e}_{r_c}$$

From dipole line $\boldsymbol{p}_l = p_l \boldsymbol{e}_x$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{p_l}{2\pi\epsilon_0 r_c^2}(\cos(\varphi)\boldsymbol{e}_{r_c} + \sin(\varphi)\boldsymbol{e}_\varphi)$$

Electrical Potential

$$\boldsymbol{E} = -\boldsymbol{\nabla} V$$

From pointsource q in $\boldsymbol{r'}$

$$V(\boldsymbol{r}) = \frac{q}{4\pi\epsilon_0 R}$$

From charge distribution

$$V(\boldsymbol{r}) = \int \frac{1}{4\pi\epsilon_0 R} dq(\boldsymbol{r}')$$

From point dipole $\boldsymbol{p} = p\boldsymbol{e}_z$

$$V(\boldsymbol{r}) = \frac{\boldsymbol{p}\cdot\boldsymbol{r}}{4\pi\epsilon_0 r^3} = \frac{p\cos(\theta)}{4\pi\epsilon_0 r^2}$$

From line charge ρ_l

$$V(\boldsymbol{r}) = \frac{\rho_l}{2\pi\epsilon_0} \ln\left(\frac{1}{r_c}\right)$$

From dipole line $\boldsymbol{p}_l = p_l\boldsymbol{e}_x$

$$V(\boldsymbol{r}) = \frac{p_l}{2\pi\epsilon_0}\frac{\cos(\varphi)}{r_c}$$

Electrical flow density

Where $\boldsymbol{D}$ is defined by $\boldsymbol{\nabla D} = \rho$

Gauss law, where $\boldsymbol{e}_n$ is the unit normal to the volume surface pointing outwards $\boldsymbol{P}, \boldsymbol{E}$ and $\boldsymbol{D}$:

$$\begin{cases} \boldsymbol{D} = \epsilon_0\boldsymbol{E} + \boldsymbol{P} & \text{(valid generally)} \\ \boldsymbol{D} = \epsilon_r\epsilon_0\boldsymbol{E} \end{cases}$$

Polarization Charge

$$\rho_p = -\boldsymbol{\nabla}\cdot\boldsymbol{P} \qquad \text{space charge density}$$

$$\rho_{p,s} = \boldsymbol{e}_{n1}\cdot(\boldsymbol{P}_1 - \boldsymbol{P}_2) \qquad \text{surface charge density}$$

where the unit normal $\boldsymbol{e}_{n1}$ is directed from 1 to 2.

Boundary Conditions

$$\begin{cases} E_t \text{ continuous} \\ \rho_s = \boldsymbol{e}_{n2}\cdot(\boldsymbol{D}_1 - \boldsymbol{D}_2) \end{cases}$$

where ρ_s is free surface charge density and $\boldsymbol{e}_{n2}$ is directed from volume 2 to volume 1.

Electrostatic Energy

$$W_e = \frac{1}{2}\sum_i Q_i V_i$$

$$W_e = \frac{1}{2}\int \rho V \, dv$$

$$W_e = \frac{1}{2}\int \boldsymbol{E}\cdot\boldsymbol{D}\, dv$$

Maxwell's voltage

$$|\boldsymbol{T}| = \frac{1}{2}\boldsymbol{E}\cdot\boldsymbol{D} \qquad \boldsymbol{E} \text{ is a bisector to } \boldsymbol{e}_n \text{ and } \boldsymbol{T}$$

Torque on Electrical Dipole

$$\boldsymbol{T}_e = \boldsymbol{p}\times\boldsymbol{E}$$

DC Current

Current Density

$$I = \int \boldsymbol{J}\cdot e_n \, dS$$

Conservation Equation

$$\boldsymbol{\Delta}\cdot\boldsymbol{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\oint \boldsymbol{J}\cdot\boldsymbol{e}_n \, dS = -\frac{dQ}{dt}$$

Conductivity

$$\boldsymbol{J} = \sigma\boldsymbol{E}$$

Effect

$$P = \int \boldsymbol{J}\cdot\boldsymbol{E}\, dv$$

Boundary Conditions

$$\begin{cases} \boldsymbol{e}_{n2}\cdot(\boldsymbol{J}_1 - \boldsymbol{J}_2) = 0 \quad \text{(no surface current)} \\ \boldsymbol{E}_{t1} = \boldsymbol{E}_{t2} \end{cases}$$

Time Constant

$$RC = \frac{\epsilon_r \epsilon_0}{\sigma}$$

Analogy Elektrostatics - DC Current

$\boldsymbol{E}, V$	$\boldsymbol{E}, V$
$\boldsymbol{D}$	$\boldsymbol{J}$
$\epsilon_r \epsilon_0$	σ
Q	I
C	G

Magnetostatics

Magnetic Flow Density

From point dipole $\boldsymbol{m} = m\boldsymbol{e}_z$:

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta \boldsymbol{e}_r + \sin\theta \boldsymbol{e}_\theta)$$

From current density $\boldsymbol{J}_{tot}(\boldsymbol{r'})$:

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int \frac{\boldsymbol{J}_{tot}(\boldsymbol{r'}) \times \boldsymbol{e}_R}{R^2} \, dv'$$

where $\boldsymbol{J}_{tot} = \boldsymbol{J} + \boldsymbol{J}_m$. From current line:

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int \frac{I \, d\boldsymbol{l'} \times \boldsymbol{e}_R}{R^2}$$

From circular thread loop:

$$\boldsymbol{B}(x = 0, y = 0, z) = \frac{\mu_0 I}{2} \frac{b^2}{(b^2 + z^2)^{3/2}} \boldsymbol{e}_z$$

From coil:

$$\boldsymbol{B} = \frac{\mu_0 N I}{\ell} \frac{\cos(\alpha_2) - \cos(\alpha_1)}{2} \boldsymbol{e}_z$$

From long straight current path:

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0 I}{2\pi r_c} \boldsymbol{e}_\varphi$$

Vector Potential

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$$

From current density $\boldsymbol{J}_{tot}(\boldsymbol{r'})$:

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int \frac{\boldsymbol{J}_{tot}(\boldsymbol{r'})}{R}\, dv'$$

From current line:

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int \frac{I\, d\boldsymbol{l'}}{R}$$

From long straight current path:

$$\boldsymbol{A} = \frac{\mu_0 I}{2\pi} \ln\left(\frac{1}{r}\right) \boldsymbol{e}_z$$

From point dipole :

$$\boldsymbol{A} = \frac{\mu_0}{4\pi} \frac{\boldsymbol{m} \times \boldsymbol{r}}{r^3}$$

Magnetic Flow

$$\Phi = \int \boldsymbol{B} \cdot \boldsymbol{e}_n\, dS = \oint \boldsymbol{A} \cdot dl$$

Linked Flow

$$\Lambda = N\Phi$$

Self-Inductance and Mutual Inductance

$$\Lambda_1 = L_1 I_1 + M I_2$$
$$\Lambda_2 = L_2 I_2 + M I_1$$

Magnetic Field Strength

Amperes Law:

$$\oint \boldsymbol{H} \cdot d\ell = \int \boldsymbol{J} \cdot \boldsymbol{e}_n\, dS = I_{\text{inside}}$$

Connection between magnetization $\boldsymbol{M}, \boldsymbol{B}$ and $\boldsymbol{H}$:

$$\begin{cases} \boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M}) & \text{(holds generally)} \\ \boldsymbol{B} = \mu_r \mu_0 \boldsymbol{H} \end{cases}$$

Equivalent Current Density

$$\boldsymbol{J}_m = \boldsymbol{\nabla} \times \boldsymbol{M} \qquad \text{volume current density}$$

$$\boldsymbol{J}_m = \boldsymbol{\nabla} \times \boldsymbol{M} \qquad \text{surface current density}$$

