Dec 21.

Equation of Motion for Quark-Antiquark Pair

I now stop reading my old paper. It was a good refresher to remember the basic problem and good warm-up to go through the derivation of action and equation of motion. Now I specialize to the equation of motion for a quark-antiquark pair.

Claim 1. *The equation of motion for a quark-antiquark pair of charge $\pm\vec{c}$ is*

$$\nabla^2 x^a(z,y) = \frac{1}{4}\frac{\partial}{\partial(x^a)^*}\left|\frac{dW}{d\vec{x}}\right|^2 + i2\pi C_a \partial_y \delta(y) \int_{-\frac{R}{2}}^{\frac{R}{2}} dz' \delta(z-z') \tag{1}$$

where the positive quark is at position $(z=-\frac{R}{2}, y=0)$ and negative quark is at position $(z=\frac{R}{2}, y=0)$, as shown in the Figure 1.

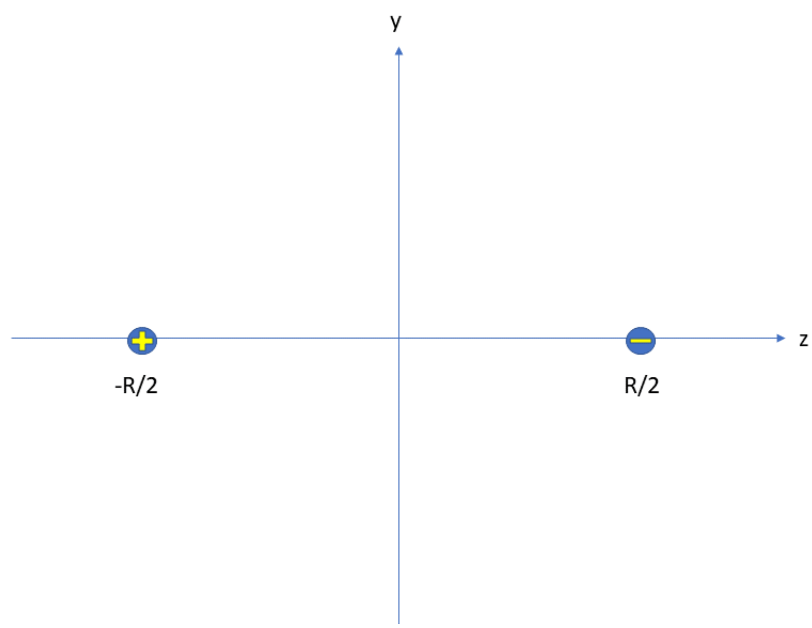

Figure 1: Schematic of Quark-Antiquark Pair

Proof. Taking the equation of motion derived from yesterday, with the exception that we make the change of variable $z \Longleftrightarrow y$, to match the usual convention of y being vertical, our generic (for arbitrary number of quarks) equation of motion is

$$\nabla^2 x^a = \frac{1}{4}\frac{\partial}{\partial(x^a)^*}\left|\frac{dW}{d\vec{x}}\right|^2 + i2\pi \sum_j (\vec{c}_j)_a \partial_y \delta(y-y_j) \int_{z_j}^{\pm\infty} dz' \delta(z-z') \tag{2}$$

Since $y_1 = y_2 = 0$, $z_1 = -\frac{R}{2}$, $z_2 = \frac{R}{2}$, we have

$$\nabla^2 x^a = \frac{1}{4}\frac{\partial}{\partial(x^a)^*}\left|\frac{dW}{d\vec{x}}\right|^2 + i2\pi C_a \partial_y \delta(y)\int_{-\frac{R}{2}}^{\infty} dz'\delta(z-z') - i2\pi C_a \partial_y \delta(y)\int_{\frac{R}{2}}^{\infty} dz'\delta(z-z')$$

Swapping upper and lower integration limit,

$$\nabla^2 x^a = \frac{1}{4}\frac{\partial}{\partial(x^a)^*}\left|\frac{dW}{d\vec{x}}\right|^2 + i2\pi C_a \partial_y \delta(y)\int_{-\frac{R}{2}}^{\infty} dz'\delta(z-z') + i2\pi C_a \partial_y \delta(y)\int_{\infty}^{\frac{R}{2}} dz'\delta(z-z')$$

Combining the two integral,

$$\nabla^2 x^a = \frac{1}{4}\frac{\partial}{\partial(x^a)^*}\left|\frac{dW}{d\vec{x}}\right|^2 + i2\pi C_a \partial_y \delta(y)\int_{-\frac{R}{2}}^{\frac{R}{2}} dz'\delta(z-z')$$

□

This will be the equation I will be solving for the next few months. Enough derivation for now! Tomorrow, my goal is to put together all of these formulas I have derived in the past few days, formulate a formal, succinct problem statement, and start coding.