Dec 26.

Math

I now review the Math module, which contains the important classes: SU class, which computes and stores important constants associated with the SU(N) group; and the Superpotential class, which represents the superpotential function, W, and computes its derivatives. I decided that this module is way too complex to rewrite. However, my fear that there is some mistake uncaught before (very small chance) could very well happen here, just due to its complexity. So I am now taking on the hard task of reviewing all the calculation here.

SU(N) Class

Reviewed the definition of weights of the fundamental representation $(\nu^A)_a$.

Reviewed the definition of the simple roots, $\vec{\alpha}_a$.

Reviewed the definition of the fundamental weights, $\vec{w}_a$.

Reviewed the definition of the Weyl's vector, $\vec{\rho} = \vec{w}_1 + \cdots + \vec{w}_{N-1}$.

As a side note, I also added these definitions to my Anki deck, so that I can memorize them.

Next, I migrated the old test_Math module to the new repository, and I ran the old tests on SU class. Since I am so confident that this part is correct from before, and I just read through the source code, I trust that the tests are correct without reading through them in detail.

Superpotential Class

Reviewed the "__call__" function, which is the most important part that everyting else depends on.

The key to solve the equation of motion is the potential term (showing the b-th component, i.e. potential term of $\nabla^2 x^b$):

$$\frac{1}{4}\frac{\partial}{\partial x^{b*}}\left|\frac{dW}{d\vec{x}}\right|^2 = \frac{1}{4}\frac{\partial W}{\partial x^a}\frac{\partial^2 W^*}{\partial x^{b*}\partial x^{a*}} \tag{1}$$

I reviewed the very complicated code whose complexity primarily lies in dealing with the shape of the array when evaluating on a 3D array (vector field on a grid). But the core mathematical formulae used are

$$\frac{dW}{d\vec{x}} = \sum_{i=1}^{N} e^{\vec{\alpha}_i\cdot\vec{x}}\vec{\alpha}_i \tag{2}$$

and

$$\frac{\partial^2 W}{\partial x^b \partial x^a} = \sum_{i=1}^{N} e^{\vec{\alpha}_i\cdot\vec{x}}(\vec{\alpha}_i)^a(\vec{\alpha}_i)^b \tag{3}$$

It took hours to analyze all the code. Now, that I believe it is correct, I will just run the test (without analyzing them because I trust the old tests written and what I just read enough. And the result is that the code passed all tests. So I conclude that the Math module is good to go for the new repository.

Final note: I am making this new repository highly organized. Inside the code folder, test belong to a subfolder, while all the "tool" modules, such as Grid and Math, that are going to be called by the main code (to be added), will be inside a "Tool" subfolder. The final main code will then be very simple and I won't need to worry at all about all the dirty math and array shape behind the scene.