Spacecraft Attitude Dynamics

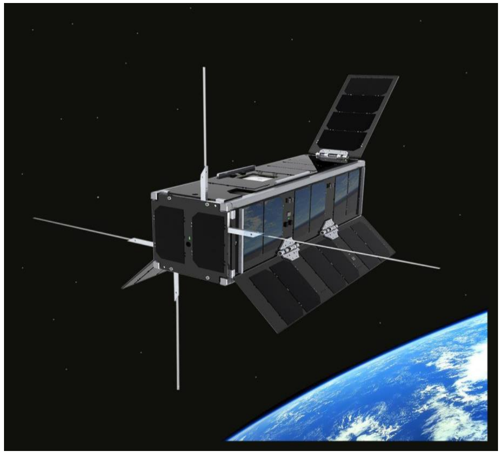
Prof. Franco Bernelli

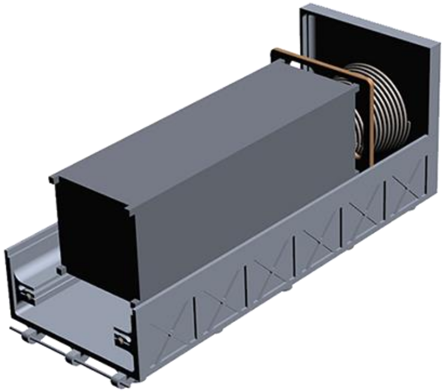
Lab 3 – Numerical Integration and the Euler equations

Task 1: Simulate the rotational motion of a 3U Cubesat

Principal moments of Inertia $I_x = 0.07kgm^2, I_y = 0.0504kgm^2, I_z = 0.0109kgm^2$

With initial conditions $\omega_x(0) = 0.45rad/s, \omega_y(0) = 0.52rad/s, \omega_z(0) = 0.55rad/s$





$$\dot{\omega}_x = \frac{I_y - I_z}{I_x}\omega_y\omega_z$$
$$\dot{\omega}_y = \frac{I_z - I_x}{I_y}\omega_x\omega_z$$
$$\dot{\omega}_z = \frac{I_x - I_y}{I_z}\omega_y\omega_x$$

i) Use a scope to analyse the output
ii) Plot the output from the workspace and label the axis and units.

Task 2: Analytic verification for the symmetric case

$$I_x = 0.0504 kgm^2, I_y = 0.0504 kgm^2, I_z = 0.0109 kgm^2$$

$$\dot{\omega}_x = \frac{I_y - I_z}{I_x}\omega_y\omega_z$$

$$\dot{\omega}_y = \frac{I_z - I_x}{I_y}\omega_x\omega_z$$

$$\dot{\omega}_z = \frac{I_x - I_y}{I_z}\omega_y\omega_x$$

Analytic Solution

$$\omega_x = \omega_{x0}\cos(\lambda t) - \omega_{y0}\sin(\lambda t)$$

$$\omega_y = \omega_{x0}\sin(\lambda t) + \omega_{y0}\cos(\lambda t)$$

$$\omega_z = \text{const} = \omega_{z0}$$

$$\lambda = \frac{(I_z - I_x)\omega_z}{I_x}$$

Does it provide a good approximation to the asymmetric case?

