Pattern Recognition

HW2: Discriminant

Part 1, Coding

Mean vectors of each 2 classes

mean vector of class 1 (label 0): [1.3559426 −1.34746216]
mean vector of class 2 (label 1): [−1.29735587 1.29096203]

Within−class scatter matrix SW

[[388.64001349 −228.92177708]
 [−228.92177708 665.56910433]]

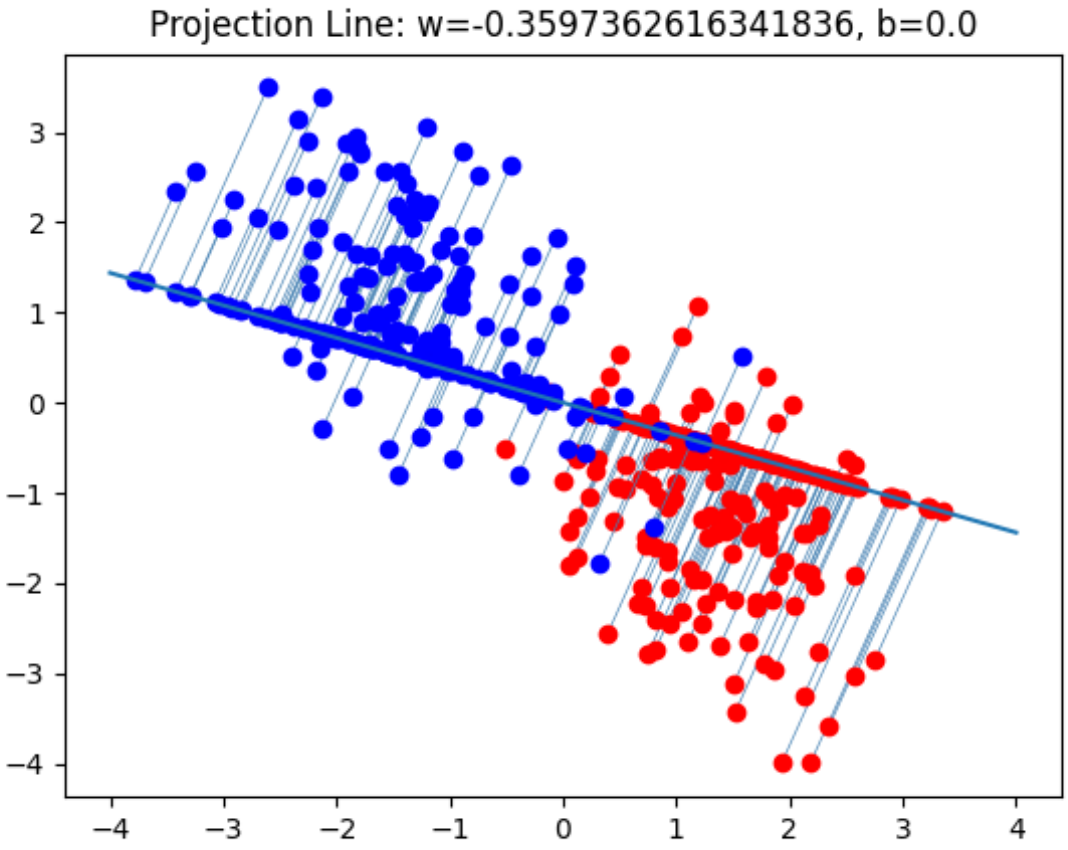
Between−class scatter matrix SB

[[7.03999279 −7.00052687]
 [−7.00052687 6.9612822]]

Fisher's linear discriminant w

[[−0.00563343]
 [0.00202655]]

Plot and results



Accuracy of test-set

0.916

Part 2, Questions

Question 1

$$L(\lambda, w) = w^T(m_2 - m_1) + \lambda(w^T w - 1)$$

$$\frac{\partial L}{\partial w} = m_2 - m_1 + 2\lambda w$$

$$w = -\frac{1}{2\lambda}(m_2 - m_1)$$

Therefore, $w \propto m_2 - m_1$

Question 2

$$\sigma(a) = \frac{1}{1+\exp(-a)}$$

$$= \frac{1}{1+e^{-a}} = \frac{e^a}{e^a+1}$$

$$\sigma(-a) = \frac{1}{1+\exp(a)}$$

$$= \frac{1}{1+e^a} = \frac{1+e^a-e^a}{1+e^a} = 1 - \frac{e^a}{1+e^a}$$

$$= 1 - \sigma(a) \ \#$$

$$y = \sigma(a) = \frac{1}{1+e^{-a}}$$

$$\frac{1}{y} = 1+e^{-a}$$

$$\frac{1}{y} - 1 = e^{-a}$$

$$\frac{1-y}{y} = e^{-a}$$

$$e^a = \frac{y}{1-y}$$

$$a = \ln\left(\frac{y}{1-y}\right)$$

Therefore, $\sigma^{-1}(y) = a = \ln\left(\frac{y}{1-y}\right)$ #