TSP–NNG Intersection Proof Pseudocode

TSP Tour

Notation:

- Index set for all possible NNG configurations. Note that $\left|S_{NNG}(n)\right| = (n-3)^n$.

  $$S_{NNG}(n) := \Big\{(\sigma_0, \ldots, \sigma_{n-1}) : \sigma_i \in \{0, \ldots, n-1\} \setminus \{j \mod n : j = i-1, i, i+1\}\Big\}.$$

- Edge swap index set. Note that $|T(n)| = 3^n - 1$.

  $$T(n) = \big\{(t_1, \ldots, t_n) : t_i \in \{-1, 0, 1\}\big\} \setminus \{(0, \ldots, 0)\}$$

Algorithm 1: TSP tour vs. NNG intersection tester for graphs on n vertices

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procedure TOURNNG (n);
Input : integer $n \geq 3$
Output: TRUE if a Hamiltonian cycle on the complement of the 1-NNG (of a complete weighted graph on n vertices with unique nearest neighbors) can always be shortened via edge exchanges with the 1-NNG; FALSE otherwise
for $(\sigma_0, \ldots, \sigma_{n-1})$ **in** $S_{NNG}(n)$ **do**
    foundEdgeSwap $\leftarrow$ FALSE;
    for t **in** $T(n)$ **do**
        $F \leftarrow \big(\{v_0, \ldots, v_{n-1}\}, \big\{(v_0, v_1), \ldots, (v_{n-2}, v_{n-1}), (v_{n-1}, v_0)\big\}\big)$;
        for t_i **in** t **do**
            if $t_i = -1$ **or** $t_i = 1$ **then**
                try:
                    $F \leftarrow F \setminus \big\{(v_i, v_{(i+t_i) \bmod n})\big\}$;
                catch:
                    pass
                end
                $F \leftarrow F \cup \big\{(v_i, v_{\sigma_i})\big\}$;
            end
        end
        if isCycle(F) **then**
            foundEdgeSwap $\leftarrow$ TRUE;
            break
        end
    end
    if not foundEdgeSwap **then**
        return FALSE;
    end
end
return TRUE;

TSP Path

Notation:

- Index set for all possible NNG configurations.

$$\begin{aligned} S_{NNG}(n) := \Big\{(\sigma_0, \ldots, \sigma_{n-1}) :\\ &\sigma_0 \in \{2, \ldots, n-1\}\\ \wedge\ &\sigma_i \in \{0, \ldots, n-1\} \setminus \{i-1, i, i+1\}, i = 1, \ldots, n-2\\ \wedge\ &\sigma_{n-1} \in \{0, \ldots, n-3\}\Big\} \end{aligned}$$

- Edge swap index set.

$$\begin{aligned} T(n) = \Big\{(t_1, \ldots, t_n) :\\ &t_0 \in \{0, 1\}\\ \wedge\ &t_i \in \{-1, 0, 1\}, i = 1, \ldots, n-2\\ \wedge\ &t_{n-1} \in \{-1, 0\}\Big\} \setminus \{(0, \ldots, 0)\} \end{aligned}$$

Algorithm 2: TSP path vs. NNG intersection tester for graphs on n vertices

procedure PATHNNG (n);

Input : integer $n \geq 3$

Output: TRUE if a Hamiltonian path on the complement of the 1-NNG (of a complete weighted graph on n vertices with unique nearest neighbors) can always be shortened via edge exchanges with the 1-NNG; FALSE otherwise

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for (σ_0, ..., σ_{n-1}) in S_{NNG}(n) do
    foundEdgeSwap ← FALSE;
    for t in T(n) do
        F ← ({v_0, ..., v_{n-1}}, {(v_0, v_1), ..., (v_{n-2}, v_{n-1})});
        for t_i in t do
            if t_i = −1 or t_i = 1 then
                try:
                    F ← F \ {(v_i, v_{i+t_i})};
                catch:
                    pass
                end
                F ← F ∪ {(v_i, v_{σ_i})};
            end
        end
        if isPath(F) then
            foundEdgeSwap ← TRUE;
            break
        end
    end
    if not foundEdgeSwap then
        return FALSE;
    end
end
return TRUE;
```