Separating propagating and non-propagating dynamics in fluid-flow equations

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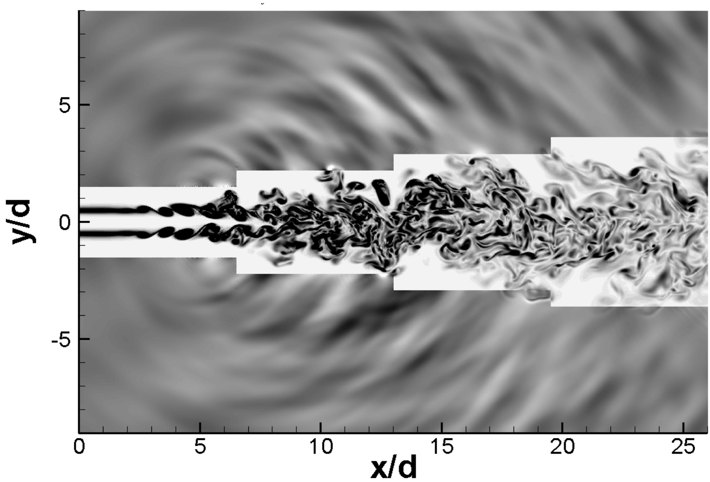

y/d
5
0
-5
0
5
10
15
20
25
x/d

Introduction

How to define the physical sources of sound?

Objectives

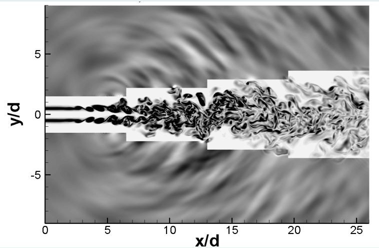
1. Derive an expression for the physical sources of sound.
2. Demonstrate that it is possible to separate the radiating and non-radiating parts of the flow.
3. Compute the physical sources of sound.

Outline

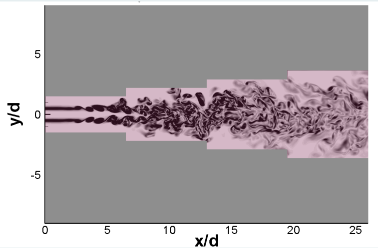
1. Defining the physical sources of sound
   - Goldstein's theory
   - Equations
2. Non-radiating filter design
   - Problem description
   - Filter defining properties
   - Local filter
   - Global filter
3. Sources of sound in an axi-symmetric jet
   - Flow description
   - Filter design
   - Sound sources

Goldstein's theory

Jet



Filtered jet



Goldstein's theory

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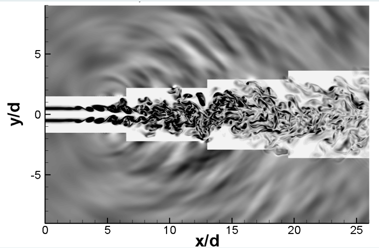

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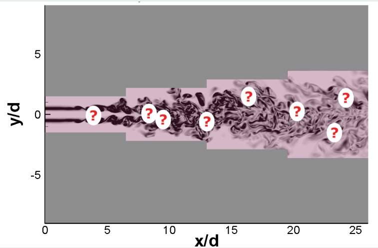
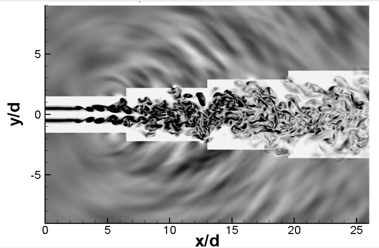
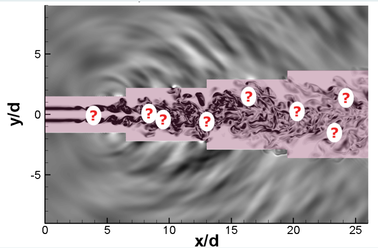


Goldstein's theory

Jet



Filtered jet



These sources should be close to the true sources of sound.

