# Experimental Study of Pressure Gradients Occurring During Continuous Two-Phase Flow in Small-Diameter Vertical Conduits

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## ABSTRACT

*A 1,500-ft experimental well was used to study the pressure gradients occurring during continuous, vertical, two-phase flow through 1-in., 1¼-in. and 1½-in. nominal size tubing.*

*The test well was equipped with two gas-lift valves and four Maihak electronic pressure transmitters as well as with instruments to measure the liquid production rate, air injection rate, temperatures and surface pressures.*

*Tests were conducted for widely varying liquid flow rates, gas-liquid ratios and liquid viscosities. From these data, an accurate pressure-depth traverse was constructed for each test in each of the three tubing sizes.*

*From the results of these tests, correlations have been developed which allow the accurate prediction of flowing pressure gradients for a wide variety of tubing sizes, flow conditions, and liquid properties. Also, the correlations and equations which are developed satisfy the necessary condition that they reduce to the relationships appropriate to single-phase flow when the flow rate of either the gas or the liquid phase becomes zero. All the correlations involve only dimensionless groups, which is a condition usually sought for in similarity analysis but not always achieved.*

*The correlations developed in this study have been used to calculate pressure gradients for pipes of larger diameter than those upon which the correlations are based. Comparisons of these calculated gradients with experimentally determined gradients for the same flow conditions obtained from the literature indicate that extrapolation to these larger pipe sizes is possible with a degree of accuracy sufficient for engineering calculations. The extent of this extrapolation can only be determined with additional data from larger pipe diameters.*

## INTRODUCTION

The accurate prediction of the pressure drop expected to occur during the multiphase flow of fluids in the flow string of a well is a widely recognized problem in the petroleum industry. The problem has been brought even more into prominence with the advent of tubingless or slim-hole completions which use small-diameter tubing. Many of the correlations which give reasonably accurate results in the larger tubing sizes are greatly in error when applied to small-diameter conduits. Small-diameter conduits are defined as 1½-in. nominal size tubing or smaller.

The study of the pressure gradients which occur during multiphase flow of fluids in pipes is exceedingly complex because of the large number of variables involved. Further difficulties relate to the possibility of numerous flow regimes of widely varying geometry and mechanism and the instabilities of the fluid interfaces involved. Consequently, a solution to the problem by the approach normally used in classical fluid dynamics based on the formulation and solution of the Navier-Stokes equation has not been forthcoming. This is primarily the result of the nonlinearities involved and the difficulty of adequately describing the boundary conditions. As a result of the foregoing, most investigators have chosen semi-empirical or purely empirical approaches in an effort to obtain a practical solution to the problem.

Much of the previous work in this area was done in short-tube models in the laboratory. A number of problems arise, however, when attempts are made to extrapolate these laboratory results to oilfield conditions where a much longer tube is encountered. In those few studies where data taken in long tubes were utilized, the data covered only a limited range of the variables, and as a result inaccuracies are introduced when the correlations are extended outside the range of the original data. Also, as a consequence of the limited amount of data available for these studies, the effects of several important variables were overlooked.

The problem of predicting the pressure drop which occurs in multiphase flow differs from that of single-phase flow in that another source of pressure loss is introduced, namely, those pressure losses arising from slippage between the phases. This slippage is a result of the difference between the integrated average linear velocities of the two phases, which in turn is due to the physical properties of the fluids involved. In contrast to single-phase flow, the pressure losses in multiphase flow do not always increase with a decrease in the size of the conduit or an increase in production rate. This is attributed to the presence of the gas phase which tends to slip by the liquid phase without actually contributing to its lift.

Many investigators have attempted to correlate both the slippage losses and the friction losses by means of a single energy-loss factor analogous to the one used in the single-phase flow problem.[1, 4-7, 16, 20] In an approach of this type, however, many of the important variables, such as

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[1]References given at end of paper.

the gas-liquid ratio, the liquid viscosity and the surface tension, are not adequately accounted for. Also, the correlations are not general, and a large number of correlations would be required to cover the range of conditions encountered in oilfield practice.

Other investigators have chosen to measure the liquid holdup, i.e., the fractional volume of the conduit actually occupied by liquid, by various means.[10,11,13,14,18] This allowed them to correct the static gradient portion of the total gradient for the effects of slippage. The remaining losses were attributed to friction, and friction factors were calculated from the test data. These friction factors were then correlated with various groups. When these correlations are applied to data taken in a long tube, however, the calculated pressure losses are much greater than those actually observed, and, in fact, in many cases those losses calculated by the corrected static gradient alone exceed the observed total pressure losses. This would indicate that the fractional volume of the tube actually occupied by liquid is smaller for the long tube than for the short tube, or that the flow is more efficient in the long tube.

The object of the present study was to obtain data from tests conducted in a long tube and utilize these data to develop correlations which would account separately for the effects of slippage and friction.

The first logical approach would be to perform the same type study on the long tubes as was done in the case of the short-tube models, i.e., measure the liquid holdup in the long tube and correlate these measurements with known flow properties. This would present a great many experimental problems, not to mention the increased expense involved.

An alternate approach to the problem would be to determine a friction-factor correlation based on an analogy with single-phase flow. The friction losses could then be determined using these friction factors, and the difference between the measured total pressure losses and the friction losses could be attributed to the static gradient as increased by slippage between the phases. Holdup factors could then be calculated from the test data. It can be argued at this point that in an approach of this type neither the liquid holdup nor the friction losses are actually measured. However, if a reasonable friction-factor correlation can be determined, a large percentage of the pressure losses calculated by this friction factor will be due to friction. Since the other major source of pressure loss is the static gradient as increased by slippage, this loss will be reflected in the holdup-factor correlation. It is true that some friction losses might be included in the holdup correlation and vice versa. These should be small, and since these factors are experimentally determined, these losses will be accounted for. It would be virtually impossible to separate all forms of losses and develop correlations for each, but an approach of this type will separate the friction losses from those due to liquid holdup to a large degree and will also account for all other losses.

The latter approach is the one which was adopted for the present study. The purpose of the experimental work described in the next section was to obtain the necessary data from long tubes which could then be used in the development of a more generalized correlation.

## EXPERIMENTAL PROCEDURE

The experimental data were taken in a test well located in Dallas, Tex. The test well was utilized to more nearly approach actual field conditions. The test well also had the advantage over an actual field well in that the liquid to be produced could be controlled from the surface, thus allowing different liquids to be tested.

Test conditions were varied in an attempt to study the effects of all the controlling groups. A complete series of tests was run on each of three pipe sizes, namely, 1-in., 1¼-in. and 1½-in. nominal diameter tubing. These three sets of data along with the data taken by Fancher and Brown[4] on 2-in. nominal size tubing made it possible to study the effect of the pipe diameter on the pressure gradients.

Four liquids of widely varying viscosity were tested in the 1¼-in. tubing, and two liquids of different viscosity were tested in the 1½-in. tubing. The effects of the liquid viscosity on the pressure gradients in the 1¼-in. tubing were reported in an earlier paper by the authors.[7] These data were used to determine the effect of liquid properties, primarily the liquid viscosity and the liquid density, on the pressure gradients. The physical properties of these liquids are given in Table 1.

