

Disturbance Calculations

Time independent disturbance:

$$\left. \begin{aligned} (H^0 + H') \psi'_m &= E'_m \psi'_m \\ H^0 \psi_n &= E_n^0 \psi_n \end{aligned} \right\} \Rightarrow$$

$$E'_m = E_m^0 + \langle m | H' | m \rangle + \sum_{n \neq m} \frac{|\langle m | H' | n \rangle|^2}{E_m^0 - E_n^0}$$

$$\psi'_m = \psi_m + \sum_{n \neq m} \frac{\int \psi_n^* H' \psi_m d^3r}{E_m^0 - E_n^0} \psi_n$$

Time dependent disturbance:

$$\left. \begin{aligned} H &= H^0 + H' \\ H^0 &\text{ Time independent} \\ H^0 \psi_n &= E_n^0 \psi_n \\ H \psi' &= i\hbar \frac{\partial}{\partial t} \psi' \end{aligned} \right\} \Rightarrow$$

$$\psi'_m = \sum_n a_{mn}(t) \psi_n$$

$$\dot{a}_{mn} = -\frac{i}{\hbar} e^{-i(E_m - E_n)t/\hbar} \cdot H'_{nm}$$

”Golden Rule”

The transition probability per unit of time $w_{f \leftarrow i}$ for a transition from the state ψ_i to a group of states $F = \{\psi_f\}$ with energy E_f^0 for a system characterized by the state density $\rho(E)$ is given by:

$$w_{f \leftarrow i} = \frac{2\pi}{\hbar} |\langle f | H' | i \rangle|^2_{E_f^0 \approx E_i^0} \cdot \rho(E_f^0)$$

Dispersion (Born Approximation)

$$\frac{d\sigma}{d\Omega} = |f(\xi, \eta)|^2$$

$$f(\xi, \eta) = \frac{m}{2\pi\hbar^2} \int e^{i(\mathbf{k}_i - \mathbf{k}_f) \cdot \mathbf{r}} \cdot v(\mathbf{r}) d^3r$$

For spherical symmetrical potential:

$$f(\xi, \eta) = \frac{2m}{\hbar^2 K} \int_0^\infty \sin(Kr) \cdot r \cdot v(r) dr, \quad |K| = 2k \cdot \sin\left(\frac{\xi}{2}\right)$$

Spherical box-potential:

$$V(r) = \begin{cases} -V_0 & r \leq a \\ 0 & r > a \end{cases}$$

$$f(\xi, \eta) = -\frac{2mV_0}{\hbar^2} \cdot \frac{\sin(Ka) - Ka \cos(Ka)}{K^3}$$

Screened Coulomb Potential:

$$v(r) = -\frac{A}{r} \cdot e^{-\alpha r}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{2mA}{\hbar^2 (\alpha^2 + 4k^2 \sin^2(\xi/2))} \right)^2$$

$$\sigma = \left(\frac{Am}{\hbar^2} \right)^2 \frac{16\pi}{\alpha^2 (\alpha^2 + 4k^2)}$$

$$\text{When } \alpha \rightarrow 0, \quad \frac{d\sigma}{d\Omega} \rightarrow \left(\frac{Am}{\hbar^2} \right)^2 \frac{1}{4(k \sin(\xi/2))^4}$$

Periodic Potential

$$V(x) = \begin{cases} 0 & n(a+b) < x < n(a+b) + a \\ V_0 & n(a+b) + a < x < (n+1)(a+b) \end{cases}$$

Continuity Requirements:

$$\cos k_1 a \cdot \cos k_2 b - \frac{k_1^2 + k_2^2}{2k_1 k_2} \sin k_1 a \cdot \sin k_2 b = \cos(k(a+b)), \quad V_0 < E$$

$$\cos k_1 a \cdot \cosh \kappa b - \frac{k_1^2 + \kappa^2}{2k_1 \kappa} \sin k_1 a \cdot \sinh \kappa b = \cos(k(a+b)), \quad V_0 < E$$

Phase and group speed:

$$v_f = \frac{\omega}{k}, \quad v_g = \frac{d\omega}{dk} = \frac{dE}{dp}$$

Effective mass:

$$m^* = \left(\frac{1}{\hbar^2} \frac{d^2 E}{dk^2} \right)^{-1}$$