$$I_x = 0.07 kgm^2, I_y = 0.0504 kgm^2, I_z = 0.0109 kgm^2$$

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t = 0 : 0.1 : 10;
x = t.^2;
plot(t,x)
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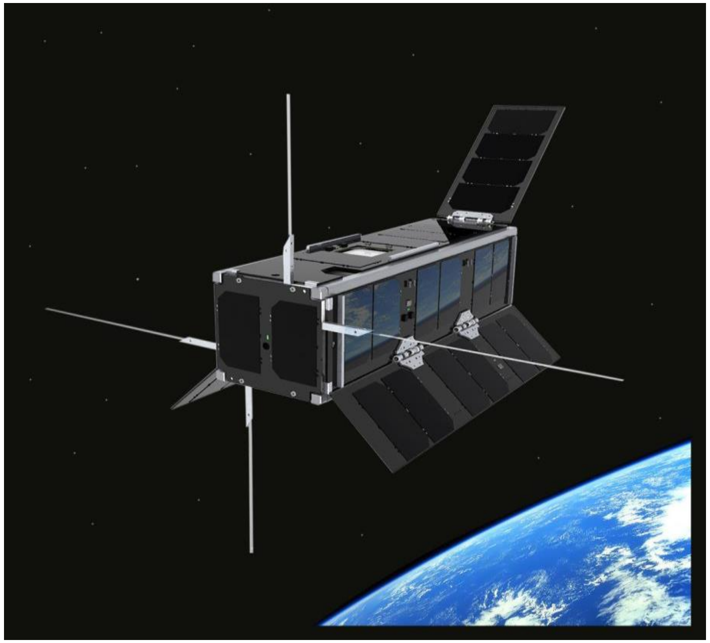
Task 3: Numerically assess the stability of the equilibrium points of the Euler equations

Principal moments of Inertia $I_x = 0.01 kgm^2, I_y = 0.05 kgm^2, I_z = 0.07 kgm^2$

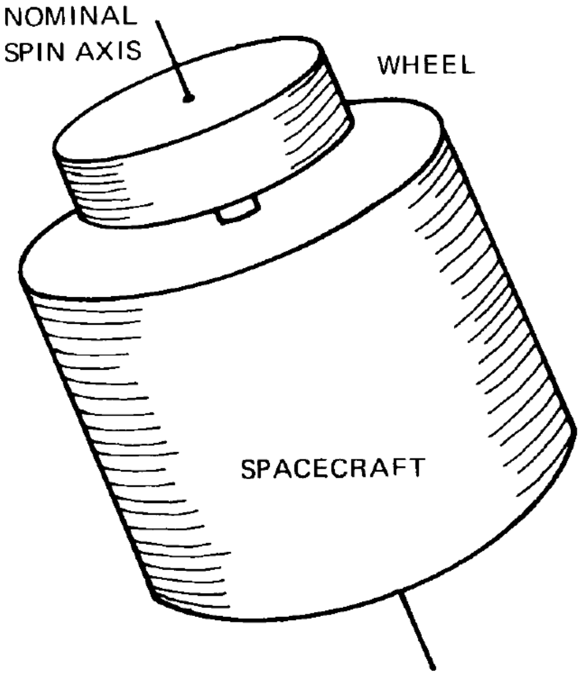


$$\dot{\omega}_x = \frac{I_y - I_z}{I_x}\omega_y\omega_z$$
$$\dot{\omega}_y = \frac{I_z - I_x}{I_y}\omega_x\omega_z$$
$$\dot{\omega}_z = \frac{I_x - I_y}{I_z}\omega_y\omega_x$$

Spin rate of the spinning axis $\omega_i(0) = 2\pi rad/\sec$

Task 4: Test the stability of a dual-spin spacecraft

$$I_x = 0.07kgm^2, I_y = 0.0504kgm^2, I_z = 0.0109kgm^2, I_r = 0.005$$



$$I_x\dot{\omega}_x = \left(I_y - I_z\right)\omega_z\omega_y - I_r\omega_r\omega_y$$
$$I_y\dot{\omega}_y = \left(I_z - I_x\right)\omega_x\omega_z + I_r\omega_r\omega_x$$
$$I_z\dot{\omega}_z = \left(I_x - I_y\right)\omega_y\omega_x - I_r\dot{\omega}_r$$
$$I_r\dot{\omega}_r = 0$$

$$\omega_x(0) = 1e - 6rad/\sec, \omega_y(0) = 1e - 6rad/\sec, \omega_z(0) = 0.02rad/\sec$$

$$\omega_r(0) = 2\pi rad/\sec$$