Governing equation for fluctuating quantities

Flow filtering

$$\mathcal{L}f = \overline{f} \tag{1}$$

Flow decomposition

$$f = \overline{f} + f' \tag{2}$$

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Conservation of mass

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_j}{\partial x_j} = 0, \tag{3}$$

$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial \overline{\rho v_j}}{\partial x_j} = 0. \tag{4}$$

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Conservation of mass for fluctuating quantities

$$\frac{\partial \rho'}{\partial t} + \frac{\partial (\rho v_j)'}{\partial x_j} = 0.$$

Governing equation for fluctuating quantities

Conservation of mass for fluctuating quantities

$$\frac{\partial \rho'}{\partial t} + \frac{\partial (\rho v_j)'}{\partial x_j} = 0. \tag{5}$$

Momentum conservation for fluctuating quantities

$$\frac{\partial (\rho v_i)'}{\partial t} + \frac{\partial (\rho v_i v_j)'}{\partial x_j} + \frac{\partial p'}{\partial x_i} = \frac{\partial \sigma'_{ij}}{\partial x_j}. \tag{6}$$

Governing equation for fluctuating quantities

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Taking $\partial(6)/\partial x_i - \partial(5)/\partial t$ gives

$$\frac{\partial^2 p'}{\partial x_i x_i} - \frac{\partial^2 \rho'}{\partial t^2} + \frac{\partial^2 (\rho v_i v_j)'}{\partial x_i \partial x_j} = \frac{\partial^2 \sigma'_{ij}}{\partial x_i \partial x_j}. \tag{7}$$

Governing equation for fluctuating quantities

$$\text{Favre averaging,}\quad \tilde{f} = \overline{\rho f}/\overline{\rho}, \tag{8}$$

Governing equation

$$\frac{\partial^2 p'}{\partial x_i \partial x_i} - \frac{\partial^2 \rho'}{\partial t^2} + \frac{\partial^2}{\partial x_i \partial x_j}(\tilde{v}_i \tilde{v}_j \rho' + \overline{\rho}\tilde{v}_j v_i' + \overline{\rho}\tilde{v}_i v_j') = \frac{\partial^2 \sigma_{ij'}}{\partial x_i \partial x_j} + s \tag{9}$$

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Governing equation

$$\frac{\partial^2 p'}{\partial x_i \partial x_i} - \frac{\partial^2 \rho'}{\partial t^2} + \frac{\partial^2}{\partial x_i \partial x_j}\left(\tilde{v}_i \tilde{v}_j \rho' + \overline{\rho}\tilde{v}_j v_i' + \overline{\rho}\tilde{v}_i v_j'\right) = \frac{\partial^2 \sigma_{ij'}}{\partial x_i \partial x_j} + s \tag{9}$$

Source definition

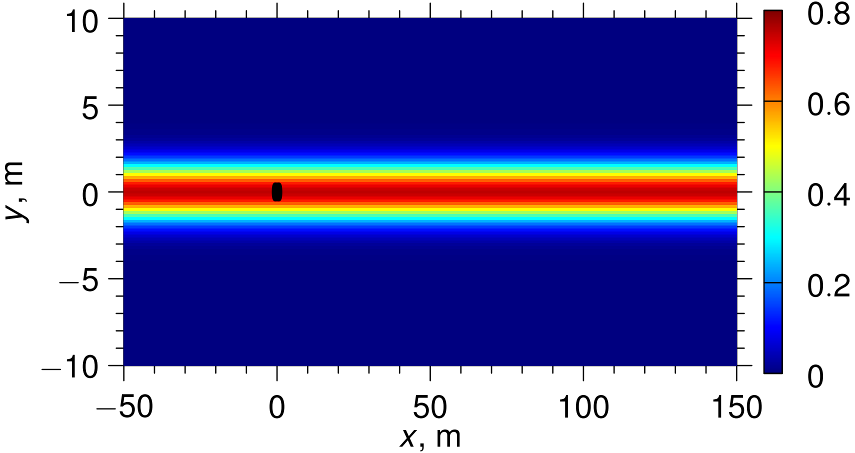
$$s = -\frac{\partial^2}{\partial x_i \partial x_j}\left(T_{ij} + \rho v_i' v_j' + \tilde{v}_i \rho' v_j' + \tilde{v}_j \rho' v_i'\right) \tag{10}$$

$$T_{ij} = -\overline{\rho}(\widetilde{v_i v_j} - \tilde{v}_i \tilde{v}_j). \tag{11}$$