Boundary Conditions

$$\begin{cases} \boldsymbol{e}_{n2} \times (\boldsymbol{H}_1 - H_2) = \boldsymbol{J}_s \\ \boldsymbol{B}_n \text{ Continuous} \end{cases}$$

Scalar Potential

From magnetic dipole $\boldsymbol{m}$:

$$V_m = \frac{1}{4\pi} \frac{\boldsymbol{m} \cdot \boldsymbol{e}_R}{R^2}$$

Magnetic Pole Density

$$\begin{cases} \rho_m = -\boldsymbol{\nabla} \cdot \boldsymbol{M} & \text{volume pole density} \\ \rho_{m,s} = e_{n1} \cdot (\boldsymbol{M}_1 - \boldsymbol{M}_2) & \text{surface pole density} \end{cases}$$

Magnetic Force Law

$$d\boldsymbol{F}_m = I d\boldsymbol{l} \times \boldsymbol{B}$$

Magetic moment for current loop

$$\boldsymbol{m} = \int I \boldsymbol{e}_n \, dS$$

Torque on Magnetic Moment

$$\boldsymbol{T}_m = \boldsymbol{m} \times \boldsymbol{B}$$

Maxwell's Voltage

$$|\boldsymbol{T}| = \frac{1}{2} \boldsymbol{B} \cdot \boldsymbol{H} \qquad \boldsymbol{B} \text{ is a bisector to } \boldsymbol{e}_n \text{ and } \boldsymbol{T}$$

Magnetic Energy

$$W_m = \frac{1}{2} \int \boldsymbol{J} \cdot \boldsymbol{A} \, dv = \frac{1}{2} \int \boldsymbol{B} \cdot \boldsymbol{H} \, dv = \frac{1}{2} \sum_i \sum_j L_{ij} I_i I_j$$

Two coils:

$$W_m = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2$$

Reluctance

$$R = \frac{1}{\mu_r \mu_0 S}$$

Electromagnetic Fields

Induced emk

$$\mathcal{E} = \oint (\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot d\ell$$

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

$$\mathcal{E} = -\frac{d\Lambda}{dt} \quad \text{(coil with multiple turns)}$$

Maxwell's equations

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\boldsymbol{\nabla} \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

$$\boldsymbol{\nabla} \cdot D = \rho$$

$$\boldsymbol{\nabla} \boldsymbol{B} = 0$$

The Conservation Equation

$$\boldsymbol{\nabla} \cdot \boldsymbol{J} + \frac{\partial \rho}{\partial t} = 0$$

Potentials

$$V(\boldsymbol{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho\left(\boldsymbol{r'}, t - \frac{R}{c}\right)}{R} \, dv' = \frac{1}{4\pi\epsilon_0} \int \frac{\rho_{ret}}{R} \, dv'$$

$$\boldsymbol{A}(\boldsymbol{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\boldsymbol{J}\left(\boldsymbol{r'}, t - \frac{R}{c}\right)}{R} \, dv' = \frac{\mu_0}{4\pi} \int \frac{\boldsymbol{J}_{ret}}{R} \, dv'$$

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$$

$$\boldsymbol{E} = -\boldsymbol{\nabla} V - \frac{\partial \boldsymbol{A}}{\partial t}$$

Magnetic Flow Density

$$\boldsymbol{B}(\boldsymbol{r},t) = \frac{\mu_0}{4\pi}\int \frac{\boldsymbol{J}_{ret} \times \boldsymbol{e}_R}{R^2}\,dv' + \frac{\mu_0}{4\pi c}\int \frac{\boldsymbol{J}'_{ret} \times \boldsymbol{e}_R}{R}\,dv'$$

Filamentuos Antenna

$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\int \frac{i(z,t-R/c)d\boldsymbol{l} \times \boldsymbol{e}_R}{R^2} + \frac{\mu_0}{4\pi c}\int \frac{i(z,t-R/c)d\boldsymbol{l} \times \boldsymbol{e}_R}{R}$$

Oscillating Electric Dipole

$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\frac{\boldsymbol{p}'(t-R/c) \times \boldsymbol{e}_R}{R^2} + \frac{\mu_0}{4\pi c}\frac{\boldsymbol{p}''(t-R/c) \times \boldsymbol{e}_R}{R}$$

Oscillating Magnetic Dipole

$$\boldsymbol{B} = -\frac{\mu_0}{4\pi}\frac{\boldsymbol{m}'(t-R/c) \times \boldsymbol{e}_R}{R^2} - \frac{\mu_0}{4\pi c}\frac{\boldsymbol{m}''(t-R/c) \times \boldsymbol{e}_R}{R}$$

Pointing's Vector

$$\boldsymbol{P}_S(\boldsymbol{r},t) = \boldsymbol{E}(\boldsymbol{r},t) \times \boldsymbol{H}(\boldsymbol{r},t)$$

Time Harmonic Fields

Planar Sinusoidal Wave

$$\boldsymbol{E} = \hat{E}\cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r} + \phi)\boldsymbol{e}_E \quad \text{instantanious value}$$

$$\boldsymbol{E} = E_0 e^{-j\boldsymbol{k}\cdot\boldsymbol{r}}\boldsymbol{e}_E \quad \text{complex value}$$

$$E_0 = \hat{E}e^{j\phi} \quad \text{top value scale}$$

$$E_0 = \frac{\hat{E}}{\sqrt{2}}e^{j\phi} \quad \text{effective value scale}$$

Propagation Rate

$$v = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}} \quad v = \frac{\omega}{k} \quad k = |\boldsymbol{k}|$$

Wave Impedance Non-Conductive Space

$$\eta = \sqrt{\frac{\mu_r\mu_0}{\epsilon_r\epsilon_0}}$$

Rule of Right-Hand Systems

$$\boldsymbol{e}_k = \boldsymbol{e}_E \times \boldsymbol{e}_H \quad E = \eta H \qquad \boldsymbol{e}_k = \boldsymbol{e}_E \times \boldsymbol{e}_B \quad E = vB$$

Planar Wave in Space with Condctivity

$$\boldsymbol{E} = E_0 e^{\gamma z} \boldsymbol{e}_x$$

Complex Propagation Constant

$$\gamma = \sqrt{j\omega\mu_r\mu_0(\sigma + j\omega\epsilon_r\epsilon_0)} \qquad \gamma = \alpha j \beta$$

Waveinpedance, Space With Given Conductivity

$$\eta = \sqrt{\frac{j\omega\mu_r\mu_0}{\sigma + j\omega\epsilon_r\epsilon_0}}$$

Penetration Depth

$$\delta = \sqrt{\frac{2}{\omega\mu_r\mu_0\sigma}}$$

Derivatives

Derivatives

$$\frac{\mathrm{d}}{\mathrm{d}x}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\arccos(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\arctan(x) = \frac{1}{1+x^2}$$

Fourier Analysis

Fourier Analysis Introduction

Fourier Sum

$$f(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} a_m \cos(m\omega t) + \sum_{m=1}^{\infty} b_m \sin(m\omega t)$$

Fourier Coefficients

$$a_0 = \frac{2}{T}\int_0^T f(t)dt$$

$$a_m = \frac{2}{T}\int_0^T f(t)\cos(m\omega t)dt$$

$$b_m = \frac{2}{T}\int_0^T f(t)\sin(m\omega t)dt$$

Rewriting with Eulers Formula

$$f(t) = \sum_{m=-\infty}^{m=\infty} c_n e^{-im\omega t}$$

$$c_m = \frac{1}{T}\int_0^T f(t)e^{im\omega t}dt$$

For Non-periodic Functions

$$f(t) = \int_{-\infty}^{\infty} g(\omega)e^{-i\omega t}d\omega$$

$$g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{i\omega t}dt$$

Fourier s integral theorem

$$f(\boldsymbol{r}) = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} A(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{r}}d^3k$$

$$A(\boldsymbol{k}) = \int_{-\infty}^{\infty} f(\boldsymbol{r})e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\, d^3r$$

