Rational solutions of Painlevé equations

Peter Miller
University of Michigan
millerpd@umich.edu

Abstract

All of the six Painlevé equations except the first have rational solutions for certain parameter values. We survey some recent results obtained in collaboration with T. Bothner, R. Buckingham, and Y. Sheng on the asymptotic behavior of rational solutions of Painlevé II, III, and IV when the parameters are large. These results are obtained by first computing the correct isomonodromy data for the Jimbo–Miwa Lax pair associated with the family of rational solutions with the help of classical special functions, their connection formulæ, and Schlesinger transformations. Then it becomes possible to apply the Deift–Zhou steepest descent method to an appropriate Riemann-Hilbert problem characterizing the rational solutions at hand. This allows the transitions between pole-free regions and regions containing regular lattices of poles to be characterized in terms of bifurcations of a suitable g-function, and provides accurate asymptotic formulæ for the rational solutions valid in both types of regions.