Motion near a manifold of quasi-equilibria

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Abstract

The dynamics of a gradient system is obviously determined by the geometric structure or, as some authors like to say, by the landscape of the graph G of the energy functional J over a Hilbert space H. In certain cases, for instance in singular perturbation problems, J depends on a small parameter ε and, for $\varepsilon \ll 1$, G exhibits special features that have a peculiar dynamical counterpart. Here we discuss the case where the dynamical system in H is not necessarily a gradient system but has a Lyapunov function related to a gradient field and the energy grows strongly moving away from a manifold M but is almost constant along M. As an example we consider the vector Allen-Cahn system.