Periodic boundaries

If the function $f(\boldsymbol{r})$ is such that

$$f(\boldsymbol{r}) = f(\boldsymbol{r} + L\boldsymbol{R})$$

[For some positive integer L and lattice vector $\boldsymbol{R}$. Then/Får något positivt heltal L och gittervektor $\boldsymbol{R}$. Då håller att]]

$$f(\boldsymbol{r}) = \frac{1}{\sqrt{V}}\sum_{\boldsymbol{k}=\frac{\boldsymbol{G}}{L}} c_{\boldsymbol{k}} \cdot e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$$

$$c_{\boldsymbol{k}} = \frac{1}{\sqrt{V}}\int_V f(\boldsymbol{r}) \cdot e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\, d^3r$$

Where $\boldsymbol{G}$ is the reciprocal lattice vector and $V = L^3|\boldsymbol{a}_1 \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3)| = L^3 V_a$. The functions $\frac{1}{\sqrt{V}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$ is a complete orthonormal basis in V.. If the volume V is large, the sum can be replaced by an integral:

$$\sum_{\boldsymbol{k}} \to \frac{V}{(2\pi)^3} \int d^3k$$

Dirac Delta Function

$$\int_A^B f(x)\delta(x - x_0)\, dx = \begin{cases} f(x_0) & \text{if } A < x_0 < B \\ 0 & \text{otherwise} \end{cases}$$

If $f(x)$ is a "nice" function.

$$\delta(f(x)) = \sum_{\forall i; f(x_i)=0, f'(x_i)\neq 0} \frac{1}{|f'(x_i)|} \delta(x - x_i)$$

$$\delta(x - x_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ik(x-x_0)}\, dk$$

Kronecker Delta

$$\delta_{n,m} = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i\phi(n-m)}\, d\phi = \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases}$$

Mechanics

Mechanics

Momentary Speed

$$v = \frac{dx}{dt}$$

Momentary Acceleration

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Momentum

$$\mathbf{p} = \mathbf{m} \cdot \mathbf{v}$$

Force

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = m \cdot \mathbf{a}$$

Gravitation

$$F = C \cdot \frac{m_1 \cdot m_2}{r^2}$$

Centripetal Acceleration

$$a_c = \frac{v^2}{r} = r\omega^2$$

Work

$$W = \int_{x_1}^{x_2} F(x)\, dx$$

Kenetic Energy

$$K = \frac{m \cdot v^2}{2}$$

Potential Energy

$$W = -\Delta U,\ F = -\frac{dU}{dx}$$

Reduced Mass

$$\frac{1}{\mu} = \frac{1}{m} + \frac{1}{M}$$

Angular Frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Quantum Mechanics

Quantum Mechanics

Schrödinger Equation

$$H\boldsymbol{\psi}(\boldsymbol{r},t) = \left[-\frac{\hbar^2}{2m}\Delta + \mathcal{U}(\boldsymbol{r})\right]\boldsymbol{\psi}(\boldsymbol{r},t) = i\hbar\frac{\partial}{\partial t}\boldsymbol{\psi}(\boldsymbol{r},t)$$

Where H is a hamiltonian operator. If H is time independent separation of variables gives:

$$\boldsymbol{\psi}(\boldsymbol{r},t) = \boldsymbol{\Phi}(\boldsymbol{r}) \cdot e^{-\frac{i}{\hbar}Et}$$
$$\left[-\frac{\hbar^2}{2m}\Delta + \mathcal{U}(\boldsymbol{r})\right]\boldsymbol{\Phi}(\boldsymbol{r}) = E\boldsymbol{\Phi}(\boldsymbol{r})$$

The general time dependent solution is:

$$\boldsymbol{\psi}(\boldsymbol{r}, t) = \sum_n a_n \cdot \boldsymbol{\Phi}(\boldsymbol{r}) e^{-\frac{i}{\hbar}Et}$$

Where a_n are found through the boundary conditions ($t = 0$):

$$a_n = \int \boldsymbol{\Phi}_n * (\boldsymbol{r}) \cdot \boldsymbol{\psi}(\boldsymbol{r}, t = 0)\, d^3r$$

Operators

Linear Operator

$$F(a\boldsymbol{\Phi}_1 + b\boldsymbol{\Phi}_2) = a \cdot F\boldsymbol{\Phi}_1 + b \cdot F\boldsymbol{\Phi}_2 \quad \forall \boldsymbol{\Phi}_1, \boldsymbol{\Phi}_2$$

Eigenvalue, Eigenfunction

$$Fu_n = f_n u_n$$

u_n is a eigen function to the operator F with corresponding eigenvalue f_n.

Hermitian Operator

$$\int (Hu) * v\, d^3r = \int u * Hv\, d^3r, \quad \forall u, v$$

A hermitian operator has real eigenvalues and corresponding eigenfunctions can be choosen to be orthonormal. Practically all operators in quantum mechanics are linear and hermitian.

Eigenfunction Expansion

$$\boldsymbol{\psi}(\boldsymbol{r}) = \sum_n a_n m \cdot u_n(\boldsymbol{r}), \quad a_n = \int u_n * \cdot \boldsymbol{\psi} \cdot d^3r$$

Expansion Postulate

At a meassurement of an observable F on a system described by a wavefunction $\boldsymbol{\psi}$ only eigenvalues of the operator F can be found. The probability of the result $F = f_n$ is given by

$$P(F = f_n) = \left| \int u_n * \boldsymbol{\psi}\, d^3r \right|^2, \quad Fu_n = f_n u_n$$

Momentum Operators

$$L^2 = -\hbar^2 \left[\frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\varphi^2} + \frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial}{\partial\theta} \right) \right]$$

$$L_z = \frac{\hbar}{i} \frac{\partial}{\partial\varphi}$$

L^2 and L_z have normalized eigenfunctions $\Upsilon_l^m(\theta,\varphi)$ for which it holds that:

$$L^2 \Upsilon_l^m = \hbar^2 l(l+1) \Upsilon_l^m$$

$$L_z \Upsilon_l^m = m\hbar \Upsilon_l^m$$

l	m	$\Upsilon_l^m(\theta,\varphi)$
0	0	$\Upsilon_0^0 = \frac{1}{\sqrt{4\pi}}$
1	0	$\Upsilon_1^0 = \sqrt{\frac{3}{4\pi}} \cos\theta$
1	±1	$\Upsilon_1^{\pm1} = \pm\sqrt{\frac{3}{8\pi}} \sin\theta e^{\pm i\varphi}$
2	0	$\Upsilon_2^0 = \sqrt{\frac{5}{16\pi}} \left(3\cos^2\theta - 1\right)$
2	±1	$\Upsilon_2^{\pm1} = \pm\sqrt{\frac{15}{8\pi}} \sin\theta \cos\theta e^{\pm i\varphi}$
2	±2	$\Upsilon_2^{\pm2} = \sqrt{\frac{15}{32\pi}} \sin^2\theta e^{\pm 2i\varphi}$

Commutators and Momentum Operators

$$\epsilon_{ijk} = \begin{cases} 1 & ijk \text{ even} \\ -1 & ijk \text{ odd} \\ 0 & \text{otherwise} \end{cases}$$

$$[x_i, p_j] = i\hbar \cdot \delta_{ij}$$

$$[x_i, L_j] = i\hbar \cdot \epsilon_{ijk} \cdot x_k$$

$$[L_i, L_j] = i\hbar \cdot \epsilon_{ijk} \cdot L_k$$

$$[x_i, x_j] = [p_i, p_j] = 0$$

$$[p_i, L_j] = i\hbar \cdot \epsilon_{ijk} \cdot p_k$$

$$J_+ = J_x + iJ_y$$
$$J_- = J_x - iJ_y$$

$$J_\pm J_\mp = J^2 - J_z^2 \pm \hbar \cdot J_z$$

$$[J_+, J_-] = 2\hbar \cdot J_z$$

$$[J_z, J_\pm] = \pm\hbar \cdot J_\pm$$

$$J_+ \phi_{j,m} = \sqrt{(j-m)(j+m+1} \cdot \hbar \cdot \phi_{j,m+1}$$

$$J_- \phi_{j,m} = \sqrt{(j+m)(j-m+1} \cdot \hbar \cdot \phi_{j,m-1}$$

$$\Upsilon_l^l(\theta, \varphi) = (-1)^l \sqrt{\frac{2l+1)}{4\pi} \frac{(2l)!}{2^{2l}(l!)^2}} \cdot \sin^l \theta \cdot e^{il\varphi}$$

