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ME 210

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Abstract

In this project, a steam boiler that transfers heat from hot combustion gases, modeled as air, to a cold water stream was investigated. For this steam boiler, subcooled water entered at 3.5 MPa and 150°C and leaves as superheated vapor at 400°C at the same pressure. Two cases were studied, one in which combustion gases maintain a constant pressure during heat exchange and another in which the combustion gas pressure drops from 200 kPa to 100 kPa. The exergy destruction rate per mass flow rate of water was investigated and the air inlet temperatures and mass flow rates that result in the minimum rate of exergy destruction were determined. It was found that for constant pressure heat exchange process, the minimum exergy destruction rate occurred when the air inlet temperature was as close to 400°C as possible for a given mass flow rate ratio, with much higher mass flow rates of air being required as the air inlet temperature became closer to 400°C. The minimum exergy destruction rate per mass flow rate of water was found to plateau to a value of around 339 kJ/kg as the mass flow rate of air increased. However, the required mass flow rate of air may not be practical when cooling water at high mass flow rates to achieve this low exergy destruction rate per mass flow rate of water in practice. For the case in which the air pressure drops during the heat exchange, it was found that the pressure drop caused increased exergy destruction. This increased exergy destruction due to pressure drop grew in magnitude as the mass flow rate of air increased. This made lower mass flow rates of air desirable for the case with pressure, though the constant pressure heat exchange process was found to always produce lower exergy destruction rates per mass flow rates of water. Providing higher air inlet temperatures than was needed to provide the required heating increased the exergy destruction rate,

so air inlet temperatures should be kept as low as possible for this case while still providing the required heating for both cases.

Introduction

A steam boiler was studied that transfers heat from hot combustion gases to a cold water stream. Subcooled liquid water enters at 3.5 MPa and 150°C and leaves as superheated vapor at 400°C and 3.5 MPa. The combustion gases, modeled as air, enter at a pressure of 200 kPa and either leave at 200 kPa or leave at 100 kPa referred to as case 1 and case 2, respectively. The temperature the combustion gases is an unknown parameter that was used to determine when the minimum exergy loss per mass flow rate of water is observed. The conditions of all inlets and outlets are shown below in Figure 1 and Figure 2 for case 1 and case 2, respectively. A MATLAB program was used to solve the equations for energy balance and exergy balance and determine the optimal temperature for minimum exergy destruction rates per mass flow rate of water.

