HOMEWORK #1

1. Find the average buffer occupancy, in messages, at the outgoing link of a concentrator for the following case (assume the infinite buffer, M/M/1 model is valid) :
   1. 10 terminals, each generating on the average of one message every 4 sec, are statistically multiplexed at the concentrator. The messages are 40 bits long, on the average. The capacity of the outgoing link is C = 1000 bps.
2. A concentrator buffer holds a maximum of N messages. Modeling it as an M/M/1 queue with finite buffer, find the probability that messages are blocked in each of the following cases. Compare results.
   1. N = 2, ρ=0.1
   2. N = 4, ρ=0.1
   3. N = 4, ρ=0.8

What is the probability the buffer is empty in each of these cases?

1. Consider a queue with state-dependent Poisson arrival rate λn and departure rate µn. Show that the equation governing the state probabilities after statistical equilibrium has set in is given by

Show the solution to this equation is given by

1. The “queue with discouragement” is one in which the arrival rates decrease with increasing queue size. Specifically, let and µn = µ. Show that the probability of state of the queue is given by

HOMEWORK #2

1. The number of programming errors made per line by the average introductory course student follows a Poisson distribution with a mean of three. What is the probability that a given line will contain exactly three errors?
2. System crashes at a certain computer center appear to follow the Poisson law, with a mean of 14 per (168hr/week) week. After bringing the system back up, one of the operators (Happy) told one of his colleagues (Grumpy) that he thought the system would remain up for the rest of the shift, 6 hours. Grumpy dissented and the ensuing discussion resulted in a case of low-calorie freeze-dried beer being wagered. What is the probability that Happy wins the freeze-dried beer?
3. A university cafeteria has a single customer queue with two equally slow servers –one for synthetic hamburger and one for imitation hamburger. When a server becomes free, the next customer is required to use that server (M/M/2). Write down the balance equations and then solve for the state probabilities.
4. Consider a pure Markovian queuing system in which
5. Find the equilibrium probabilities Pk for the number in the system.
6. What relationship must exist among the parameters of the problem in order that the system be stable and, therefore, that this equilibrium solution in fact be reached?
7. Consider a Markovian queuing system in which
   1. Find the equilibrium probability Pk of having k customers in the system. Express your answer in terms of Po.
   2. Give an expression for Po.
8. Consider an M/M/2 queuing system where the average arrival rate is λ customers per second and the average service time is 1/µ sec, where λ < 2µ.
   1. Find the differential equations that govern the time-dependent probabilities Pk(t).
   2. Find the equilibrium probabilities

HOMEWORK #3

1. Eight data terminals statistically share the capacity of an outgoing link. Traffic is combined from the eight terminals and then served first-come first-served. An (M/M/1) queue with infinite buffer is assumed. Find the expected number of packets in the buffer and the expected waiting time in the following cases:
   1. A terminal generates a packet every 10 seconds, the link speed is 1024/bits/second, and packet length = 512 bits.
   2. Repeat (a) using finite buffer k = 10 and compare your results to the infinite buffer case.
2. In a finite population with M customers, m servers, and K finite storage queue, the arrival and service rates are given by

Assume Poisson arrivals, exponential service times, and M ≥ K = m. Sketch the state transition diagram. Find the equilibrium distribution of the number of customers in the queue, the average delay, and the blocking probability.

1. The M/M/m queue arises in circuit switching applications where Poisson call arrivals are served by a maximum of m exponentially distributed servers. The (m+1) call is blocked. The arrival and service rates are given by

Find the steady-state distribution of the number of active calls on the system Pn, the blocking probability Bt, and the expected call waiting time E(W).