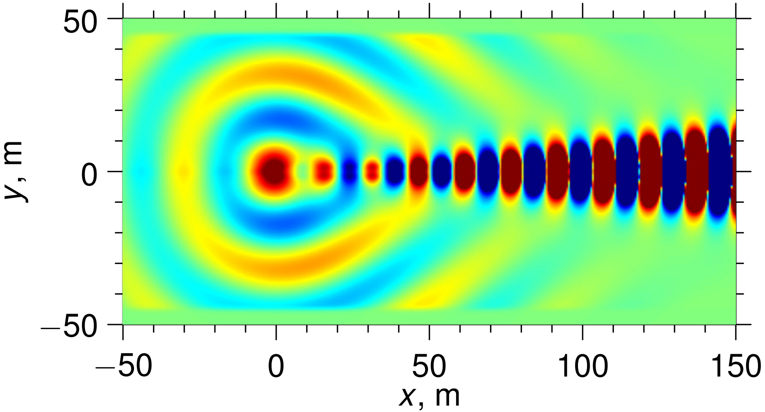
Outline

Problem description

Parallel flow



Problem description
Pressure field



Defining properties

Fourier transform

$$f(\mathbf{x}, t) \rightarrow F(\mathbf{k}, \omega)$$

$$\overline{f}(\mathbf{x}, t) \rightarrow \overline{F}(\mathbf{k}, \omega)$$

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Fourier transform

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Non-radiating condition

$$\overline{F}(\mathbf{k}, \omega) = 0 \quad \text{for} \quad |\mathbf{k}| = \frac{|\omega|}{c_\infty}$$

Defining properties

Fourier transform

$$f(\mathbf{x}, t) \to F(\mathbf{k}, \omega)$$
$$\overline{f}(\mathbf{x}, t) \to \overline{F}(\mathbf{k}, \omega)$$

Non-radiating condition

$$\overline{F}(\mathbf{k}, \omega) = 0 \quad \text{for} \quad |\mathbf{k}| = \frac{|\omega|}{c_\infty}$$

Additional requirement

$$\overline{F}(\mathbf{k}, \omega) = F(\mathbf{k}, \omega) \quad \text{for} \quad |\mathbf{k}| \neq \frac{|\omega|}{c_\infty}$$

Local filter

Filter definition

D'Alembertian filter

$$\bar{f}(\mathbf{x}, t) = \left(\frac{1}{c_\infty^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) f(\mathbf{x}, t),$$

Local filter
Filter definition

D'Alembertian filter

$$\overline{f}(\mathbf{x}, t) = \left(\frac{1}{c_\infty^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) f(\mathbf{x}, t),$$

Frequency domain

$$\overline{F}(\mathbf{k}, \omega) = \left(|\mathbf{k}|^2 - \frac{\omega^2}{c_\infty^2} \right) F(\mathbf{k}, \omega)$$

Local filter

Filter definition

D'Alembertian filter

$$\overline{f}(\mathbf{x}, t) = \left(\frac{1}{c_\infty^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) f(\mathbf{x}, t),$$

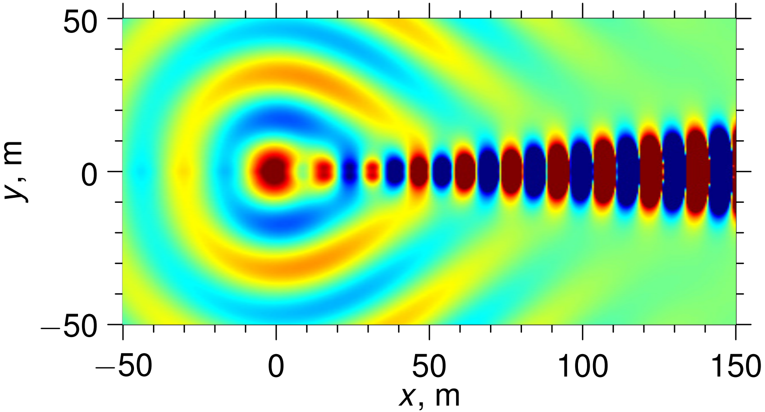
Frequency domain

$$\overline{F}(\mathbf{k}, \omega) = \left(|\mathbf{k}|^2 - \frac{\omega^2}{c_\infty^2} \right) F(\mathbf{k}, \omega)$$

$$\Rightarrow \overline{F}(\mathbf{k}, \omega) = 0 \quad \text{for} \quad |\mathbf{k}| = \frac{|\omega|}{c_\infty}$$

