Jan 12.

While preparing for the talk next week, I learned two things related to this research, the first thing should probably go into the thesis.

Precise Relation of $\vec{\sigma}$ and $\vec{\phi}$ to 4D

Recall from our paper the definition

$$\frac{g^2}{4\pi L}\partial_\mu \phi^a = F^a_{\mu 3} \tag{1}$$

$$\frac{g^2}{4\pi L}\epsilon_{\mu\nu\lambda}\partial^\lambda \sigma^a = F^a_{\mu\nu} \tag{2}$$

where $\mu, \nu = 0, 1, 2$, and x^3 is the direction of $\mathbb{S}^1$.

Suppressing the constant factor, if $\mu = 0$ and $\nu = i$, then in the second equation, we have

$$E^a_i = F^a_{0i} = \epsilon_{0i\lambda}\partial^\lambda \sigma^a$$

which implies

$$E^a_1 = F^a_{01} = \epsilon_{012}\partial^2 \sigma^a = \partial^2 \sigma^a \tag{3}$$

$$E^a_2 = F^a_{02} = \epsilon_{021}\partial^1 \sigma^a = -\partial^1 \sigma^a \tag{4}$$

So the spatial derivative of the dual photon is the electric field rotated by 90 degrees.

If $\mu, \nu = 1, 2$, then

$$B^a_3 = F^a_{12} = \epsilon_{120}\partial^0 \sigma^a = \partial^0 \sigma^a \tag{5}$$

So the time derivative of the dual photon is the magnetic field in the $\mathbb{S}^1$ direction.

In the first equation, if $\mu = 0$,

$$E^a_3 = F^a_{03} = \partial_0 \phi^a \tag{6}$$

which means the time derivative of $\vec{\phi}$ is the electric field in the $\mathbb{S}^1$ direction.

If $\mu = 1, 2$, then

$$B^a_2 = -F^a_{13} = -\partial_1 \phi^a \tag{7}$$

$$B^a_1 = F^a_{23} = \partial_2 \phi^a \tag{8}$$

So the spatial derivative of $\vec{\phi}$ is just the magnetic field rotated by 90 degrees.

In summary, the spatial derivative of $\vec{\sigma}$ is the rotated electric field; the spatial derivative of $\vec{\phi}$ is the rotated magnetic field; the time derivative of $\vec{\sigma}$ is the perpendicular magnetic field; and the time derivative of $\vec{\phi}$ is the perpendicular electric field. This is depicted in figure 1.

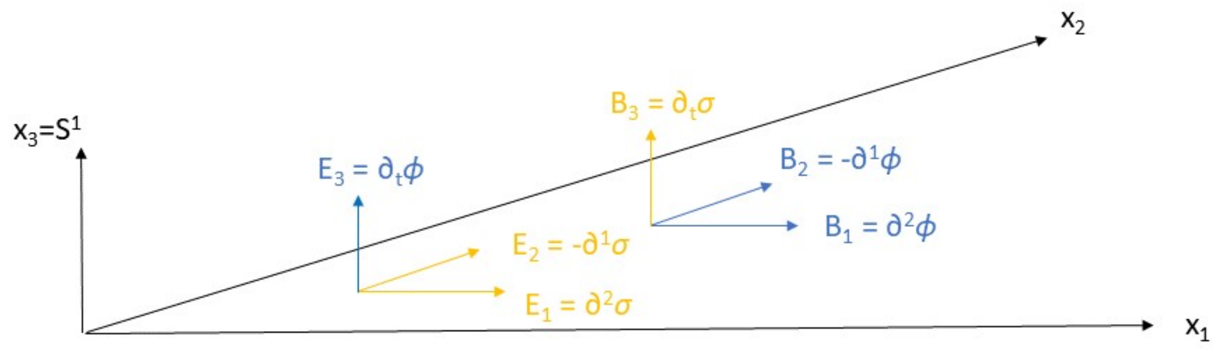

Figure 1: Relation of $\vec{\sigma}$ and $\vec{\phi}$ to 4D

Special Case of My Magnetless Soliton Formula

Recall my magnetless finding from the summer: if N is even, there are always at least 2 magnetless soltions. Their boundaries are given by

$$i2\pi \left(\underbrace{\vec{w}_1 + \vec{w}_3 + \cdots + \vec{w}_{N-1}}_{\text{all odd terms}} \right) \to \vec{x}_{N/2} \tag{9}$$

$$i2\pi \left(\underbrace{\vec{w}_2 + \vec{w}_4 + \cdots + \vec{w}_{N-2}}_{\text{all even terms}} \right) \to \vec{x}_{N/2} \tag{10}$$

I proved these in general. Now let's look at the case for $N = 2$. In $SU(2)$, there are $\vec{w}_1$ only, and there is only $k = 1$ wall. So a soliton can either go from 0 to $\vec{x}_1$, or from $\vec{w}_1$ to $\vec{x}_1$. The second one is described by the first formula as all odd terms. But what about the soliton starting at the origin? It seems not to be in any of the formula for magnetless boundaries!

Is this a mistake? No! The answer is that $\vec{w}_N$ should be identified as 0. In that case, the only time the all even terms boundaries will be 0 is for $SU(2)$. Then this case is taken care of. In the paper, we also gave a consistency check of this formula by showing that these two fluxes form a size 2 orbit under the center symmetry. But recall that in the formalism we developed for the center symmetry, we must identify $\vec{w}_N$ with 0, exactly what we have here. Hence,this formula describes $SU(2)$ as well. It is remarkable that I proved this formula in all generality, without noticing the pit fall with $SU(2)$, but it turns out $\vec{w}_N = 0$ is built into the math.