

Derivación - Problema 8.

A) Polinomio de interpolación de Newton

$$P(X) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$

$$\text{con: } a_0 = f(x_0)$$

$$a_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$a_2 = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

b) Derivat $P(x)$

$$P'(x) = a_1 + a_2((x - x_1) + (x - x_0))$$

$$P'(x_0) = a_1 + a_2(x_0 - x_1)$$

$$\Rightarrow \text{si: } x_0 = x, x_1 = x + h, x_2 = x + 2h$$

$$a_1 = \frac{f(x+h) - f(x)}{h}$$

$$\begin{aligned} a_2 &= \frac{\frac{f(x+2h) - f(x+h)}{h} - \frac{f(x+h) - f(x)}{h}}{2h} \\ &= \frac{f(x) - 2f(x+h) + f(x+2h)}{2h^2} \end{aligned}$$

$$\Rightarrow P'(x) = \frac{f(x+h) - f(x)}{h} + \frac{f(x) - 2f(x+h) + f(x+2h)}{2h^2}(-h)$$

$$= \frac{2f(x+h) - 2f(x) - f(x) + 2f(x+h) - f(x+2h)}{2h}$$

$$= \frac{1}{2h} (-3f(x) + 4f(x+h) - f(x+2h))$$