For each liquid in a given pipe size, the liquid rate was varied, and for each liquid rate the gas-injection rate was varied over the complete range made possible by the experimental equipment. These data made it possible to study the effects of the flow parameters on the pressure gradients.

The assumption was made that the amount of air which went into solution during the tests was negligible. This is certainly true in the case of water. The assumption is also believed to be valid in the case of the oils because of the low pressures encountered and the very short contact time even at the highest of these pressures. The widely differing compositions of the air and the oils also tend to decrease the amount of air going into solution. This assumption implies that there was essentially no effect of pressure on the viscosity of the oils. In the application of the correlations, however, these solubility effects are taken into account.

No attempt was made to study specifically the effect of surface tension. The surface tension is included in several of the groups, but additional experiments are needed to determine if it is sufficiently accounted for. In most oilfield situations, as in the case of this study, the surface tension will vary only approximately two-fold, whereas the viscosity of the liquid in this study varied over a hundredfold. Since the present work is directed primarily toward oilfield conditions, the surface tension is not considered to be one of the more important variables. Nemet has also indicated that in short tubes the effects of the surface tension, density ratio, and gas introduction are more important than in long tubes.[15]

The quantity of data taken in an installation of this type is not as great as might be taken in a laboratory model. Nevertheless, the 475 tests run in the test well along with 106 tests reported by Fancher and Brown[4] provided 2,905 pressure points over a wide range of conditions. The data taken as part of this study have been reported elsewhere.[8]

The surface equipment utilized during these tests is shown in Fig. 1. The downhole equipment for the tests involving the 1¼-in. tubing is shown in Fig. 2. The only changes for the other tubing sizes were the locations of the

TABLE 1—PHYSICAL PROPERTIES OF LIQUIDS

| Liquid | Specific Gravity | Surface Tension (dynes/cm) | Viscosity (cp @ 80F) |
|---|---|---|---|
| Water | 1.000 | 72.0 | 0.86 |
| Oil | 0.856 | 33.5 | 10.00 |
| Oil | 0.875 | 34.8 | 35.00 |
| Oil | 0.900 | 36.2 | 110.00 |
| Oil | 0.870 | 34.4 | 30.30 |

downhole equipment, and these locations are given in Table 2. The testing procedure has been described previously.[7,8]

## DEVELOPMENT OF CORRELATIONS

### THE GRADIENT EQUATION

The development of the gradient equation used in the evaluation of the data taken as part of the present study is given in the Appendix. The form of the equation used in the calculation is:

$$144\frac{\Delta p}{\Delta h} = \bar{\rho}_m + \frac{f q_L^2 M^2}{2.9652 \times 10^{11} D^5 \bar{\rho}_m} + \bar{\rho}_m \frac{\Delta\left(\frac{v_m^2}{2g_c}\right)}{\Delta h}, \quad \ldots \ldots \ldots \quad (1)$$

where

$$\bar{\rho}_m = \bar{\rho}_L H_L + \bar{\rho}_g (1 - H_L) \quad \ldots \ldots \ldots \quad (2)$$

The gradient equation does not neglect the contribution of the acceleration gradient to the total pressure gradient. Lubinski has cited a practical example in a discussion of Poettmann and Carpenter's paper for which the loss of pressure due to a change in kinetic energy was appreciable and could not be neglected.[16] Calculations made during the course of this work indicate that under conditions of high mass flow rates and low tubing pressures, the pressure losses as a result of the acceleration gradient may constitute as much as 10 per cent of the total pressure drop near the top of the well. Under these conditions, the change in kinetic energy should not be neglected.

Eqs. 1 and 2 contain two factors which must be determined—the friction factor $f$ and the holdup factor $H_L$. To determine these factors, it is necessary to fix the value of one by some means, and then calculate the value for the other by using Eqs. 1 and 2 and experimental data. The approach taken by previous investigators has been to measure the liquid holdup in the laboratory and correlate it with known fluid, pipe and flow properties. The friction

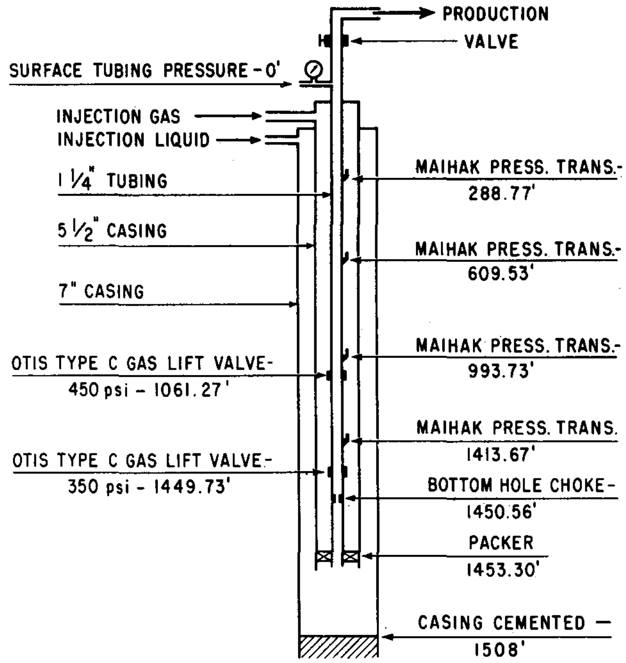


FIG. 2—OTIS EXPERIMENTAL TEST WELL WITH DOWN-HOLE EQUIPMENT.

factors could then be calculated from the experimental data. These values of the holdup, however, appear to be too high when applied to the long tubes encountered in oilfield practice, particularly for small values of the liquid holdup.

The approach used in this study was to develop a means for determining the friction factor and then use this friction factor and Eqs. 1 and 2 to calculate values of the holdup factor from the experimental data. The development of the friction-factor correlation and the holdup-factor correlation will be presented in that order.

