Ill-posedness of truncated series models of water waves

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Abstract

Some numerical methods for water waves, such as the Craig-Sulem method, involve expanding terms in the water wave evolution equations as series, truncating those series, and then simulating the resulting equations. For one such scheme, we present analytical evidence that the truncated system is in fact ill-posed; this involves further reducing the evolution equations to a model for which we can prove ill-posedness. We then present numerical evidence that the full truncated system is ill-posed, showing that arbitrarily small data can lead to arbitrarily fast growth. We present this numerical evidence for multiple levels of truncation. We are able to prove that by adding a viscosity to the system, we instead arrive at a well-posed initial value problem. If time allows, we will discuss progress towards a full proof of ill-posedness, and will also discuss how higher-order dispersion can regularize the system. This is joint work with Jerry Bona, Shunlian Liu, David Nicholls, and Michael Siegel.