Rigidity of three-dimensional internal waves with constant vorticity

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Abstract

In this talk, we will discuss some recent results on the structural implications of constant vorticity for steady three-dimensional internal water waves. It is known that in many physical regimes, water waves beneath vacuum that have constant vorticity are necessarily two dimensional. The situation is more subtle for internal waves that traveling along the interface between two immiscible fluids. When the layers have the same density, there is a large class of explicit steady waves with constant vorticity that are three-dimensional in that the velocity field and pressure depend on one horizontal variable while the interface is an arbitrary function of the other.

Our main theorem states that every three-dimensional traveling internal wave with bounded velocity for which the vorticity in the lower layer $\boldsymbol{\omega}_1$ and vorticity in the upper layer $\boldsymbol{\omega}_2$ are nonzero, constant, and parallel must belong to this family. If the densities in each layer are distinct, then in fact the flow is fully two dimensional. This result is obtained using a novel but fairly elementary argument based on unique continuation, the maximum principle, and an analysis of streamline patterns.

This is joint work with R. M. Chen, L. Fan, and M. H. Wheeler.