Applications

0.0.1 Low potential with infinitely rigid walls in one dimension

$$\mathcal{U}(x) = \begin{cases} \infty & x \leq 0,\ a \leq x \\ 0 & 0 < x < a \end{cases}$$

$$\Phi_n(x) = \begin{cases} 0 & \text{for } x \leq 0 \text{ and } a \leq x \\ \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a} & \text{for } 0 < x < a \end{cases}$$

$$E_n = \frac{\pi^2 \hbar^2 n^2}{2ma^2}$$

Harmonic Oscillator 1D

$$\mathcal{U}(x) = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}kx^2$$

$$N_n = (2^n n!)^{-1/2}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}$$

Hermites polynom:

$$H_n(\xi) = (-1)^n \cdot e^{\xi^2} \cdot \frac{d^n e^{-\xi^2}}{d\xi^n}$$

$$\boldsymbol{\Phi}_n(x) = N_n \cdot e^{-\frac{m\omega}{2\hbar}x^2} \cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right)$$

$$E_n = \hbar\omega \cdot \left(n + \frac{1}{2}\right)$$

The wave equations can alternatively be written:

$$u_n(x) = N\left(\frac{\partial}{\partial x} - ax\right)^n \cdot u_0(x)$$

$$u_0(x) = e^{-ax^2/2}$$

Spherical Symmetric Potential

$$\mathcal{U}(\boldsymbol{r}) = \mathcal{U}(r)$$

$$H = -\frac{\hbar}{2mr^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial}{\partial r}\right] + \frac{L^2}{2mr^2} + \mathcal{U}(r)$$

$$H\boldsymbol{\psi}_{nlm}(\boldsymbol{r}) = E_{nlm}\boldsymbol{\psi}_{nlm}(\boldsymbol{r})$$

$$\boldsymbol{\psi}_{nlm}(\boldsymbol{r}) = \frac{G_{nl}(r)}{r}\Upsilon_l^m(\theta,\phi)$$

Radial equation:

$$-\frac{\hbar^2}{2m}\frac{d^2}{dr^2}G(r) + \left[\frac{l(l+1)\hbar^2}{2mr^2} + \mathcal{U}(r)\right]G(r) = EG(r)$$

Hydrogen-like Atom

$$\mathcal{U}(r) = -\frac{Ze^2}{4\pi\epsilon_0 r}$$

The Schrödinger equation simplifies to:

$$\left[\Delta + \frac{2Z}{a_0 r} + \frac{2mE}{\hbar^2}\right]\mathbf{\Phi}(r) = 0$$

Radial wave functions of hydrogenic atoms:

n	l	$R_{nl}(r)$
1	0	$R_{10}(r) = 2\left(\frac{Z}{a_0}\right)^{3/2} e^{-\rho/2}$
2	0	$R_{20}(r) = \frac{1}{2\sqrt{2}}\left(\frac{Z}{a_0}\right)^{3/2}(2-\rho)e^{-\rho/2}$
2	1	$R_{21}(r) = \frac{1}{2\sqrt{6}}\left(\frac{Z}{a_0}\right)^{3/2}\rho e^{-\rho/2}$
3	0	$R_{30}(r) = \frac{1}{9\sqrt{3}}\left(\frac{Z}{a_0}\right)^{3/2}\left(6-6\rho+\rho^2\right)e^{-\rho/2}$
3	1	$R_{31}(r) = \frac{1}{9\sqrt{6}}\left(\frac{Z}{a_0}\right)^{3/2}\rho(4-\rho)e^{-\rho/2}$
3	2	$R_{32}(r) = \frac{1}{9\sqrt{30}}\left(\frac{Z}{a_0}\right)^{3/2}\rho^2 e^{-\rho/2}$

$$E-n = -\frac{mZ^2e^4}{32\pi^2\epsilon_0^2\hbar^2n^2} = -\frac{Z^2\hbar^2}{2a_0^2mn^2} = -13.6\frac{Z^2}{n^2}\text{eV}$$

$$S(x,t) = \frac{\hbar}{2im}\left[\boldsymbol{\psi}^*\cdot\frac{\partial\boldsymbol{\psi}}{\partial x} - \boldsymbol{\psi}\frac{\partial}{\partial x}\boldsymbol{\psi}^*\right]$$

Disturbance Calculations

Time independent disturbance:

$$\left.\begin{array}{l}\left(H^0+H'\right)\boldsymbol{\psi}'_m = E'_m\boldsymbol{\psi}'_m \\ H^0\boldsymbol{\psi}_n = E_n^0\boldsymbol{\psi}_n\end{array}\right\} \implies$$

$$E'_m = E_m^0 + \langle m|H'|m\rangle + \sum_{n\neq m}\frac{|\langle m|H'|n\rangle|^2}{E_m^0 - E_n^0}$$

$$\boldsymbol{\psi}'_m = \boldsymbol{\psi}_m + \sum_{n\neq m}\frac{\int\boldsymbol{\psi}_n^* H'\boldsymbol{\psi}_m\, d^3r}{E_m^0 - E_n^0}\boldsymbol{\psi}_n$$

Time dependent disturbance:

$$\left.\begin{array}{l} H = H^0 + H' \\ H^0 \text{ Time independent} \\ H^0 \boldsymbol{\psi}_n = E_n^0 \boldsymbol{\psi}_n \\ H\boldsymbol{\psi}' = i\hbar \frac{\partial}{\partial t}\boldsymbol{\psi}' \end{array}\right\} \Longrightarrow$$

$$\boldsymbol{\psi}'_m = \sum_n a_{mn}(t)\boldsymbol{\psi}_n$$

$$\dot{a}_{mn} = -\frac{i}{\hbar} e^{-i(E_m - E_n)t/\hbar} \cdot H'_{nm}$$

"Golden Rule"

The transition probability per unit of time $w_{f\leftarrow i}$ for a transition from the state $\boldsymbol{\psi}_i$ to a group of states $F = \{\boldsymbol{\psi}_f\}$ with energy sin E_i^0 for a system characterized by the state density $\rho(E)$ is given by:

$$w_{f\leftarrow i} = \frac{2\pi}{\hbar} |\langle f|H'|i\rangle|^2_{E_i^0 \approx E_f^0} \cdot \rho(E_f^0)$$

Dispersion (Born Approximation)

$$\frac{d\sigma}{d\Omega} = |f(\xi,\eta)|^2$$

$$f(\xi,\eta) = \frac{m}{2\pi\hbar^2} \int e^{i(\boldsymbol{k}_i - \boldsymbol{k}_f)\cdot \boldsymbol{r}} \cdot v(\boldsymbol{r})\, d^3r$$

For spherical symmetrical potential:

$$f(\xi,\eta) = \frac{2m}{\hbar^2 K} \int_0^\infty \sin(Kr) \cdot r \cdot v(r) dr, \qquad |K| = 2k \cdot \sin\left(\frac{\xi}{2}\right)$$

