Finite difference solutions of pseudo-parabolic problem with finite delay

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One dimensional initial-boundary delay pseudo-parabolic problem is being considered. To solve this problem numerically, we construct higher order difference method for approximation to the considered problem and obtain the error estimate for its solution. Based on the method of energy estimate the fully discrete scheme is shown to be convergent of order four in space and of order two in time. Numerical example is presented.

Growing stripes, with and without wrinkles

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We present results on selection of stripe patterns in the Swift–Hohenberg equation with an inhomogeneity modeling directional quenching. Stripes are grown in the wake of a propagating parameter step, and we analyze the selection of the orientation of stripes by the quenching speed and the lateral aspect ratio. We observe stripes perpendicular to the quenching line, stripes at an oblique angle, and periodic wrinkles in an otherwise oblique stripe pattern. Technically, we study these patterns analytically and numerically as traveling wave solutions to reduced Cahn–Hilliard and Newell–Whitehead–Segel equations, and focus on a singular perturbation problem for a traveling wave ODE. We also study these phenomena in the Swift–Hohenberg equation via numerical continuation and direct simulation.

Snakes and lattices: Understanding the bifurcation structure of localized solutions to lattice dynamical systems

Jason Bramburger

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A wide variety of spatially localized steady-state solutions to partial differential equations (PDEs) are known to exhibit a bifurcation phenomenon termed snaking. That is, these solutions bounce between two different values of the bifurcation parameter while expanding the region of localization and hence ascending in norm. The mechanism that drives snaking in PDEs has been understood by analyzing the evolution of the ordinary differential equation in the spatial variable governing steady-state solutions to the PDE. This posters focuses on how we can extend this theory to lattice dynamical systems by showing that the associated steady-state equations in this context can be written as a discrete dynamical system. We can then interpret localized solutions to the lattice system as homoclinic orbits of the associated discrete dynamical system, and show that the bifurcation structure is determined by bifurcations of nearby heteroclinic orbits. We supplement these results with examples from a well-studied bistable lattice differential equation which has been the focus of many works to date.

Stationary distributions and convergence rates for piecewise deterministic Markov processes

James Broda

Bowdoin College

We study processes that consist of deterministic evolution punctuated at random times by disturbances with random severity. Under appropriate assumptions such a process admits a unique stationary distribution. We develop a technique for establishing bounds on the rate of convergence to equilibrium. An important example of such a process is the carbon content of a forest or grassland ecosystem, whose deterministic growth is interrupted by natural disasters, such as fires, droughts, or insect outbreaks.

Rational decay of a canonical structural acoustic PDE dynamics

Paula Egging
University of Nebraska - Lincoln

A rate of rational decay is obtained for solutions of a PDE model which has been used in the literature to describe structural acoustic flows. This structural acoustics PDE consists partly of a wave equation which is invoked to model the interior acoustic flow within a given cavity. Moreover, a structurally elastic equation is invoked to describe time-evolving displacements along the flexible portion of the cavity walls. The coupling between these two distinct dynamics will occur across a boundary interface. We obtain this rational decay rate by establishing certain a priori inequalities for the PDE which will eventually allow the invocation of an abstract resolvent criterion for rational decay.

Uniform L^2 bounds for semigroups generated by Hamiltonian linearizations

Harrison Gaebler
University of Kansas

We consider linearized operators of the form JL where J is anti-self-adjoint and L is self-adjoint (and diagonal). We will investigate their spectra and hope to demonstrate appropriate resolvent estimates for the application of Gomilko's Lemma on the semigroup $\exp(tJL)$.

Asymptotics of planar spiral wave solutions

Ang Li
Brown University

Both spiral waves and target patterns to planar reaction-diffusion equations converge asymptotically to one-dimensional wave trains with a $O(\log r)$ shift. Despite their difference in shape, both solutions share the same leading order coefficient for the shift. We study the higher order coefficients for the shift and explore their connection to the kinematical theory of spiral waves.

Reversal permanent charge and reversal potential with unequal diffusion coefficients via classical Poisson-Nernst-Planck models

Hamidreza Mofidi

University of Kansas

We apply techniques from singular perturbation theory to understand the effects of a simple profile of permanent charges on ionic flows based on geometric singular perturbation analysis of a quasi-one dimensional Poisson–Nernst–Planck model for ionic flows. Our focus is on two ion species, one positively charged (cation) and one negatively charged (anion), with not necessarily equal diffusion coefficients. Under the setting in the paper, we are able to identify two governing equations for the existence and uniqueness and the value of the permanent charge for a current reversal. A number of new features are established.