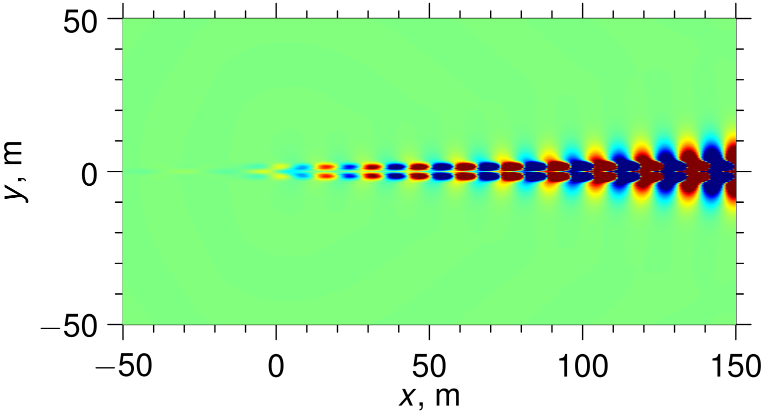
Local filter

Results



Local filter

Results



Global filter

Filter definition

Time-domain

$$\overline{f} = w * f \tag{12}$$

Frequency-domain

$$\overline{F} = WF \tag{13}$$

Global filter

Filter definition

Time-domain

$$\overline{f} = w * f \tag{12}$$

Frequency-domain

$$\overline{F} = WF \tag{13}$$

$$f(\mathbf{x}, t) \xrightarrow{\text{F.T.}} F(\mathbf{k}, \omega)$$

Filter window

$$\begin{cases} 0 & \text{if} \quad |\mathbf{k}| = |\omega|/c_\infty \\ 1 & \text{otherwise} \end{cases}$$

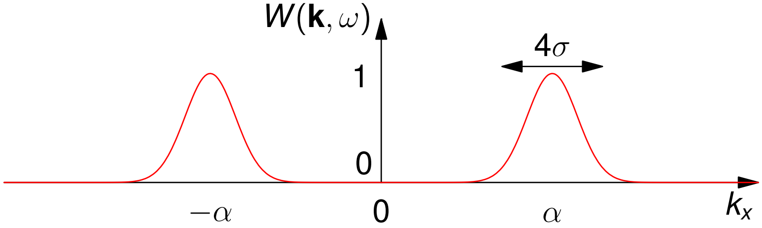
$$\overline{f}(\mathbf{x}, t) \xleftarrow{\text{I.F.T.}} \overline{F}(\mathbf{k}, \omega)$$

Global filter
Filter definition

Gaussian filter

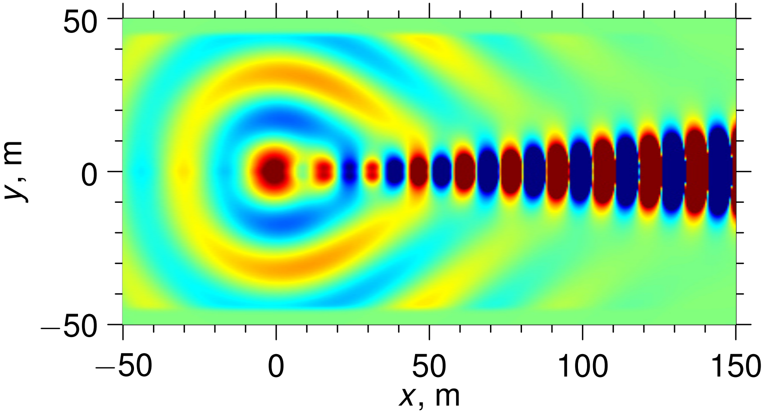
$$W(\mathbf{k},\omega) = \exp\left(-\frac{(k_x-\alpha)^2}{2\sigma^2}\right) + \exp\left(-\frac{(k_x+\alpha)^2}{2\sigma^2}\right)$$

