Pattern Recognition

HW4: Support Vector Machine

Part 1, Coding

Question 1

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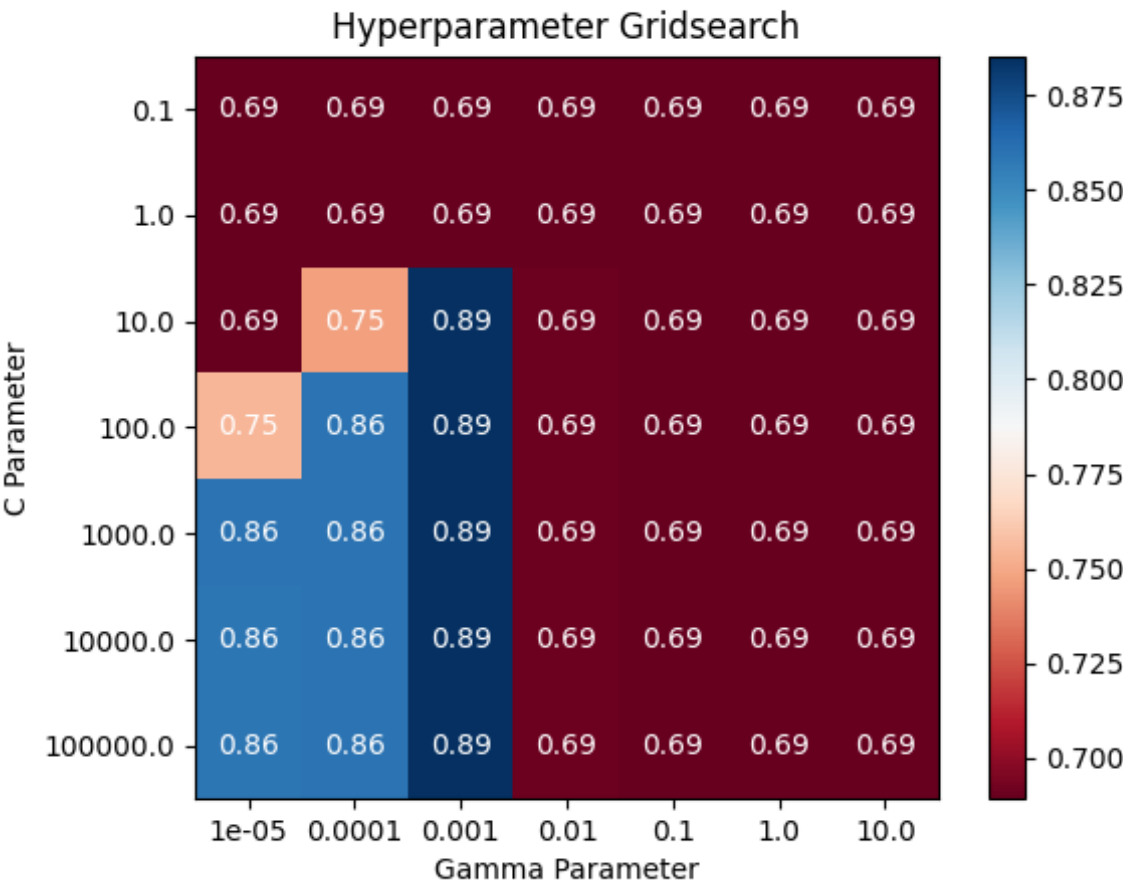
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Question 2