Stability of multi-pulse solutions to nonlinear wave equations

Ross Parker

Brown University

Higher order nonlinear wave equations, such as the fifth-order Korteweg–de Vries equation (KdV5) and the Chen–McKenna suspension bridge equation, are used to model phenomena such as capillary-gravity water waves and traveling waves on a suspended beam. For certain parameter regimes, these equations exhibit multi-pulse traveling wave solutions. Linear stability of these multi-pulse solutions is determined by eigenvalues near the origin representing the interaction between the individual pulses. We locate these small eigenvalues using spatial dynamics techniques such as Lin's method and the Krein matrix. We are able to give analytical criteria for the stability of these multi-pulse solutions. We also present numerical results to support our analysis.

Modulational instability of viscous fluid conduit periodic waves

Wesley Perkins
University of Kansas

The Whitham modulation equations are widely used to describe the behavior of modulated periodic waves on large space and time scales; hence, they are expected to give insight into the stability of spatially periodic structures. However, the derivation of these equations are based on formal asymptotic (WKB) methods, thus removing a layer of rigor that would otherwise support their predictions. In this study, we aim at rigorously verifying the predictions of the Whitham modulation equations in the context of the so-called conduit equation, a nonlinear dispersive PDE governing the evolution of the circular interface separating a light, viscous fluid rising buoyantly through a heavy, more viscous, miscible fluid at small Reynolds numbers. In particular, using rigorous spectral perturbation theory, we connect the predictions of the Whitham modulation equations to the rigorous spectral (in particular, modulational) stability of the underlying wave trains. This makes rigorous recent formal results on the conduit equation obtained by Maiden and Hoefer.

Geometric quantization of classical mechanical systems with focus-focus singularities

Mahesh Sunkula
University of Oklahoma

We use Geometric quantization to construct quantum Hilbert space of classical mechanical systems with singularities. While we consider all types of singularities, our primary focus is on systems such as spherical pendulum (mathematical pendulum) which have a type of singularity called focus-focus singularity. To construct the quantum theory using geometric quantization procedure one needs to compute sheaf cohomologies. We use a de Rham like resolution to compute the cohomology groups. We first show that the resolution that we defined is a fine resolution and compute the cohomology groups on an open neighborhood of the singular fiber

Loss of time reversibility in the Nonlinear Schrödinger Equation

Amir Sagiv
Tel Aviv University

Light propagation in absorption-free media is considered to be a reversible process. Indeed, as a prevalent model, the focusing Nonlinear Schrödinger equation (NLS) is invariant under phase-conjugation and time reversal. Here, however, we present theoretical limitations to reversibility of the NLS dynamics under small perturbations. Fundamentally, these limitations reveal a preferred "arrow of time" in nonlinear optics. First, we prove that given a noisy initial condition, a solution's phase becomes uniformly random with propagation. This "loss of phase" phenomenon inherently limits one's ability to recover the input phase at moderately noisy settings. Next, we show numerically that reversibility is lost in common processes such as soliton fusion and wave collapse, i.e., that the recovery of the input profiles becomes highly unlikely if the output data is even mildly perturbed. This loss of reversibility is not due to energy or Hamiltonian loss, but rather due to the convergence to an attractor state, and the interactions between the radiation and the attractor state in backward propagation.

This is a joint work with Adi Ditkowski and Gadi Fibich.

A vanishing moment method for second-order linear elliptic PDEs in non-divergence form

Stefan Schnake
University of Oklahoma

This poster will focus on the vanishing moment method for second order linear elliptic PDEs in non-divergence form whose coefficients are only continuous. These PDEs present themselves in the nonlinear Hamilton–Jacobi–Bellman equations, which have applications in stochastic optimal control and mathematical finance, as well as the linearization of the Monge–Ampère equations. The vanishing moment method seeks to approximate the second order PDE by a family of fourth order PDEs created by the addition of a small bilaplacian term. Uniform H^1 and H^1 estimates will be given implying the convergence of the method. In addition, error estimates in the L^2 and H^1 norms will be shown.

KAM stability of the Kepler problem with a general relativistic correction term

Majed Sofiani
University of Kansas

In this work, we will be investigating a specific Hamiltonian system, namely, the Kepler problem with a correction term $\frac{\delta}{r^3}$ added to the potential energy. Our objective is to show that the system is stable in the sense of the KAM theorem. An informal statement of the KAM theorem is that if the unperturbed Hamiltonian system H_0, expressed in the action variable J, is non-degenerate, then under sufficiently small perturbation ϵH_1 we have that

$$H(J, \Phi) = H_0(J) + \epsilon H_1(J, \Phi)$$

for $\epsilon \ll 1$, most of the quasiperiodic orbits persist under the small perturbation ϵH_1. The system under which the perturbation will be acting is the following

$$H_0(r, \theta, \phi, p_r, p_\theta, p_\phi) = \frac{1}{2m}(p_r^2 + \frac{1}{r^2}p_\theta^2 + \frac{1}{r^2 \sin^2\theta}p_\phi^2) - \frac{\gamma}{r} \pm \frac{\delta}{r^3},$$

for $\gamma > 0$.