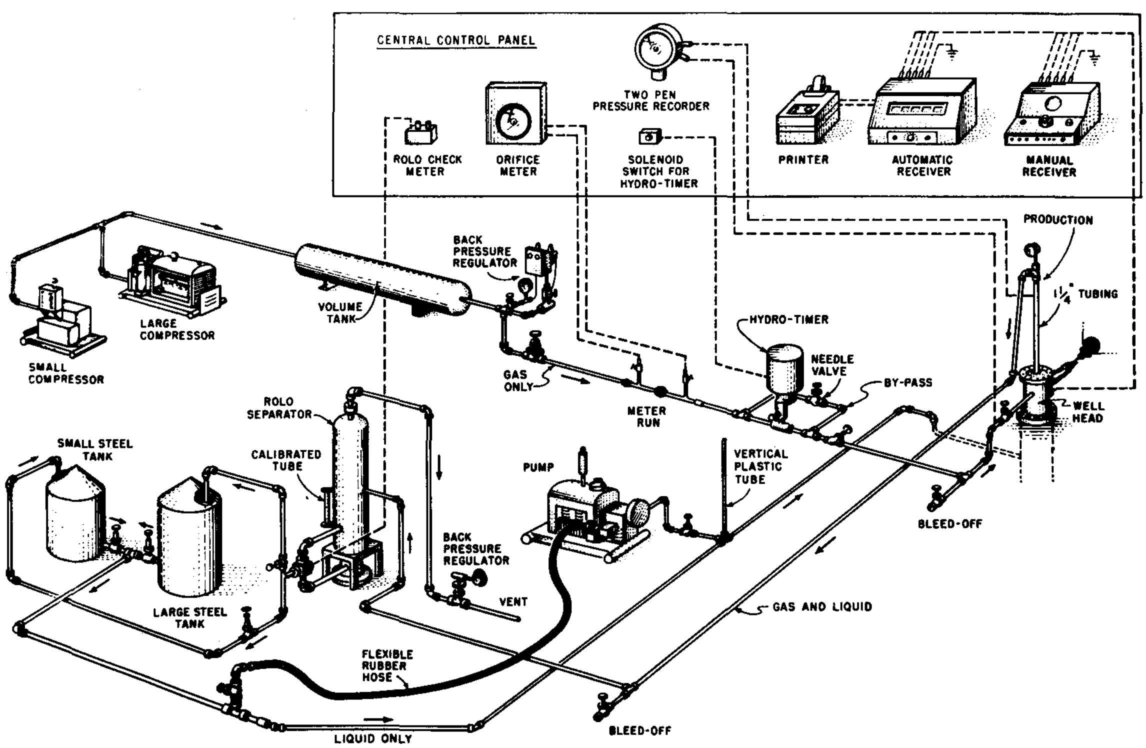


FIG. 1—SURFACE TESTING EQUIPMENT.

TABLE 2—LOCATION OF DOWN-HOLE EQUIPMENT

| Tubing Size (in.) | Maihak Instruments No. 1 (ft) | No. 2 (ft) | No. 3 (ft) | No. 4 (ft) | Gas-Lift Valves No. 1 (ft) | No. 2 (ft) | Bottom-Hole Choke (ft) |
|---|---|---|---|---|---|---|---|
| 1 | 398.01 | 650.10 | 1018.83 | 1428.06 | 1084.37 | 1463.37 | 1499.47 |
| 1¼ | 288.77 | 609.53 | 993.73 | 1413.67 | 1061.27 | 1449.73 | 1450.56 |
| 1½ | 228.43 | 491.58 | 917.91 | 1346.56 | 986.79 | 1383.23 | 1417.18 |

## FRICTION-FACTOR CORRELATION

The friction-factor correlation used in this work is the standard one for single-phase flow in pipes in which the friction factor $f$ is given by

$$f = \frac{2gD\,\Delta W_f}{\bar{v}^2\,\Delta h} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This friction factor is commonly plotted as a function of the Reynolds number $N_{Re}$ with the relative roughness, $\varepsilon/D$, as a parameter. It should be noted that the friction factor $f$ used herein is given by the Darcy-Weisbach equation and differs from the friction factor $f'$ in the Fanning equation by a factor of four, i.e., $f = 4f'$.

Since the assumption was made that over a finite interval, the mixture of liquid and gas can be treated as a homogeneous mixture, the Reynolds number for the mixture can be written as

$$(N_{Re})_{TP} = C_1 \frac{v_m\,\rho_m\,D}{\mu_m} \quad , \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where

$$v_m = \frac{C_2\,q_L + C_3\,q_g}{A_t} \quad , \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$= v_{SL} + v_{SG},$$

$$\rho_m = \rho_L\,H_L + \rho_g(1-H_L) \quad , \quad . \quad . \quad . \quad . \quad (6)$$

and $C_1$, $C_2$ and $C_3$ are the necessary conversion constants for dimensional consistency.

The problem then arises as how best to represent the viscosity of the gas-liquid mixture, $\mu_m$. The simplest assumption would be that the viscosities of the two components should be additive:

$$\mu_m = x\mu_1 + (1-x)\mu_2, \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where $\mu_1$ and $\mu_2$ are the viscosities of the components. $x$ has been expressed as a volume fraction, weight fraction, and molar fraction with no apparent justification for any of these. This would assume an ideal mixture, and no ideal mixtures have been found which will follow this law no matter which way the concentration is expressed.

It has been noted that in real mixtures, the viscosity-concentration curve is convex toward the concentration axis. This behavior was noted also by Uren in his work on the absolute viscosity of a gas-liquid mixture.[21] As the gas-liquid ratio is increased, the viscosity of the mixture rapidly decreases from the viscosity of the liquid and approaches the viscosity of the gas at very high gas-liquid ratios. A similar type of curve is observed when the viscosity of an oil is plotted as a function of the gas in solution.

It would thus seem that to represent the viscosity of a gas-liquid mixture, an equation is necessary which will produce this "sag" in the viscosity-concentration curve. A relationship which exhibits this characteristic behavior is an empirical equation proposed by Arrhenius[9] which appears as follows:

$$\mu_m = \mu_1^{x}\,\mu_2^{1-x} \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where the logarithms of the viscosities of the components are assumed to be additive. A comparison of this equation with the linear equation for the same component viscosities is shown in Fig. 3. The shape of the curve for Eq. 8 is similar to that for the curve determined by Uren. Eq. 8 was used to represent the viscosity of the mixture with $\mu_1 = \mu_L$, $\mu_2 = \mu_g$, and $x = H_L$. In most situations encountered in oilfield practice, a deviation between the actual viscosity of the mixture and the viscosity given by Eq. 8 will not affect the friction-factor determination a great deal, since the Reynolds numbers are in the turbulent region of the friction-factor curve. Only in the case of high liquid viscosities and low liquid rates and gas-liquid ratios would the friction-factor determination be affected by any devation between the actual and calculated viscosities of the mixture. Even here the deviation is small for most practical situations. Any deviation, however, will be included in the calculated holdup factor.

Combining Eqs. 4, 5, 6 and 8, the Reynolds number for the two-phase mixture becomes

$$(N_{Re})_{TP} = \frac{C_1 D}{\mu_L^{H_L}\,\mu_g^{1-H_L}}\,[\rho_L H_L + \rho_g(1-H_L)]\,\frac{(C_2 q_L + C_3 q_g)}{A_t} \quad . \quad . \quad . \quad . \quad (9)$$

If the limit is taken of the Reynolds number for the mixture as $H_L \to 0$, $q_L \to 0$ and $H_L \to 1$, $q_g \to 0$, it reduces to the Reynolds number for single-phase gas or single-phase liquid flow, respectively,

$$\lim_{\substack{H_L \to 0\\ q_L \to 0}} (N_{Re})_{TP} = \lim_{\substack{H_L \to 0\\ q_L \to 0}} \frac{C_1 D}{\mu_L^{H_L}\,\mu_g^{1-H_L}}[\rho_L H_L + \rho_g(1-H_L)]\,\frac{(C_2 q_L + C_3 q_g)}{A_t}, \quad . \quad . \quad . \quad (10)$$

$$= \frac{C_1 D \rho_g (C_3 q_g)}{\mu_g \quad A_t}$$

$$= C_1\,\frac{v_g\,\rho_g D}{\mu_g} = (N_{Re})_g,$$

and

$$\lim_{\substack{H_L \to 1\\ q_g \to 0}} (N_{Re})_{TP} = \lim_{\substack{H_L \to 1\\ q_g \to 0}} \frac{C_1 D}{\mu_L^{H_L}\,\mu_g^{1-H_L}}\,[\rho_L H_L + \rho_g(1 - H_L)]\,\frac{(C_2 q_L + C_3 q_g)}{A_t}$$

$$= \frac{C_1 D \rho_L (C_2 q_L)}{\mu_L \quad A_t}$$

$$= C_1\,\frac{v_L \rho_L D}{\mu_L} = (N_{Re})_L. \quad . \quad . \quad . \quad . \quad (11)$$

