6.3 Controls

> "A control system is an interconnection of components forming a system configuration that will provide a desired system response." *(Ogata, 2010).*

A closed loop control system uses feedback and measurements to compare the system with the desired output. By modelling a system and developing the appropriate controls system we may generate the desired behaviour from the system.

A closed loop control system, see Figure 3, can be summarised as the interaction of (Ogata, 2010):

- Controllers: which are informed of the desired behaviour while receiving feedback from sensors on the actual behaviour of the system;
- Actuators: which modulate their behaviour based on instructions from the controllers;
- A process to be controlled; and,
- Sensors: which measure the behaviour of the system (the actual output of the process) and inform the controllers.

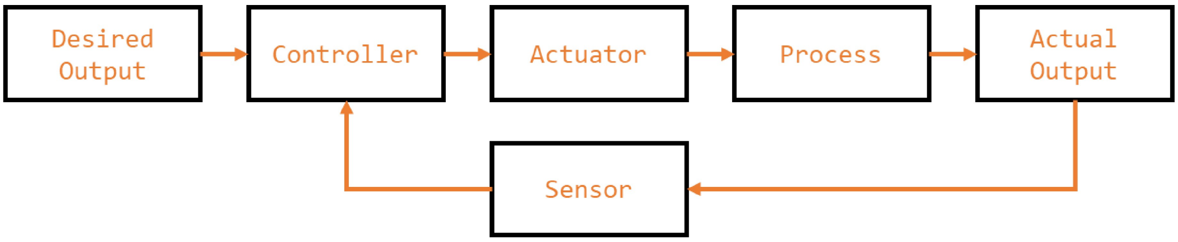

Figure 3: Closed Loop Control System

One method of conceptualizing the behaviour of a system is to create block diagrams of the system (Golnaraghi & Kuo, 2010). Block diagrams may be used to communicate comparators, transfer function components, input and output signal, feedback loops, etc. We may describe the state of a closed loop system via block diagrams, see Figure 4.

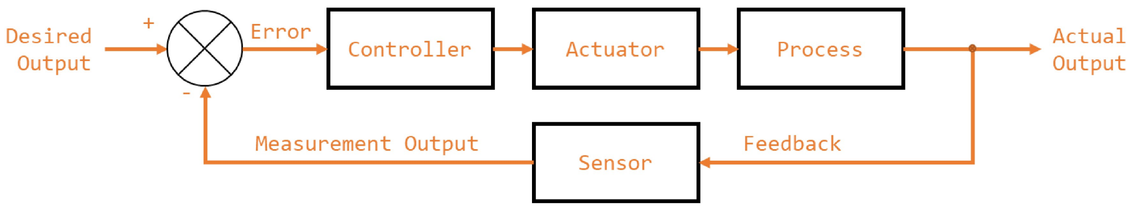

Figure 4: Closed Loop Control System Block Diagram

Here the error of the system is the difference between the desired state and the measured state. The (usual) goal of controls is to reduce the error to zero. This may be accomplished by numerous methods, which include the transfer function method[1].

6.3.1 Transfer Function Approach to Modelling and Control

> *"The transfer function of a linear, time-invariant differential-equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial conditions are zero." (Otaga, 2004)*

We may define the transfer function, $G(s)$ for a system as given by Equation 1; where $X(s)$ is the Laplace transform of the input and $Y(s)$ is the laplace transform of the output. This may be seen diagrammatically in Figure 5. The transfer function allows us to map from a set of inputs to the outputs of system.

$$G(s) = \frac{X(s)}{Y(s)}$$

Equation 1: Transfer Function

Figure 5: Transfer Function

For a closed loop system, we may say that the behaviour of the system, as seen in Figure 6, may be described by the relationship of the control system and the system process with respect to the error (Golnaraghi & Kuo, 2010).

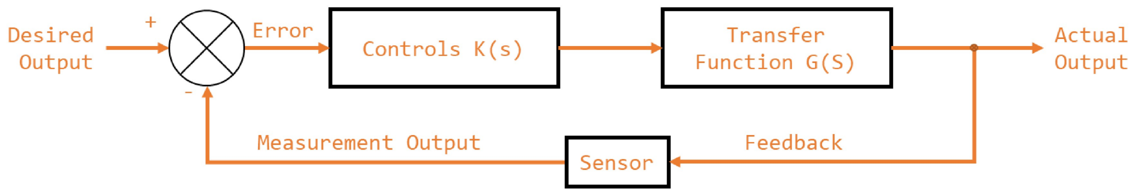

Figure 6: Controlled Closed Loop System

We may minimise the error of the system by correctly curating the control system (K). This is to say, given a system described by G(s), by controlling K(s) (the controllers and actuators) we can ensure that the actual output of the system is the desired output of the system (Otaga, 2004). The transfer function for this system, based on the block diagram in Figure 7, is given by Equation 2.