Spherical box-potential:

$$V(r) = \begin{cases} -V_0 & r \leq a \\ 0 & r > a \end{cases}$$

$$f(\xi,\eta) = -\frac{2mV_0}{\hbar^2} \cdot \frac{\sin(Ka) - Ka\cos(Ka)}{K^3}$$

Screened Coulomb Potential:

$$v(r) = -\frac{A}{r} \cdot e^{-\alpha r}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{2mA}{\hbar^2\left(\alpha^2 + 4k^2\sin^2(\xi/2)\right)}\right)^2$$

$$\sigma = \left(\frac{Am}{\hbar^2}\right)^2 \frac{16\pi}{\alpha^2\left(\alpha^2+4k^2\right)}$$

$$\text{When } \alpha \to 0, \quad \frac{d\sigma}{d\Omega} \to \left(\frac{Am}{\hbar^2}\right)^2 \frac{1}{4\left(k\sin(\xi/2)\right)^4}$$

Periodic Potential

$$V(x) = \left.\begin{matrix} 0 & n(a+b) < x < n(a+b)+a \\ V_0 & n(a+b)+a < x < (n+1)(a+b) \end{matrix}\right\}$$

Continuity Requirements:

$$\cos k_1 a \cdot \cos k_2 b - \frac{k_1^2+k_2^2}{2k_1k_2} \sin k_1 a \cdot \sin k_2 b = \cos(k(a+b)), \quad V_0 < E$$

$$\cos k_1 a \cdot \cosh \kappa b - \frac{k_1^2+\kappa^2}{2k_1\kappa} \sin k_1 a \cdot \sinh \kappa b = \cos(k(a+b)), \quad V_0 < E$$

Phase and group speed:

$$v_f = \frac{\omega}{k}, \quad v_g = \frac{d\omega}{dk} = \frac{dE}{dp}$$

Effective mass:

$$m^* = \left(\frac{1}{\hbar^2}\frac{d^2E}{dk^2}\right)^{-1}$$

Atomic Physics

Atomic Physics

Rydberg

$$\tilde{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right) Z^2$$

$$E_n = -hcR\frac{Z^2}{n^2}$$

$$hcR_\infty = \frac{m_e(e^2/4\pi\epsilon_0)^2}{2\hbar^2} = 13.606 \text{ eV}$$

$$R = R_\infty \cdot \frac{M_N}{m_e + M_N}$$

Alkaline-like System

$$n^* = n - \delta_l$$

$$E = -hcR_\infty \frac{Z_0^2}{n^{*2}}$$

$$\Delta E_{FS} = -\frac{Z_i^2 Z_0^2}{n^{*3} l(l+1)} \alpha^2 hcR_\infty$$

Hydrogen-like Atoms

$$V(r) = -\frac{Ze^2}{4\pi\epsilon_0 r} = -\frac{Ze_0^2}{r}$$

$$a_0 = \frac{\hbar^2 4\pi\epsilon_0}{m_e e^2}$$

In Bohrs model of the atom: $r_n = a_0 n^2 / Z$

Radial Functions for Hydrogen-like Systems

$$R_{1,0} = \left(\frac{Z}{a_0}\right)^{3/2} 2e^{-Zr/a_0}$$

$$R_{2,0} = \left(\frac{Z}{2a_0}\right)^{3/2} 2\left(1 - \frac{Zr}{2a_0}\right) e^{-Zr/2a_0}$$

$$R_{2,0} = \left(\frac{Z}{2a_0}\right)^{3/2} \frac{2}{\sqrt{3}} \frac{Zr}{2a_0} e^{-Zr/2a_0}$$

Spherical Surface Functions

l	m	$\Upsilon_l^m(\theta,\varphi)$
0	0	$\Upsilon_0^0 = \frac{1}{\sqrt{4\pi}}$
1	0	$\Upsilon_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$
1	±1	$\Upsilon_1^{\pm1} = \pm\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\varphi}$
2	0	$\Upsilon_2^0 = \sqrt{\frac{5}{16\pi}}\left(3\cos^2\theta - 1\right)$
2	±1	$\Upsilon_2^{\pm1} = \pm\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{\pm i\varphi}$
2	±2	$\Upsilon_2^{\pm2} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{\pm 2i\varphi}$

Hamilton Operator for Multi-electron Systems

$$\boldsymbol{H} = \sum_{i=1}^{N}\left(-\frac{\hbar^2}{2m}\nabla_i^2 - \frac{Ze^2/4\pi\epsilon_0}{r_i} + \sum_{j>i}^{N}\frac{e^2/4\pi\epsilon_0}{r_{ij}}\right)$$

$$\langle LM_L|l_1|LM_L\rangle = \frac{\langle l_1\cdot\boldsymbol{L}\rangle}{L(L+1)}\langle LM_L|\boldsymbol{L}|LM_L\rangle$$

LS coupling

$$\text{Terms:}\quad \begin{cases} L = |l_1 - l_2|, ..., l_1 + l_2 \\ S = |s_1 - s_2|, ..., s_1 + s_2 \end{cases}$$

$$\text{Levels:}\quad J = |L - S|, ..., L + S$$

Zeeman Effect

$$E_{ZE} = \begin{cases} g_J\mu_B BM_J & \text{(fine structure)} \\ g_F\mu_B BM_F & \text{(weak field, hfs)} \\ g_J\mu_B BM_J + AM_IM_J & \text{(strong field, } \mu_B B > A\text{)} \end{cases}$$

Connection between magnetic moment and momentum

$$g_S = 2$$

$$g_J = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$$

$$g_F = g_J \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)}$$

$$\boldsymbol{\mu}_I = g_I \mu_N \boldsymbol{I}$$

Doppler Width

$$\frac{\Delta\omega_D}{\omega_0} = 2\sqrt{\ln 2}\frac{u}{c} \approx 1.7\frac{u}{c}$$

Most Probable Speed

$$u = 2230\sqrt{\frac{T}{300M}} \quad \text{m/s}$$

Dopplershift

$$\delta = kv = \frac{\omega v}{c}$$

Natural Width

$$\Delta\omega_N = \Gamma = A_{21} = 1\frac{1}{\tau}$$

$$\Delta f_N = \frac{\Delta\omega_N}{2\pi}$$

Hyper Fine Structure

$$H = -\boldsymbol{\mu}_I \cdot \boldsymbol{B}_e = A\boldsymbol{I} \cdot \boldsymbol{J}$$

For S-electrons in Hydrogen-like Systems

$$A = \frac{2}{3}\mu_0 g_S \mu_B g_I \mu_N \frac{Z^3}{\pi a_0^3 n^3}$$

Boltzman Distribution

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-\Delta E/(kT)}$$

Integrals

$$\int_0^\infty x^n e^{-\alpha x}\,dx = \frac{n!}{\alpha^{n+1}}$$

$$\int_0 \infty x^{2n+1} e^{-\alpha x^2}\,dx = \frac{n!}{2\alpha^{n+1}}$$

$$\int_0^\infty e^{-\alpha x^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

$$\int_0^\infty x^{2n} e^{-\alpha x^2}\,dx = \frac{(2n-1)!!}{2(2\alpha)^n}\sqrt{\frac{\pi}{\alpha}}$$

Operators

$$\boldsymbol{p} = -i\hbar\nabla$$

$$\boldsymbol{L} = -i\hbar\boldsymbol{r}\times\nabla$$

$$\boldsymbol{H} = -\frac{\hbar^2}{2m}\nabla^2 + V \qquad \text{(standard)}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{r^2}\left(\frac{\partial^2}{\partial\theta^2} + \frac{1}{\tan\theta}\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right)$$

Dirac Notation

$$<\boldsymbol{H}> = <\psi|\boldsymbol{H}|\psi> = \int_{\mathbb{R}} \psi^* \boldsymbol{H}\psi\,dv$$

Commutators

$$[A,B] = AB - BA$$
$$[A,B] = -[B,A]$$
$$[A,B+C] = [A,B] + [A,C]$$
$$[AB,C] = A[B,C] + [A,C]B$$

Schrödinger Equation

$$\boldsymbol{H}\psi = E\psi \qquad \text{(time independent)}$$

$$\boldsymbol{H}\psi = i\hbar\frac{\partial}{\partial t}\psi \qquad \text{(time dependent)}$$

Configuration	$\prod n_i l_i^{\omega_i}$
Terms	L and $S(^{2S+1}L)$
Levels	J
States (ZE-sublevels)	M_J
Hyperfinivåer	F

1	$\Delta J = 0, \pm 1$	$(J = 0 \nleftrightarrow J' = 0)$	level
2	$\Delta M_J = 0, \pm 1$	$(M_J = 0 \nleftrightarrow M_{j'} = 0$ if $\Delta J = 0)$	state
3	break parity		configuration
4	$\Delta l = \pm 1$		
5	$\Delta L = 0, \pm 1$	$(L = 0 \nleftrightarrow L' = 0)$	term
6	$\Delta S = 0$		term

$1, 2$ are replaced for similar formulas for F and M_F if F is a good quantum number. $5, 6$ only hold if L and S are good quantum numbers.