For the quantities and units used in the present study, the Reynolds number for the two-phase mixture becomes

$$(N_{Re})_{TP} = 2.2 \times 10^{-2}\,\frac{q_L M}{D\,\mu_L^{H_L}\,\mu_g^{1-H_L}} \quad . \quad . \quad . \quad . \quad (12)$$

The friction-factor correlation[12] used to determine $f$ appears in Fig. 4. The relative roughness is also accounted for in this correlation, although the effect of the relative roughness appears to be very small in two-phase flow.[18]

## HOLDUP-FACTOR CORRELATION

To determine the effect of including the slippage losses in the friction factor, the friction factors for each pressure increment were calculated from the test data assuming no slippage between the phases. The value of "pseudo" holdup factor $H'_L$ can be calculated directly under this condition. These friction factors were then plotted vs the two-phase Reynolds number. The results for water in the 1¼-in. tubing are shown in Fig. 5.

Several interesting observations can be made on a plot of this type. First, at high values of the Reynolds number, or at high mass flow rates, the points approach the curve for single-phase flow through a pipe of the same diameter. At these high mass flow rates, most of the energy losses are the result of friction and could be correlated with the Reynolds number. Second, at the lower mass flow rates where slippage between the phases becomes significant, the points begin to deviate from the curve for single-phase flow. The deviation of the points from the solid curve was thus attributed to the increased liquid holdup as a consequence of slippage. The values of the holdup factor $H_L$ necessary to make the friction factors determined from Fig. 4 and those calculated from Eq. 1 identical were then calculated from the experimental data.

It can be argued at this point that $H_L$ might not represent the actual fraction of the pipe occupied by liquid. This may be true, and the only way to resolve this point would be to measure the liquid holdup in place. To do so would be quite expensive and was therefore not attempted. It is recognized, however, by most investigators in this area that the pressure losses in two-phase flow are the result of two primary causes—friction and liquid holdup. Consequently, if the friction factors as determined from Fig. 4 are a reasonable approximation of the friction losses

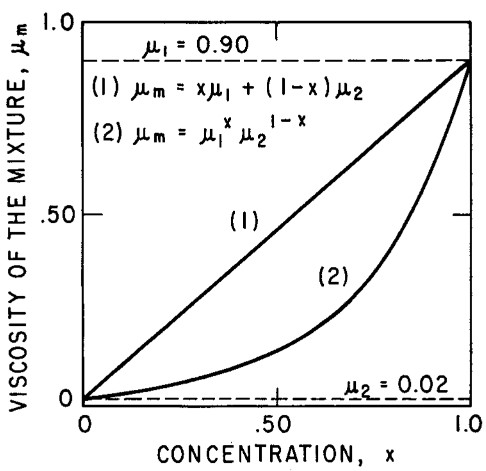


Fig. 3—Comparison of Relationships Predicting Viscosities of Mixtures.

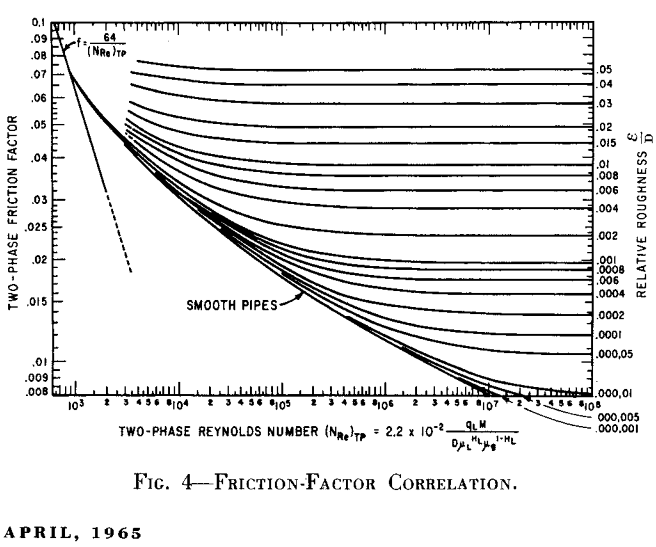


Fig. 4—Friction-Factor Correlation.

then a large percentage of the remaining losses are due to the increased static gradient because of slippage, and should be correlative through the use of a holdup factor.

The holdup factors calculated from the test data were then correlated with known flow, pipe and fluid properties. It can be shown by dimensional analysis[18] that the liquid holdup is related principally to four dimensionless parameters:

$$\left.\begin{array}{lr} N_{LV} = v_{SL}\sqrt[4]{\rho_L/g\sigma} & \text{liquid velocity number} \\ N_{GV} = v_{SG}\sqrt[4]{\rho_L/g\sigma} & \text{gas velocity number} \\ N_D = D\sqrt{\rho_L g/\sigma} & \text{pipe diameter number} \\ N_L = \mu_L\sqrt[4]{g/\rho_L\sigma^3} & \text{liquid viscosity number} \end{array}\right\} \quad (13)$$

For the units given in the nomenclature, these groups become

$$\left.\begin{array}{l} N_{LV} = 1.938\, v_{SL}\sqrt[4]{\rho_L/\sigma} \\ N_{GV} = 1.938\, v_{SG}\sqrt[4]{\rho_L/\sigma} \\ N_D = 120.872\, D\sqrt{\rho_L/\sigma} \\ N_L = 0.15726\, \mu_L\sqrt[4]{1/\rho_L\sigma^3}. \end{array}\right\} \quad . \; . \; . \; . \; . \quad (14)$$

The holdup factor should then be correlative with these four dimensionless groups.

The first step was to find a correlation for water in the 1¼-in. tubing. This was done to determine the effect of the flow parameters $N_{LV}$ and $N_{GV}$. After many attempts, it was determined that the function $N_{LV}/N_{GV}^{\alpha}$ would best correlate the holdup factor. The best value of $\alpha$ was then determined and found to be 0.575.

It was noted that the pressure level was still having some effect on the scatter of data points so a new dimensionless parameter $p/p_a$ was introduced and the correlating function assumed the form

$$(N_{LV}/N_{GV}^{.575})\ (p/p_a)^{\beta}.$$

The best value of $\beta$ was then determined and found to be 0.1. The resulting correlation for 1¼-in. tubing is shown in Fig. 6.

This group would not, however, correlate the data for the 35-cp and 110-cp oil in the 1¼-in. tubing and for the water in the 1-in. tubing. The increased scatter noted in these cases appeared to be dependent on the liquid viscosity, the in-place gas velocity, and the pipe diameter. The observations indicate that a change in the flow pattern in the pipe might be the cause of the deviations.

