Dec 22.

Energy

The energy of a field configuration is given by

$$E = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dzdy \left(\left| \frac{\partial x^a}{\partial z} \right|^2 + \left| \frac{\partial x^a}{\partial y} \right|^2 + \frac{1}{4} \left| \frac{\partial W}{\partial x^a} \right|^2 \right)$$

This is a generalization of the one dimensional BPS energy we derived earlier. It also makes sense since in classical EM, the energy is $|E|^2$, and the $E = \nabla \Phi$. Since the dual photon has the same unit as the electric potential, the energy should also contains the gradient square of x^a.

Problem Statement

The action of a static quark-antiquark pair of charge $\pm\vec{C}$, separated by a distance R, with the positive quark at position $(z = -\frac{R}{2}, y = 0)$ and negative quark at position $(z = \frac{R}{2}, y = 0)$, as shown in Figure 1, is given by

$$S = \frac{g^2}{16\pi^2 L} \int dt \int dz \int dy \left(|\partial_0 x^a|^2 - |\partial_i x^a|^2 - \frac{1}{4} \left| \frac{dW}{dx^a} \right|^2 - 4\pi C_a \partial_y \delta(y) \int_{-\frac{R}{2}}^{\frac{R}{2}} dz' \delta(z - z') Im(x^a(z, y)) \right) \tag{1}$$

where $W(\vec{x})$ is the holomorhpic superpotential

$$W(\vec{x}) = \sum_{a=1}^{N} e^{\vec{\alpha}_a \cdot \vec{x}} \tag{2}$$

and $\vec{\alpha}_1, \ldots, \vec{\alpha}_{N-1}$ are the simple roots, and $\vec{\alpha}_N = -\sum_{a=1}^{N-1} \vec{\alpha}_a$.

The static equation of motion is

$$\nabla^2 x^a(z, y) = \frac{1}{4} \frac{\partial}{\partial (x^a)^*} \left| \frac{dW}{d\vec{x}} \right|^2 + i2\pi C_a \partial_y \delta(y) \int_{-\frac{R}{2}}^{\frac{R}{2}} dz' \delta(z - z') \tag{3}$$

Upon solving for the field, the energy of the configuration is given by

$$E = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dzdy \left(\left| \frac{\partial x^a}{\partial z} \right|^2 + \left| \frac{\partial x^a}{\partial y} \right|^2 + \frac{1}{4} \left| \frac{dW}{d\vec{x}} \right|^2 \right) \tag{4}$$

It is also known that for large R, the energy depends linearly on the distance, and that the coefficient of proportionality, f, also called string tension, depends only on the number of fundamental color indices (N-ality p), and the number of color, i.e. the rank of gauge group (N):

$$E = f(p, N) R \tag{5}$$

The goal is to find the unit-less ratio

$$F(p, N) = \frac{f(p, N)}{f(1, N)} \tag{6}$$

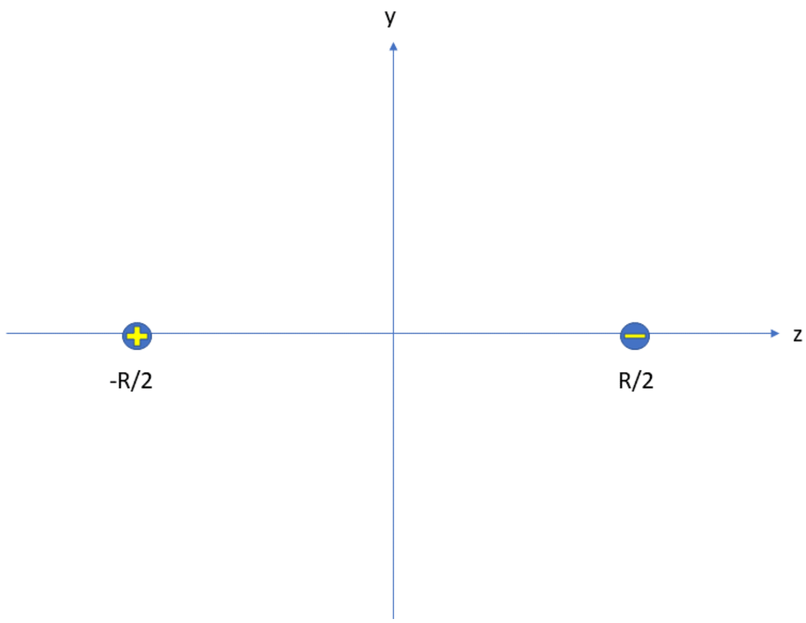

Figure 1: Schematic of Quark-Antiquark Pair

Grid

In starting to write code, the first step is to review the code from the summer project and move them to the new repository. The reason to carefully review the code is that the old version has become way too cluttered, with many different versions of similar class written in the same file. For the new project, I am only simulating confining pair, so I can remove all the unnecessary code with Baryon, deconfinement, etc. Also, I have this constant worry that there might be some small mistake, so I want to check over everything again. In addition, I want to do certain things differently this time around. For example, I want to implement the half grid calculation to speed things up.

The first thing is to review the Grid class and its related classes stored in the Grid module. I read over the general Grid class (except for the plot empty grid, which is exclusively used for testing). Everything is very basic common sense and looks good. There is no need to change anything.

Secondly, I reviewed the children class, Grid_Dipole. I added the important comment that this only works if the input parameters has the 0 y-axis right at the middle of the grid. In fact, this is so important that I am adding a function that enforces this.

I made some small modifications to the Standard_Dipole class, as well as some tests, but overall, this module looks good. I copied it to the new repository. Eventually, I want to add a half-grid class, but I will do that later and it seems the current code is perfectly compatible with it.