Best hyperparameters found: C = 10.0, gamma = 0.001

Question 3

Hyperparameter Gridsearch

Question 4

Accuracy score: 0.8958333333333334

Question 5

Procedure 1

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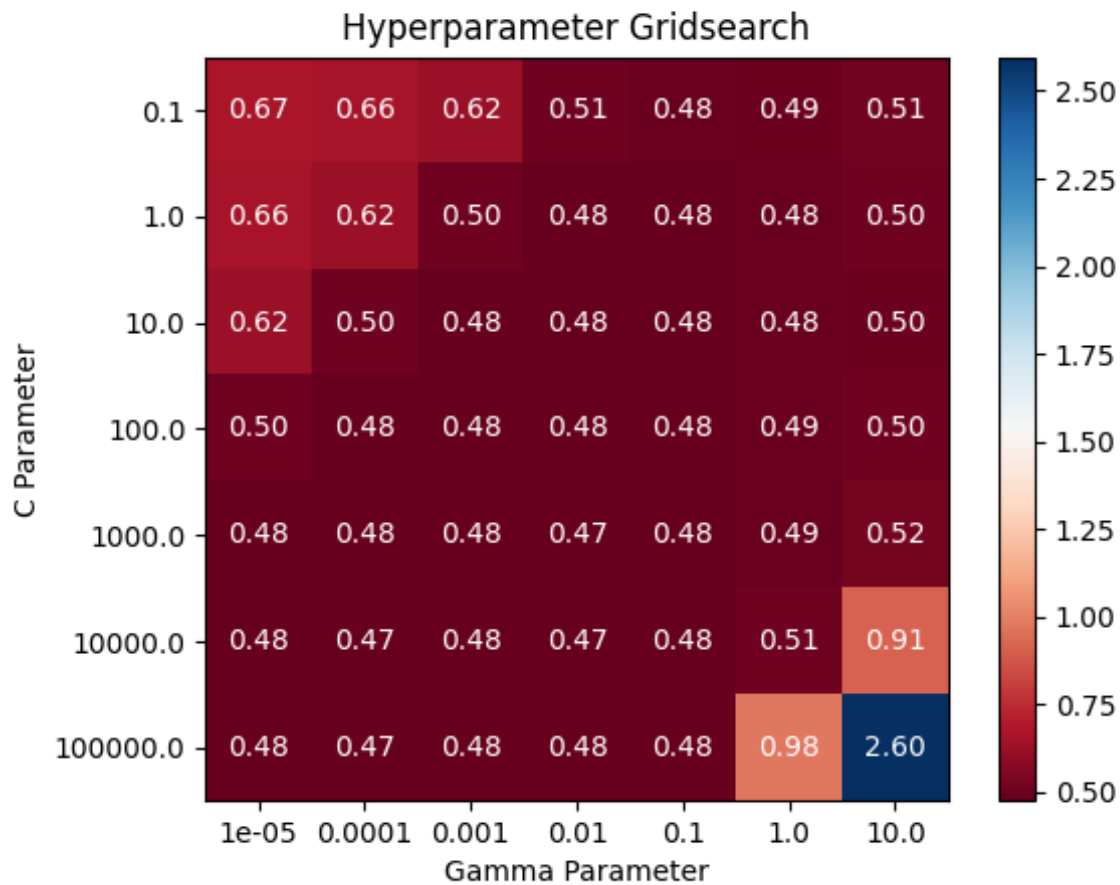
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Procedure 2

Best hyperparameters found: C = 100000.0, gamma = 0.0001

Procedure 3

Hyperparameter Gridsearch

Procedure 4

Mean error of SVR: 0.49258440370635614
Mean error of MSE: 0.4917001061329206

Part 2, Questions

Question 1

Since $k_1(x, x')$ is a valid kernel,

suppose ϕ_1 is the feature map of $k_1(x, x')$

1) Consider feature map $\phi: x \to \sqrt{c}\, \phi_1(x)$

We have

$$\phi(x)^T \phi(x') = \sqrt{c}\, \phi_1(x)^T \sqrt{c}\, \phi_1(x')$$

$$= c\, \phi_1(x)^T \phi_1(x')$$

$$= c\, k_1(x, x')$$

$$= k(x, x')$$

$k(x, x')$ is expressed by an inner product

Therefore, $k(x, x')$ is valid

2) Consider feature map $\phi: x \to \phi_1(x) f(x)$

We have

$$\phi(x)^T \phi(x') = f(x)^T \phi_1(x)^T \phi_1(x') f(x')$$

$$= f(x)\, \phi_1(x)^T \phi_1(x') f(x')$$

$$= f(x)\, k_1(x, x') f(x')$$

$$= k(x, x')$$

$k(x, x')$ is expressed by an inner product

Therefore, $k(x, x')$ is valid