Flow Filtering and the Physical Sources of Aerodynamic Sound

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Introduction

Objective

To understand the physical sources of jet noise.

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Motivation

- By-pass ratio is limited
- We need alternative strategies

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Methods

- Goldstein's theory
- Direct Numerical Simulation

Outline

How to define sound sources?

- Navier–Stokes equations

$$\mathbf{N}\mathbf{q} = 0 \tag{1}$$

- Choose base flow $\overline{\mathbf{q}}$

$$\mathbf{q} = \overline{\mathbf{q}} + \mathbf{q}' \tag{2}$$

- Rearrange equation for $\mathbf{q}'$:

$$\mathbf{L}\mathbf{q}' = \mathbf{s} \tag{3}$$

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$$\mathbf{L}\mathbf{q}' = \mathbf{s} \tag{3}$$

- Scalar wave equation

$$\mathcal{N}\mathbf{q} = 0 \tag{4}$$

- Choose base flow $\overline{\mathbf{q}}$

$$\mathbf{q} = \overline{\mathbf{q}} + \mathbf{q}' \tag{5}$$

- Rearrange equation for $\mathbf{q}'$:

$$\mathcal{L}\mathbf{q}' = s \tag{6}$$

Acoustic analogy sources

- Continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_j) = 0 \qquad (7)$$

- Time averaged base flow, $\mathbf{q} = \overline{\mathbf{q}} + \mathbf{q}'$
- Rearrange equation for $\mathbf{q}'$:

$$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_j)' = 0 \qquad (8)$$

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$$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_j}(\rho'\overline{v_j} + \overline{\rho} v_j') = -\frac{\partial}{\partial x_j}(\rho' v_j')' \tag{11}$$

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Acoustic analogy sources

Physical interpretation

$$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_j}\left(\overline{v_j}\rho' + \overline{\rho} v_j'\right) = -\frac{\partial}{\partial x_j}\left(\rho' v_j'\right)' \tag{9}$$

Propagation operator

Linearised Euler operator

$\longrightarrow$ well defined

Acoustic analogy sources

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Propagation operator

Linearised Euler operator

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Sources

- Depend on acoustic variables
- Only a small portion produces sound
- Include propagation effects

$\rightarrow$ ambiguous

Physical sound sources

Flow decomposition

Flow decomposition

$$\mathbf{q} = \widetilde{\mathbf{q}} + \mathbf{q}''$$

$\widetilde{\mathbf{q}} \rightarrow$ non-radiating base flow, $\mathbf{q}'' \rightarrow$ radiating components

Physical sound sources

Flow decomposition

Flow decomposition

$$\mathbf{q} = \widetilde{\mathbf{q}} + \mathbf{q}''$$

$\widetilde{\mathbf{q}} \rightarrow$ non-radiating base flow, $\quad \mathbf{q}'' \rightarrow$ radiating components

Fourier transform:

$$\mathrm{q}(\mathbf{x}, t) \rightarrow \mathrm{Q}(\mathbf{k}, \omega), \quad \widetilde{\mathrm{q}}(\mathbf{x}, t) \rightarrow \widetilde{\mathrm{Q}}(\mathbf{k}, \omega)$$

Non-radiating condition

$$\widetilde{\mathrm{Q}}(\mathbf{k}, \omega) = 0 \qquad \text{if} \quad |\mathbf{k}| = |\omega|/c_\infty$$

$$\widetilde{\mathrm{Q}}(\mathbf{k}, \omega) = \mathrm{Q}(\mathbf{k}, \omega) \qquad \text{if} \quad |\mathbf{k}| \neq |\omega|/c_\infty$$

