# Samuel Walsh

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# **Employment**

#### University of Missouri, Columbia, MO

2019- Associate Professor 2013-2019 Assistant Professor

Courant Institute of Mathematical Sciences, New York University, New York, NY

2010-2013 Courant Instructor

#### Education

2010 Рн.D., Applied Mathematics, Brown University, Providence, RI

Dissertation: Stratified steady periodic water waves

Thesis Advisor: Walter Strauss

2005 B.S. (with High Honors), Mathematical Sciences, Carnegie Mellon University, Pittsburgh, PA

## Research interests

Nonlinear PDE, water waves, mathematical fluid mechanics, nonlinear dispersive equations, infinite-dimensional dynamical systems

#### **Publications**

Submitted

Desingularization and global continuation for hollow vortices (with R. M. Chen and M. H. Wheeler), submitted.

arXiv:2303.03570.

#### APPEARED AND IN PRESS

Rigidity of three-dimensional internal waves with constant vorticity, (with R. M. Chen, L. Fan, and M. H. Wheeler), to appear in J. Math. Fluid Mech.

arXiv:2208.06477.

*Global bifurcation for monotone fronts of elliptic equations*, (with R. M. Chen and M. H. Wheeler), to appear in J. Eur. Math. Soc.

arXiv:2005.00651.

Smooth stationary water waves with exponentially localized vorticity, (with M. Ehrnström and C. Zeng), J. Eur. Math. Soc., **25**(3) (2023), 1045–1090.

DOI:10.4171/jems/1204, arXiv:1907.07335.

*Orbital stability of internal waves*, (with R. M. Chen), Commun. Math. Phys., **391** (2022), 1091–1141. DOI:10.1007/s00220-022-04332-x, arXiv:2102.13590.

*Traveling water waves* — *The ebb and flow of two centuries*, (with S. V. Haziot, V. M. Hur, W. Strauss, J. F. Toland, E. Wahlén, and M. H. Wheeler), Q. Appl. Math., **80**(2) (2022), 317–401.

DOI:10.1090/qam/1614, arXiv:2109.09208.

Center manifolds without a phase space for quasilinear problems in elasticity, biology, and hydrodynamics, (with R. M. Chen and M. H. Wheeler), Nonlinearity, **35**(4) (2022) 1927–1985.

DOI:10.1088/1361-6544/ac5096, arXiv:1907.04370.

Global bifurcation of anti-plane shear fronts, (with R. M. Chen and M. H. Wheeler), J. Nonlinear Sci., 31(28) (2021).

DOI:10.1007/s00332-021-09684-7, arXiv:2008.09453.

*Large-amplitude internal fronts in two-fluid systems*, (with R. M. Chen and M. H. Wheeler), Comptes Rendus. Mathématique, **358**(9-10) (2020), 1073–1083.

DOI:10.5802/crmath.128, arXiv:2007.16055.

On the stability of solitary water waves with a point vortex, (with K. Varholm and E. Wahlén), Commun. Pure Appl. Math., 73(12) (2020), 2634–2684.

DOI:10.1002/cpa.21891, arXiv:1811.08024.

Existence, nonexistence, and asymptotics of deep water solitary waves with localized vorticity, (with R. M. Chen and M. H. Wheeler), Arch. Rational Mech. Anal.,  $\bf 234(2)$  (2019),  $\bf 595-633$ .

DOI:10.1007/s00205-019-01399-0, arXiv:1706.00147.

Solitary water waves with discontinuous vorticity, (with A. Akers), J. Math. Pures Appl., 124 (2019), 220–272.

DOI:10.1016/j.matpur.2018.06.008, arXiv:1709.09918.

Existence and qualitative theory for stratified solitary water waves, (with R. M. Chen and M. H. Wheeler), Ann. Inst. H. Poincaré Anal. Non Linéaire, **25**(2) (2018), 517–576.

DOI:10.1016/j.anihpc.2017.06.003, arXiv:1601.05130.

*Unique determination of stratified steady water waves from pressure*, (with R. M. Chen), J. Differential Equations, **264**(1) (2018), 115–133.

DOI:10.1016/j.jde.2017.09.002, arXiv:1502.07775.

*Pressure transfer functions for interfacial fluid problems*, (with R. M. Chen and V. M. Hur), J. Math. Fluid Mech., **19**(1) (2017), 59–76.

