NO.

DATE 4/20:

問 2.22 $(\log x)^2 - 5\log x - 6 < 0$.

$\log x = y$ とおく. $\iff$ $y^2 - 5y - 6 < 0$.

$-1 < y < 6$.

$y = \log x$ ~~とおく~~ なので. $e^{-1} < x < e^6$.

問 2.23.

$f(x) = e^x - e^{-x}$

(1) $f'(x) = e^x + e^{-x}$

(2). $-\frac{1}{2}\log x \leqq x \leqq \frac{1}{2}\log x$.

$f(-\frac{1}{2}\log 2) = e^{-\frac{1}{2}\log 2} - e^{\frac{1}{2}\log 2}$
$= -2$

$f(\frac{1}{2}\log 2) = e^{\frac{1}{2}\log 2} - e^{-\frac{1}{2}\log 2}$
$= 2$

(3) $t = e^x - e^{-x}$

$$I = \int_{-\frac{1}{2}\log 2}^{\frac{1}{2}\log 2} (e^x - e^{-x} + \sqrt{2})(e^x + e^{-x})\,dx$$

$dt = e^x + e^{-x}dx \iff dx = \frac{dt}{e^x + e^{-x}}$

$x \quad -\frac{1}{2}\log 2 \to \frac{1}{2}\log 2$

$t \quad -2 \to 2$

$\iff$

$$I' = \int_{-2}^{2} (t + \sqrt{2})\cdot dt \iff = \left[\frac{1}{2}t^2 + \sqrt{2}t\right]_{-2}^{2}$$

$= 4\sqrt{2}t$.

(4) $e^{3x} - 3e^x + 3e^{-x} - e^{-3x} = (e^x - e^{-x})^3$.

(4) (5) $e^{x} - e^{-x} = t$.

$$J = \int_{-\frac{1}{2}\log 2}^{\frac{1}{2}\log 2} \left| e^{3x} - 3e^{x} + 3e^{-x} - e^{-3x} \right| (e^{x} + e^{-x})\, dx$$

(3),(4)よりそれぞれ置換するとJは.

$$J = \int_{-2}^{2} |t^3|\, dt$$ と書ける.

$$= \int_{-2}^{0} -t^3\, dt + \int_{0}^{2} t^3\, dt.$$

これを計算して. $J = 4+4 = 8$. よって $J = 8$ □.

問2.24.

$$Y_1 = \mu_1 + \sigma_1 X_1, \quad Y_2 = \mu_2 + \sigma_2 (\rho X_1 + \sqrt{1-\rho^2}\, X_2).$$

(1) 系2.34より.

$$Y_1 = \sigma_1 X_1 + \mu_1 \iff f_{Y_1}(y_1) = \frac{1}{|\sigma_1|} f_{X_1}\left(\frac{y-\mu_1}{\sigma_1}\right).$$

よって.

$$f_{Y_1}(y_1) = \frac{1}{|\sigma_1|}\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(y_1-\mu_1)^2}{2\sigma_1^2}\right) = \frac{1}{\sqrt{2\pi}|\sigma_1|} \exp\left(-\frac{(y_1-\mu_1)^2}{2\sigma_1^2}\right).$$

(2) $$f_{X_1X_2}(x_1, x_2) = \frac{1}{2\pi} \exp\left\{-\frac{1}{2}(x_1^2 + x_2^2)\right\}$$

(3). $$A = \begin{pmatrix} \sigma_1 & 0 \\ \sigma_2\rho & \sigma_2\sqrt{1-\rho^2} \end{pmatrix}, \quad (b_1, b_2)^T = (\mu_1, \mu_2)^T.$$

(4)

系 2.36 より.

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = A\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}.$$

(3) より. $A = \begin{pmatrix} \sigma_1 & 0 \\ \sigma_2\rho & \sigma_2\sqrt{1-\rho^2} \end{pmatrix}$, $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$

$$c = \det A = \sigma_1\sigma_2\sqrt{1-\rho^2}.$$

(2) より $f_{X_1X_2}(x_1, x_2) = \frac{1}{2\pi}\exp\left\{-\frac{1}{2}(x_1^2+x_2^2)\right\}.$

$\Leftrightarrow$ このとき

$$f_{Y_1Y_2}(y_1, y_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\exp\left\{-\frac{1}{2}\left(\frac{\left[\sigma_2\sqrt{1-\rho^2}(y_1-\mu_1)\right]^2}{\sigma_1^2\sigma_2^2(\sqrt{1-\rho^2})^2} + \frac{\sigma_2^2\rho^2(y_1-\mu_1)^2}{\sigma_1^2\sigma_2^2(\sqrt{1-\rho^2})^2}\right.\right.$$

$$\left.\left. + \frac{-2\sigma_1\sigma_2\rho(y_1-\mu_1)(y_2-\mu_2)}{\sigma_1^2\sigma_2^2(\sqrt{1-\rho^2})^2} + \frac{\sigma_1^2(y_2-\mu_2)^2}{\sigma_1^2\sigma_2^2(\sqrt{1-\rho^2})^2}\right)\right\}.$$

$$\Leftrightarrow f_{Y_1Y_2}(y_1, y_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\exp\left\{-\frac{\left(\frac{y_1-\mu_1}{\sigma_1}\right)^2 - 2\rho\left(\frac{y_1-\mu_1}{\sigma_1}\right)\left(\frac{y_2-\mu_2}{\sigma_2}\right) + \left(\frac{y_2-\mu_2}{\sigma_2}\right)^2}{2(1-\rho^2)}\right\}.$$