$$\alpha = 0.68\mathrm{m}^{-1}, \quad \sigma = 0.1\mathrm{m}^{-1}.$$



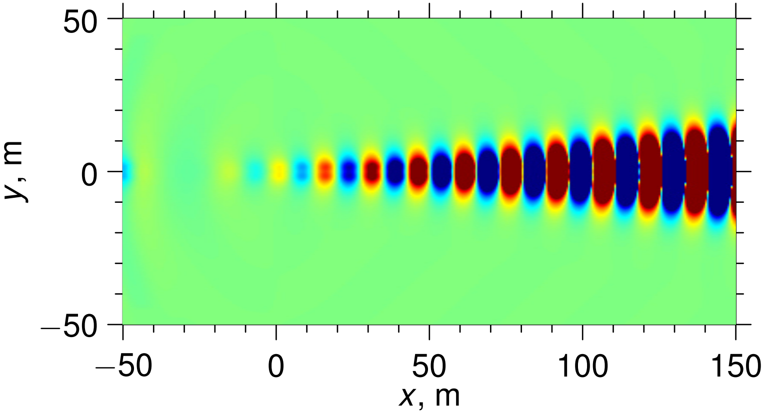
Global filter

Results



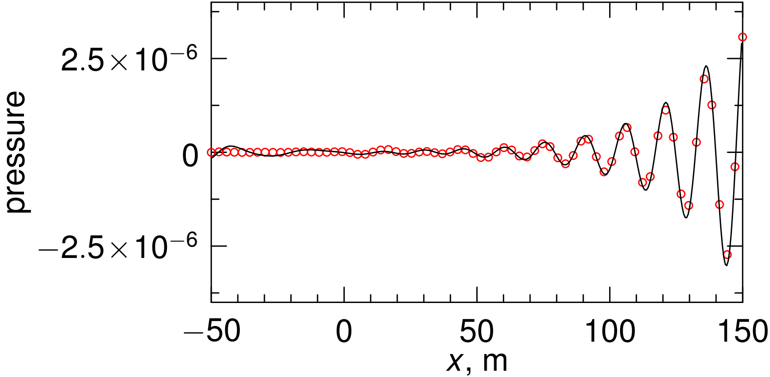
Global filter

Results



Global filter
Validation

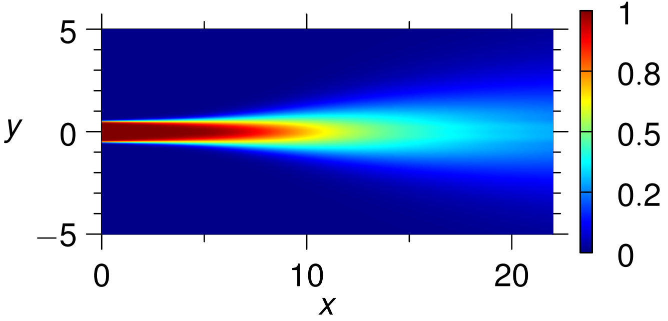
Comparison with analytical result along profile $y = 15$m



Outline

Flow description

Mean flow

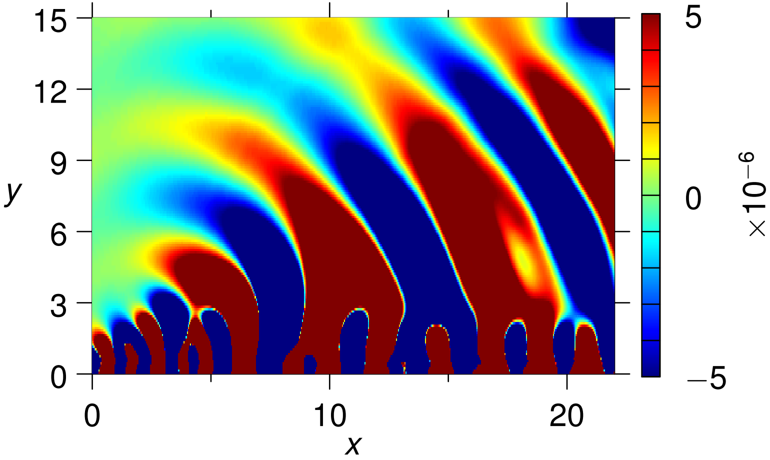


Mean flow excited at two frequencies:

