Given $(x_0, \cdots, x_{2^N})$, we wish to find:

$$\tilde{x}_k = \sum_{j=0}^{2^N-1} \exp\left(-i\frac{\tau k}{2^N} j\right) x_j$$

Notice that we can decompose:

$$\tilde{x}_k = \sum_{j=0}^{2^{N-1}-1} \exp\left(-i\frac{\tau \boxed{2} k}{\boxed{2^N}} j\right) \left(x_{2j} + \exp\left(-i\frac{\tau k}{2^N}\right) x_{2j+1}\right) \tag{1}$$

$$= \tilde{y}_k + \exp\left(-i\frac{\tau k}{2^N}\right) \tilde{z}_k \tag{2}$$

Here, $y_j = x_{2j}$ and $z_j = x_{2j+1}$ are alternating subsequences of our original inputs. The second equality follows when we notice the $\boxed{\text{boxed}}$ 2's cancel. Also note that there are only 2^{N-1} total $\tilde{y}$'s; after that they wrap around. Same with $\tilde{z}$'s.