DOI:10.1007/s00021-016-0265-6, arXiv:1511.05550.

On the wind generation of water waves, (with O. Bühler, J. Shatah, and C. Zeng), Arch. Rational Mech. Anal., 222(2) (2016), 827–878.

DOI:10.1007/s00205-016-1012-0, arXiv:1505.02032.

On the existence and qualitative theory for stratified solitary water waves, (with R. M. Chen and M. H. Wheeler), C. R. Acad. Sci. Paris, Ser. I., 354(6) (2016), 601–605.

DOI:10.1016/j.crma.2016.03.004.

Continuous dependence on the density for stratified steady water waves, (with R. M. Chen), Arch. Rational Mech. Anal., **219**(2) (2016), 741–792.

DOI:10.1007/s00205-015-0906-6, arXiv:1408.5030.

Nonlinear resonances with a potential: Multilinear estimates and an application to NLS, (with P. Germain and Z. Hani), Internat. Math. Res. Notices, **2015**(18) (2015), 8484–8544.

DOI:10.1093/imrn/rnu195, arXiv:1303.4354.

Steady stratified periodic gravity waves with surface tension I: Local bifurcation, Discrete Cont. Dyn. Syst. Ser. A, **8** (2014), 3241–3285.

DOI:10.3934/dcds.2014.34.3241.

Steady stratified periodic gravity waves with surface tension II: Global bifurcation, Discrete Cont. Dyn. Syst. Ser. A, 8 (2014), 3287–3315.

DOI:10.3934/dcds.2014.34.3287.

Travelling water waves with compactly supported vorticity, (with J. Shatah and C. Zeng), Nonlinearity, **26** (2013), 1529–1564.

DOI:10.1088/0951-7715/26/6/1529, arXiv:1211.3314.

Steady water waves in the presence of wind, (with O. Bühler and J. Shatah), SIAM J. Math. Anal., 45 (2013), 2182–2227.

DOI:10.1137/120880124, arXiv:1211.3308.

Some criteria for the symmetry of stratified water waves, Wave Motion, **46** (2009), 350–362. DOI:10.1016/j.wavemoti.2009.06.008, arXiv:0903.0908.

Stratified steady periodic water waves, SIAM J. Math. Anal., **41** (2009), 1054–1105. DOI:10.1137/080721583, arXiv:0807.0474v3.

#### INTERDISCIPLINARY WORKS

S. Balkissoon, N. Fox, A. Lupo, S. E. Haupt, Y. C. Li, P. Market, S. Walsh, *Determining chaotic characteristics and forecasting tall tower wind speeds in Missouri using Empirical Dynamical Modeling (EDM)*, Renewable Energy, **170** (2021), 1292–1307.

DOI:10.1016/j.renene.2021.01.108.

# Honors, research awards, and visiting appointments

2023-2026 NSF DMS-2306243, "Non-perturbative interfacial waves."

2022-2023 Simons Foundation, Mathematics and Physical Sciences-Collaboration Grants for Mathematicians, "Existence, stability, and instability of steady water waves."

2022 Chancellor's Award for Outstanding Research and Creative Activity in the Physical and Mathematical Sciences, University of Missouri.

- 2020 Richard F. and Sharon A. Kiester Faculty Enhancement Award, University of Missouri.
- Research-in-Teams Grant, "Global bifurcation techniques for traveling waves on non-compact domains", Erwin Schrödinger Institute, Vienna, Austria.
  - US Junior Oberwolfach Fellow.
  - NSF DMS-1844731, conference grant to support the 2019 KUMUNU Conference on PDE, Dynamical Systems, and Application.
- NSF DMS-1812436, "Existence and energetic stability of traveling waves in the presence of symmetry."
  - 2017 Longterm Visitor, Institute for Computational and Experimental Research Mathematics, Providence, RI.
    - Richard F. and Sharon A. Kiester Faculty Enhancement Award, University of Missouri.
  - 2016 NSF DMS-1549934, conference grant to support the 2016 KUMU Conference on PDE, Dynamical Systems, and Applications.
- 2015-2018 NSF DMS-1514910, "Existence, stability, and qualitative theory of traveling water waves."
  - 2013 Travelling water waves with compactly supported vorticity officially recognized as a "Highlight of 2013" by the journal Nonlinearity.
  - 2010 Dunmu Ji Award, recognizing a doctoral thesis as particularly original and independent, Division of Applied Mathematics, Brown University.

