

## Thu, Jan 2.

## Finished Axion Side-Project

Today was a very productive day for the Axion side-project. I worked most of the day at McDonald and Chatime. I found those environments quite productive and made a mental note to work at these places more.

I went back and re-derive the equation of motion. The original one I had was wrong basically because I forgot the Lorentzian metrics give the spatial derivative a negative sign in the Lagrangian. Once I take that into account, everything else I did was correct.

One trouble I had was figuring out how Erich re-scaled the units and derived the Hamiltonian (with very simple unit) from the Lagrangian (with complicated pre-factors and units). But since I now understand why it is equivalent to vary the energy when there is no time dependence, I just apply the Euler-Lagrange equation on the expression of the energy given by Erich; the resulting equation of motion must be in the simplified unit where the mass of dual-photon is set to unity. In short, I understood everything except for the unit, but it didn't prevent me from getting the right equation of motion, which is all I needed.

Then I made the appropriate sign change in my code and everything just went smoothly. I ran the soliton solution for all different values of the scale  $\alpha$ . Nothing that when  $\alpha$  gets large, the axion's profile becomes so large that it does not approach a flat line at the boundary of the grid, signalling the grid is too small. I made sure the grid is large enough for the lowest  $\alpha$ , but didn't bother to increase them further for the larger one since I now know that the solution is essentially the same, and increasing the grid significantly seems to be too time-expensive.

In addition, I noticed some interesting "bumps" in the dual photons solutions. I summarized all of these results and send all the images to Erich in the evening.