Waves and Optics

Oscillations

Simple harmonic occilations are described by

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

With real solutions on the form

$$y = A\sin(\omega t + \alpha)$$

Angular Frequency

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Energy for Elastic Pendulum

$$W_{pot} = \frac{ky^2}{2}$$

$$W_{tot} = \frac{m}{2}A^2\omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

Angular Frequency

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Wave Number

$$k = \frac{2\pi}{\lambda}$$

Wave Equation

Progressive Plane Wave

$$s = s_o \sin[2\pi(\frac{t}{T} \pm \frac{x}{\lambda}) + \alpha]$$

Standing Wave Equation

$$s = A\cos\left(2\pi\frac{x}{\lambda} + \frac{\phi}{2}\right)\sin\left(2\pi\frac{t}{T} + \frac{\phi}{2}\right)$$

where ϕ is the phase shift at origo. Node distance is $\frac{\lambda}{2}$

The General Wave Equation

$$\frac{\partial^2 s}{\partial t^2} = v^2\frac{\partial^2 s}{\partial x^2}$$

Occilation Frequency

$$f_{\text{occilation}} = |f_1 - f_2|$$

Sound and Doppler Effect

Doppler Effect

$$f_m = f_s\frac{v - v_m}{v - v_s}$$

Supersonic Speed

$$\sin\theta = \frac{v_{sound}}{v_{[planar/[plan]]}} = \frac{1}{M\alpha}$$

Compressibility coefficient

$$\kappa = -\frac{1}{\Delta P}\cdot\frac{\Delta V}{V}$$

Sound Pressure

$$p = -\frac{1}{\kappa}\cdot\frac{\partial s}{\partial x}$$

$$p = \mp p_0\cos\left[2\pi\left(\frac{t}{T} \pm \frac{x}{\lambda}\right)\right]$$

Pressure Amplitude

$$p_0 = \frac{2\pi s_0}{\kappa\lambda} = Zs_0\omega$$

Acoustic Impedance

$$Z = \rho v$$

Speed of Sound (Fluid and Gas)

$$v = \frac{1}{\sqrt{\kappa\rho}}$$

$$v = \sqrt{\frac{c_p RT}{c_v M}}$$

Speed of Sound (String and Rod)

$$v = \sqrt{\frac{F}{\mu}}$$

$$v = \sqrt{\frac{E}{\rho}}$$

Sound Intensity

$$I = \frac{Z}{2} s_0^2 \omega^2$$

$$I = \frac{p_0^2}{2Z}$$

Sound Intensity Level

$$L_I = 10lg\frac{I}{I_0}$$

$$\text{med } I_0 = 1,0 \cdot 10^{-12}\, W/m^2$$

Refraction and Transmittance of Sound

$$R \equiv \frac{I_r ef}{I_i n} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

$$T \equiv \frac{I_t r}{I_i n} = 1 - R$$

Harmonics (Strings and Open Cylinders)

$$f_m = m \cdot f_1 \quad m = 2, 3, 4, ...$$

Harmonics (Half Open Cylinders)

$$f_m = (2m-1) \cdot f_1 \quad m = 2, 3, 4, ...$$

Light

Speed of Light

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$v = \frac{c}{\sqrt{\mu_r \epsilon_r}}$$

Intensity EM-Wave

$$I = \frac{1}{2}\sqrt{\frac{\epsilon_0 \epsilon_r}{\mu_0 \mu_r}} E_0^2 \ , \quad B_z = \frac{E_y}{v}$$

Intensity when two waves are added

$$I_{tot} = I_1 + I_2 + 2\sqrt{I_1 I_2} < \cos\delta >$$

where δ is the relative phase between the waves.

Refractive Index

$$n \equiv \frac{c}{v} = \sqrt{\mu_r \epsilon_r}$$

Snell's Law

$$\frac{\sin\alpha_1}{\sin\alpha_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

Boundary Angle for Total Reflection

$$\alpha_g = \arcsin\left(\frac{n_2}{n_1}\right)$$

Prism

$$\sin\left(\frac{A+\delta}{2}\right) = n \cdot \sin\left(\frac{A}{2}\right)$$

Where A is the prisms top angle and δ the reflection angle.