	Fine Structure - LS	Hyper Fine Structure - IJ
interaction	$\beta \boldsymbol{L} \cdot \boldsymbol{S}$	$A \boldsymbol{I} \cdot \boldsymbol{J}$
moment	$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$	$\boldsymbol{F} = \boldsymbol{I} + \boldsymbol{J}$
eigen-states	$\lvert LSJM_J \rangle$	$\lvert IJFM_F \rangle$
energy	$\beta/2(J(J+1) - L(L+1) - S(S+1))$	$A/2(F(F+1) - I(I+1) - J(J+1))$
interval	$E_J - E_{J-1} = \beta J$	$E_F - E_{F-1} = AF$
	(if $E_{S-O} \ll E_{re}$)	(if $A \gg \Delta E_{quadrupole}$)

Waves and Optics

Oscillations

Simple harmonic occilations are described by

$$\frac{d^2 y}{dt^2} + \omega^2 y = 0$$

With real solutions on the form

$$y = A \sin(\omega t + \alpha)$$

Angular Frequency

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Energy for Elastic Pendulum

$$W_{pot} = \frac{ky^2}{2}$$

$$W_{tot} = \frac{m}{2}A^2\omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

Angular Frequency

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Wave Number

$$k = \frac{2\pi}{\lambda}$$

Wave Equation

Progressive Plane Wave

$$s = s_o \sin[2\pi(\frac{t}{T} \pm \frac{x}{\lambda}) + \alpha]$$

Standing Wave Equation

$$s = A\cos\left(2\pi\frac{x}{\lambda} + \frac{\phi}{2}\right)\sin\left(2\pi\frac{t}{T} + \frac{\phi}{2}\right)$$

where ϕ is the phase shift at origo. Node distance is $\frac{\lambda}{2}$

The General Wave Equation

$$\frac{\partial^2 s}{\partial t^2} = v^2\frac{\partial^2 s}{\partial x^2}$$

Occilation Frequency

$$f_{\text{occilation}} = |f_1 - f_2|$$

Sound and Doppler Effect

Doppler Effect

$$f_m = f_s \frac{v - v_m}{v - v_s}$$

Supersonic Speed

$$\sin\theta = \frac{v_{sound}}{v_{[planar/[plan]]}} = \frac{1}{M\alpha}$$

Compressibility coefficient

$$\kappa = -\frac{1}{\Delta P} \cdot \frac{\Delta V}{V}$$

Sound Pressure

$$p = -\frac{1}{\kappa} \cdot \frac{\partial s}{\partial x}$$

$$p = \mp p_0 \cos\left[2\pi\left(\frac{t}{T} \pm \frac{x}{\lambda}\right)\right]$$

Pressure Amplitude

$$p_0 = \frac{2\pi s_0}{\kappa\lambda} = Z s_0 \omega$$

Acoustic Impedance

$$Z = \rho v$$

Speed of Sound (Fluid and Gas)

$$v = \frac{1}{\sqrt{\kappa\rho}}$$

$$v = \sqrt{\frac{c_p RT}{c_v M}}$$

Speed of Sound (String and Rod)

$$v = \sqrt{\frac{F}{\mu}}$$

$$v = \sqrt{\frac{E}{\rho}}$$

Sound Intensity

$$I = \frac{Z}{2} s_0^2 \omega^2$$

$$I = \frac{p_0^2}{2Z}$$

Sound Intensity Level

$$L_I = 10 lg \frac{I}{I_0}$$

$$\text{med } I_0 = 1,0 \cdot 10^{-12} \, W/m^2$$

Refraction and Transmittance of Sound

$$R \equiv \frac{I_r ef}{I_i n} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2$$

$$T \equiv \frac{I_t r}{I_i n} = 1 - R$$

Harmonics (Strings and Open Cylinders)

$$f_m = m \cdot f_1 \quad m = 2, 3, 4, ...$$

Harmonics (Half Open Cylinders)

$$f_m = (2m - 1) \cdot f_1 \quad m = 2, 3, 4, ...$$

Light

Speed of Light

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$v = \frac{c}{\sqrt{\mu_r \epsilon_r}}$$

Intensity EM-Wave

$$I = \frac{1}{2} \sqrt{\frac{\epsilon_0 \epsilon_r}{\mu_0 \mu_r}} E_0^2 \ , \quad B_z = \frac{E_y}{v}$$

Intensity when two waves are added

$$I_{tot} = I_1 + I_2 + 2\sqrt{I_1 I_2} < \cos \delta >$$

where δ is the relative phase between the waves.

Refractive Index

$$n \equiv \frac{c}{v} = \sqrt{\mu_r \epsilon_r}$$

Snell's Law

$$\frac{\sin \alpha_1}{\sin \alpha_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

Boundary Angle for Total Reflection

$$\alpha_g = \arcsin\left(\frac{n_2}{n_1}\right)$$

Prism

$$\sin\left(\frac{A+\delta}{2}\right) = n \cdot \sin\left(\frac{A}{2}\right)$$

Where A is the prisms top angle and δ the reflection angle.

Fiber Optics, Numerical Aperture

$$N.A. \equiv n_0 \sin \theta_m$$

$$N.A. = \sqrt{n_1^2 - n_2^2}$$

Material Properties for Sound and Light

Material Properties for Sound and Light

Speed of Sound at 1 atm and 20 °C:

Iron	5950 m/s
Glass (Approx)	5600 m/s
Copper	4760 m/s
Lead	2160 m/s
Rubber	1550 m/s
Water	1461 m/s
Mercury	1407 m/s
Methanol	1143 m/s
Ether	1032 m/s
Hydrogen	1286 m/s
Helium	1008 m/s
Air	343 m/s
Oxygen	326 m/s
Carbon dioxide	269 m/s

Aucustic Impedance at 1 atm and 20 °C:

Hydrogen Gas	111 Ns/m^3
Air	412 Ns/m^3
Water	$1,46 \cdot 10^6$ Ns/m^3
Rubber	$1,47 \cdot 10^6$ Ns/m^3
Glycerin	$2,42 \cdot 10^6$ Ns/m^3
Quarts	$13,1 \cdot 10^6$ Ns/m^3
Glass (Approx)	$14 \cdot 10^6$ Ns/m^3
Aluminum	$17,3 \cdot 10^6$ Ns/m^3
Mercury	$19,1 \cdot 10^6$ Ns/m^3
Copper	$33,9 \cdot 10^6$ Ns/m^3
Steel	$46,4$ Ns/m^3
Tungsten	$101 \cdot 10^6$ Ns/m^3

Vacuum Wavelengths and Frequencies of Light:

Color	Wavelength	Frequency
Violet	$400 - 440$ nm	$749 - 681$ THz
Blue	$440 - 480$ nm	$681 - 625$ THz
Green	$480 - 560$ nm	$625 - 535$ THz
Yellow	$560 - 590$ nm	$535 - 508$ THz
Orange	$590 - 620$ nm	$508 - 484$ THz
Red	$620 - 700$ nm	$484 - 428$ THz