Physical sound sources

Flow decomposition: convolution filters

Space-time domain

$$\widetilde{\mathrm{q}} = w * \mathrm{q}$$

Wavenumber-frequency domain

$$\widetilde{\mathrm{Q}} = W \times \mathrm{Q}$$

Physical sound sources

Flow decomposition: convolution filters

Space-time domain

$$\tilde{q} = w * q$$

Wavenumber-frequency domain

$$\tilde{Q} = W \times Q$$

$$q(\mathbf{x}, t) \xrightarrow{\text{F.T.}} Q(\mathbf{k}, \omega)$$

Filter window

$$\begin{cases} 0 & \text{if} \quad |\mathbf{k}| = |\omega|/c_\infty \\ 1 & \text{otherwise} \end{cases}$$

$$\tilde{q}(\mathbf{x}, t) \xleftarrow{\text{I.F.T.}} \tilde{Q}(\mathbf{k}, \omega)$$

Physical sound sources

- Continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_j) = 0 \tag{10}$$

- Non-radiating base flow, $\mathbf{q} = \widetilde{\mathbf{q}} + \mathbf{q}''$
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$$\rho v_j = \widetilde{\rho}\,\widetilde{v_j} + \rho''\widetilde{v_j} + \widetilde{\rho} v_j'' + \rho'' v_j'' \tag{12}$$

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$$(\rho v_j)'' = \underbrace{(\widetilde{\rho}\,\widetilde{v_j})''}_{\text{source}} + \underbrace{(\rho''\widetilde{v_j} + \widetilde{\rho} v_j'')''}_{\text{propagation}} + \underbrace{(\rho'' v_j'')''}_{\approx 0} \tag{13}$$

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$$\frac{\partial \rho''}{\partial t} + \frac{\partial}{\partial x_j}(\rho''\widetilde{v_j} + \widetilde{\rho} v_j'')'' = -\frac{\partial}{\partial x_j}(\widetilde{\rho}\widetilde{v_j})'' \tag{14}$$

Physical sound sources

- Continuity equation

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_j) = 0} \qquad (15)$$

- Non-radiating base flow, $\boxed{\mathbf{q} = \widetilde{\mathbf{q}} + \mathbf{q}''}$
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Physical sound sources

Physical interpretation

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Propagation operator

Depends on the filter → a bit more complex

Physical sound sources

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$$\frac{\partial \rho''}{\partial t} + \frac{\partial}{\partial x_j}\left(\widetilde{v_j}\,\rho'' + \widetilde{\rho} v_j''\right)'' = -\frac{\partial}{\partial x_j}\left(\widetilde{\rho}\widetilde{v_j}\right)'' \qquad (20)$$

Propagation operator

Depends on the filter $\longrightarrow$ a bit more complex

Sources

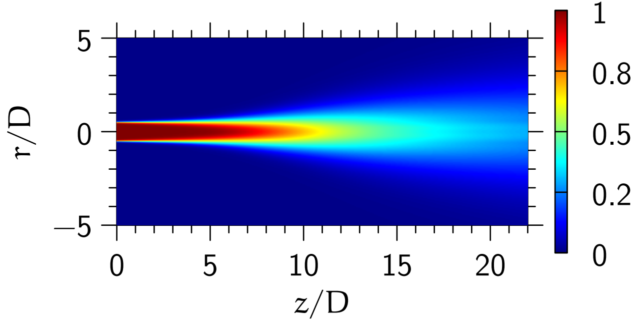
- Depends on non-radiating flow only
- Is purely radiating
- No propagation effect

$\longrightarrow$ well defined

Outline

Flow description

Mean flow

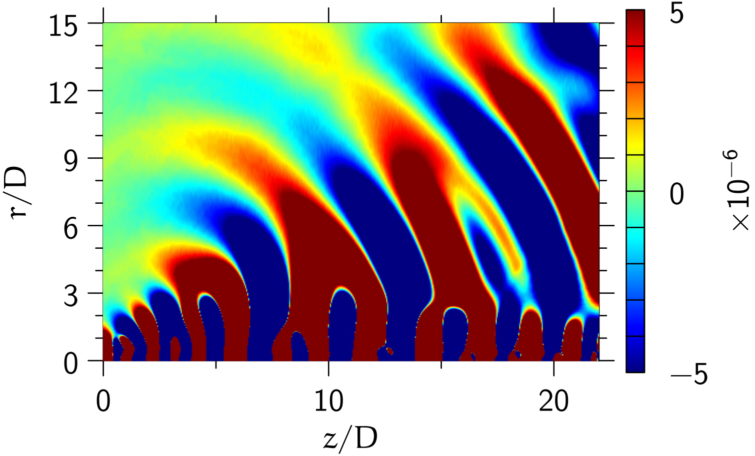


Mean flow excited at two frequencies:

$$\omega_1 = 2.2, \qquad \omega_2 = 3.4, \qquad \Delta\omega = 1.2.$$

Flow description

Pressure field



Flow description

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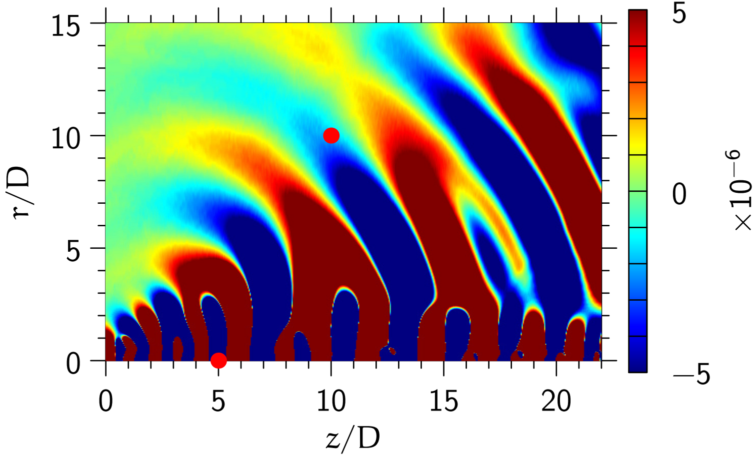
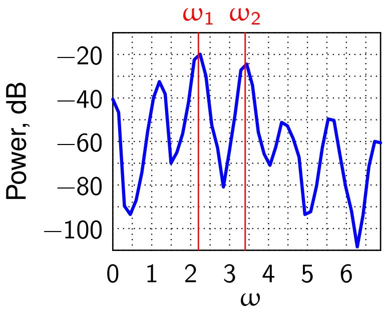


Flow description

Frequency analysis

Hydrodynamic region



Flow description

Frequency analysis

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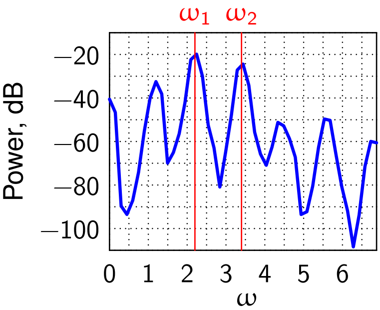
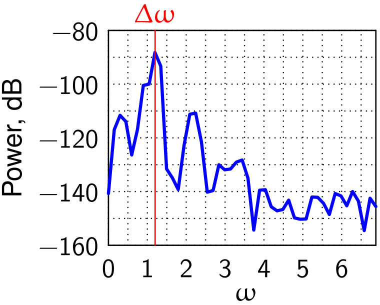


Acoustic region



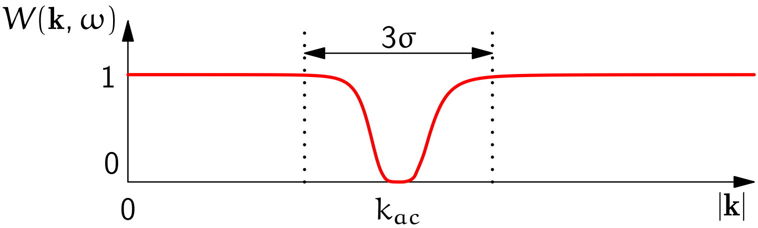
Flow decomposition

Filter definition

Butterworth filter

$$W(\mathbf{k}, \omega) = \left(1 + \frac{|\mathbf{k}|\sigma}{|\mathbf{k}|^2 - k_{ac}^2}\right)^{-4}$$