## Selected invited talks

- Apr. 2023 Desingularization and global continuation for hollow vortices, Workshop on Nonlinear Dispersive Waves, University College Cork, Ireland.
- Mar. 2023 Desingularization and global continuation for hollow vortices, Analysis Seminar, University of Alabama, Tuscaloosa, AL.
- Jul. 2022 *Orbital stability of internal waves*, New Directions in Water Waves, Workshop and Summer School, University of Bath, Bath, UK.
- Jun. 2022 *Orbital stability of internal waves*, Workshop on Nonlinear Waves in Discrete and Continuum Systems, University of Pittsburgh, Pittsburgh, PA.
- Mar. 2022 Global bifurcation of hydrodynamic bores, Continuum Mechanics Seminar, University of Nebraska–Lincoln, Online.
- Nov. 2021 *Global bifurcation for monotone fronts of elliptic equations*, Computational and Applied Mathematics Seminar, University of Kansas, Lawrence, KS.

- Oct. 2021 Global bifurcation for monotone fronts of elliptic equations, Special Session on Progress in Nonlinear Waves, AMS Central Virtual Sectional Meeting, Online.
  - *Global bifurcation of hydrodynamic bores*, Seminar on Analysis, Geometry, and PDEs, Lund University, Online.
- May 2021 Orbital stability of internal waves, Differential Equations and Numerical Analysis Seminar, Norwegian University of Science and Technology, Online.
- Apr. 2021 Orbital stability of internal waves, Analysis Seminar, University of Kansas, Online.
- Feb. 2021 Large fronts in hydrodynamics and nonlinear elasticity, Analysis and PDE Seminar, University of Pittsburgh, Online.
- Oct. 2020 Global bifurcation for monotone fronts of elliptic equations, Analysis Seminar, University of Oklahoma, Online.
  - Water waves with density stratification or localized vorticity, ONEPAS Seminar, Online.
- Sep. 2020 Global bifurcation for monotone fronts of elliptic equations, PDE Seminar, Brown University, Online.
- Mar. 2020 *Global bifurcation for monotone fronts of elliptic equations*, Analysis and PDE Seminar, University of Pittsburgh, Pittsburgh, PA.
- Oct. 2019 Center manifolds without a phase space for quasilinear PDE from elasticity, biology, and hydrodynamics, Analysis and PDE Seminar, University of Pittsburgh, Pittsburgh, PA.
- Sep. 2019 *Capillary-gravity water waves with exponentially localized vorticity*, Mathematics Colloquium, Missouri S&T, Rolla, MO.
  - Center manifolds without a phase space for quasilinear PDE from elasticity, biology, and hydrodynamics, Analysis in Missouri: A Midwestern Symposium, Columbia, MO.
- Jul. 2019 *Orbital stability and instability of fractional KdV solitary waves*, Mathematisches Forschungsinstitut Oberwolfach, Oberwolfach, Germany.
- May 2019 Water waves with localized vorticity, Mathematical Colloquium, University of Vienna, Austria.
- Mar. 2019 Capillary-gravity water waves with exponentially localized vorticity, Midwest Partial Differential Equations Seminar, Indiana University, Bloomington, IN.
- Nov. 2018 Capillary-gravity water waves with exponentially localized vorticity, Applied Math/PDE Seminar, Drexel University, Philadelphia, PA.
- Oct. 2018 *Capillary-gravity water waves with exponentially localized vorticity*, Workshop on Nonlinear Differential Equations, Dynamical Systems and Applications, University of Kansas, Lawrence, KS.
- Jun. 2018 Capillary-gravity water waves with exponentially localized vorticity, Lund Workshop on Fluid Dynamics and Dispersive Equations, Lund, Sweden.