$$\omega_1 = 2.2, \qquad \omega_2 = 3.4, \qquad \Delta\omega = 1.2.$$

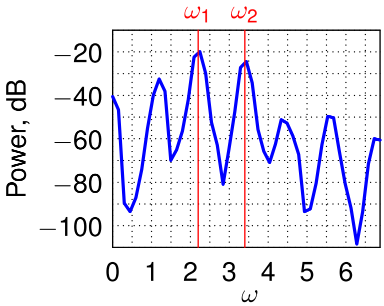
Flow description

Pressure field



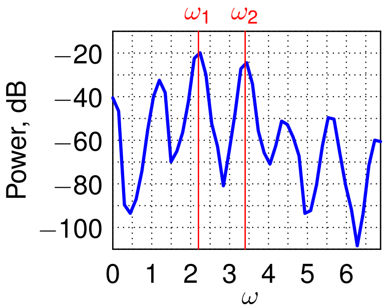
Flow description
Frequency analysis

Hydrodynamic region

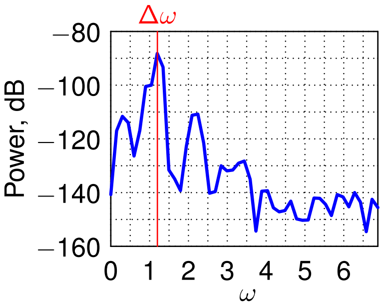


Flow description
Frequency analysis

Hydrodynamic region



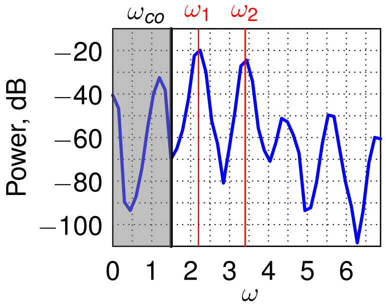
Acoustic region



Flow description

Frequency analysis

Hydrodynamic region



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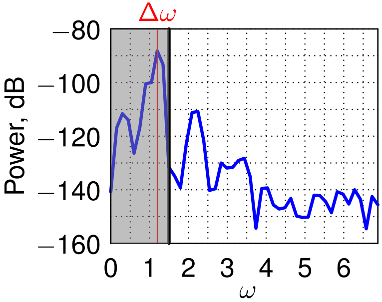
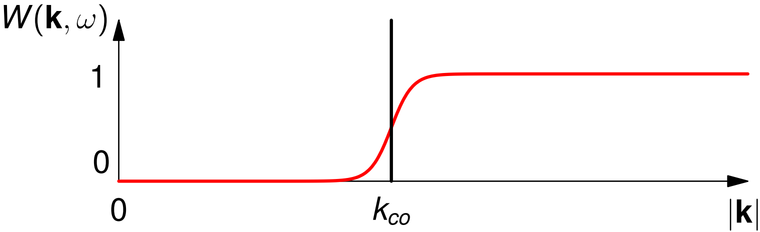


Filter design
Definition

Tanh filter

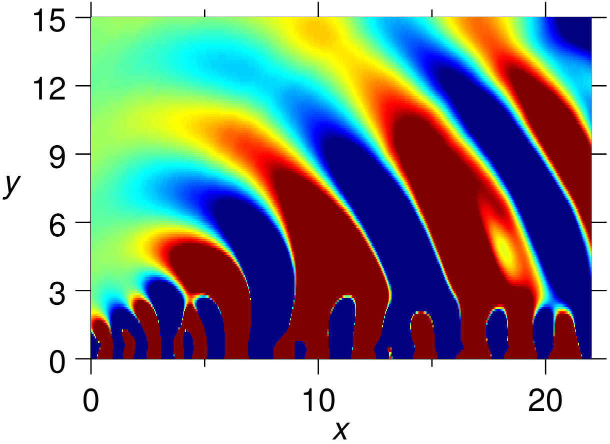
$$W(\mathbf{k},\omega) = \frac{1}{2}\left[1 + \tanh\left(\frac{|\mathbf{k}| - k_{co}}{\sigma}\right)\right],$$