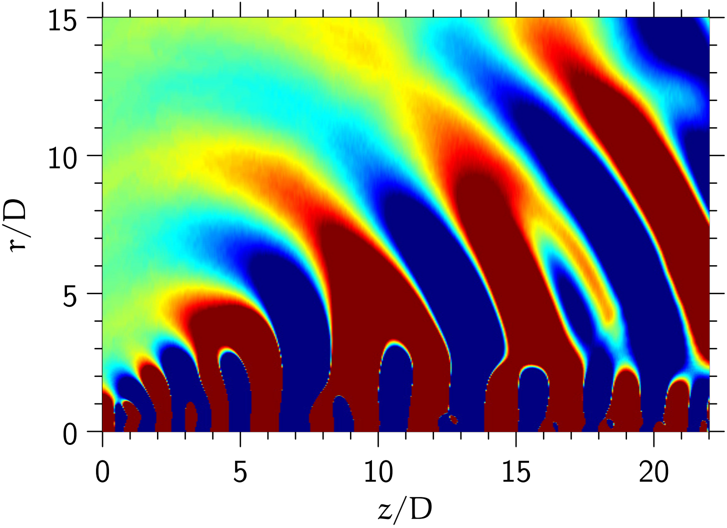
$$k_{ac} = \Delta\omega / c_\infty = 1.08, \quad \sigma = 0.25.$$



Flow decomposition

Results

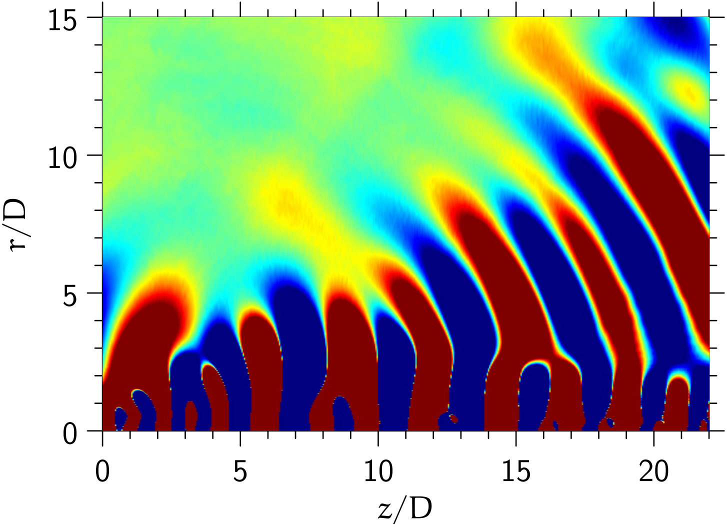
Pressure field p



Flow decomposition

Results

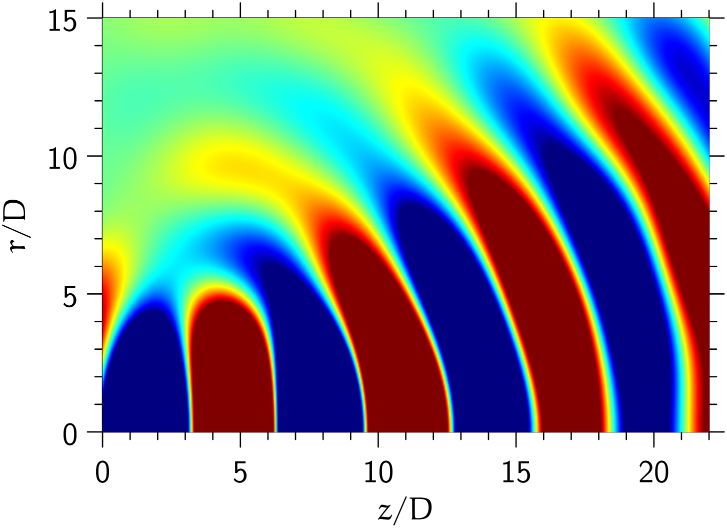
Filtered pressure $\widetilde{p}$



Flow decomposition

Results

Radiating pressure p''