Fiber Optics, Numerical Aperture

$$N.A. \equiv n_0 \sin\theta_m$$

$$N.A. = \sqrt{n_1^2 - n_2^2}$$

Material Properties for Sound and Light

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Speed of Sound at 1 atm and 20 °C:

Iron	5950 m/s
Glass (Approx)	5600 m/s
Copper	4760 m/s
Lead	2160 m/s
Rubber	1550 m/s
Water	1461 m/s
Mercury	1407 m/s
Methanol	1143 m/s
Ether	1032 m/s
Hydrogen	1286 m/s
Helium	1008 m/s
Air	343 m/s
Oxygen	326 m/s
Carbon dioxide	269 m/s

Aucustic Impedance at 1 atm and 20 °C:

Hydrogen Gas	111 Ns/m^3
Air	412 Ns/m^3
Water	$1,46 \cdot 10^6$ Ns/m^3
Rubber	$1,47 \cdot 10^6$ Ns/m^3
Glycerin	$2,42 \cdot 10^6$ Ns/m^3
Quarts	$13,1 \cdot 10^6$ Ns/m^3
Glass (Approx)	$14 \cdot 10^6$ Ns/m^3
Aluminum	$17,3 \cdot 10^6$ Ns/m^3
Mercury	$19,1 \cdot 10^6$ Ns/m^3
Copper	$33,9 \cdot 10^6$ Ns/m^3
Steel	$46,4$ Ns/m^3
Tungsten	$101 \cdot 10^6$ Ns/m^3

Vacuum Wavelengths and Frequencies of Light:

Color	Wavelength	Frequency
Violet	$400 - 440$ nm	$749 - 681$ THz
Blue	$440 - 480$ nm	$681 - 625$ THz
Green	$480 - 560$ nm	$625 - 535$ THz
Yellow	$560 - 590$ nm	$535 - 508$ THz
Orange	$590 - 620$ nm	$508 - 484$ THz
Red	$620 - 700$ nm	$484 - 428$ THz

Geometrical Optics

Refraction in spherical surface

$$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R}$$

Gauss Formula

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

Lateral Enlargement

$$M \equiv \frac{y_b}{y_a} \qquad M = -\frac{b}{a}$$

Focal Length Curved Mirror

$$f = -\frac{R}{2}$$

Refractive Power (Lens)

$$B \equiv \frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Lens

Lens with refractive index n_1 in medium with refractive index n_2:

$$B \equiv \frac{1}{f} = \left[\frac{n_1}{n_2} - 1\right] \cdot \left[\frac{R_2 - R_1}{R_1 \cdot R_2}\right]$$

Aparture Number

$$b_t \equiv \frac{f}{D}$$

Depth of Field

$$s \approx \frac{a^2}{1000f} b_t$$

Angular Magnification of Magnifier

$$G = \frac{d_0}{f} \quad \text{where,} \quad d_0 = 25\,\text{cm}$$

Angular Magnification of Microscope

$$G = |M_{ob}| \cdot G_{ok} = \frac{L}{f_{ob}} \frac{d_0}{f_{ok}}$$

where the tube length $L = 16\,\text{cm}$

Angle magnification of the Kepler and Galileo binoculars

$$G = \left| \frac{f_{ob}}{f_{ok}} \right|$$

Refraction in a spherical surface

Positive if: C is to the right of O
- Positive if: A is to the left of O
- Positive if: B is to the right of O
- Positive if: F_A is to the left of O
- Positive if: F_B is on the right of O
- Image with thin lens in air
- Positive if: the lens is convex (gathers light)
- Positive if: the object is to the left of the lens
- Positive if: the image is to the right of the lens
- Positive if: the object is above the optical axis
- Positive if: the image is above the optical axis
- Positive if: the image is upside up
- Image with a curved mirror
- Positive if: C is to the right of O (convex)
- Positive if: F is to the left of O (concave)
- Positive if: A is to the left of O
- Positive if: B to the left of O
- Positive if: the image is upside up