- Dec. 2017 Existence, nonexistence, and asymptotics of deep water solitary waves with localized vorticity, "Non-linear water waves an interdisciplinary interface" workshop, Erwin Schrödinger Institute, Vienna, Austria.
- Oct. 2017 *Stability of traveling waves with a point vortex*, Analysis, Dynamics, and Applications Seminar, University of Arizona, Tucson, AZ.
- Aug. 2017 Stability of traveling waves with a point vortex, Nonlinear Water Waves Workshop, Isaac Newton Institute for Mathematical Sciences, Cambridge, UK.
- Apr. 2017 Stability of traveling waves with a point vortex, Water Waves Workshop, ICERM, Providence, RI.
- Mar. 2017 Water waves with localized vorticity, Conference on Nonlinear Waves: Analysis and Applications University of Pittsburgh, Pittsburgh, PA.
- Oct. 2016 Existence and qualitative theory for solitary stratified water waves, Theoretical and Computational Aspects of Nonlinear Surface Waves, Banff International Research Station for Mathematical Innovation and Discovery, Banf, Alberta, Canada.
  - Existence and qualitative theory for solitary stratified water waves, 2nd Annual Meeting of SIAM Central States Section, Little Rock, AR.
- Jul. 2016 Existence and qualitative theory for solitary stratified water waves, 11th AIMS Conference on Dynamical Systems, Differential Equations and Applications, Orlando, FL.
- Dec. 2015 Existence and qualitative theory for solitary stratified water waves, SIAM Conference on Analysis of Partial Differential Equations, Scottsdale, AZ.
- Jun. 2015 Mathematical theory of wind-generated water waves, Differential Equations and Numerical Analysis Seminar, Norwegian University of Science and Technology (NTNU), Trondheim, Norway.
- Apr. 2015 Continuous dependence on the density for stratified steady water waves, Workshop on Mathematical Theory of Water Waves, Mathematisches Forschungsinstitut Oberwolfach, Oberwolfach, Germany.
- Apr. 2015 Continuous dependence on density for stratified steady water waves, IMACS International Conference on Nonlinear Evolution Equations and Wave Phenomena: Computation and Theory, Athens, GA.
- Mar. 2015 Instability of traveling water waves with a point vortex, Conference on PDEs and Free Boundary Problems, University of Pittsburgh, Pittsburgh, PA.
- Nov. 2014 On the wind-generation of water waves, Harmonic Analysis and Differential Equations Seminar, University of Illinois Urbana-Champaign, Urbana-Champaign, IL.
- Jul. 2014 On the wind-generation of water waves, AIMS Conference on Dynamical Systems, Differential Equations and Applications, Madrid, Spain.
- Dec. 2013 Mathematical theory of wind-generated water waves, SIAM Conference on Analysis of PDEs, Orlando, FL.

- Nov. 2013 Resonance for nonlinear dispersive equations with a potential, Computational and Applied Mathematics Seminar, University of Kansas, Lawrence, KS.
- Oct. 2013 Resonance for nonlinear dispersive equations with a potential, PDE and Analysis Seminar, University of Pittsburgh, Pittsburgh, PA.
- Jun. 2013 Resonance for nonlinear dispersive equations with a potential, Chinese–Norwegian Mathematics Workshop, Trondheim, Norway.
- Jun. 2013 *Resonance for nonlinear dispersive equations with a potential*, 26th Nordic and 1st European–Nordic Congress of Mathematicians, Lund, Sweden.
- Mar. 2013 Resonance for nonlinear dispersive equations with a potential, IMACS International Conference on Nonlinear Evolution Equations and Wave Phenomena, Athens, GA.
- Jan. 2013 Steady water waves with compactly supported vorticity, Joint Mathematics Meetings, San Diego, CA.
- Nov. 2012 *Steady waves with compactly supported vorticity*, Applied Mathematics Seminar, University of Maryland, College Park, MD.
- Jun. 2012 Steady water waves with compactly supported vorticity, Nonlinear Waves and Interface Problems Workshop, University of Lund.
- Apr. 2012 Steady water waves in the presence of wind, Georgia Institute of Technology, Atlanta, GA.
- Apr. 2011 Theory of steady stratified water waves, IMACS International Conference on Nonlinear Evolution Equations and Wave Phenomena, Athens, GA.
- Nov. 2010 Traveling waves in stratified water, Applied Mathematics Seminar, Courant Institute, NYU, New York, NY.
- Apr. 2010 An existence theory for steady stratified water waves, Analysis Seminar, University of Pennsylvania, Philadelphia, PA.
- Dec. 2009 Stratified steady water waves, SIAM Conference on Analysis of PDEs, Miami, FL.
- Feb. 2009 *Stratified steady water waves*, Mathematisches Forschungsinstitut Oberwolfach, Oberwolfach, Germany.