Figure 1

Diagram of the Heat Exchanger for Case 1

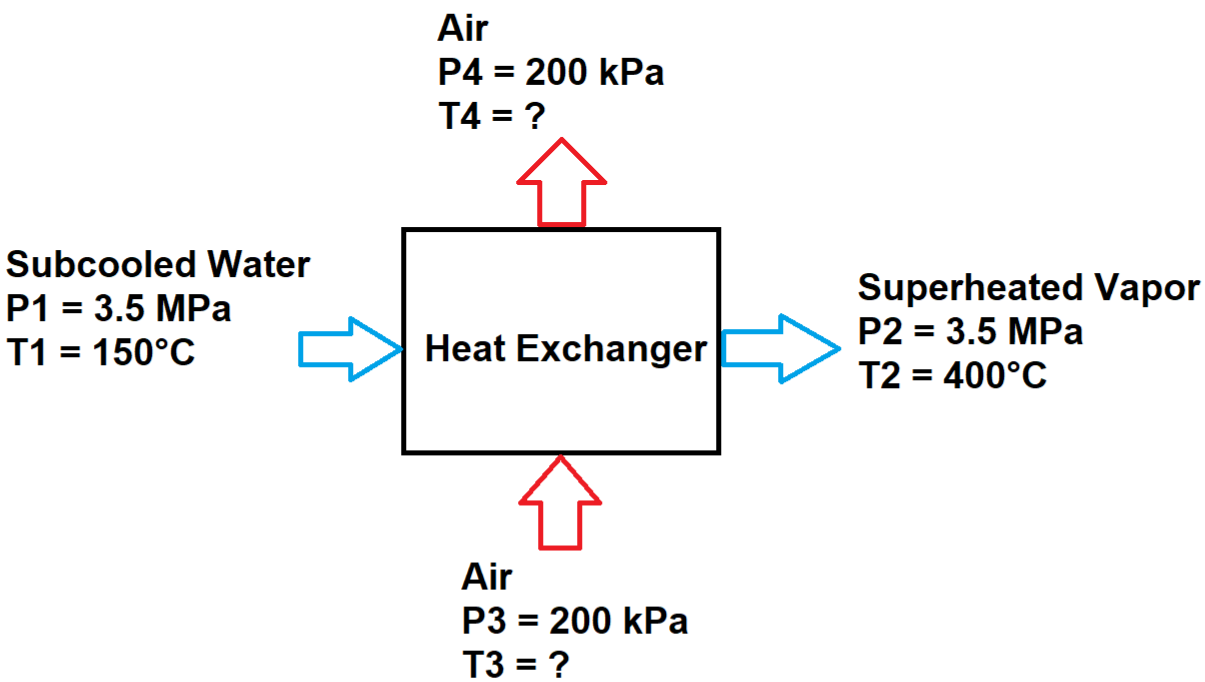
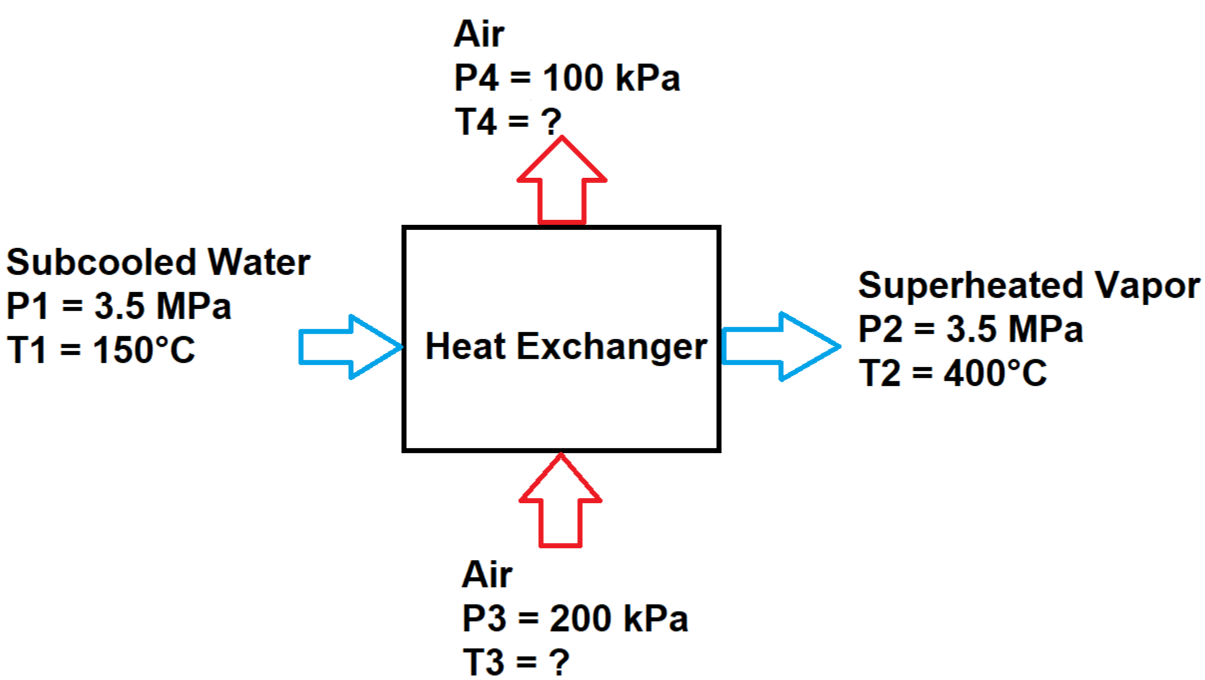


Figure 2

Diagram of the Heat Exchanger for Case 2



Nomenclature

$\dot{m}_w$ = mass flow rate of water

$\dot{m}_{air}$ = mass flow rate of air

$\dot{E}_{in}$ = rate of energy entering the control volume

$\dot{E}_{out}$ = rate of energy entering the control volume

h = enthalpy

$c_{p,air}$ = specific heat of air at constant pressure

$\bar{c}_{p,air}$ = molar specific heat of air at constant pressure

Methodology

Energy Balance

In order to analyze the heat exchanger, the principles of energy balance and exergy balance must be used to ultimately determine the minimum exergy loss per mass flow rate. The enthalpy value for state 1 and state 2 of the water were found to be 631.66 and 3223.2 kJ/kg respectively. Starting with the energy balance:

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\dot{m}_w h_1 + \dot{m}_{air} h_3 - \dot{m}_w h_2 - \dot{m}_w h_4 = 0$$

$$\dot{m}_w (h_1 - h_2) + \dot{m}_{air} (h_3 - h_4) = 0$$

Substituting in the enthalpy values for the water:

$$\dot{m}_w (631.66 - 3223.2) + \dot{m}_{air} (h_3 - h_4) = 0$$

$$\dot{m}_w (-2591.54) + \dot{m}_{air} (h_3 - h_4) = 0$$

Since air is an ideal gas, the change in enthalpy can be calculated by:

$$dh = c_p dT$$

For air the specific heat at constant pressure depends on temperature and can be related to temperature via a third-degree polynomial as shown:

$$\bar{c}_{p,air} = 28.11 + 0.001967T + 0.4003 \times 10^{-5}T^2 - 1.966 \times 10^{-9}T^3$$