Figs. 7 and 8 were then prepared for the 35-cp and the 110-cp oils, respectively. The data in Fig. 7 represent a liquid rate of 60 B/D, and the data in Fig. 8 are for a liquid rate of 54 B/D. Similar plots were prepared for other liquid rates, but only the gas velocity in place and not the liquid rates seemed to be affecting the scatter of the points. It is apparent in these figures that the curves for the various gas rates approach some limiting curve at high pressures and low gas rates, each of which reduces the in-place gas velocity, and that this limiting curve was parallel to those for water and the 10-cp oil in 1¼-in. tubing. Also, as the gas rate becomes very large or the pressure becomes very small, i.e., the in-place gas velocity becomes very large, these curves must approach zero since the liquid holdup becomes very small. Consequently, these individual curves must also approach the limiting curve as the in-place gas velocity increases to very large values.

It is postulated that these deviations are the result of the formation of a ring-type or annular flow pattern. As the in-place gas velocity is increased, the gas breaks through the liquid phase with the result that the liquid phase forms a concentric cylinder around the gas phase. As a consequence, slippage is increased. This breakthrough of the gas would explain the rapid increase of the deviations shown in Figs. 7 and 8 as the in-place gas velocity is increased. As the gas velocity is increased even further, the thickness of the liquid cylinder diminishes until the mist flow pattern is formed, where the gas is the continuous phase with the liquid dispersed in the form of small droplets. At this point the deviations would disappear. Cromer has observed the formation of the annular flow pattern during visual studies of vertical two-phase flow in pipes.[3] The pattern was reportedly formed under the same conditions as described above.

The secondary correction factors $\psi$ necessary to account for these deviations were calculated and correlated with the group

$$N_{GV} N_L^{.38} / N_D^{2.14}$$

as shown in Fig. 9. The curve is consistent with the observation that $\psi = 1$ for all data except the three cases upon which this secondary correction factor is based.

The use of this secondary correction factor where conditions make it necessary results in a reversal of the curvature in the calculated pressure-depth traverses near the top of the well. This reversal actually appears in measured traverses,[1,4,7] and the correction factor $\psi$ makes it possible to predict this reversal with a high degree of accuracy.

The curves for water in the four pipe sizes were then plotted as shown in Fig. 10 to determine the effect of pipe diameter. The curves were essentially parallel and by including the pipe diameter number, $N_D$, in the denominator of the correlating function, the curves were shifted the necessary amount to make them coincident.

The curves for the four liquids tested in the 1¼-in. tubing were then plotted as shown in Fig. 11 in an effort to determine the effect of the fluid properties, primarily the liquid viscosity and liquid density. It was apparent that no simple function of the viscosity number would make the correlating function independent of the viscosity number. The curves in Fig. 11 are essentially parallel, so by multiplying each curve by a constant it is possible to make them coincident. The constant, however, is a function of the viscosity number. Therefore, the term $CN_L$ was included in the correlating function. The curve for water was arbitrarily chosen as the base curve, and $C$ was defined to be 1 for water. The values of $C$ necessary to shift each of the curves for the oils until they were coincident with the curve for water were then calculated. The term $CN_L$ was then plotted as a function of $N_L$ as shown in Fig. 12. This curve indicates that for low values

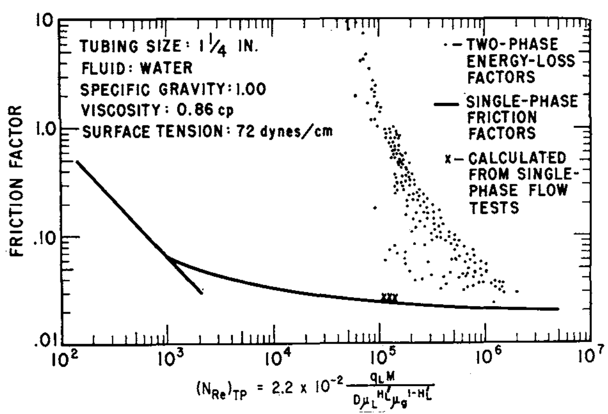


FIG. 5—COMPARISON OF TWO-PHASE ENERGY LOSS FACTORS WITH SINGLE-PHASE FRICTION FACTORS.

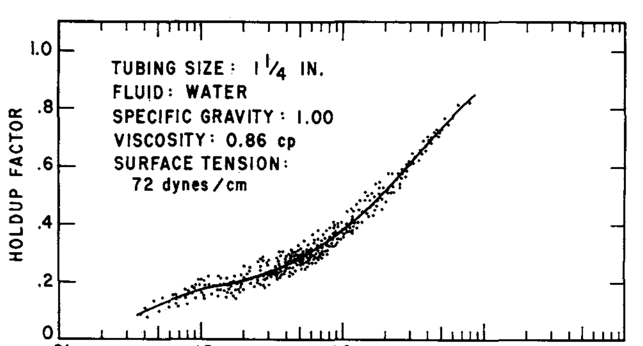


FIG. 6—HOLDUP-FACTOR CORRELATION, 1¼-IN. TUBING.

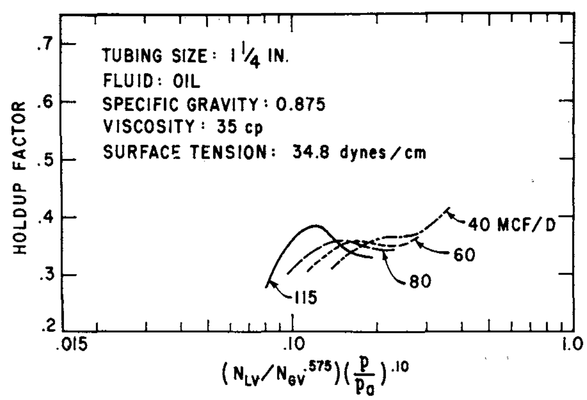


FIG. 7—EFFECT OF GAS VELOCITY ON HOLDUP FACTOR IN HIGH-VISCOSITY OIL.

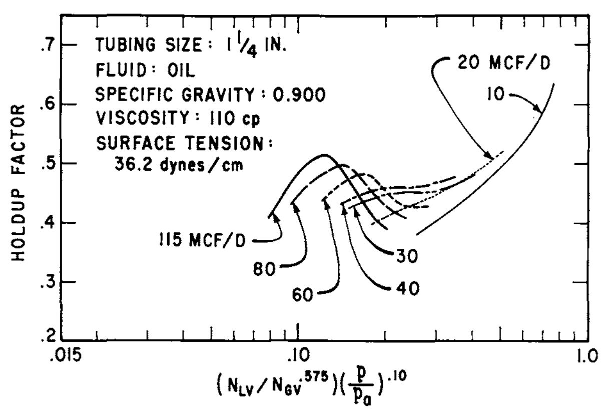


FIG. 8—EFFECT OF GAS VELOCITY ON HOLDUP FACTOR IN HIGH-VISCOSITY OIL.

of the liquid viscosity, the viscosity has very little effect. This has been noted previously by others.[7,18]

The final holdup-factor correlation is presented in Fig. 13. Several important features of the holdup-factor correlation should be noted. First, if the gas rate approaches zero, the value of the correlating function becomes very large, and the value of the holdup factor approaches one. Similarly, as the liquid rate approaches zero, the value of the correlating function becomes very small, and the holdup factor approaches zero. Therefore, as was previously shown, the two-phase Reynolds number becomes the Reynolds number for a single phase, and the gradient equation reduces to the gradient equation describing single-phase pressure gradients as either the liquid or the gas flow rate becomes zero. The holdup-factor correlation is thus consistent with the arguments presented during the development of the gradient equation and the friction-factor correlation.

