Existence and symmetry of small-amplitude solitary water waves with discontinuous vorticity

Adelaide Akers

University of Missouri-Columbia

In this project, we consider a two-dimensional body of water with constant density which lies below a vacuum. The ocean bed is assumed to be impenetrable, while the boundary which separates the fluid and the vacuum is assumed to be free. Following the work of M. D. Groves and E. Wahlen (2008), we use the Hamiltonian structure of the system coupled with center manifold reduction techniques to establish the existence of small-amplitude solitary water waves with discontinuous vorticity. Furthermore, we utilize a modified moving planes method to show that such solitary waves possess even symmetry.

Efficient and rigorous computation of the Evans function

Blake Barker

Brown University

We describe recent results pertaining to efficient and rigorous computation of the Evans function. In particular, we discuss computing the Evans function as a boundary value problem with an eye geared toward large dimensional systems, and we detail a method for rigorous computation of the Evans function that can be used in computer aided proof.

Modeling of mRNA localization in Xenopus oocytes

Veronica Ciocanel

Brown University

Messenger RNA (mRNA) localization is essential for Xenopus oocyte development and embryo patterning. This accumulation of RNA at the cell periphery is not well understood, but is thought to depend on diffusion, bidirectional movement and anchoring mechanisms. Our goal is to test these proposed mechanisms using PDE models and dynamical systems analysis, informed by numerical parameter estimation. Our results yield approximate traveling wave solutions and different parameter estimates in various regions of the cell cytoplasm.

These represent the currently available abstracts and titles for poster presenters at the 2016 KUMU Conference on Dynamical Systems, PDE, and Applications. If you are planning to present but do not see your abstract listed here, please email it as soon as possible to walshsa@missouri.edu.

A rotation-induced magnetic effect

Graham Cox
Pennsylvania State University

We prove a general averaging theorem that describes the dynamics of a particle in a rapidly varying time-periodic potential. This includes as special cases rotating potentials and oscillating potentials of the form $a(t)U(x)$. The rapid time variation is found to produce an effective magnetic force on the particle, acting perpendicular to its direction of motion. This phenomenon is studied in detail for the case of a tethered satellite moving in the gravitational field of a planet. (Joint work with Mark Levi.)

On the spectral stability of periodic waves of the coupled Schrödinger equations

Aslihan Demirkaya
University of Hartford

In this present work we (Sevdzhan Hakkaev and I) consider the periodic standing wave solutions for the coupled nonlinear Schrödinger equations. We restrict our problem to two cases and construct the solutions explicitly. Then we make the stability analysis for each case. We show that the relevant stationary solutions are spectrally stable for certain parameters. For those parameters, we also present our numerical results in the spectral analysis.

Mathematical analysis of ivermectin as a malaria control method

Robert Doughty
Miami University

The spread of malaria is detrimental the general health of people in many regions today. Due to this, we seek a new method of malaria control. We modify a previous malaria model to study the effect of an anti-parasitic medication, ivermectin, on two threshold parameters, which can determine the spread of malaria. The model takes the form of a system of nonlinear ordinary differential equations which model the dynamics of malaria transmission while taking into account the behavior and life cycle of the mosquito and its interaction with the human population. The model is studied using some applied dynamical systems techniques and we find that in certain cases, careful use of ivermectin can curtail the spread of malaria without harming the mosquito population. More often, the ivermectin either eradicates the mosquito population, or has little to no effect on the spread of malaria. We suggest that ivermectin be used as a control method in conjunction with other control methods such as reduction of breeding effects.

Stability of standing wave solutions for 4th-order NLS and Klein–Gordon Equations

Wen Feng
University of Kansas

We consider four different equations–fourth order NLS and fourth order Klein-Gordon as well as the fractional NLS and Klein-Gordon equations. Our goal is to investigate the linear stability of the standing wave solutions for each equation using the index theory for evolution equations of the form $u_t = JLu$, where J is an anti-symmetric operator and L is a self-adjoint operator with certain spectral properties. We prove the stability of the explicit solutions for the fourth-order equations and we give exact values of the frequencies for which the standing wave solutions of the fractional equations are stable.