Defining a scalar source term

Start with a wave-like equation

$$\frac{\partial^2 p}{\partial x_i \partial x_i} - \frac{\partial^2 \rho}{\partial t^2} + \frac{\partial^2 \rho v_i v_j}{\partial x_i \partial x_j} = \frac{\partial^2 \sigma_{ij}}{\partial x_i \partial x_j} \tag{21}$$

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Acoustic analogy source

$$s_2 = \partial^2 T_{ij} / \partial x_i \partial x_j$$
$$T_{ij} = -(\overline{\rho} v_i' v_j' + \overline{v_i} \rho' v_j' + \overline{v_j} \rho' v_i')'$$

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Physical sound source

$$s_1 = \partial^2 S_{ij} / \partial x_i \partial x_j$$

$$S_{ij} = -(\widetilde{\rho}\, \widetilde{v_i}\, \widetilde{v_j})'' = -(\widetilde{\rho}\, \widetilde{v_i}\, \widetilde{v_j} - \widetilde{\widetilde{\rho}\, \widetilde{v_i}\, \widetilde{v_j}})$$

Defining a scalar source term

Comparison with Goldstein's definition

Physical sound source:

$$S_{ij} = -(\widetilde{\rho}\,\widetilde{v_i}\,\widetilde{v_j})'' = -(\widetilde{\rho}\,\widetilde{v_i}\,\widetilde{v_j} - \widetilde{\widetilde{\rho}\,\widetilde{v_i}\,\widetilde{v_j}})$$

Goldstein's sound source:

$$G_{ij} = -(\widetilde{\rho}\,\widetilde{v_i}\,\widetilde{v_j} - \widetilde{\rho\, v_i\, v_j})$$

Defining a scalar source term

Comparison with Goldstein's definition

Physical sound source:

$$S_{ij} = -(\tilde{\rho}\,\tilde{v}_i\,\tilde{v}_j)'' = -(\tilde{\rho}\,\tilde{v}_i\,\tilde{v}_j - \widetilde{\tilde{\rho}\,\tilde{v}_i\,\tilde{v}_j})$$

Goldstein's sound source:

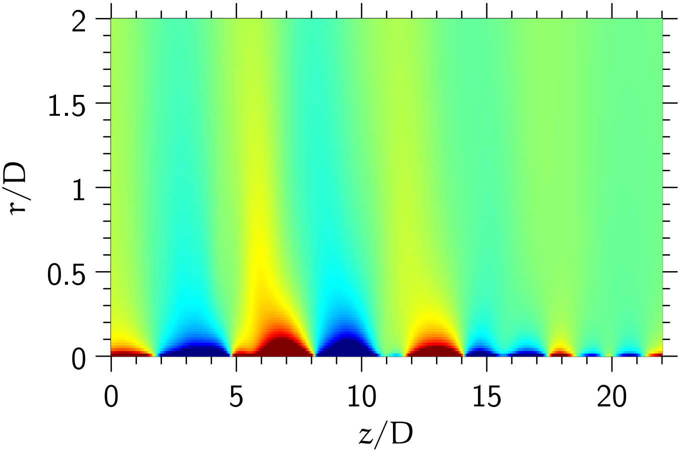
$$G_{ij} = -(\tilde{\rho}\,\tilde{v}_i\,\tilde{v}_j - \widetilde{\rho\,v_i\,v_j})$$

Comparison:

$$G_{ij} = S_{ij} + \underbrace{\widetilde{\tilde{v}_i\,\tilde{v}_j\,\rho'} + \widetilde{\tilde{\rho}\,\tilde{v}_j\,v_i'} + \widetilde{\tilde{\rho}\,\tilde{v}_i\,v_j'}}_{\text{non-radiating terms}}$$

