

- VBAP in 2D
- VBAP in 3D
- VBAP in 3D in cartesian
- VBAP en termes de cosinus (3D)

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In[55]:= eq1 = (cp1 == g1 + g2 c12 + g3 c13);
eq2 = (cp2 == g1 c12 + g2 + g3 c23);
eq3 = (cp3 == g1 c13 + g2 c23 + g3);
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In[61]:= so = FullSimplify[Solve[{eq1, eq2, eq3}, {g1, g2, g3}]]
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$$\text{Out[61]} = \left\{ \left\{ \begin{aligned} g1 &\rightarrow \frac{(-1 + c23^2) cp1 + c13 (-c23 cp2 + cp3) + c12 (cp2 - c23 cp3)}{-1 + c12^2 + c13^2 - 2 c12 c13 c23 + c23^2}, \\ g2 &\rightarrow \frac{-c13 c23 cp1 - cp2 + c13^2 cp2 + c23 cp3 + c12 (cp1 - c13 cp3)}{-1 + c12^2 + c13^2 - 2 c12 c13 c23 + c23^2}, \\ g3 &\rightarrow \frac{c13 cp1 - c12 c23 cp1 - c12 c13 cp2 + c23 cp2 + (-1 + c12^2) cp3}{-1 + c12^2 + c13^2 - 2 c12 c13 c23 + c23^2} \end{aligned} \right\} \right\}$$

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In[87]:= re = {c12 → Cos[a12], c13 → Cos[a13], c23 → Cos[a23]};
re4 = {s12 → Sin[a12], s13 → Sin[a13], s23 → Sin[a23]};
re2 = {cp1 → Cos[ap1], cp2 → Cos[ap2], cp3 → Cos[ap3]};
re3 = {(-1 + c12^2) → -s12*s12, (-1 + c13^2) → -s13*s13, (-1 + c23^2) → -s23*s23};
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In[63]:= so2 = FullSimplify[so /. re]
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$$\text{Out[63]} = \left\{ \left\{ \begin{aligned} g1 &\rightarrow \frac{\cos[a13] (cp3 - cp2 \cos[a23]) + \cos[a12] (cp2 - cp3 \cos[a23]) - cp1 \sin[a23]^2}{-1 + \cos[a12]^2 + \cos[a13]^2 - 2 \cos[a12] \cos[a13] \cos[a23] + \cos[a23]^2}, \\ g2 &\rightarrow \frac{\cos[a12] (cp1 - cp3 \cos[a13]) + (cp3 - cp1 \cos[a13]) \cos[a23] - cp2 \sin[a13]^2}{-1 + \cos[a12]^2 + \cos[a13]^2 - 2 \cos[a12] \cos[a13] \cos[a23] + \cos[a23]^2}, \\ g3 &\rightarrow \frac{-cp3 + cp3 \cos[a12]^2 + cp1 \cos[a13] + cp2 \cos[a23] - \cos[a12] (cp2 \cos[a13] + cp1 \cos[a23])}{-1 + \cos[a12]^2 + \cos[a13]^2 - 2 \cos[a12] \cos[a13] \cos[a23] + \cos[a23]^2} \end{aligned} \right\} \right\}$$

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In[68]:= so3 = FullSimplify[so2 /. re2]
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$$\text{Out[68]} = \left\{ \left\{ \begin{aligned} g1 &\rightarrow \left(\cos[a13] (-\cos[a23] \cos[ap2] + \cos[ap3]) + \cos[a12] (\cos[ap2] - \cos[a23] \cos[ap3]) - \right. \\ &\quad \left. \cos[ap1] \sin[a23]^2 \right) / (-1 + \cos[a12]^2 + \cos[a13]^2 - 2 \cos[a12] \cos[a13] \cos[a23] + \cos[a23]^2), \\ g2 &\rightarrow \left(\cos[a23] (-\cos[a13] \cos[ap1] + \cos[ap3]) + \cos[a12] (\cos[ap1] - \cos[a13] \cos[ap3]) - \right. \\ &\quad \left. \cos[ap2] \sin[a13]^2 \right) / (-1 + \cos[a12]^2 + \cos[a13]^2 - 2 \cos[a12] \cos[a13] \cos[a23] + \cos[a23]^2), \\ g3 &\rightarrow \left(\cos[a23] (-\cos[a12] \cos[ap1] + \cos[ap2]) + \cos[a13] (\cos[ap1] - \cos[a12] \cos[ap2]) - \right. \\ &\quad \left. \cos[ap3] \sin[a12]^2 \right) / (-1 + \cos[a12]^2 + \cos[a13]^2 - 2 \cos[a12] \cos[a13] \cos[a23] + \cos[a23]^2) \end{aligned} \right\} \right\}$$