Geometrical Optics

Refraction in spherical surface

$$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R}$$

Gauss Formula

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

Lateral Enlargement

$$M \equiv \frac{y_b}{y_a} \qquad M = -\frac{b}{a}$$

Focal Length Curved Mirror

$$f = -\frac{R}{2}$$

Refractive Power (Lens)

$$B \equiv \frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Lens

Lens with refractive index n_1 in medium with refractive index n_2:

$$B \equiv \frac{1}{f} = \left[\frac{n_1}{n_2} - 1\right] \cdot \left[\frac{R_2 - R_1}{R_1 \cdot R_2}\right]$$

Aparture Number

$$b_t \equiv \frac{f}{D}$$

Depth of Field

$$s \approx \frac{a^2}{1000f} b_t$$

Angular Magnification of Magnifier

$$G = \frac{d_0}{f} \text{ where,} \quad d_0 = 25\,\text{cm}$$

Angular Magnification of Microscope

$$G = |M_{ob}| \cdot G_{ok} = \frac{L}{f_{ob}} \frac{d_0}{f_{ok}}$$

where the tube length $L = 16\,\text{cm}$

Angle magnification of the Kepler and Galileo binoculars

$$G = \left| \frac{f_{ob}}{f_{ok}} \right|$$

Refraction in a spherical surface

Positive if: C is to the right of O
Positive if: A is to the left of O
Positive if: B is to the right of O
Positive if: F_A is to the left of O
Positive if: F_B is on the right of O
Image with thin lens in air
Positive if: the lens is convex (gathers light)
Positive if: the object is to the left of the lens
Positive if: the image is to the right of the lens
Positive if: the object is above the optical axis
Positive if: the image is above the optical axis
Positive if: the image is upside up
Image with a curved mirror
Positive if: C is to the right of O (convex)
Positive if: F is to the left of O (concave)
Positive if: A is to the left of O
Positive if: B to the left of O
Positive if: the image is upside up

Refractive Index for Some Materials

Refractive Index with $\lambda = 589\,\mathrm{nm}$ at 20 °C:

Water	$1,333$
Diethyl Ether	$1,353$
Ethanol	$1,361$
Glycerin	$1,455$
Benzene	$1,501$
Carbon Sulfur	$1,628$
Is (0 °C)	$1,31$
NaCl	$1,544$
Polystyrene	$1,59$
Crown Glass (FK5)	$1,487$
Crown Glass (BK7)	$1,517$
Canada balsam	$1,542$
Flint Glass (F2)	$1,620$
Flint Glass (SF10)	$1,728$
Flint Glass (SFS1)	$1,922$
Quarts	$1,458$
Plexiglass	$1,49 - 1,52$
Diamond	$2,417$

Diffraction and Interferance

Intensity when Diffraction

$$I = I_0 \left(\frac{\sin\beta}{\beta} \right)^2 \quad \text{with} \quad \beta = \frac{\pi}{\lambda} b \sin\theta$$

Diffraction minimun of slit

$$b \sin\theta = m\lambda \quad \text{where} \quad m = \pm 1, \pm 2, \pm 3, ...$$

Diffraction minimum of round opening

$$D \sin\theta = k\lambda$$

$$\text{where} k = 1,22 \quad 2,23 \quad 3,24 \quad 4,25 \quad 5,25...$$

Rayleigh's Resolution Criterion

Central top for the first point over the first min for the second point

Interference if Diffraction is neglected

$$I = I_0 \left(\frac{\sin N\gamma}{\sin \gamma} \right) \qquad \text{där} \qquad \gamma = \frac{\pi}{\lambda} d \sin\theta$$

Interference gives main max if

$$d \sin\theta = m\lambda \qquad \text{där} \qquad m = \pm1, \pm2, \pm3, ...$$

Visibility

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Grating, transmission or reflection

$$d(\sin\alpha_2 + \sin\alpha_1) = m\lambda$$
$$d(\sin\alpha_2 - \sin\alpha_1) = m\lambda$$

Max or min in case of interference in thin layers

$$2n_2 d \cos\alpha_2 = m\lambda \qquad \text{där} \qquad m = 0, \pm1, \pm2, ...$$

Finesse in Fabry-Perot interferometer

$$F = \frac{\Delta f}{\delta f} \qquad \text{where} \qquad \Delta f = \frac{c}{2d}$$

Airy Function

$$T = \frac{1}{1 + \left[\frac{4r^2}{(1-r^2)^2}\right] \sin^2\left(\frac{\delta}{2}\right)}$$

Fresnel Diffraction

Fresnel-Kirchhoff

$$E_p = \frac{-ik}{2\pi} E_s e^{-i\omega t} \iint_{Obstacle} F(\theta) \frac{e^{ik(r+r')}}{rr'} \, dA$$

Skewness Factor

$$F(\theta) = \frac{1 + \cos\theta}{2}$$

Raius of Fresnel Zones

$$R_n \approx \sqrt{nL\lambda} \quad \text{where} \quad \frac{1}{L} = \frac{1}{p} + \frac{1}{q}$$

Polarization

Malus Law

$$I = I_0 \cos^2 \theta$$

Phase difference in birefringent material

$$\phi = \frac{2\pi}{\lambda} d|n_e - n_o|$$

Reflection at normal incidence

$$R \equiv \frac{I_{ref}}{I_{in}} = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2$$

Brewster's Angle in Air

$$\theta_{air} = \arctan n$$

Wiens Displacement Law

$$\lambda_{max} T = 2,898 \cdot 10^3 \, \mu m \cdot K$$

Thermodynamics

Thermodynamics

Heat Expansion

$$\frac{\Delta L}{L} = \alpha \Delta T, \quad \frac{\Delta V}{V} = \beta \Delta T, \quad \beta = 3\alpha$$

Heat

$$Q = mc\Delta T, \quad l_s = \frac{Q_s}{m}, \quad l_{\text{å}} = \frac{Q_{\text{å}}}{m}$$

Fluid Pressure

$$p_{tot} = p_{\text{fluid}} + p_{\text{air}} = \rho g h + p_{\text{air}}$$

Ideal Gas Law

$$pV = NkT \quad \text{or} \quad pV = nRT$$

$$\text{where} \quad n = \frac{m_{tot}}{M} = \frac{N}{N_A} \quad \text{and} \quad R = kN_A$$

Gas Density and Particle Density

$$\rho = \frac{m_{tot}}{V} = \frac{pM}{RT}, \quad n_o = \frac{N}{V} = \frac{p}{kT}$$

Barometric Height Formula

$$p = p_0 e^{-\rho_0 g h / p_0}, \quad h = \frac{p_0}{\rho_0 g} \ln \frac{p_0}{p}$$

Relative Moisture

$$R_M = \frac{p_{\text{water}}}{p_{\text{saturation}}}$$

Van der Waal's Equation

$$\left(p + a\frac{n^2}{V^2}\right)(V - nb) = nRT$$

Critical Point

$$V_k = 3nb, \quad T_k = \frac{8a}{27Rb}, \quad p_k = \frac{a}{27b^2}$$

Molecule Radius

$$r = \left(\frac{3b}{16\pi N_A}\right)^{1/3}$$

Vapor Pressure Curve

$$p = Ae^{-Ml_{\text{å}}/(RT)}$$

Reynolds Number

$$Re = \frac{\rho v d}{\eta}, \quad Re < 2300 \text{ laminar}$$

Volume Flow

$$\Phi = \frac{\mathrm{d}V}{\mathrm{d}t} = A_1 v_1 = A_2 v_2$$

Bernoullis Equation

$$p_1 + \frac{\rho v_1^2}{2} + \rho g y_1 = p_2 + \frac{\rho v_2^2}{2} + \rho g y_2$$

