POLITECNICO
MILANO 1863

Spacecraft Attitude Dynamics

Prof. Franco Bernelli

Lab 4 - Attitude Kinematics

Task 1: Simulate the dynamics and kinematics using the DCM using standard integration

Dynamics

$$\dot{\omega}_1 = \frac{I_2 - I_3}{I_1}\omega_2\omega_3$$
$$\dot{\omega}_2 = \frac{I_3 - I_1}{I_2}\omega_1\omega_3$$
$$\dot{\omega}_3 = \frac{I_1 - I_2}{I_3}\omega_2\omega_1$$

$\longrightarrow$

Kinematics

$$\frac{dA(t)}{dt} = -[\omega]^{\wedge}A(t)$$

$$[\omega^\wedge] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

Dynamics $\longrightarrow$ Kinematics

Test the orthonormaility of the solution

Task 2: Orthonormalize the matrix at each sampling period

Iterative formulas for orthonormalization

$A_{k+1}(t)= A_k(t)*3/2- A_k(t)*A_k^T(t)*A_k(t)/2$

converges rapidly, with increasing k, to the exact value of A. In a first order approximation it is possible to adopt a single step iteration

$A(t)= A_0(t)*3/2- A_0(t)*A_0^T(t)*A_0(t)/2$

Test the orthonormality of the solution

Task 3: Simulate the kinematics using quaternion

Dynamics

$$\dot{\omega}_1 = \frac{I_2 - I_3}{I_1}\omega_2\omega_3$$
$$\dot{\omega}_2 = \frac{I_3 - I_1}{I_2}\omega_1\omega_3$$
$$\dot{\omega}_3 = \frac{I_1 - I_2}{I_3}\omega_2\omega_1$$

Kinematics

$$\frac{dq(t)}{dt} = \frac{1}{2}[\Omega]q(t)$$

$$[\Omega] = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix}$$

$$\boxed{\text{Dynamics}} \longrightarrow \boxed{\text{Kinematics}}$$

Test the normality of the solution