## Service

Past students: Thomas Hogancamp (PhD 2023), Daniel Sinambela (PhD 2022), Sebastian Henderson (MS, 2022), Hung Le (PhD, 2019), Max Highsmith (MS, 2018), Adelaide Akers (PhD, 2017), Jessie Bleile (MS, 2016), Evan Datz (MS, 2016).

Undergraduate research advising: Elijah Good (Discovery Research Fellow, MU, 2021–); Noah Casey (Undergraduate Summer Research Project, MU, 2022); Michael Dotzel (Undergraduate Thesis, MU, 2016); Yungjoo Lee, Timothy Mok, and Haochuan Wang (S.U.R.E. Program, NYU, 2012).

Editorial board member for Communications in Analysis and Mechanics 2023-.

Reviewer for AMS Mathematical Reviews.

Reviewer for Simons Foundation.

Refereed submitted articles for: Advances in Mathematics, Archive Rational Mechanics and Analysis; Communications in Mathematical Physics; Communications on Pure and Applied Mathematics; Discrete and Continuous Dynamical Systems A; Fluid Dynamics Research; Journal of Differential Equations; Journal of Fluid Mechanics; Journal of Mathematics Analysis and Applications; Nonlinearity; Philosophical Transactions of the Royal Society A; Proceedings of the Royal Society of Edinburgh, Section A; SIAM Journal on Mathematical Analysis; Studies in Applied Mathematics; and the Transactions of the AMS.

2023 Organizer of the MU Differential Equations Seminar.

Math Competition Advisor and faculty advisor to the undergraduate Math Club, University of Missouri.

Awards Committee, Mathematics Department, University of Missouri.

Curriculum Committee, Mathematics Department, University of Missouri

2022 Co-organizer 86th Midwest PDE Seminar

Co-organizer mini-symposium "Recent advances in nonlinear PDEs modeling fluids and other non-linear system" at Twelfth IMACS International Conference on Nonlinear Evolution Equations and Wave Phenomena: Computation and Theory.

Co-organizer of the MU-MST Joint Analysis Seminar.

Organizer of the MU Differential Equations Seminar.

Math Competition Advisor and faculty advisor to the undergraduate Math Club, University of Missouri

Awards Committee, Mathematics Department, University of Missouri.

2021 Co-organizer of the MU-MST Joint Analysis Seminar.

Organizer of the MU Differential Equations Seminar.

Math Competition Advisor and faculty advisor to the undergraduate Math Club, University of Missouri

Awards Committee, Mathematics Department, University of Missouri.

2020 Invited talk for Mathematics Honors Society at Hickman High school.

Co-organizer of the MU-MST Joint Analysis Seminar.

Organizer of the MU Differential Equations Seminar.

Math Competition Advisor and faculty advisor to the undergraduate Math Club, University of Missouri

Awards Committee, Mathematics Department, University of Missouri.

2019 Co-organizer of the 2019 KUMUNU Conference on Partial Differential Equations, Dynamical Systems, and Applications.

Co-organizer of mini-symposium "Stability and traveling waves" at the 11th IMACS International Conference on Nonlinear Evolution Equations and Wave Phenomena: Computation and Theory.

Organizer of the MU Differential Equations Seminar.

Math Competition Advisor and faculty advisor to the undergraduate Math Club, University of Missouri

Awards Committee, Mathematics Department, University of Missouri.

2018 Organizer of the MU Differential Equations Seminar.

Math Competition Advisor and faculty advisor to the undergraduate Math Club, University of Missouri

Awards Committee, Mathematics Department, University of Missouri.

Served on Library Committee, Mathematics Department, University of Missouri.

2017 Served as Math Competition Advisor for MU and faculty advisor to the undergraduate Math Club.

Organizer of the MU Differential Equations Seminar.

Workload Policy Committee, Mathematics Department, University of Missouri.

Co-organizer of mini-symposium "Traveling waves and spectral theory" at the 10th IMACS International Conference on Nonlinear Evolutions Equations and Wave Phenomena: Computation and Theory.