Universal wavenumber selection in apical growth

Ryan Goh
University of Minnesota – Twin Cities

We study pattern-forming dissipative systems in growing domains. We characterize classes of boundary conditions that allow for defect-free growth and derive universal scaling laws for the wavenumber in the bulk of the domain. Scalings are based on a description of striped patterns in semi-bounded domains via strain-displacement relations and a multiple time scale analysis. Our predictions compare well with direct simulations in the Swift–Hohenberg, the Complex Ginzburg–Landau, the Cahn–Hilliard, and reaction-diffusion equations. Such systems have been used to study patterns in many areas such as phyllotaxis and animal coats, and could also suggest novel ways for fabricating self-organized and self-mediated functionalized structures. Joint with A. Scheel, R. Beekie, D. Matthias, and J. Nunley.

Pointwise Green's function bounds for a viscoelastic Burgers equation

Rachel Jennings

University of Wyoming

Interestingly, viscoelastic Burgers equation, which incorporates an ad hoc reduction of a standard constitutive law for viscoelastic fluids into Burgers-type framework, supports traveling-wave solutions (e.g., "double" shocks). Others have established the existence of families of traveling-wave solutions, but the investigations of stability were confined to observations in the context of numerical simulations. Here, we present a complete nonlinear stability analysis. Although Zumbrun and collaborators have treated a great variety of systems by such techniques, we believe several features of this system (relatively simple form, lack of conservation form, structure of source terms, ...) suggest that the detailed derivation of estimates on the Green function (and consequent stability analysis) will serve as a valuable case study.

Numerical computation of a connecting orbit based on the principle of Wazewski

John Jinho Kim

North Carolina State University

The principle of Wazewski states that if there is a bounding region for the differential equation $x' = f(x)$ that satisfies certain hypotheses, then there exists a solution inside the region all the time. This has been a major tool to show the existence of traveling waves in many areas of mathematics such as predator-prey models, chemical reactions, and thermodynamics.

The purpose of this research is twofold. The first part is to develop a shooting method based on the principle of Wazewski to numerically compute the heteroclinic and homoclinc solutions of the dynamical system, and determine the speed of convergence in terms of eigenvalues and the angles of intersection between stable and unstable manifolds of the equilibria.

The second part is to prove an inverse of the principle of Wazewski for some specific cases. Given the differential equation $x' = f(x)$ in $\mathbb{R}^3$, if there is a heteroclinic orbit formed by the intersection between the unstable manifold of $E-$ and the stable manifold of $E+$, then the trapping region containing such an orbit can be constructed.

This poster is based on the thesis in North Carolina State University under direction of Professor Xiao-Biao Lin.

Nonlinear modulational instability of dispersive PDE models

Shasha Liao
Georgia Tech

We prove the nonlinear modulational instability for both periodic and localized perturbations of periodic traveling waves of several dispersive PDEs, such as the KdV-type equation (e.g. Whitham equation) and the BBM equation. The main ingredients of our proof are: construction of higher order approximate solution by the wave packets; energy methods; the semi-group estimates by the Hamiltonian structures of the models. This is a joint work with Jiayin Jin and Zhiwu Lin.

Polynomial energy decay for periodic damped beam equation on $\mathbb{R}^n$

Satbir Malhi
University of Kansas

We prove that energy of the solution for the beam equation with damping term decay at a rate $1/t^2$. Our approach is based on the asymptotic theory of C_0-semigroup, in particular on a result by Borichev and Tomilov, in which they relate the decay rate of energy in term of the the resolvent growth of the semigroup generator. We estimate the resolvent of the semigroup via the observability estimate of bi-Laplacian on $\mathbb{R}^n$.

Towards metastability in the Burgers equation with periodic boundary conditions

Kelly McQuighan
Boston University

Roughly speaking, metastable solutions capture transient behavior which persists for long times. Recent work on Burgers equation on the real line and on Navier-Stokes equation with periodic boundary conditions has provided some insight into various mechanisms for metastability. In this talk we discuss a candidate metastable solution for the viscous Burgers equation with periodic boundary conditions. We construct the "frozen-time" spectrum for this solution using ideas from singular perturbation and Melnikov theory. Finally, we indicate future directions in which this spectrum can be used to understand metastability for the full PDE.

Unstable periodic wavetrains in a shallow water model

Ashish Pandey
University of Illinois-Urbana Champaign

We propose bi-directional Whitham, or Boussinesq–Whitham, equations for shallow water waves and demonstrate the instability of the Benjamin–Feir kind.