## CALCULATIONAL PROCEDURE

To construct a pressure-depth traverse for a specific set of flow conditions, it is necessary to solve the finite-difference form of the gradient equation given by Eq. 1. The right side of this equation is a function of both the pressure and the length of the increment of the tube over which the incremental pressure drop $\Delta p$ occurs. If it can be determined that the pressure drop due to a change in kinetic energy is negligible, the last term can be neglected and the solution is simplified to a single trial-and-error; otherwise, it is a double trial-and-error solution. The method of solution presented here is for the form of the gradient equation as it appears in Eq. 1. The step-by-step procedure is as follows:

1. Determine a suitable temperature-depth traverse. A simplified approach assumes a straight-line relationship. A more accurate method involves the calculation of the temperature distribution in the wellbore using the method proposed by Ramey.[17]

2. Beginning with a known pressure and elevation, assume a value for $\Delta p$ and a value for $\Delta h$.

3. Calculate the average pressure and temperature for the assumed increment, and determine $\bar{\rho}_g$ at these conditions. Also, calculate the velocity of the mixture at both ends of the increment using the pressures at those two points and the ratio of the flow rates as measured at the out-flow end of the tube.

4. Calculate a value for $N_L$, and determine a value for $CN_L$ from Fig. 12. In the calculation of $N_L$, the viscosity of the gas-free oil at the average temperature for the increment must be corrected for the effect of solution gas.

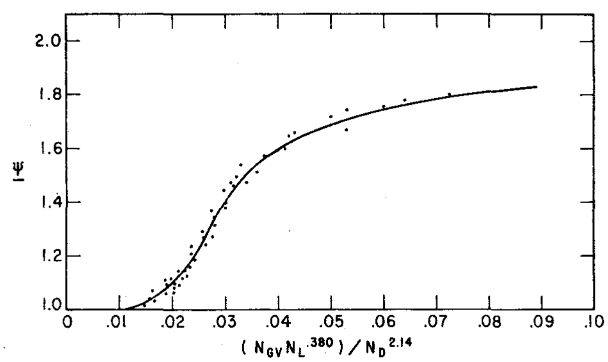


FIG. 9—CORRELATION FOR SECONDARY CORRECTION FACTOR.

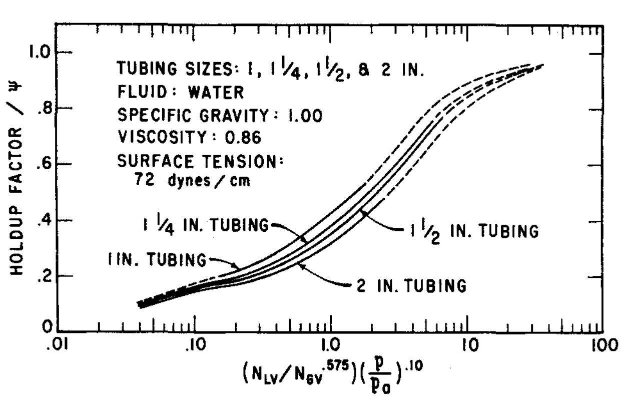


FIG. 10—EFFECT OF PIPE DIAMETER ON HOLDUP-FACTOR CORRELATION.

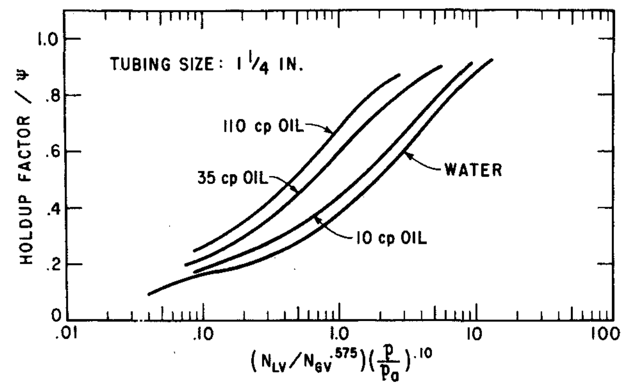


FIG. 11—EFFECT OF FLUID PROPERTIES ON HOLDUP-FACTOR CORRELATION.

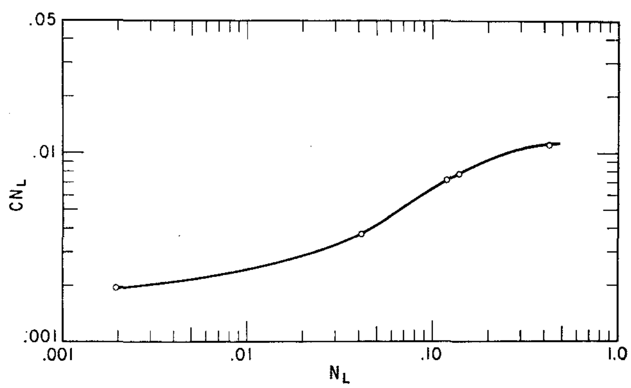


FIG. 12—CORRELATION FOR VISCOSITY NUMBER COEFFICIENT $C$.

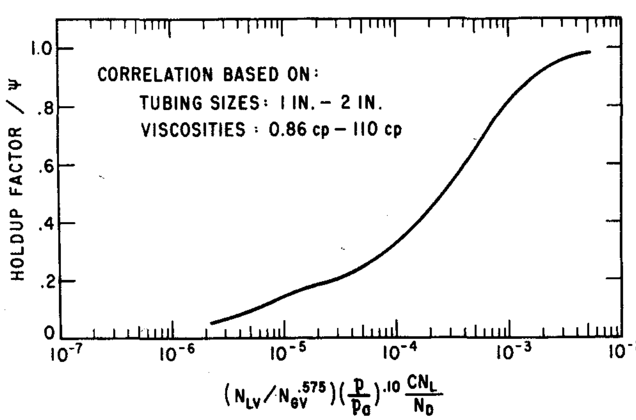


FIG. 13—HOLDUP-FACTOR CORRELATION.

If these data are not available for the specific oil, correlations such as the one developed by Chew and Connally[3] can be used.

5. Calculate the value of the correlating function, and determine the value of the holdup-factor/$\psi$ from Fig. 13. When calculating the superficial liquid and gas velocities, the effect of solution gas must be accounted for. The liquid rate must be multiplied by the formation volume factor for the liquid, and the gas-liquid ratio is decreased by the amount of gas in solution at the average pressure for the increment. This must be done since only free gas should be considered in calculating the superficial gas velocity. The correlating function can be expressed in field units as follows:

$$(N_{LV}/N_{GV}^{.575})(p/p_a)^{.1}(CN_L/N_D) = 0.00326 \times$$

$$\frac{q_L^{.425}\left[\left(\frac{1}{1+\text{WOR}}\right)\text{B}_o+\left(\frac{\text{WOR}}{1+\text{WOR}}\right)B_w\right]\bar{p}^{.675}\,\sigma^{.394}CN_L}{D^{1.850}(\bar{T}\,\bar{Z})^{.575}\bar{\rho}_L^{.394}\left[\text{GLR}-R_s\left(\frac{1}{1+\text{WOR}}\right)\right]^{.575}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

6. Calculate a value for $N_{GV}N_L^{.380}/N_D^{2.14}$. Obtain a value for $\psi$ from Fig. 9 and multiply the value for the holdup-factor/$\psi$ obtained in Step 5 by $\psi$ to obtain the value for the holdup factor.