Then dividing by the molar mass of air, 28.9647 kg/kmol:

$$c_{p,air} = 0.9705 + 67.91 \times 10^{-6}T + 138.20 \times 10^{-9}T^2 - 67.876 \times 10^{-12}T^3$$

Plugging this into the equation for change in enthalpy gives:

$$dh = 0.9705 + 67.91 \times 10^{-6}T + 138.20 \times 10^{-9}T^2 - 67.876 \times 10^{-12}T^3 dT$$

Integrating this equation between states 3 and 4 results in:

$$(h_3 - h_4) = 0.9705(T_3 - T_4) + 33.955 \times 10^{-6}({T_3}^2 - {T_4}^2) + 46.067 \times 10^{-9}({T_3}^3 - {T_4}^3) - 19.969 \times 10^{-12}({T_3}^4 - {T_4}^4)$$

This value can then be plugged into the equation for enthalpy change and divided by the mass flow rate of water:

$$(-2591.54) + \frac{\dot{m}_{air}}{\dot{m}_w}(h_3 - h_4) = 0 \tag{1}$$

This is the final form of the energy equation that will be used for case 1. Note this equation can be rearranged to solve for the mass flow rate of air such that the mass flow rate of air can be substituted out in terms of the difference of air enthalpies, mass flow rate of water, and the difference in enthalpies of water in the exergy balance equation. This was not done but similar results were obtained regarding what the value of the mass flow rate of air must be to satisfy the energy and exergy balance. Without substituting out the mass flow rate of air, the inlet temperature and mass flow rate of air could be used as parameters that are more realistic to what engineers might have control over when designing the system.

Exergy Balance

Next, the exergy balance equation must be used to determine the rate of exergy destruction.

$$\frac{dX_{CV}}{dt} = \sum \dot{Q}\left(1 - \frac{T_0}{T}\right) + \left(\dot{W} + P_0\frac{dV_{CV}}{dt}\right) + \sum \dot{m}_i\psi_i - \sum \dot{m}_o\psi_o - \dot{X}_{dest}$$

Noting that the flow is steady-state and assuming the heat exchanger is adiabatic and there is no work transfer to the surroundings, the equation can be written as:

$$0 = \dot{m}_w\psi_1 + \dot{m}_{air}\psi_3 - \dot{m}_w\psi_2 - \dot{m}_{air}\psi_4 - \dot{X}_{dest}$$

This can then be rearranged to be:

$$\dot{X}_{dest} = \dot{m}_w(\psi_1 - \psi_2) + \dot{m}_{air}(\psi_3 - \psi_4)$$

The difference in flow exergies can then be determined to be:

$$(\psi_1 - \psi_2) = (h_1 - h_2) - T_0(s_1 - s_2)$$

$$(\psi_3 - \psi_4) = (h_3 - h_4) - T_0(s_3 - s_4)$$

The heat exchanger will be assumed to be surrounded by an environment at 25°C or 298 K. The values for the entropy of the water at state 1 and state 2 are 1.8418 kJ/kgK and 6.8428 kJ/kgK, resulting in:

$$(s_1 - s_2) = -5.001\ \frac{kJ}{kgK}$$

For air, the change in entropy can be calculated as:

$$ds = \frac{c_p}{T}dT - \left(\frac{\partial v}{\partial T}\right)_P dP$$

For case 1, the change in pressure is 0, resulting in the following equation:

$$ds = \frac{c_p}{T}dT$$

Plugging in the equation for the specific heat of air determine earlier results in:

$$ds = 0.9705T^{-1} + 67.91 \times 10^{-6} + 138.20 \times 10^{-9}T - 67.876 \times 10^{-12}T^2 dT$$

Integrating this equation and plugging in the values for state 3 and state 4 results in:

$$(s_3 - s_4) = 0.9705\ln\left(\frac{T_3}{T_4}\right) + 67.91 \times 10^{-6}(T_3 - T_4) + 69.10 \times 10^{-9}({T_3}^2 - {T_4}^2)$$

$$- 22.625 \times 10^{-12}({T_3}^3 - {T_4}^3)$$

Plugging all of these values into the exergy equations results in:

$$(\psi_1 - \psi_2) = -1101.2\ \frac{kJ}{kg}$$

$$(\psi_3 - \psi_4) = (h_3 - h_4) - 298(s_3 - s_4)$$

$$\dot{X}_{dest} = -1101.2\dot{m}_w + \dot{m}_{air}[(h_3 - h_4) - 298(s_3 - s_4)]$$

And then dividing by the mass flow rate of water:

$$\frac{\dot{X}_{dest}}{\dot{m}_w} = -1101.2 + \frac{\dot{m}_{air}}{\dot{m}_w}[(h_3 - h_4) - 298(s_3 - s_4)] \qquad (2)$$

