A simplified proof of the Gohberg-Sigal-Rouché theorem

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Abstract

The GSR theorem generalizes the classical Rouché theorem of complex analysis, concerning the stability of the total multiplicity of "roots minus poles" for finitely meromorphic functions taking values in the space of Fredholm operators on a Hilbert space. This theorem has been a very useful tool in the stability analysis of nonlinear waves. Proofs based on logarithmic integrals are usually considerably complicated by the discontinuous nature of the trace operator in infinite dimensions. We describe how these proofs can be significantly simplified, by establishing continuity and analyticiy of trace based only on pointwise bounds for rank. This is a preliminary report on joint work with Zachary Stone and Jürgen Leiterer.