



# Sun, Jan 5.

## Official End to Axion Project

I got an email from Andrew that he redid his calculation and now his result agrees with me, which is expected by me. Erich also told us that there is no need for the new axion domain wall simulation now. I am officially done with it.

## An “Eureka” Idea: Direct Test of Edge Effect

I didn’t actually get any concrete work done today because I spent half the day travelling back to Toronto. However, something quite remarkable happened today. In the morning, while thinking about the overall direction of the research, I had a “Eureka” moment in which I realized a new way to solve the confining string problem!

In summary, we already have all the technology to solve the confining string (at least Andrew does; I do in principle, but my something about my  $SU(3)$  solution looks strange. It is probably some bug or some convergence issue; nevertheless, I have everything I need to solve in principle). The last few days, I was just reviewing and rewriting the old code to make sure they are right and optimizing them for mass computation.

However, the main obstacle is the edge effect. During the summer, we got stuck because we decided to study the grid size dependence (edge effect) to the simulation. By running the simulation on a grid, we assumed that the edge of the grid is far enough from the center (the location of interest) that it can be approximated to infinity. But when we consider the classical electrodynamics, we found that by solving the grounded box - image charge problem, the edge effect is very strong! In fact, the expected behaviour of logarithmic dependence of the energy gets destroyed by the edge effect at about 20% or 30%. This is a huge computational cost.

It occurred to me, while waking up and still in bed, that we didn’t need to do any of this. There is a much better method to study the grid effect, both in terms of time cost and accuracy. The idea is super simple. We run the same simulation twice: once on a large grid, once a small grid. Presumably, the large grid is a few times larger than the small grid. The ratio of the size is such that the small grid total area covers a region in the center of the large grid such that it is so far away from the edge that we are confident there is no edge effect in that region. Then we run a range of quark distance  $R$  in both grid and compare their slope, to see how much does the edge effect change anything.

This is much better in terms of accuracy because there is zero doubt in whether the two simulation are equivalent: they are literally the same simulation, just different grid. The previous method has to make the assumption that the grid effect in a theory without superpotential somehow translates to a theory with one.

Furthermore, this method is quite time efficient because we only have to run the large grid once, for each

$N$  at least. Presumably, the grid effect does not depend on the charge, and I suspect not even  $N$ , which is the number of grid component. This idea is a huge game changer to the project! Baring some completely unforeseen problems, I believe the project will finish successfully just by running large scale simulation.