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In[72]:= so4 = FullSimplify[so /. re3]
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$$\text{Out[72]} = \left\{ \left\{ \begin{aligned} g1 &\rightarrow \frac{c12 \, cp2 - c13 \, c23 \, cp2 + c13 \, cp3 - c12 \, c23 \, cp3 - cp1 \, s23^2}{c13^2 - 2 \, c12 \, c13 \, c23 + c23^2 - s12^2}, \\ g2 &\rightarrow \frac{c12 \, cp1 - c13 \, c23 \, cp1 - cp2 + c13^2 \, cp2 - c12 \, c13 \, cp3 + c23 \, cp3}{c13^2 - 2 \, c12 \, c13 \, c23 + c23^2 - s12^2}, \\ g3 &\rightarrow \frac{c13 \, cp1 - c12 \, c23 \, cp1 - c12 \, c13 \, cp2 + c23 \, cp2 - cp3 \, s12^2}{c13^2 - 2 \, c12 \, c13 \, c23 + c23^2 - s12^2} \end{aligned} \right\} \right\}$$

This is mariconada for real time purposes.

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In[75]:= den = Denominator[g1 /. so4[[1]]]
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$$\text{Out[75]} = c13^2 - 2 \, c12 \, c13 \, c23 + c23^2 - s12^2$$

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In[79]:= nums = ({g1, g2, g3} /. so4[[1]]) * den
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$$\text{Out[79]} = \left\{ \begin{aligned} &c12 \, cp2 - c13 \, c23 \, cp2 + c13 \, cp3 - c12 \, c23 \, cp3 - cp1 \, s23^2, \\ &c12 \, cp1 - c13 \, c23 \, cp1 - cp2 + c13^2 \, cp2 - c12 \, c13 \, cp3 + c23 \, cp3, \\ &c13 \, cp1 - c12 \, c23 \, cp1 - c12 \, c13 \, cp2 + c23 \, cp2 - cp3 \, s12^2 \end{aligned} \right\}$$

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In[81]:= norm = FullSimplify[nums.nums]
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$$\text{Out[81]} = \left(-c12 \, cp1 + c13 \, c23 \, cp1 + cp2 - c13^2 \, cp2 + c12 \, c13 \, cp3 - c23 \, cp3 \right)^2 + \\ \left(-c13 \, cp1 + c12 \, c23 \, cp1 + c12 \, c13 \, cp2 - c23 \, cp2 + cp3 \, s12^2 \right)^2 + \\ \left(-c12 \, cp2 + c13 \, c23 \, cp2 - c13 \, cp3 + c12 \, c23 \, cp3 + cp1 \, s23^2 \right)^2$$

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In[92]:= norm2 = Simplify[norm /. re /. re2 /. re4]
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$$\text{Out[92]} = \left(\cos[a13] \cos[a23] \cos[ap1] + \cos[ap2] - \cos[a13]^2 \cos[ap2] - \right. \\ \left. \cos[a23] \cos[ap3] + \cos[a12] (-\cos[ap1] + \cos[a13] \cos[ap3]) \right)^2 + \\ \left(\cos[a12] \cos[a23] \cos[ap1] - \cos[a23] \cos[ap2] + \cos[a13] (-\cos[ap1] + \cos[a12] \cos[ap2]) + \right. \\ \left. \cos[ap3] \sin[a12]^2 \right)^2 + \left(\cos[a13] (\cos[a23] \cos[ap2] - \cos[ap3]) + \right. \\ \left. \cos[a12] (-\cos[ap2] + \cos[a23] \cos[ap3]) + \cos[ap1] \sin[a23]^2 \right)^2$$