These equations will then be solved using MATLAB to find the minimum value of exergy destruction per mass flow rate of water for a given inlet temperature of combustion gases for case 1. For case 2, the equations for change in entropy and flow exergy must be modified to account for the changes in pressure. Starting with the equation for change in entropy:

$$ds = \frac{c_p}{T}dT - \left(\frac{\partial v}{\partial T}\right)_P dP$$

Using the ideal gas law, it can be shown that:

$$Pv = RT$$

$$v = \frac{RT}{P}$$

$$\left(\frac{\partial v}{\partial T}\right)_P = \frac{R}{P}$$

$$ds = \frac{c_p}{T}dT - \frac{R}{P}dP$$

Next substituting in the equation for specific heat of air:

$$ds = 0.9705T^{-1} + 67.91 \times 10^{-6} + 138.20 \times 10^{-9}T - 67.876 \times 10^{-12}T^2 dT - \frac{R}{P}dP$$

Then integrating the equations between the temperatures and pressures of the inlet and outlet of the combustion gas and noting that R for air is 0.2870 kJ/kgK:

$$s_3 - s_4 = 0.9705\ln\left(\frac{T_3}{T_4}\right) + 67.91 \times 10^{-6}(T_3 - T_4) + 69.10 \times 10^{-9}\left({T_3}^2 - {T_4}^2\right)$$

$$- 22.625 \times 10^{-12}\left({T_3}^3 - {T_4}^3\right) - 0.2870 \ln\left(\frac{P_3}{P_4}\right)$$

This value for entropy change can then be plugged into the earlier equations for change in flow exergy and the exergy destruction like in case 1, resulting in similar equations with additional pressure terms. In order to solve the equations determined above, a MATLAB program was used to solve the equations numerically. The inlet temperature was used as a parameter and the mass flow rate of air was set to a multiple of the mass flow rate of air. The exergy destroyed per mass flow rate of water was determined as a function of the inlet temperature of the combustion gases for a variety of different air mass flow rates.