Sound sources

Sources distribution



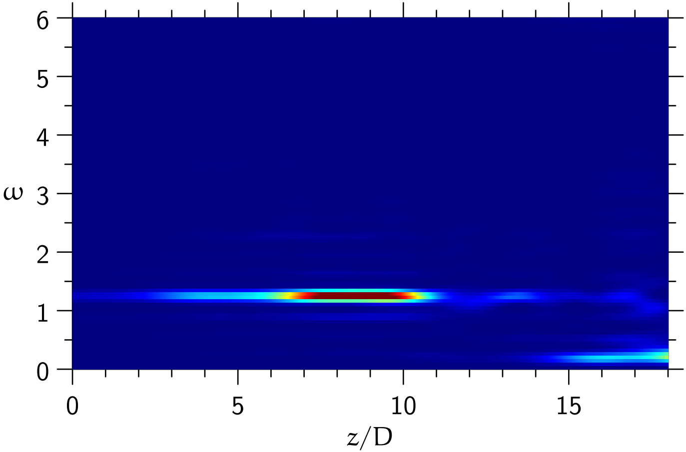
Sound sources

Movie

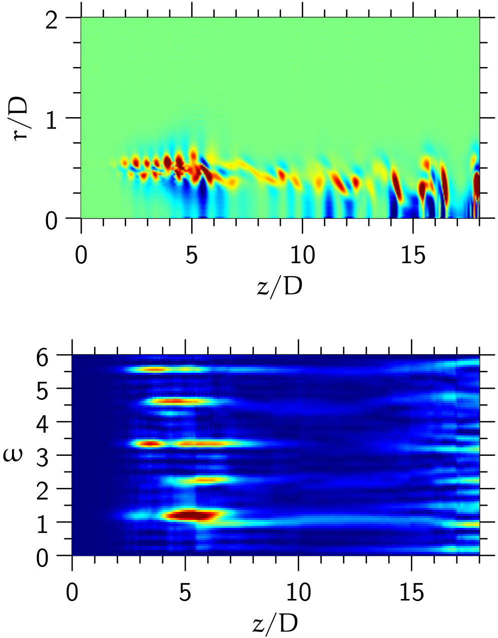
(source)

Physical sound sources

Power spectrum



Acoustic analogy sources



Conclusion

Results

1. Flow decomposition is possible using convolution filters
2. Sources obtained in a laminar jet
3. Goldstein's sources can be further decomposed

Conclusion

Results

1. Flow decomposition is possible using convolution filters
2. Sources obtained in a laminar jet
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Future work

- Reconstruct the sound field
- Filter the flow in 3 dimensions (ω, k_z, k_r),
- Sound sources in a mixing layer
- Sound sources in a turbulent jet
- Understand the physical mechanisms

Acknowledgements

EPSRC
Engineering and Physical Sciences Research Council

Rolls-Royce

Thank you!

Scalar wave equation

Continuity equation, $$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_j}{\partial x_j} = 0 \tag{22}$$

Momentum equation, $$\frac{\partial \rho v_i}{\partial t} + \frac{\partial \rho v_i v_j}{\partial x_j} + \frac{\partial p}{\partial x_i} = \frac{\partial \sigma_{ij}}{\partial x_j} \tag{23}$$

Taking $\partial(23)/\partial x_i - \partial(22)/\partial t$ gives

$$\frac{\partial^2 p}{\partial x_i \partial x_i} - \frac{\partial^2 \rho}{\partial t^2} + \frac{\partial^2 \rho v_i v_j}{\partial x_i \partial x_j} = \frac{\partial^2 \sigma_{ij}}{\partial x_i \partial x_j} \tag{24}$$

Return 1

Return 2

Acoustic analogy sources

The governing equation for fluctuating quantities is

$$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_j}(\overline{\rho} v_j' + \overline{v_j}\rho') = m \tag{25}$$

$$\frac{\partial}{\partial t}(\overline{\rho} v_i' + \overline{v_i}\rho') + \frac{\partial}{\partial x_j}(\overline{v_i}\,\overline{v_j}\,\rho' + \overline{\rho}\,\overline{v_j}\,v_i' + \overline{\rho}\,\overline{v_i}\,v_j') + \frac{\partial}{\partial x_i}p' = f_i, \tag{26}$$

where

$$m = -\frac{\partial}{\partial x_j}(\rho' v_j')' \tag{27}$$

$$f_i = -\frac{\partial}{\partial t}(\rho' v_i')' - \frac{\partial}{\partial x_j}(\overline{\rho} v_i' v_j' + \overline{v_i}\rho' v_j' + \overline{v_j}\rho' v_i')' \tag{28}$$