Counting spectrum via the Maslov index for one dimensional θ-periodic Schrödinger operators

Selim Sukhtaiev
University of Missouri-Columbia

We study the spectrum of the Schrödinger operators with $n \times n$ matrix valued potentials on a finite interval subject to θ-periodic boundary conditions. For two such operators, corresponding to different values of θ, we compute the difference of their eigenvalue counting functions via the Maslov index of a path of Lagrangian planes. In addition we derive a formula for the derivatives of the eigenvalues with respect to θ in terms of the Maslov crossing form.

The Morse and Maslov indices for Schrödinger operators: Hadamard-type formulas

Alim Sukhtayev
Indiana University Bloomington

An important ingredient in the analysis of a dynamical system is the stability of a given equilibrium state. In many cases this is determined by the eigenvalues of an elliptic boundary value problem. For the one dimensional scalar case the situation is completely understood, with Sturm–Liouville theory relating the (in)stability of a steady state to its underlying geometric features, such as the number of zeros or critical points. However, for the systems or multiple spatial dimensions much less is known. We describe new methods for computing spectra of Schrödinger operators using the Maslov index, a symplectic invariant counting signed intersections of Lagrangian subspaces. In particular, we derive an analogue of the Morse-Smale Index Theorem for the Schrödinger operators. We also derive Hadamard-type formulas via the Maslov form.

Quasiperiodic solutions to nonhomogeneous elliptic equations in $\mathbb{R}^{N+1}$

Dario Valdebenito
University of Minnesota

We establish sufficient conditions to obtain solutions of $-\Delta u + u_{yy} + a_1(x)u + bf(x,u) = 0$, $(x,y) \in \mathbb{R}^{N+1}$, which are quasiperiodic in y and decaying as $|x| \to \infty$, uniformly in y. Here $f(x,u) = O(u^2)$ as $u \to 0$. Such solutions are found using a center manifold reduction and KAM theory. These conditions are satisfied for a large class of sufficiently smooth radial potentials a_1 and radial nonlinearities f. Joint work with P. Polacik.

Arnold's stability results for the 2D α-Euler equations

Shibi Vasudevan
University of Missouri-Columbia

We shall present analogues of the celebrated Arnold stability theorems in the context of the 2D α-Euler equations.

Existence, bifurcation, and geometric evolution of quasi-bilayers in the multicomponent functionalized Cahn-Hilliard equation

Qiliang Wu
Michigan State University

Multicomponent bilayer structures arise as the ubiquitous plasma membrane in cellular biology and as blends of amphiphilic copolymers used in electrolyte membranes, drug delivery, and emulsion stabilization within the context of synthetic chemistry. We present the multicomponent functionalized Cahn-Hilliard (mFCH) free energy as a model which allows competition between bilayers with distinct composition and between bilayers and higher codimensional structures, such as co-dimension two filaments and co-dimension three micelles. We investigate the stability and slow geometric evolution of multicomponent bilayer interfaces within the context of an H^{-1} gradient flow of the mFCH, addressing the impact of aspect ratio of the amphiphile (lipid or copolymer unit) on the intrinsic curvature and the codimensional bifurcation. In particular we derive a Canham-Helfrich sharp interface energy whose intrinsic curvature arises through a Melnikov parameter associated to amphiphile aspect ratio. We construct asymmetric homoclinic bilayer profiles via a billiard limit potential and show that co-dimensional bifurcation is driven by the experimentally observed layer-by-layer pearling mechanism.

Stability of time independent solutions to a class of reaction diffusion systems in a multidimensional space

Xinyao Yang
University of Missouri-Columbia

This joint work with Ghazaryan and Latushkin proves the stability of time independent solutions to a class of reaction diffusion systems in a multidimensional space. This work is a generalization of Ghazaryan and Latushkin's previous results of planar in a weighted one-dimensional space and Kapitula's stability results of planar in an unweighted multidimensional space. The left end is stable given that its perturbation exponentially decays in the weighted norm and bounded in the unweighted norm. The travelling wave solution is stable if its perturbation algebraically decays in the weighted norm and bounded in the unweighted norm. A decomposition of the variables that yield a triangular structure for the linearization at the left end and the diffusion matrix is an identity matrix are pre-assumed.