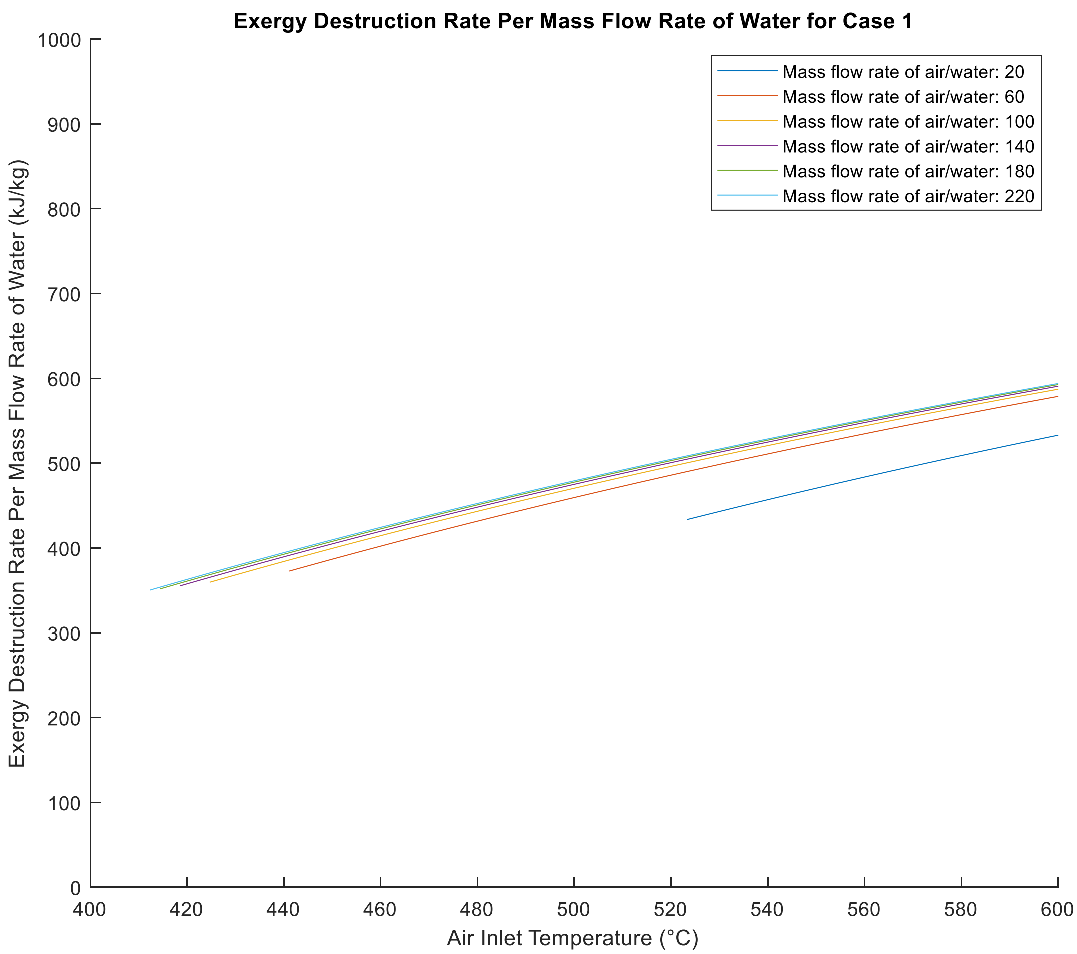
Results and Discussion

Case 1: Constant Air Pressure

For the first case, multiple mass flow rates and inlet temperatures were used to determine the minimum rate of exergy destruction. Plots of exergy destruction versus inlet temperature were generated for each mass flow rate of air tested, as shown in Figure 3. As the mass flow rate of air was increased, it was found that the inlet temperature of air could be decreased and the rate of exergy destruction could be decreased further. The heat exchanger was assumed to be a crossflow heat exchanger, meaning that the outlet temperature for the air could not be lower than the outlet temperature of the water.

Figure 3

Plot of the Exergy Destruction Rate Per Mass Flow Rate of Water for Case 1



It was found the exergy destruction rate reached an asymptotic value as the ratio of the mass flow rate of air to mass flow rate of water continued to increase and the inlet temperature continually decreased and became closer to the 400°C water outlet temperature. As the air inlet temperature decreases, a larger air mass flow rate is needed to achieve the heat transfer needed to increase the temperature of the water. As the mass flow rate of air becomes closer to the mass flow rate of water, the required inlet temperature of air and the minimum exergy destruction increases. The minimum exergy destruction at each air mass flow rate is shown in Table 1, along with the corresponding inlet and outlet temperatures for that case. The plot of exergy destruction at various mass flow rates of air is shown below in Figure 4. Using a very large mass flow rate ratio of 100,000 results in an exergy destruction rate of 339.1348 kJ/kg with an inlet temperature of air of 400.0349°C. For case 1, the minimum exergy loss per mass flow rate of water occurs at the inlet temperature of air closest to 400°C that can be achieved for the ratio of mass flow rates used for the steam boiler. While it is ideal to have the inlet temperature of air as close to 400°C to minimize the exergy destroyed, the required mass flow rate of air necessary to achieve this may be too high for a realistic design in some cases.