2016 Co-organizer of the KUMU Conference on PDE, Dynamical Systems, and Applications.

Organizer of the MU Differential Equations Seminar.

Library Committee, Mathematics Department, University of Missouri.

- 2015 Co-organizer of mini-symposium "Water waves" at the SIAM Conference on Analysis of Partial Differential Equations.
- External sensor evaluating the master's thesis for a student (K. Varholm) at NTNU (Norwegian University of Science and Technology)

Analysis Qualification Exam Committee, Mathematics Department, University of Missouri.

Referee for Missouri Research Board grant application.

- Speaker at *cSplash*, an annual one-day festival of mathematics and computer science talks aimed at high school students
- Faculty advisor to three participants in the Courant Institute Summer Undergraduate Research Experience (S.U.R.E.) program, Summer 2012.
- 2011 Speaker at cSplash.
- 2008-2010 Organizer, Brown University Working Seminar in PDEs.

## Teaching experience

#### 2013- University of Missouri

Calculus III

Introduction to multivariate calculus, with topics spanning the geometry of three-dimensional space to Stokes' Theorem.

Differential Equations

Introductory undergraduate course on ordinary differential equations: explicit solution methods for first- and second-order ODEs, numerical methods, phase plane analysis, and applications.

Numerical Linear Algebra

Advanced undergraduate/beginning graduate course on numerical methods for linear algebra.

Mathematical Modeling

Advanced undergraduate/beginning graduate course introducing methods of mathematical modeling and applied mathematics.

Applied Analysis

Advanced undergraduate/beginning graduate course introducing PDEs, with an emphasis on Fourier Analysis methods.

Topics in Applied Math: Bifurcation Theory

Advanced graduate-level topics course on applications of local and global bifurcation theory to problems in hydrodynamics and elliptic PDE.

Topics in Applied Math: Nonlinear Dispersive Equation

Advanced graduate-level topics course on nonlinear dispersive PDEs.

Topics in Applied Math: Nonlinear Dispersive Equation II

Continuation of Topics in Applied Math: Nonlinear Dispersive Equation.

Topics in Applied Math: Mathematical Theory of Water Waves

Advanced graduate-level topics course in theoretical fluid mechanics and water waves.

Topics in Applied Math: Advanced Topics in Partial Differential Equations

Advanced graduate-level topics covering Schauder theory, variational methods for PDEs, and semi-groups.

Partial Differential Equations I

Graduate-level introductory course on partial differential equations.

Partial Differential Equations II

Second semester graduate-level course on partial differential equations.

Advance Ordinary Differential Equations

Graduate-level introductory course on ODEs and dynamical systems.

#### 2010-2013 Courant Institute

Analysis I

Introductory undergraduate course in real analysis: limits, point-set topology, compactness, metric spaces.

Discrete Mathematics

Introduction to proof-based mathematics, with material drawn from a survey of Discrete Mathematics topics.

Partial Differential Equations

Advanced undergraduate course in PDEs covering the highlights of linear theory, as well as some topics in nonlinear PDE.

Linear Algebra

Introduction to basic tools of Linear Algebra: vector and matrix manipulation, eigenvalues, vector spaces.

Calculus III

Introduction to multivariate calculus, with topics spanning the geometry of three-dimensional space to Stokes' Theorem.

### 2006-2009 Brown University

#### PRIMARY INSTRUCTOR

Intermediate Calculus (Physics/Engineering)

Introduction to multivariate calculus with emphasis on applications in electrostatics and fluid mechanics.

TEACHING ASSISTANT

Essential Statistics

First course in statistics, giving introductory treatment of probability, data analysis and common statistical methods.

Methods of Applied Mathematics I

Theory of first- and second-order ordinary differential equations, including analytic and numerical methods.

Methods of Applied Mathematics II

Systems of ODEs, stability analysis, introduction to partial differential equations.

## 2004-2005 Carnegie Mellon University

TEACHING ASSISTANT

 $Concepts\ of\ Mathematics$ 

Introduction to proof-based mathematics with material drawn from number theory and discrete mathematics.

Integration, Differential Equations, and Approximation Second course in differential and integral calculus, covering methods of integration, sequences and series and basic theory of ODEs.

# Related skills

Python, C/C++/Objective-C, Lisp/Scheme, Matlab, Maple, OpenGL.