Refractive Index for Some Materials

Refractive Index with $\lambda = 589\,\text{nm}$ at 20 °C:

Water	1,333
Diethyl Ether	1,353
Ethanol	1,361
Glycerin	1,455
Benzene	1,501
Carbon Sulfur	1,628
Is (0 °C)	1,31
NaCl	1,544
Polystyrene	1,59
Crown Glass (FK5)	1,487
Crown Glass (BK7)	1,517
Canada balsam	1,542
Flint Glass (F2)	1,620
Flint Glass (SF10)	1,728
Flint Glass (SFS1)	1,922
Quarts	1,458
Plexiglass	1,49 − 1,52
Diamond	2,417

Diffraction and Interferance

Intensity when Diffraction

$$I = I_0 \left(\frac{\sin\beta}{\beta} \right)^2 \quad \text{with} \quad \beta = \frac{\pi}{\lambda} b \sin\theta$$

Diffraction minimun of slit

$$b \sin\theta = m\lambda \quad \text{where} \quad m = \pm1, \pm2, \pm3, ...$$

Diffraction minimum of round opening

$$D \sin\theta = k\lambda$$

$$\text{where} k = 1,22 \quad 2,23 \quad 3,24 \quad 4,25 \quad 5,25...$$

Rayleigh's Resolution Criterion

Central top for the first point over the first min for the second point

Interference if Diffraction is neglected

$$I = I_0 \left(\frac{\sin N\gamma}{\sin \gamma} \right) \qquad \text{där} \qquad \gamma = \frac{\pi}{\lambda} d \sin\theta$$

Interference gives main max if

$$d \sin\theta = m\lambda \qquad \text{där} \qquad m = \pm 1, \pm 2, \pm 3, ...$$

Visibility

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Grating, transmission or reflection

$$d(\sin\alpha_2 + \sin\alpha_1) = m\lambda$$
$$d(\sin\alpha_2 - \sin\alpha_1) = m\lambda$$

Max or min in case of interference in thin layers

$$2n_2 d \cos\alpha_2 = m\lambda \qquad \text{där} \qquad m = 0, \pm 1, \pm 2, ...$$

Finesse in Fabry-Perot interferometer

$$F = \frac{\Delta f}{\delta f} \qquad \text{where} \qquad \Delta f = \frac{c}{2d}$$

Airy Function

$$T = \frac{1}{1 + \left[\frac{4r^2}{(1-r^2)^2} \right] \sin^2\left(\frac{\delta}{2}\right)}$$

Fresnel Diffraction

Fresnel-Kirchhoff

$$E_p = \frac{-ik}{2\pi} E_s e^{-i\omega t} \iint_{Obstacle} F(\theta) \frac{e^{ik(r+r')}}{rr'}\, dA$$

Skewness Factor

$$F(\theta) = \frac{1 + \cos\theta}{2}$$

Raius of Fresnel Zones

$$R_n \approx \sqrt{nL\lambda} \qquad \text{where} \qquad \frac{1}{L} = \frac{1}{p} + \frac{1}{q}$$

Polarization

Malus Law

$$I = I_0 \cos^2\theta$$

Phase difference in birefringent material

$$\phi = \frac{2\pi}{\lambda} d |n_e - n_o|$$

Reflection at normal incidence

$$R \equiv \frac{I_{ref}}{I_{in}} = \left(\frac{n_2 - n_1}{n_2 + n_1} \right)^2$$

Brewster's Angle in Air

$$\theta_{air} = \arctan n$$

Wiens Displacement Law

$$\lambda_{max} T = 2,898 \cdot 10^3 \ \mu m \cdot K$$