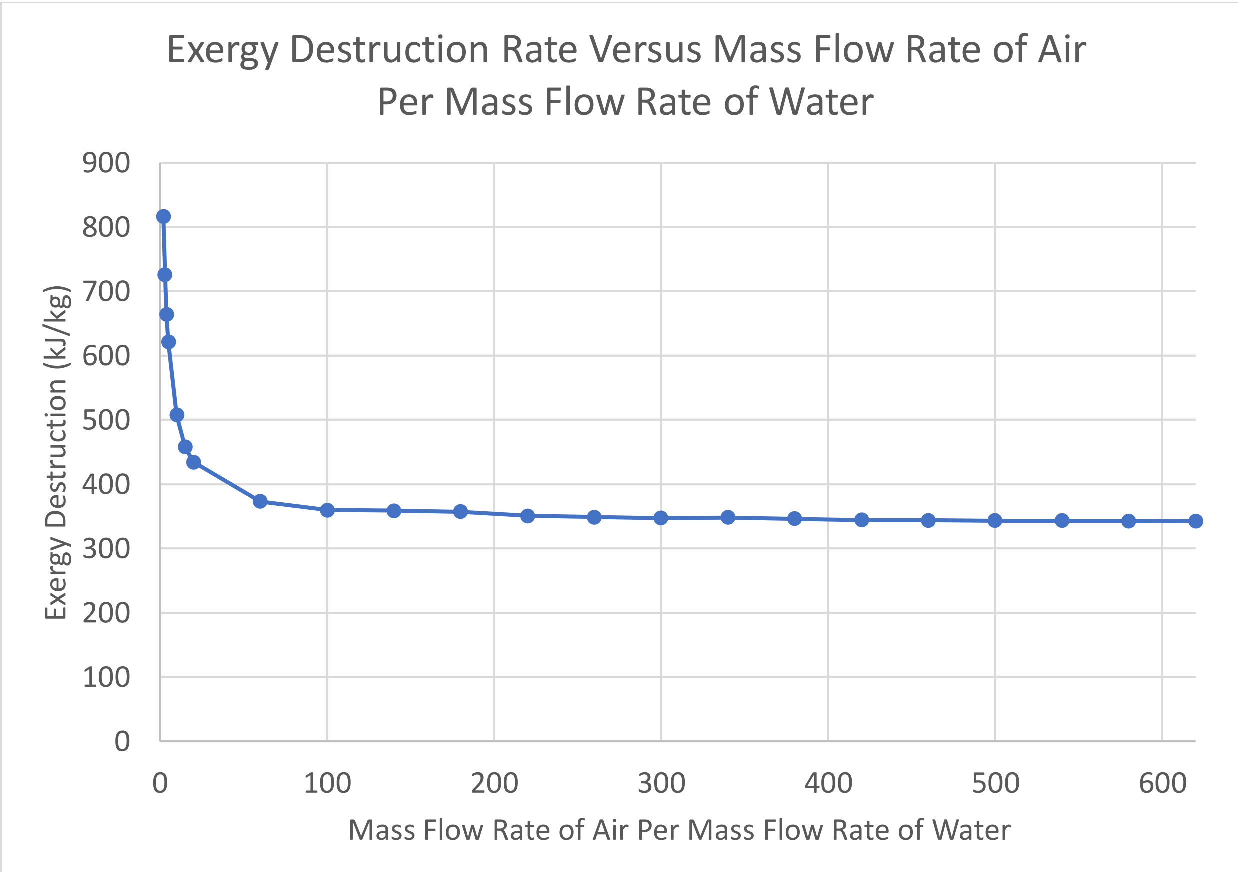
Table 1

Table of the Minimum Exergy Destruction and the Corresponding Inlet Temperature, Outlet Temperature, and Air Mass Flow Rate for Case 1

T_3 (C)	T_4 (C)	$\frac{\dot{m}_a}{\dot{m}_w}$	$\frac{\dot{X}_{dest}}{\dot{m}_w}$
1583.263	401.7285	2	816.1877
1180.728	402.5379	3	725.4527
995.62	401.7337	4	663.7135
880.461	403.0218	5	620.993
643.767	400.129	10	507.808
562.391	400.3198	15	458.244
524.1203	402.2803	20	434.6441
441.3734	400.5298	60	373.1952
424.8241	400.2898	100	359.8534
420.6867	403.1607	140	358.8731
416.5494	402.9139	180	357.3492
412.412	401.2513	220	350.5222
411.2399	400.0774	260	348.982
409.6342	400.188	300	347.3336
409.6342	401.4482	340	348.388
408.0825	400.8045	380	346.5147
406.1177	400.2687	420	344.4705
405.438	400.0973	460	343.7574
405.0981	400.1846	500	343.5461
404.7582	400.2086	540	343.2814
404.4184	400.1823	580	342.9743
404.0785	400.1157	620	342.6328

Figure 4

Plot of the Exergy Destruction Rate Versus Mass Flow Rate of Air Per Mass Flow Rate of Water for Case 1