$$k_{co} = 1.3, \quad \sigma = 0.2.$$



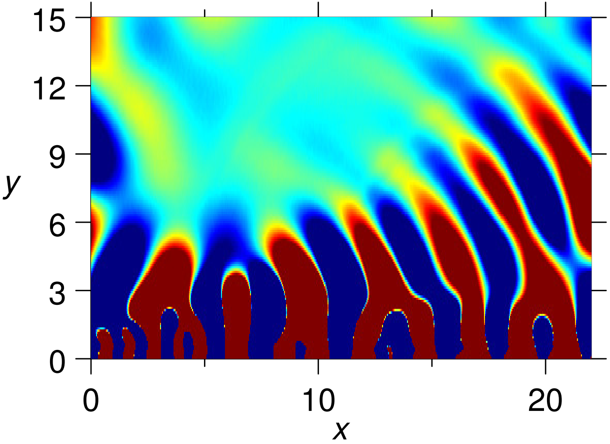
Filter design
Validation

Pressure field p



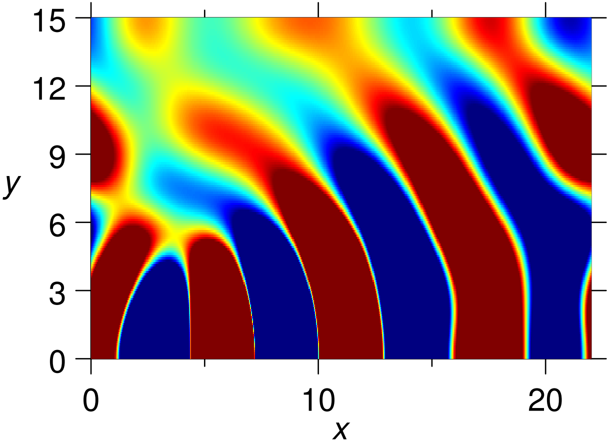
Filter design
Validation

Filtered pressure $\overline{p}$



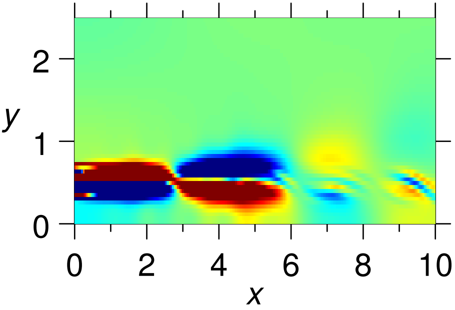
Filter design
Validation

Fluctuating pressure p'



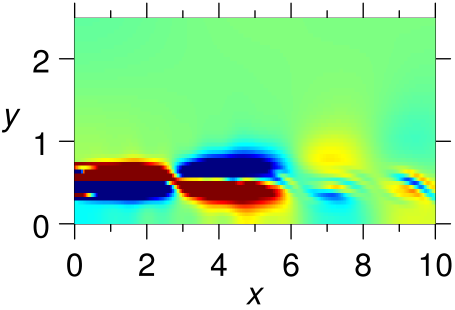
Sound sources
Using non-radiating filter

Sound source s

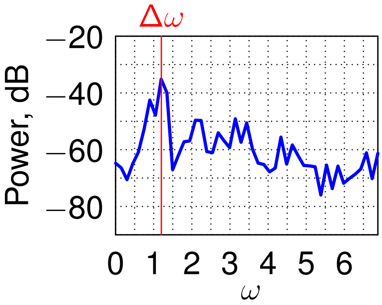


Sound sources
Using non-radiating filter

Sound source s



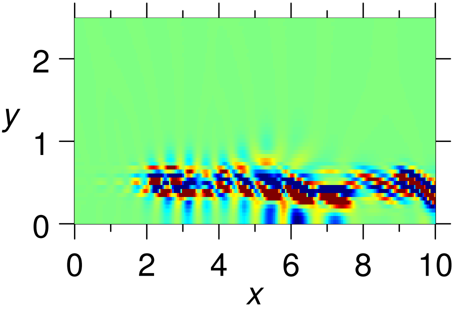
Spectrum at $(4.0, 0.55)$



Sound sources

Using time average filter

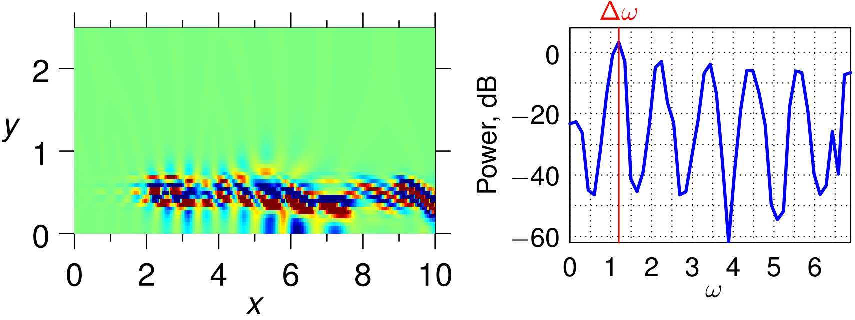
Sound source s



Sound sources
Using time average filter

Sound source s

Spectrum at $(5.5, 0.5)$



Sound sources

Evolution in time

(source)

Conclusion and future work

Results

- Sound source definition
- Separation possible with convolution filters.
- Clearer physical interpretation of the sources.

Conclusion and future work

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Future work

- Mixing-layer and a two-dimensional jet.
- Physical mechanism behind the sound sources.

Acknowledgements

EPSRC
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