7. Using the holdup factor from Step 6, calculate a value for the two-phase Reynolds number and the relative roughness ratio $\varepsilon/D$ and obtain a value for the friction factor $f$ from Fig. 4.

8. Calculate $M$ and $\bar{\rho}_m$.

9. Calculate $\Delta p/\Delta h$ from Eq. 1.

10. Calculate $\Delta h$ by dividing the assumed $\Delta p$ by the value of $\Delta p/\Delta h$ from Step 9. If the calculated $\Delta h$ is not the same as the originally assumed $\Delta h$, assume a new value and repeat Steps 3 through 10 until the two values of $\Delta h$ agree with the required accuracy.

The pressure $p+\Delta p$ occurs at depth $h+\Delta h$. A new $\Delta p$ is then assumed, and the procedure is repeated. A pressure-depth traverse can then be plotted for the particular flow conditions, and the pressure at any depth is determined from the curve. These procedures were used to obtain the results which are discussed in the next section.

## RESULTS

To obtain a measure of the accuracy of the correlations developed in the preceding sections, a statistical analysis was performed on the results of the calculations utilizing the data obtained as part of the present study as well as those data reported by Fancher and Brown. The latter were included because the data represented higher tubing pressures and greater gas-liquid ratios than were obtainable in the test well.

All the pressure traverses measured by Baxendell[1] in 2⅞-in. and 3½-in. nominal size tubing, except those which appeared to be heading, were also included in the analysis. These data included production rates as high as 5,082 B/D with a gas-liquid ratio of 723 scf/bbl. Baxendell has also recorded bottom-hole pressures for 29 field wells with depths to 10,774 ft and tubing pressures to 1,000 psia. These data were included in the analysis to see if the correlations could predict the results for conditions so far removed from the test conditions from which they were developed. The results indicate the correlations can be extrapolated to these conditions with a high degree of accuracy.

Gaither, *et al* reported pressure measurements at the ends of 1,000-ft vertical sections of 1-in. and 1¼-in. nominal size tubing over a wide range of flow conditions for gas and water.[5] The bottom-hole pressures were also calculated for the reported conditions using the correlations developed in this paper and compared with the measured bottom-hole pressures.

Since the calculation of the pressure-depth traverses involves an iteration process, any error made in one increment is carried over to all succeeding increments. The maximum error should occur, in most cases, at the deepest calculated point in the well for the particular test. All values, therefore, should be maxima. A negative per cent error indicates the calculated value is too low.

The algebraic average per cent error, or bias, as well as the standard deviation from the algebraic average per cent error were calculated for each set of data as well as for the combined data. The results are shown in Table 3.

For the data presented by Gaither, *et al,* the calculated values are lower than the measured experimental values. The maximum deviations occurred in the 1-in. tubing at very high total fluid production rates. The deviations also increased with decreasing tubing pressure. The possibility exists, however, that the measured bottom-hole pressures are too high as a consequence of end effects at the point of measurement.

## CONCLUSIONS

As a result of the present work, the following conclusions have been reached:

1. Friction factors for two-phase flow can be determined from a conventional friction-factor diagram by defining a Reynolds number for two-phase flow, provided a suitable definition of the holdup factor is made.

2. It is not necessary to separate two-phase flow into the various flow patterns and develop correlations for each. The generalized correlations developed in this work in which no attempt was made to determine the flow patterns provide sufficient accuracy for engineering purposes.

3. In many instances, the pressure loss due to a change in the kinetic energy can account for an appreciable percentage of the total pressure losses, particularly near the top of the well when low tubing pressures are encountered. Under these conditions, the change in kinetic energy should be taken into consideration.

4. The correlations developed as part of this work

TABLE 3—STATISTICAL ANALYSIS OF RESULTS

| Source | Pipe Diameter (in.) | Liquid | Average Per Cent Error | Standard Deviation |
|---|---|---|---|---|
| Hagedorn | 1 | Water | 1.166 | 5.516 |
| Hagedorn | 1¼ | Water | −2.373 | 6.231 |
| Hagedorn | 1¼ | 10 cp Oil | 0.804 | 5.071 |
| Hagedorn | 1¼ | 35 cp Oil | 0.767 | 4.591 |
| Hagedorn | 1¼ | 110 cp Oil | 0.261 | 4.181 |
| Hagedorn | 1½ | Water | −2.329 | 5.154 |
| Hagedorn | 1½ | 30 cp Oil | 1.549 | 5.564 |
| Gaither, *et al.* | 1<br>1¼ | Water | −5.782 | 7.531 |
| Fancher & Brown | 2 | 95% Water<br>5% Oil | 0.538 | 3.697 |
| Baxendell (test data) | 2⅞<br>3½ | 34° API Oil | 1.727 | 4.346 |
| Baxendell (field data) | 2⅞<br>3½ | Oil | −1.373 | 8.801 |
| Combined Data | All above | All above | −1.101 | 6.469 |

appear to be quite general and can be applied over a much wider range of conditions than most correlations presented previously. The correlations involve only dimensionless groups and are consistent with the requirement that the gradient equation for two-phase flow reduce to the gradient equation for single-phase flow when either the flow rate of the gas or the liquid is allowed to approach zero.

## NOMENCLATURE

$A_t$ = cross-sectional area of tubing, sq ft
$B$ = formation-volume factor, bbl/bbl
$C$ = coefficient for liquid viscosity number, –
$D$ = pipe diameter, ft
$f$ = Darcy-Weisbach friction factor, –
$g$ = acceleration of gravity, ft/sec$^2$
$g_c$ = conversion constant equal to 32.174 $lb_m ft/lb_f sec^2$
GLR = gas-liquid ratio, scf/bbl
$h$ = depth, ft
$H_L$ = liquid-holdup factor, –
$M$ = total mass of oil, water and gas associated with 1 bbl of liquid flowing into and out of the flow string, $lb_m$/bbl
$N_D$ = pipe diameter number, –
$N_{GV}$ = gas velocity number, –
$N_L$ = liquid viscosity number, –
$N_{LV}$ = liquid velocity number, –
$N_{Re}$ = Reynolds number, –
$p$ = pressure, psia
$\bar{p}$ = average pressure for increment, psia
$q_g$ = gas production rate, scf/day
$q_L$ = total liquid production rate, B/D
$R_s$ = solution gas-oil ratio, scf/bbl
$T$ = temperature, °R
$\bar{T}$ = average temperature for increment, °R
$v$ = velocity, ft/sec
$\bar{v}$ = average velocity at flowing conditions, ft/sec
$V$ = specific volume of fluid at flowing conditions, cu ft/$lb_m$
$\bar{V}$ = average specific volume at flowing conditions, cu ft/$lb_m$
$V_e$ = volume of pipe element, cu ft
$W_n$ = external work done by the flowing fluid, $lb_f ft/lb_f$
$W_f$ = irreversible energy losses, $lb_f ft/lb_f$
WOR = water-oil ratio, bbl/bbl
$Z$ = compressibility factor for gas, –

SUBSCRIPTS

$a$ = atmospheric
$b$ = base
$g$ = gas
$L$ = liquid
$m$ = mixture
$o$ = oil
$SG$ = superficial gas
$SL$ = superficial liquid
$TP$ = two-phase
$w$ = water

GREEK SYMBOLS

$\alpha$ = arbitrary constant, –
$\beta$ = arbitrary constant, –
$\gamma$ = specific gravity, –
$\Delta$ = difference, –
$\varepsilon$ = absolute roughness, ft
$\mu$ = viscosity, cp
$\rho$ = density, $lb_m$/cu ft
$\bar{\rho}$ = integrated average density at flowing conditions, $lb_m$/cu ft
$\sigma$ = surface tension of liquid-air interface, dynes/cm
$\psi$ = secondary correction factor, –


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## APPENDIX

### GRADIENT EQUATION

The basic flow equation in symbolic differential form based on 1 lb of the flowing fluid is

$$144\frac{g_c}{g}Vdp + dh + \frac{vdv}{g} + dW_f + dW_e = 0. \quad \text{(A-1)}$$

This equation assumes only steady flow and can be made the basis of any fluid-flow relationship.

