Almost-sure exponential mixing in the stochastic Navier–Stokes equations

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Abstract

I will present recent joint work with Alex Blumenthal and Sam Punshon-Smith, in which we deduce almost-sure exponentially fast mixing of passive scalar densities advected by solutions of the stochastically-forced 2D Navier–Stokes equations and 3D hyper-viscous Navier–Stokes equations in T^d subject to non-degenerate Sobolev regular noise. That is, for all $s > 0$, there is a deterministic exponential decay rate such that all mean-zero H^s passive scalar densities decay in H^{-s} at this same rate with probability 1.

The main results follow from geometric ergodicity of the Markov process associated to two particles being advected simultaneously— this geometric ergodicity is proved via a drift condition designed to use the positive Lyapunov exponent deduced in our previous work to capture the high likelihood that the particles separate at an exponential rate when close together. The methods significantly simplify when applied to velocity fields evolving according to finite-dimensional models, for example Galerkin truncations of Navier–Stokes or the Stokes equations with very degenerate forcing. For all $0 < k < \infty$, we exhibit examples of $C_t^k C_x^\infty$ statistically stationary velocity fields that are almost-sure exponentially fast mixers.