[1] The root locus method could be used to develop the controls for the system. However, that would just be a recount of Ogata and as such we refer to Modern Control Engineering (Ogata, 2010), rather than repeating it. Chapter 6 addresses design by the Root-Locus Method and Chapter 10 speaks directly regarding servo system design (a perhaps more apt framing of the 'actuators side' of the project).

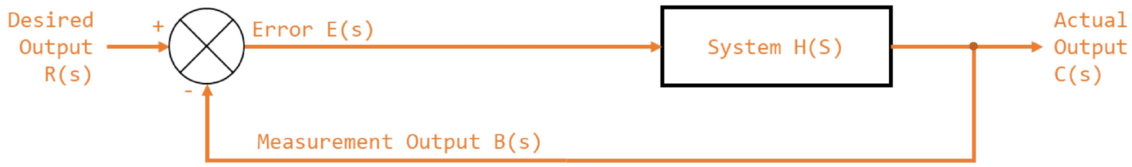

Figure 7: Controlled Closed Loop System (Parameterised)

$$H(s) = K(s)G(s)$$

$$C(s) = E(s)H(s)$$

$$E(s) = R(s) - B(s) = R(s) - C(s)$$

$$C(s) = H(s)\big(R(s) - C(s)\big)$$

$$TF = \frac{C(s)}{R(s)} = \frac{H(s)}{1 + H(s)}$$

Equation 2: Controlled Closed Loop Control System Transfer Function

6.3.2 Proportional–Integral–Derivative controller

A proportional–integral–derivative controller (PID) is a method of developing the controls parameters (and K(s)) for a system.

One approach to controls is to amplify the actuators response proportionally to the error. This is known as proportional control and is shown in Figure 8.

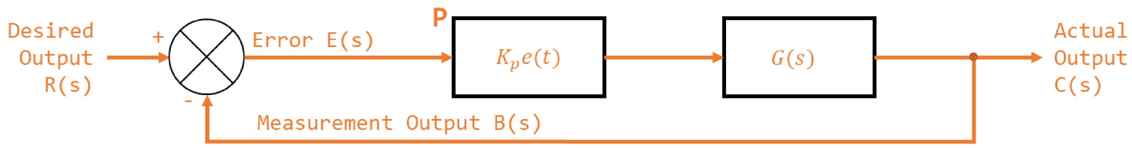

Figure 8: P Control

If the proportional gain of a system is too small the response time of the system may be too slow. If the value is to high the system may overcompensate, resulting in oscillations and overshooting of the target values. If the proportional gain is sufficiently high the system may oscillate so wildly that it becomes unstable (Otaga, 2004). Unstable meaning the system never reaches the desired output and the error of the system increases over time. (National Instruments, 2018).

Proportional control is effective at addressing the gross present error in the system. To create a fast response time for a system, without increasing the proportional gain to unstable values often a derivative term is employed.

Figure 9: PD Control depicts a system with a proportional (K_p) and a derivative gain (K_d). K_d changes in response to the rate at which error is changing. By incorporating a proper K_d value the system can effective pre-empt change in error, leading to a faster response time (Ogata, 2010).

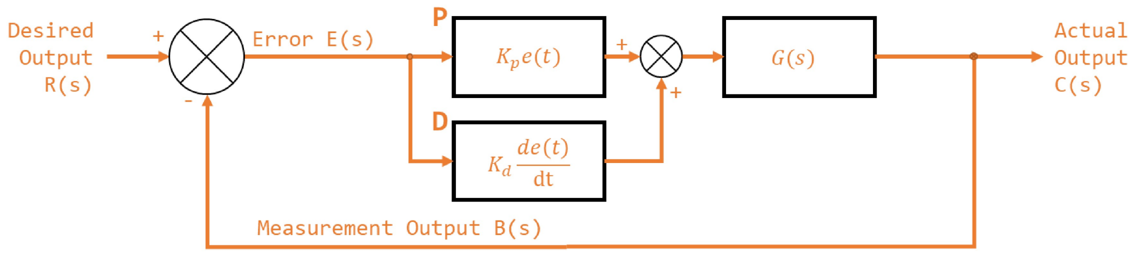

Figure 9: PD Control

A K_d value that is too low will result in a system that doesn't react strongly to changes in error. A K_d value that is correctly calibrated will result in a system that reacts strongly to sudden changes in the system. A K_d value that is too high will reacts too strongly to changes in error and will become highly sensitive to signal noise. Small changes in absolute error which would be ignored by the proportional term may illicit an unwarranted response from an overly sensitive K_d.

