Instability, index theorems, and exponential dichotomy of Hamiltonian PDEs

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Abstract

In this talk, we start with a general linear Hamiltonian system $u_t = JLu$ in a Hilbert space X – the energy space. The main assumption is that the energy functional $\frac{1}{2}\langle Lu, u\rangle$ has only finitely many negative dimensions – $n^-(L) < \infty$. Our first result is an index theorem related to the linear instability of e^{tJL}, which gives some relationship between $n^-(L)$ and the dimensions of spaces of generalized eigenvectors of eigenvalues of JL. Under some additional non-degeneracy assumption, for an eigenvalue $\lambda \in iR$ of JL we also construct special "good" choice of generalized eigenvectors which both realize the corresponding Jordan canonical form corresponding to λ and work well with L. Our second result is the linear exponential trichotomy of the group e^{tJL}. This includes the nonexistence of exponential growth in the finite co-dimensional invariant center subspace and the optimal bounds on the algebraic growth rate there. If time permits we consider the structural stability of this type of systems under perturbations. Finally we discuss applications to examples of nonlinear Hamiltonian PDEs such as BBM, GP, and 2-D Euler equations, including the construction of some local invariant manifolds near some coherent states (standing wave, steady state, traveling waves etc.). This is a joint work with Zhiwu Lin.