COMPARISON

**Non-Linear Equations:**

Bisection: Always converges, but is slow.

Newton-Raphson: More efficient than Bisection, but a derivate is required.

False-Position: Always converges, but can get stuck.

Secant Method: More efficient than Newton Method, but if the initial values are not close to the root, then it might not converge.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Bisection** | **Regular Falsi** | **Newton** | **Secant** |
| **Type** | Bracket | Bracket | Open | Open |
| **Num of Initial Guess** | 2 | 2 | 1 | 2 |
| **Rate of Convergence** | Slow but Steady | Slow | Faster | Faster |
| **Accuracy** | Great | Good | Good | Good |
| **Convergence** | Linear | Linear | Quadratic | Super Linear |
| **Method of Approach** | MidPoint | Interpolation | Taylor Series | Interpolation |
| **Prog Effect** | Easy | Tedious | Easy | Tedious |

**Interpolation:**

**Numerical Integration:**

* Midpoint rule is one of the least accurate methods and can lead to very wrong result.
* Trapezoid rule is similar to midpoint rule, but instead of taking rectangles, we use trapezoid. In other words, we approximate by inscribing polygonal chain in the graph of the function, taking separate segment for each subinterval.
* Simpson’s rule is the most accurate method and the fastest convergent

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | a | b | n | The exact value | Simpson’s rule | Trapezoid rule | Midpoint rule |
| *sin*(*x*) | 0 | 0.77 | 1 | 0.28208933000000003 | 0.178675 | 0.26801206700000002 | 0.28918053500000002 |
|  | 0 | 0.77 | 4 | 0.28208933000000003 | 0.28209099999999998 | 0.28121769400000002 | 0.28252535000000001 |
|  | 0 | 0.77 | 10 | 0.28208933000000003 | 0.28208899999999998 | 0.28194994099999998 | 0.28215902999999998 |
| *x*3 | 0 | 2.5 | 1 | 9.765625 | 13.020799999999999 | 19.53125 | 4.8828125 |
|  | 0 | 2.5 | 4 | 9.765625 | 9.7656299999999998 | 10.37597656 | 9.4604492189999991 |
|  | 0 | 2.5 | 10 | 9.765625 | 9.7656299999999998 | 9.86328125 | 9.716796875 |
| *ex* | 0 | 2 | 1 | 6.3890560990000003 | 6.4207278040000002 | 8.3890560989999994 | 5.4365636569999998 |
|  | 0 | 2 | 4 | 6.3890560990000003 | 6.3891937250000002 | 6.5216101100000001 | 6.3229855329999998 |
|  | 0 | 2 | 10 | 6.3890560990000003 | 6.3890596439999996 | 6.4103387679999999 | 6.3784200819999999 |
| *log*(*x*) | 1 | 3 | 1 | 1.2958368659999999 | 1.2904003369999999 | 1.0986122890000001 | 1.2969442799999999 |
|  | 1 | 3 | 4 | 1.295836867 | 1.2957983500000001 | 1.2821045820000001 | 1.3026452340000001 |
|  | 1 | 3 | 10 | 1.2958368680000001 | 1.2958358109999999 | 1.2936188740000001 | 1.2969442799999999 |
| *x* | 0 | 2 | 1 | 2 | 2 | 2 | 2 |
|  | 0 | 2 | 4 | 2 | 2 | 2 | 2 |
|  | 0 | 2 | 10 | 2 | 2 | 2 | 2 |
| *x*3− *x* | 0 | 2 | 1 | 2 | 4 | 6 | 0 |
|  | 0 | 2 | 4 | 2 | 2 | 2.25 | 1.875 |
|  | 0 | 2 | 10 | 2 | 2 | 2.04 | 1.9799924799999999 |

Table I: Comparison results for selected methods

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | a | b | n | Error for Simpson’s rule | Error for trapezoid rule | Error for midpoint rule |
| *sin*(*x*) | 0 | 0.77 | 1 | 0.3666013537144141 | 4.9903565900870923E-2 | 2.5138153565741946E-2 |
|  | 0 | 0.77 | 4 | 5.9187314800328924E-6 | 3.0899296609271398E-3 | 1.5456808262322172E-3 |
|  | 0 | 0.77 | 10 | 1.1712211858265657E-6 | 4.9413208506619789E-4 | 2.4708439425509267E-4 |
| *x*3 | 0 | 2.5 | 1 | 0.33332991999999995 | 1 | 0.5 |
|  | 0 | 2.5 | 4 | 5.1199999998061685E-7 | 6.25E-2 | 3.125E-2 |
|  | 0 | 2.5 | 10 | 5.1199999998061685E-7 | 0.01 | 5.0000000000000001E-3 |
| *ex* | 0 | 2 | 1 | 4.9571806194195298E-3 | 0.31303528549466747 | 0.1490818717605904 |
|  | 0 | 2 | 4 | 2.1540908057286479E-5 | 2.0747041271432749E-2 | 1.0341209174399794E-2 |
|  | 0 | 2 | 10 | 5.5486599967827115E-7 | 3.3311132255614338E-3 | 1.6647242702880168E-3 |
| *log*(*x*) | 1 | 3 | 1 | 4.195380717004561E-3 | 0.1521986155624622 | 8.5459355281236604E-4 |
|  | 1 | 3 | 4 | 2.9723648848697756E-5 | 1.0597232838259647E-2 | 5.2540306157380376E-3 |
|  | 1 | 3 | 10 | 8.1568909350790468E-7 | 1.7116305723136754E-3 | 8.5459200808910485E-4 |
| *x* | 0 | 2 | 1 | 0 | 0 | 0 |
|  | 0 | 2 | 4 | 0 | 0 | 0 |
|  | 0 | 2 | 10 | 0 | 0 | 0 |
| *x*3− *x* | 0 | 2 | 1 | 1 | 2 | 1 |
|  | 0 | 2 | 4 | 0 | 0.125 | 6.25E-2 |
|  | 0 | 2 | 10 | 0 | 2.0000000000000018E-2 | 1.0003759765624953E-2 |

Table II: The results of the comparison of the error value.

**Ordinary Differential Equations:**

The 4 R-K Method gives the best approximate result as the error is very small as compared to Heun’s, Midpoint or Modified Euler. Heun’s method is slightly better than Midpoint Formula which is slightly better in approximation than Modified Euler.

As far as computation is concerned, the Midpoint formula is the easiest to compute, followed by Modified Euler and Heun’s Method. The 4-RK method requires most computation.