For small errors the proportional gain may be insufficient for correction and increasing the proportional gain may result in instability. For constant small errors the change in error may be too small for the derivative gain to correct and increasing the derivative gain may result in instability. Small persistent error in the system, otherwise known as steady state error, may exist in a system with pure PD control.

To compensate and correct the steady state error in a system an integral gain term may be introduced. A system with an integral gain may be seen in Figure 10. The integral gain in practical terms accumulates historical error in the system. This way, even small errors can slowly increase the integral term to an actionable magnitude.

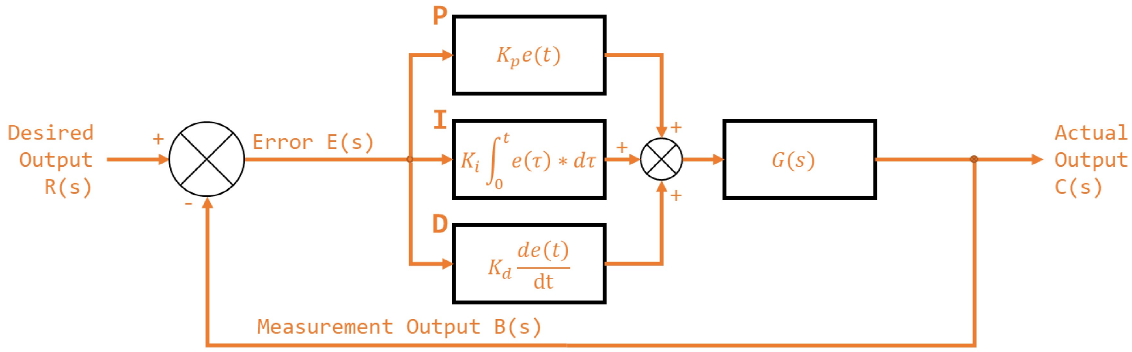

Figure 10: PID Control

An integral gain that is too small will take a very long to correct steady state error within the system. An integral gain that is too large will suffer from integral windup, specifically excessive overshooting the target value. Integral windup is phenomena where a sudden and large change in error cause the integral term to accumulate a massive error that saturates the entire PID response leading to overcompensations. An integral gain that is too large may result in integral wind up of sufficient magnitude that the system becomes unstable.

The transfer function for a PID controller is given by Equation 3.

$$K(s) = K_p + K_d s + \frac{K_i}{s}$$

Equation 3: PID Transfer Function

6.4 PID Tuning

There is a plethora of methods for finding the correct range of values for a systems PID to achieve the desired controls, excellent sources include: Automatic Control Systems (Golnaraghi & Kuo, 2010), Modern Control Engineering (Ogata, 2010), Springer Handbook of Robotics (Siciliano & Khatib, 2016), and Modern Control Systems (Dorf & Bishop, 2011). Two were used in this thesis:

- Software tools, specifically MATLAB's PID tuner pidTuner; and,
- The second Ziegler-Nichols method for empirical PID Tuning.

6.4.1 Ziegler-Nichols PID Tuning

The second Ziegler-Nichols (ZN) method entails (Ogata, 2010):

Initialise the K_d and K_i terms to 0 (Equation 4). Note, that in the literature (Ogata, 2010), K_d and K_i are expressed as the T_d and T_i in terms of the K_p (Equation 5) to generate a control transfer function of Equation 6.

$$K_d = 0 \;\&\; K_i = 0$$

Equation 4: ZN Initial Values

$$T_d = \frac{K_d}{K_p} \;\&\; T_i = K_i K_p$$

Equation 5: ZN Convention

$$K_{\text{Ziegler-Nichols}}(s) = K_p\left(1 + T_d s + \frac{1}{T_i s}\right)$$

Equation 6: ZN TF

1. Beginning at 0, increase K_p slowly to the *critical value*, K_{cr}. Where K_{cr} is the lowest K_p "at which the output of the system exhibits sustained oscillation" (Ogata, 2010);
2. Determine the corresponding period (P_{cr}) of K_{cr}; and,
3. Using P_{cr}, K_{cr}, and the values given in kt find the approximate PID values for the system.

Table 2: Ziegler-Nichols Tuning Rules

Controller Type	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$0.833^{\cdot}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Note that the Ziegler-Nichols parameters attained are not be taken as absolutes. They are systematically determined estimates of the optimal values. Once found empirical testing should be conducted to refine the values and ensure stability.