1.

- (a) By looking at each relation in the schema:
 - (i) BH+=BHADGEFC, thus BH is a superkey and BH->AD doesn't violate BCNF
 - (ii) D+=DBHAGEFC, thus D is also a superkey and D->BH doesn't violate BCNF
 - (iii) BCE⁺=BCEF, therefore BCE->F violates BCNF
 - (iv) F⁺=FC, therefore F->C violates BCNF
 - (v) A+=AGEFC, therefore A->GEF violates BCNF

(b)

(i)Decompose R using A->GEF where A⁺=AGEFC This yields R1=ACEFG and R2=ABDH Project the FDs onto R1=ACEFG

А	С	E	F	G	Closure	FDs
1					A⁺=ACEF G	A->CEFG
	1				C+=C	Nothing
		1			E+=E	Nothing
			✓		F ⁺ =CF	F->C violates BCNF

- (ii)Decompose R1 using F->C. This yields R3=CF and R4=AEFG
- (iii)Project the FDs onto R3=CF

С	F	Closure	FDs
✓		C+=C	Nothing
	1	F ⁺ =CF	F->C. F is a superkey of R3
1	1	CF ⁺ =CF	Nothing

This relation satisfies BCNF.

(iv)Project the FDs onto R4 = AEFG

Α	E	F	G	Closure	FGs
✓				A ⁺ =AGEFC	A->GEF. A is a superkey of R4
	1			E+=E	nothing
		√		F ⁺ =FC	nothing
			√	G⁺=G	nothing
Supersets of A			Irrelevant	Can only generate weaker FDs than what we have already	
	1	√		EF⁺=EFC	nothing
	✓		√	EG⁺=EG	nothing
		√	√	FG⁺=FGC	nothing
	1	1	1	EFG⁺=EFGC	nothing

This relation satisfies BCNF.

(v) R2=ABDH, project the FDs onto R2=ABDH

Α	В	D	Н	Closure	FDs
✓				A⁺=ACGEF	Nothing
	1			B⁺=B	Nothing
		1		D+=DBHA	D->ABH. D is a superkey of R2
			1	H⁺=H	Nothing
1	✓			AB⁺=ABGEF C	Nothing
1			1	AH⁺=AHGEF C	Nothing
	1		1	BH ⁺ =BHADG EFC	BH->AD. BH is a superkey of R2
Supersets of D			Irrelevant	Can only generate weaker FDs than what we have already.	

This relation satisfies BCNF

(vi) Final decomposition:

R2=ABDH with FDs D->ABH, BH->AD R3=CF with FD F->C R4=AEFG with A->GEF

2.

(a)

Left	Middle	Right
	ABCDF	EG

 $A^+=A$

B+=B

C+=C

D⁺=ABCDEFG (key)

 $F^+=F$

AB⁺=AB

AC+=AC

AD⁺=ABCDEFG (key)

AF⁺=AF

BC⁺=BC

BD⁺=ABCDEFG (key)

BF⁺=BF

CD⁺=ABCDEFG (key)

CF⁺=CF

DF⁺=ABCDEFG (key)

ABC⁺=ABC

ABD⁺=ABCDEFG (key)

ABF⁺=ABF

BCD⁺=ABCDEFG (key)

BCF⁺=BCF

CDF⁺=ABCDEFG (key)

ABCD⁺=ABCDEFG (key)

ABCF⁺=ABCF

BCDF⁺=ABCDEFG (key)

ABCDF⁺=ABCDEFG (key)

Therefore the keys are D, AD, BD, CD, DF, ABD, BCD, CDF, ABCD, BCDF, and ABCDF.

(b) (i)Set S1 (Simplify to singleton RHS):

FD	Closure	Decision
DBE->F	DBE+=DBECAGF	discard
DBE->C	DBE+=DBEAGCF	discard
CD->A	CD*=CDFABGE	discard
CD->F	CD*=CDABGFE	discard
D->A	D ⁺ =DBGF, there is no way to get A without this FD.	keep
D->B	D ⁺ =DGAF, there is no way to get B without this FD	keep
D->G	D ⁺ =DABFE	keep
BADE->C	BADE ⁺ =BADEFB	keep
ABD->E	ABD+=ABDFG	keep
D->F	D ⁺ =ABDGEC	keep
EF->B	EF ⁺ =EF	keep

```
(ii) Remaining FDs S2
```

1.D->A

2.D->B

3.D->G

4.BADE->C

5.ABD->E

6.D->F

7.EF->B

4.BADE->C

D⁺=ABCDEFG so we can reduce the LHS to D.

5.ABD->E

D⁺=ABCDEFG so we can reduce the LHS to E

7.EF->B

We saw that $E^+=E$ and $F^+=F$, so this FD remains as it is.

(iii) Remaining FDs S3

1.D->A

2.D->B

3.D->G

4.D->C

5.D->E

6.D->F

7.EF->B

FD	Closure	Decision
D->A	There is no way to get A without this FD.	keep
D->B	D⁺=ADGCEFB	discard
D->G	There is no way to get G without this FD.	keep
D->C	There is no way to get C without this FD.	keep
D->E	There is no way to get E without this FD.	keep
D->F	There is no way to get F without this FD.	keep
EF->B	There is no way to get B without this FD.	keep

(iv) No further simplifications are possible, so the following set S4 is a minimal basis. Remaining FDs S4 $\,$

1.D->A

2.D->C

3.D->E

4.D->F

5.D->G

6.EF->B

(c)

D->ACEFG, EF->B

R1(A, C, D, E, F, G), R2(B, E, F)

D is a key of R, thus there is no need to add another relation that includes a key. Therefore the final set of relations is:

(d)

No because there are no FDs that violate BCNF, thus redundancy is not allowed.