Return

Physical sound sources

The governing equation for fluctuating quantities is

$$\frac{\partial \rho''}{\partial t} + \frac{\partial}{\partial x_j}(\widetilde{\rho} v_j'' + \widetilde{v_j}\rho'') = m \tag{29}$$

$$\frac{\partial}{\partial t}(\widetilde{\rho} v_i'' + \widetilde{v_i}\rho'') + \frac{\partial}{\partial x_j}(\widetilde{v_i}\,\widetilde{v_j}\,\rho'' + \widetilde{\rho}\,\widetilde{v_j}\,v_i'' + \widetilde{\rho}\,\widetilde{v_i}\,v_j'') + \frac{\partial}{\partial x_i}p'' = f_i, \tag{30}$$

where

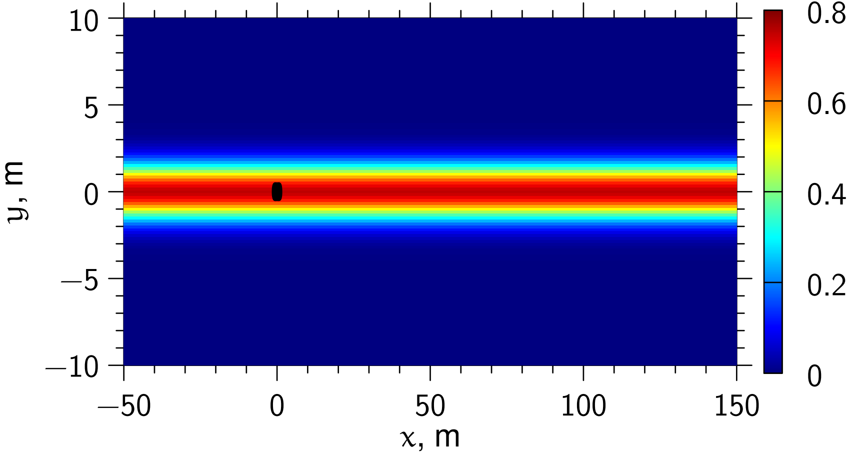
$$m = -\frac{\partial}{\partial x_j}(\widetilde{\rho}\widetilde{v_j})'' \tag{31}$$

$$f_i = -\frac{\partial}{\partial t}(\widetilde{\rho}\widetilde{v_i})'' - \frac{\partial}{\partial x_j}(\widetilde{\rho}\,\widetilde{v_i}\,\widetilde{v_j})'' \tag{32}$$

◂ Return

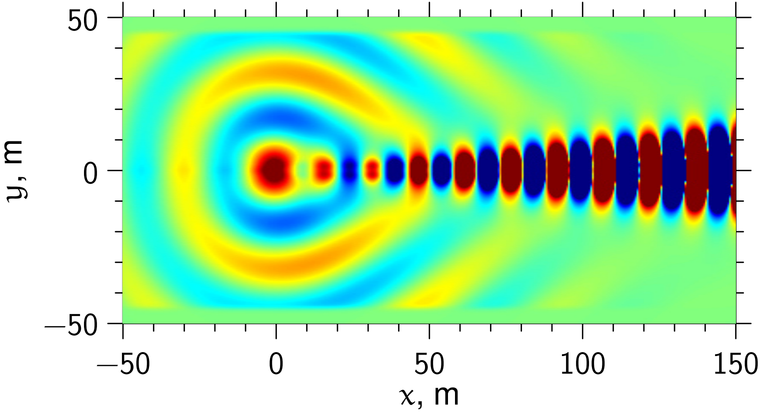
Filtering of a two-dimensional shear layer

