Notes from *Topological Vector Spaces, Distributions and Kernels*

by Francois Treves

taken by Samuel T. Wallace

Publisher's Description

This text for upper-level undergraduates and graduate students focuses on key notions and results in functional analysis. Extending beyond the boundaries of Hilbert and Banach space theory, it explores aspects of analysis relevant to the solution of partial differential equations.

The three-part treatment begins with topological vector spaces and spaces of functions, progressing to duality and spaces of distribution, and concluding with tensor products and kernels. The archetypes of linear partial differential equations (Laplace's , the wave, and the heat equations) and the traditional problem (Dirichlet's and Cauchy's) are the volume's main focus. Most of the basic classical results appear here. There are 390 exercises, several of which contain detailed information that will enable readers to reconstruct the proofs of some important results.

A Note From the Transcriber

I have owned this book for several months but have never put dedicated study time into it. Now is the time. I have just entered graduate school, and plan to devote some time for studying. I have previous exposure to some topics in functional analysis. I know enough Banach space theory to prove the uniqueness of fixed points of contraction mappings, and enough Hilbert space theory to understand the spectral theorem in some form. I have also seen distributions before in some depth as a way to derive fundamental solutions and Green's functions for linear constant-coefficient PDEs. I think it's time for a dedicated advancement of these topics.

0.1 Filters. Topological Spaces. Continuous Mappings

Skipped, as nothing outside of a standard point-set topology class is covered.

0.2 Vector Spaces. Linear Mappings

Skipped, as nothing outside of a standard linear algebra class is covered.

0.3 Topological Vector Spaces. Definition

Let E be a vector space over the complex numbers. Let

$$A : E \times E \to E;\ (x, y) \mapsto x + y$$

$$M : \mathbb{C} \times E \to E;\ (\lambda, x) \mapsto \lambda x$$

be the basic vector operations on E . A topology $\mathcal{T}$ in E is said to be *compatible with the linear structure* of E if the two maps are continuous in the product topology on each domain. We call E a *topological vector space* if it has a topology compatible with the linear structure.

Note that the topology is "translation-invariant," i.e. neighborhoods uniformly shifted are still neighborhoods. Thus is it only necessary to study the topology near the origin.

Theorem 0.3.1. *A filter $\mathcal{F}$ on a vector space E is the filter of neighborhoods of the origin compatible with the linear structure of E if and only if the following hold:*

1. *The origin belongs to every $U \in \mathcal{F}$*
2. *For every $U \in \mathcal{F}$ there is $V \in \mathcal{F}$ such that $V + V \subset U$*
3. *For every $U \in \mathcal{F}$ and every $\lambda \in \mathbb{C}$ nonzero, we have that $\lambda U \in \mathcal{F}$*
4. *Every $U \in \mathcal{F}$ is absorbing*
5. *Every U contains some $V \in \mathcal{F}$ which is balanced*

Definition 0.3.1. *A subset A of a vector space E is said to be absorbing if for every $x \in E$ there exists $c_x > 0$ such that for all $\lambda \in \mathbb{C}$, $|\lambda| \leq c_x$ $\lambda x \in A$.*

Definition 0.3.2.