In this study, the mixture of gas and liquid is treated as a homogeneous mixture of combined properties. Assuming no external work is done by the fluid between Points 1 and 2 of the flow string, the symbolic equation becomes

$$144\frac{g_c}{g}Vdp + dh + \frac{vdv}{g} + dW_f = 0, \quad \text{(A-2)}$$

where $v$ is based on the ratio of fluids entering or leaving the system. By defining a two-phase friction factor similar to the one used in single-phase flow, the two-phase friction factor is given by

$$f = \frac{2gD}{\bar{v}_m^2}\frac{dW_f}{dh}. \quad \text{(A-3)}$$

Substituting Eq. A-3 into Eq. A-2, the basic flow equation for the mixture becomes

$$144\frac{g_c}{g}V_m dp + dh + \frac{v_m dv_m}{g} + \frac{f\bar{v}_m^2 dh}{2gD} = 0, \quad \text{(A-4)}$$

where $\bar{v}_m$ is an average velocity of the mixture whose existence is guaranteed by the theorem of the mean for integrals on the pressure range from $p_1$ to $p_2$. Eq. A-4 can now be integrated from Point 1 to Point 2 to get

$$144\frac{g_c}{g}\int_{p_2}^{p_1} V_m dp + (h_2 - h_1) + \frac{(v_{m_2}^2 - v_{m_1}^2)}{2g} + \frac{f\bar{v}_m^2(h_2 - h_1)}{2gD} = 0. \quad \text{(A-5)}$$

Since $V_m$ is an approximately linear function of pressure over fairly large increments of pressure, the average integrated specific volume of the mixture $\bar{V}_m$ between pressure limits $p_1$ and $p_2$ can be approximated by

$$\bar{V}_m = \frac{\int_{p_1}^{p_2} V_m dp}{\int_{p_1}^{p_2} dp} \quad \text{(A-6)}$$

After substitution, Eq. A-5 becomes

$$144\frac{g_c}{g}\bar{V}_m\Delta p + \Delta h + \Delta\left(\frac{v_m^2}{2g}\right) + \frac{f\bar{v}_m^2\Delta h}{2gD} = 0. \quad \text{(A-7)}$$

The average integrated velocity between Points 1 and 2, $\bar{v}_m$ can be calculated from

$$\bar{v}_m = \bar{v}_{SL} + \bar{v}_{SG} \quad \text{(A-8)}$$

and the velocity of the mixture, $v_m$ at a point is given by

$$v_m = v_{SL} + v_{SG}, \quad \text{(A-9)}$$

The following relationship for the average integrated density of the mixture between Points 1 and 2 is given by the definition of the density of the mixture

$$\bar{\rho}_m = \frac{1}{\bar{V}_m} \quad \text{(A-10)}$$

By employing this substitution, Eq. A-7 becomes

$$144\frac{g}{g}\frac{\Delta p}{\bar{\rho}_m} + \Delta h + \Delta\left(\frac{v_m^2}{2g}\right) + \frac{f\bar{v}_m^2\Delta h}{2gD} = 0. \quad \text{(A-11)}$$

Eq. A-11 may be solved for the pressure gradient, 144 $\Delta p/\Delta h$, and expressed in terms of quantities normally measured in the field as

$$144\frac{\Delta p}{\Delta h} = \bar{\rho}_m + \frac{fq_L^2M^2}{2.9652 \times 10^{11}D^5\bar{\rho}_m} + \bar{\rho}_m\frac{\Delta\left(\frac{v_m^2}{2g}\right)}{\Delta h}, \quad \text{(A-12)}$$

where $g$ is assumed numerically equal to $g_c$ and $\Delta p = p_1 - p_2$. The total mass associated with each barrel of produced liquid is given by

$$M = \left(\frac{1}{1 + \text{WOR}}\right) \times (\gamma_o)(5.61 \times 62.4) + (0.0764)(\gamma_g)(\text{GLR}) + \left(\frac{\text{WOR}}{1 + \text{WOR}}\right)(\gamma_w)(5.61 \times 62.4). \quad \text{(A-13)}$$

Since the average density in-place cannot be calculated directly in view of the slippage which occurs between the phases, it is necessary to introduce the concept of a holdup factor. The holdup factor is theoretically the fractional volume of the conduit actually occupied by the liquid phase. The average density of the mixture in an element of the pipe is then described by

$$\bar{\rho}_m = \frac{\gamma_L\rho_w H_L V_e + \gamma_g\rho_{air}\left(\frac{\bar{p}}{p_b}\frac{T_b}{\bar{T}}\frac{1}{\bar{Z}}\right)V_e(1 - H_L)}{V_e}$$

or

$$\bar{\rho}_m = \bar{\rho}_L H_L + \bar{\rho}_g(1 - H_L). \quad \text{(A-14)}$$

Eq. A-14 can be substituted into Eq. A-11 to give

$$144\frac{\Delta p}{\Delta h} = \frac{g}{g_c}\left[\bar{\rho}_L H_L + \bar{\rho}_g(1 - H_L)\right]\left\{1 + \frac{f(\bar{v}_{SL} + \bar{v}_{SG})^2}{2gD} + \frac{\Delta[(v_{SL} + v_{SG})^2]}{2g\ \Delta h}\right\} \quad \text{(A-15)}$$

Taking the limit as $H_L \to 1$, $\bar{v}_{SG} \to 0$, and $v_{SG} \to 0$, *i.e.*, as the gas rate becomes zero, Eq. A-15 reduces to

$$144\frac{\Delta p}{\Delta h} = \frac{g}{g_c}\bar{\rho}_L + \frac{f\bar{\rho}_L\bar{v}_L^2}{2g_cD} + \bar{\rho}_L\frac{\Delta\left(\frac{v_L^2}{2g_c}\right)}{\Delta h}. \quad \text{(A-16)}$$

since the superficial velocity is the real velocity when only one phase is present.

Eq. A-16 may be recognized as the equation describing the pressure gradients occurring in single-phase liquid flow. Similarly, if the limit of Eq. A-16 is taken as $H_L \to 0$, $\bar{v}_{SL} \to 0$, and $v_{SL} \to 0$, *i.e.*, as the liquid rate becomes zero, the result is the equation describing the pressure gradients which occur in the single-phase gas flow. ★★★