Poiseuilles Law

$$\Phi = \frac{\pi R^4}{8\eta} \frac{(p_1 - p_2)}{L}$$

Pressure (Microscopic)

$$p = \frac{2}{3} n_o \frac{m_{\text{en}}}{2} \langle v^2 \rangle = \frac{2}{3} n_o \left\langle W_{\text{kin}} \right\rangle_{\text{en}}$$

Temperature (Microscopic)

$$\left\langle W_{\text{kin}} \right\rangle_{\text{en}} = \frac{3}{2} kT$$

Inner Energy (change)

$$\Delta U = \frac{f}{2} N k \Delta T = \frac{f}{2} n R \Delta T$$

First Theorem

$$Q = \Delta U + W \quad \text{with} \quad W = \int_1^2 p dV$$

Isokor

$$W \equiv 0$$

Isobar

$$W = p\,(V_2 - V_1)$$

Isotherm

$$W = nRT \ln\left(\frac{V_2}{V_1}\right)$$

Adiabat

$$W = -\Delta U$$

Molar Heat Capacity

$$C = Mc, \quad C_V = \frac{f}{2} R, \quad C_p = C_V + R$$

Adiabat(Poissons Equations)

$$T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)}$$
$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

Quotient

$$\gamma \equiv \frac{C_p}{C_V} = \frac{c_p}{c_V} = 1 + \frac{2}{f}$$

Circuit Process

$$Q_{\text{net}} = W_{\text{net}} = \oint p dV$$

Efficiency

$$\eta = \frac{W_{\text{net}}}{Q_{\text{in}}} = \frac{Q_{\text{in}} - |Q_{\text{out}}|}{Q_{\text{in}}} = 1 - \frac{|Q_{\text{out}}|}{Q_{\text{in}}}$$

Ideal Efficiency

$$\eta = \frac{T_{\text{warm}} - T_{\text{cold}}}{T_{\text{warm}}} = 1 - \frac{T_{\text{cold}}}{T_{\text{warm}}}$$

Cold Factor (def. and Ideal)

$$K_f \equiv \frac{Q_{\text{in}}}{|W_{\text{net}}|}, \quad K_f = \frac{T_{\text{cold}}}{T_{\text{warm}} - T_{\text{cold}}}$$

Heat Factor (def. and Ideal)

$$V_f \equiv \frac{Q_{\text{out}}}{|W_{\text{net}}|}, \quad V_f = \frac{T_{\text{warm}}}{T_{\text{warm}} - T_{\text{cold}}}$$

Gauss Distribution

$$f(v_z) = \sqrt{\frac{m_{\text{en}}}{2\pi kT}} e^{-m_{\text{en}} v_z^2/(2kT)}$$

Maxwell-Boltzmann Distribution

$$f(v) = 4\pi v^2 \left(\frac{m_{\text{en}}}{2\pi kT}\right)^{3/2} e^{-m_{\text{en}} v^2/(2kT)}$$

Averages

$$\langle v \rangle = \sqrt{\frac{8kT}{\pi m_{\text{en}}}}, \quad \langle v \rangle = 2\langle |v_x| \rangle$$
$$\langle W_{\text{kin}} \rangle = \left\langle \frac{m_{\text{en}} v^2}{2} \right\rangle = \frac{m_{\text{en}}}{2} \langle v^2 \rangle = \frac{3}{2} kT$$

Collision Number (per second and square meter)

$$n^* = \frac{n_o}{4} \langle v \rangle$$

Mean Free Path

$$l = \frac{1}{n_o \pi d^2 \sqrt{2}}$$

Heat Conduction (General and Rod)

$$P = -\lambda A \frac{\mathrm{d}T}{\mathrm{d}x}, \quad P = \lambda A \frac{T_1 - T_2}{L}$$

Heat Transfer

$$P = \alpha A \Delta T$$

Radiation

$$P_{\text{ideal}} = \sigma A T^4, \quad P_{\text{real}} = e P_{\text{ideal}}$$

Tables

Saturation Pressure for Water

$t/°\mathrm{C}$	Water/kPa
-30	0.0381
-20	0.103
-15	0.165
-10	0.260
-5	0.401
0	0.610
5	0.872
10	1.23
15	1.70
20	2.34
25	3.17
30	4.24
35	5.64
40	7.37
50	12.3
60	19.9
70	31.2
80	47.3
90	70.1
100	101.3
110	143.2
120	198.4
130	270.0

Length expansion coefficient at 20 °C and normal air pressure.

Substance	$\alpha/(10^-6\text{K}^-1)$	Substance	$\alpha/(10^-6\text{K}^-1)$
Aluminum	23	Glass (typical)	6.0
Silver	19	Tungsten	4.3
Brass (Cu + Zn)	19	Marble (typical)	2.5
Copper	17	Invar (Fe + Ni)	2.0
Iron	12	Graphite	2.0
Steel	11	Diamond	1.2
Platinum	9.0	Quartz	0.4

Constants

Constants

Constants

Name	Variable	Value	Unit
Speed of light in a vacuum	c	$299\,792\,458$	m/s
Planks Constant	$\hbar$	$6.626\,070\,15 \cdot 10^{-34}$	Js
Planks Constant	h	$4.135\,667\,87 \cdot 10^{-15}$	eVs
The Elemental Charge	e	$1.602\,176\,634 \cdot 10^{-19}$	C
Bohr Radius	a_0	$5.291\,772\,109\,03 \cdot 10^{-11}$	m
Electron Mass	m_e	$9.109\,383\,7015 \cdot 10^{-31}$	kg
Electron Mass	m_e	$0.510\,998\,954$	MeV/c^2
Proton Mass	m_p	$1.672\,621\,923\,69 \cdot 10^{-27}$	kg
Proton Mass	m_p	$938.272\,096$	MeV/c^2
Proton Mass	m_p	$1836.152\,673\,43$	m_e
Neutron Mass	m_n	$1.674\,927\,498\,04 \cdot 10^{-27}$	kg
Neutron Mass	m_n	$939.565\,428$	MeV/c^2
Neutron Mass	m_n	$1838.683\,661\,73$	m_e
Boltzmanns Constant	k	$1.380\,649 \cdot 10^{-23}$	J/K
Boltzmanns Constant	k	$8.617\,333\,6333 \cdot 10^{-5}$	eV/K
Avogadros Constant	N_A	$6.022\,140\,76 \cdot 10^{23}$	mol^{-1}
Rydbergs Constant	R_y	$\frac{\hbar^2}{2ma_0^2}$	
Rydbergs Constant	R_y	13.6057	eV
Rydbergs Constant	R_y	$109\,737.32$	cm^{-1}
The General Gas Constant	R	$8.314\,462\,618$	J/(mol $\cdot$ K)
The Fine Structure Constant	α	$\frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{137.036}$	
Dielectric Constant for Vacuum	ε_0	$0.885\,419 \cdot 10^{-11}$	As/Vm
Permeability of Vacuum	μ_0	$1.256\,637\,062\,12 \cdot 10^{-6}$	Vs/Am
Permeability of Vacuum	μ_0	$4\pi \cdot 10^{-7}$	Vs/Am
The Bohr Magnetone	μ_B	$\frac{e\hbar}{2m} = 9.274\,010\,0783 \cdot 10^{-24}$	Am^2

Prefix

Prefix

SI-prefix

SI-prefix	Symbol	Decimal
Yotta	Y	$1e24$
Zetta	Z	$1e21$
Exa	E	$1e18$
Peta	P	$1e15$
Tera	T	$1e12$
Giga	G	$1e9$
Mega	M	$1e6$
Kilo	k	$1e3$
Hecto	h	$1e2$
Deca	da	$1e1$
Deci	d	$1e-1$
Centi	c	$1e-2$
Milli	m	$1e-3$
Micro	µ	$1e-6$
Nano	n	$1e-9$
Pico	p	$1e-12$
Femto	f	$1e-15$
Atto	a	$1e-18$
Zepto	z	$1e-21$
Yocto	y	$1e-24$

Periodic Table

Unit Conversion