High-frequency stability of detonation waves and block diagonalization of ODEs

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Abstract

A delicate aspect of numerical stability investigations for ZND (inviscid) detonations is truncation of the computational domain via high-frequency asymptotics, amounting to an ordinary differential semi-classical limit problem $h(dW/dx) = A(x,h)W$, where $h > 0$ is a small parameter tending toward 0^+. This leads to interesting and general questions related to WKB expansion, turning points, and block-diagonalization/separation of modes. In particular, as we shall describe, it highlights the distinction between "spectral gap" and "spectral separation" in asymptotic ODE theory, revealing essential differences between C^∞ coefficient and analytic coefficient theory. These differences are in turn related to oscillatory integrals and differences in stationary phase estimates for C^∞ vs. analytic symbols. The results we describe are an essential part of our recent proof of high-frequency detonation stability, completing a program pioneered by J.J. Erpenbeck in the 1960's. However, no knowledge of detonation theory is assumed in the talk and the analysis, though motivated by physical considerations, concerns basic questions in asymptotic ordinary differential equations. This is joint work with Olivier Lafitte (Paris 13), and Mark Williams (UNC).