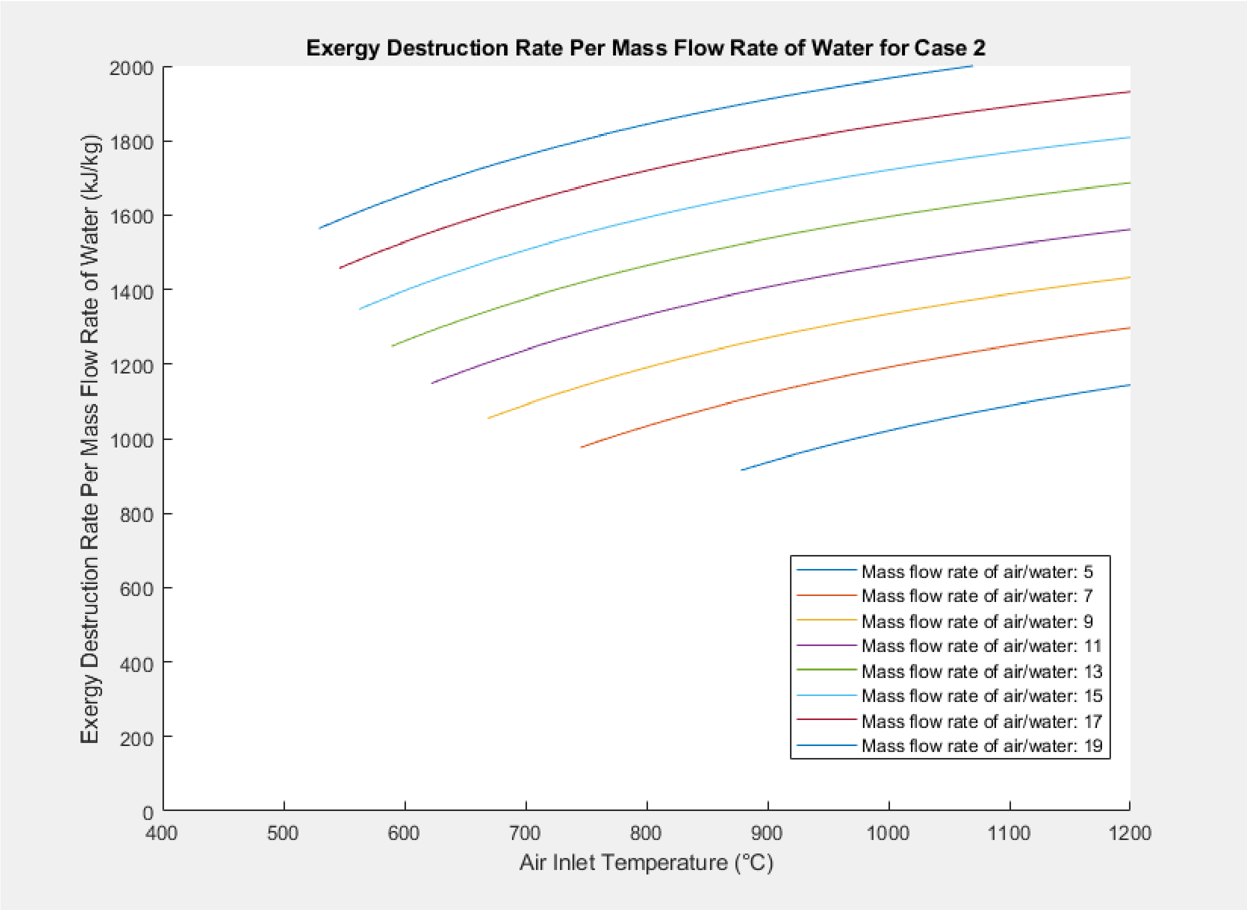


Case 2: Air Pressure Decreases from 200 kPa to 100 kPa

For case 2, the change in pressure decreases the difference in entropy for air between temperatures. This increases the rate of exergy destruction, since the difference in air entropy is subtracted in Equation 2. The air pressure change was found to significantly increase the minimum rate of exergy destruction than the constant pressure case. At higher mass flow rates, lower air temperatures could be used to provide the required heating but the exergy destruction rate increased. An example plot is shown in Figure 5, with lower mass flow rates requiring higher air inlet temperatures but producing lower rates of exergy destruction.

Figure 5

Plot of the Exergy Destruction Rate Per Mass Flow Rate of Water for Case 2



It was found that the minimum exergy destruction rate occurred at lower mass flow rates of air than in case 1. The minimum exergy destruction rate for a given flow rate always occurred at the lowest possible air inlet temperature for that mass flow rate, with values tabulated in Table 2. Notably, these air inlet temperatures are higher than in case 1 at the same flow rates. The optimal mass flow rate ratios are much lower with lower mass flow rates of air being required to achieve the minimum exergy destruction rate for case 2. At higher mass flow rate of airs, lower air inlet temperatures can be used for the heating process but a higher rate of exergy destruction is required. Values for the exergy destruction rate per mass flow rate of water plateau to a value of around 900 kJ/kg at low mass flow rates, but significantly higher inlet temperatures are needed as the mass flow rate of air decreases.