Flow description



Filtering of a two-dimensional shear layer problem

Pressure field



Filtering of a two-dimensional shear layer

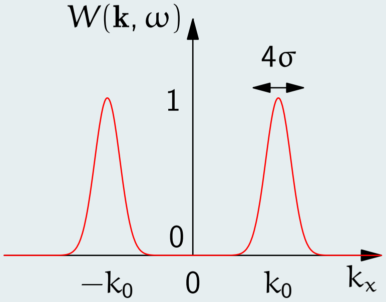
Filter design

Gaussian filter

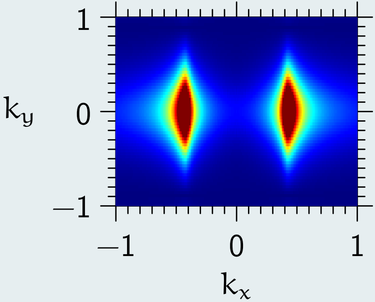
$$W(\mathbf{k}, \omega) = \exp\left(-\frac{(k_x - k_0)^2}{2\sigma^2}\right) + \exp\left(-\frac{(k_x + k_0)^2}{2\sigma^2}\right)$$

$k_0 = 0.41459, \quad \sigma = 0.1$

Filter window

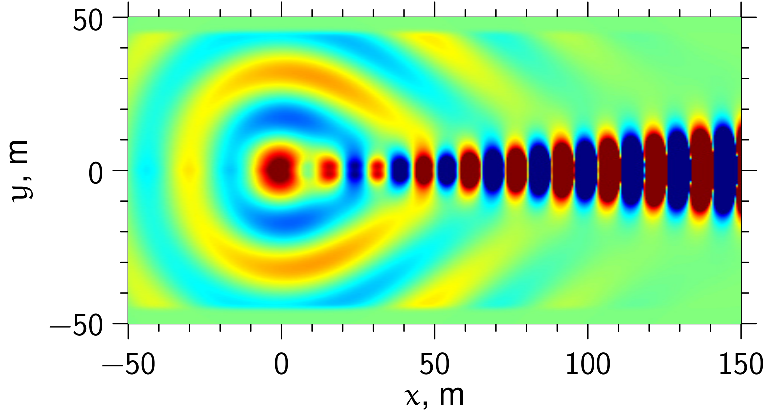


Fourier transform of pressure



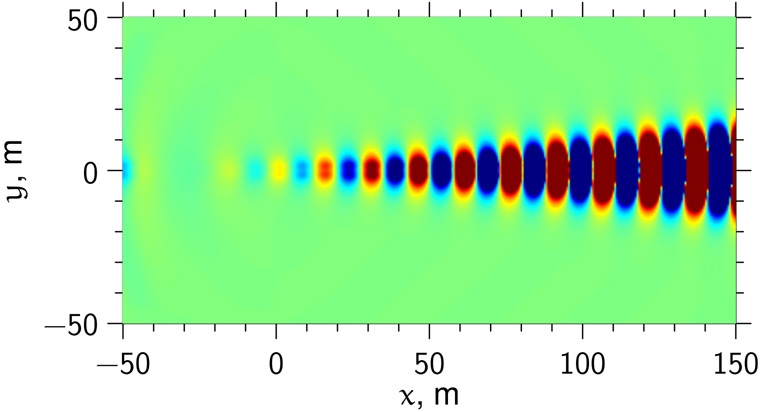
Filtering of a two-dimensional shear layer

Results



Filter of a two-dimensional shear layer problem

Results

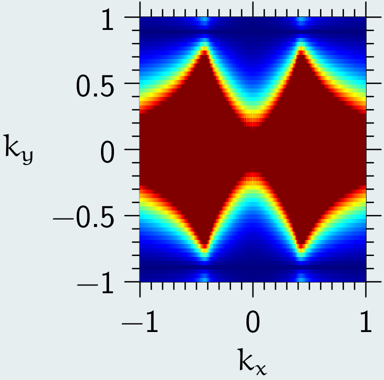


Filtering of a two-dimensional shear layer

Effect of windowing

Radiating components: $k_\infty = 0.219\mathrm{m}^{-1}$

Fourier transform with no windowing



Fourier transform with windowing