Exercise Set 2

This report contains my answers for exercise set 2.

Problem 8

Task a

Coefficients for penguins train data fit.

```
      (Intercept)    bill_length_mm     bill_depth_mm flipper_length_mm
   -380.81882000       14.30428414      -11.68758439        0.48138045
     body_mass_g
     -0.03094901
```

Accuracy on training and test sets without regularisation:

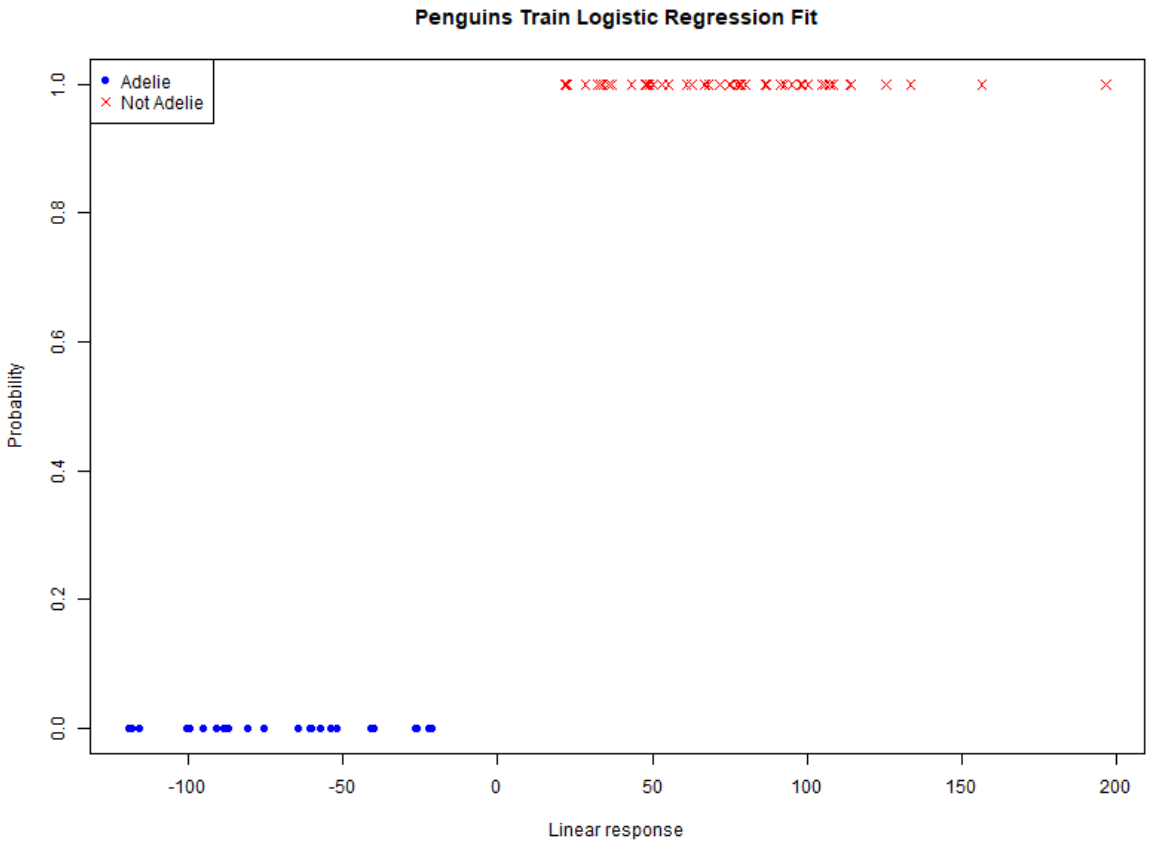
```
[1] "Train accuracy:"
```

```
[1] 1
```

```
[1] "Test accuracy:"
```

```
[1] 0.9466667
```

Plot of penguins train data predictions.



Task b

Coefficients for penguins train data fit.

```
Loading required package: Matrix

5 x 1 sparse Matrix of class "dgCMatrix"
                              s0
(Intercept)       -16.082633193
bill_length_mm      0.662337141
bill_depth_mm      -0.733855638
flipper_length_mm   0.004030675
body_mass_g         .
```

Accuracy on training and test sets using lasso with regularisation:

```
[1] "Train accuracy:"
```

```
[1] 1
```

```
[1] "Test accuracy:"
```

```
[1] 0.9866667
```

Task c

R gives warnings because the model is fitting the data too well and because there's no regulation, it will try to push probabilities closer to 0 or 1 forever if the programme didn't stop itself and give the warning.

Problem 9

According to the textbook:

The discriminant function is linear when p=1 and all classes have a shared variance:

$$\delta_k(x) = \frac{x \cdot \mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k) \tag{4.18}$$

When each class has its own variance the discriminant function becomes quadratic (non-linear):

$$\delta_k(x) = -\frac{1}{2}(x-\mu_k)^\top \Sigma_k^{-1}(x-\mu_k) - \frac{1}{2}\log|\Sigma_k| + \log \pi_k \tag{4.28}$$

Expanded form:

$$\delta_k(x) = -\frac{1}{2}x^\top \Sigma_k^{-1} x + x^\top \Sigma_k^{-1}\mu_k - \frac{1}{2}\mu_k^\top \Sigma_k^{-1}\mu_k - \frac{1}{2}\log|\Sigma_k| + \log \pi_k \tag{4.28}$$

Problem 10

Task a

Tables of each attribute's means, standard deviations and class probabilities using Laplace smoothing:

species	variable	name	value
Adelie	mean	bill_length_mm	38.1240000
notAdelie	mean	bill_length_mm	47.8180000
Adelie	mean	bill_depth_mm	18.3360000
notAdelie	mean	bill_depth_mm	15.8900000
Adelie	mean	flipper_length_mm	188.8800000
notAdelie	mean	flipper_length_mm	211.3000000
Adelie	mean	body_mass_g	3576.0000000
notAdelie	mean	body_mass_g	4657.0000000
Adelie	sd	bill_length_mm	2.7815284
notAdelie	sd	bill_length_mm	3.5994722
Adelie	sd	bill_depth_mm	1.2041179
notAdelie	sd	bill_depth_mm	1.9654412
Adelie	sd	flipper_length_mm	6.3200738
notAdelie	sd	flipper_length_mm	11.7928550
Adelie	sd	body_mass_g	461.3431478
notAdelie	sd	body_mass_g	787.5310166
Adelie	laplace	class_prob	0.3418255
notAdelie	laplace	class_prob	0.6581745

Task b

The posterior probability:

$$P(Y = k \mid X = x) = \frac{\pi_k \prod_{j=1}^{p} f_{kj}(x_j)}{\sum_{l=1}^{K} \pi_l \prod_{j=1}^{p} f_{lj}(x_j)}$$

For this specific case:

$$\widehat{p}(y = \text{Adelie} \mid x) = \frac{\pi_{\text{Adelie}} \prod_{j=1}^{4} f_{\text{Adelie},j}(x_j)}{\pi_{\text{Adelie}} \prod_{j=1}^{4} f_{\text{Adelie},j}(x_j) \; + \; \pi_{\text{notAdelie}} \prod_{j=1}^{4} f_{\text{notAdelie},j}(x_j)}$$

Task c

The posterior probabilities for the first 3 penguins:

```
[1] 0.9964977 0.8015094 0.9987976
```

The accuracy score of the classifier:

```
[1] 0.92
```