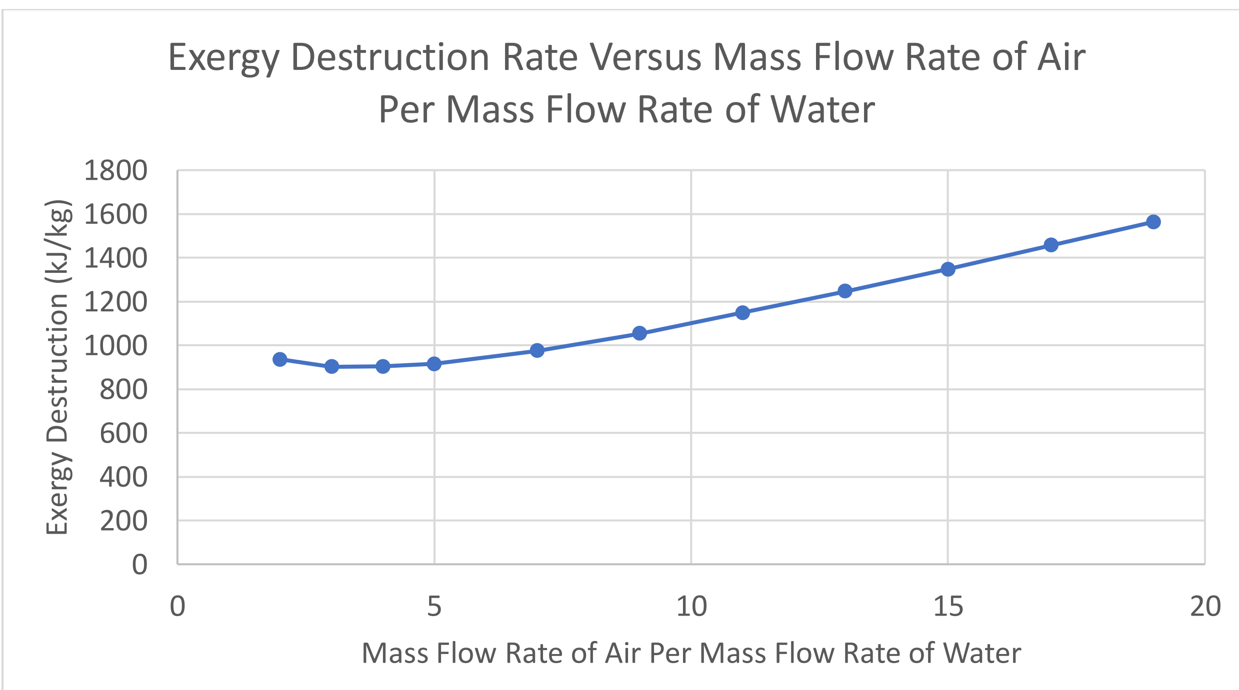
Table 2

Table of the Minimum Exergy Destruction and the Corresponding Inlet Temperature, Outlet Temperature, and Air Mass Flow Rate for Case 2

$T_3\ (C)$	$T_4\ (C)$	$\frac{\dot{m}_a}{\dot{m}_w}$	$\frac{\dot{X}_{dest}}{\dot{m}_w}$
1583.263	401.7285	2	934.7519
1187.664	400.3544	3	901.6922
997.9649	404.2144	4	902.9613
878.1783	400.6197	5	915.0608
745.351	401.9077	7	975.529
668.9753	400.6477	9	1054.323
622.4587	402.3469	11	1148.122
589.2789	402.6233	13	1247.241
562.7135	400.6496	15	1347.919
546.11	402.9812	17	1457.519
529.5066	401.2908	19	1563.886

Figure 6

Plot of the Exergy Destruction Rate Versus Mass Flow Rate of Air Per Mass Flow Rate of Water for Case 2



Like in Case 1, Case 2 also showed that higher mass flow rates allowed for lower air inlet temperatures to achieve the necessary heating. Both cases showed that the most optimal outlet temperature of air was to have the outlet air temperature as close to 400°C as possible. However, the trends of exergy destruction compared to the ratio of mass flow rates is very different for the two cases. The pressure drop associated with case 2 made a large difference due to the associated exergy destruction rate caused by the pressure drop making it not desirable to have such high air mass flow rates. While the exergy destruction rate due to the air inlet temperature required being lower at higher mass flow rates decreases, the exergy destruction rate due to the pressure drop increases as the air mass flow rate increases. Case 1 and Case 2 had identical temperature results at the same mass flow rate ratios, but the additional exergy destruction caused by the pressure drop made higher air mass flow rates undesirable.

Summary and Conclusions

The results for Case 1 and Case 2 were found to show that reducing the pressure of the air as it passes through the steam boiler significantly affects the exergy destruction rate. For case 1, it was found that the air inlet temperature should be kept as close to 400°C as possible to reduce the exergy destruction rate as much as possible. However, the ratios of mass flow rate of air to mass flow rate of water required are very high and it may not be feasible to heat water at high flow rates due to the incredibly high flow rates of air required for minimum exergy destruction at low air temperatures. The minimum exergy destruction rate per mass flow rate of water was found to plateau to around 339 kJ/kg as the ratio of mass flow rates increased and the air inlet temperatures became closer to 400°C.

Case 2 produced generally higher exergy destruction rates and did not require such high mass flow rate ratios to produce these results. It was found that lower mass flow rates of air were actually desirable. While increasing the mass flow rate of air allows for lower air inlet temperatures, the associated exergy destruction rate due to the pressure drop increases with mass flow rate. Mass flow rates of air should be kept closer to the mass flow rate of water for case 2. Air inlet temperatures should be kept as low as possible to provide the heating required at that particular air mass flow rate.

Providing a higher air inlet temperature than needed increases the difference between air inlet and outlet temperature, increasing the difference in flow exergies between the states of the air and the exergy destruction rate. The optimal air inlet temperatures at each mass flow rate are similar for both cases. This makes sense given that the pressure drop did not affect the energy balance at all so the energy required to heat the water does not change between the two cases. At incredibly air mass flow rates, case 1 is capable of producing far lower exergy destruction and even outperforms case 2 at lower mass flow rate ratios. Case 1 is the recommended design for all mass flow rate ratios and air inlet temperatures and is capable of producing exergy destruction rates per mass flow rate of water of around 340 kJ/kg at incredibly high mass flow rate ratios. Case 2 produces exergy destruction rates per mass flow rate of water of around 900 kJ/kg at best.

References

Cengel, Y. A. (2015). *Thermodynamics: An Engineering Approach* (Eighth Edition). McGraw-Hill Education.