Partial Differential Equations

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Chapter 1

Basics

A PDE is an equation relating the partial derivatives of some unknown function
https://www.youtube.com/watch?v=atvw5iseoGQ&list=PLF6061160B55B0203&index=3

Chapter 2

Partial Differential Equations

2.1 Notation

First order differential of u w.r.t t

$$\frac{du}{dt} \tag{2.1}$$

Second order differential of u w.r.t t

$$\frac{d^2u}{dt^2} \tag{2.2}$$

Laplace function

$$\Delta u(x) = \sum_{i=0}^{n} \frac{d^2u}{dx_i^2}(x) = 0 \tag{2.3}$$

2.2 Definitions

2.3 Equations

2.3.1 Heat Equation

$$\frac{du}{dt} = \frac{d^2u}{dx^2} \tag{2.4}$$

Example

Is he following equation a solution to the heat equation

$$u = \frac{1}{2}x^2 + t$$

Check

$$\frac{du}{dt} = \frac{d}{dt}\frac{1}{2}x^2 + \frac{d}{dt}t$$

$$\frac{du}{dt} = 1$$

$$\frac{du}{dx} = \frac{d}{dx}\frac{1}{2}x^2 + \frac{d}{dx}t$$

$$\frac{du}{dx} = x$$

$$\frac{d^2u}{dx^2} = 1$$

$$\therefore \frac{du}{dt} = \frac{d^2u}{dx^2}$$

Family of Solutions

The following equation can generate an infinite number of solutions to the heat equation.

$$u = e^{ax+bt}$$

We can determine what a and b can represent as follows

$$\begin{aligned}\frac{du}{dt} &= e^{ax+bt}\frac{d(ax+bt)}{dt} \\ &= e^{ax+bt} \cdot b\end{aligned}$$

$$\begin{aligned}\frac{du}{dx} &= e^{ax+bt}\frac{d(ax+bt)}{dx} \\ &= e^{ax+bt} \cdot a \\ \frac{d^2u}{dx^2} &= e^{ax+bt} \cdot a^2\end{aligned}$$

$$\begin{aligned}e^{ax+bt} \cdot b &= e^{ax+bt} \cdot a^2 \\ \therefore b &= a^2\end{aligned}$$

2.3.2 Wave Equation

$$\frac{d^2u}{dt^2} = t^2\frac{d^2u}{dx^2} \tag{2.5}$$

Derivation

Model a string being moved up and down in a flat plane. Due to the nature of the string the force acting upon it to return to the y=0 axis is proportional to the "concavity".

Force = F = ma

Since acceleration is the 2nd derivative to position

F = m $\cdot \frac{d^2u}{dt^2}$

Concavity can be modelled as the 2nd derivative of the shape of the string, since we are describing the degree of the curve the string is creating

$\therefore \frac{d^2u}{dx^2}$

Since we now have both sides of the equation but we are missing c^2. We can compare the units each side so far.

where c2 is the velocity of the string

2.3.3 Laplace Equation

2.3.4 Transport Equation

2.4 Basics

? Does a solution exist?
? Is the solution unique?
? Does the solution depend continuously on the data?
? How regular is the solution? Is it continuously differentiable? Or even smooth?