TSP–NNG Enumeration Algorithm Pseudocode

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TSP Tour

Notation:

- Index set for all possible NNG configurations. Note that $\left|S_{NNG}(n)\right| = (n-3)^n$.

$$S_{NNG}(n) := \Big\{(\sigma_0, \ldots, \sigma_{n-1}) : \sigma_i \in \{0, \ldots, n-1\} \setminus \{j \mod n : j = i-1, i, i+1\}\Big\}.$$

- Edge swap index set. Note that $|\mathcal{E}(n)| = 3^n - 1$.

$$\mathcal{E}(n) = \big\{(e_0, \ldots, e_{n-1}) : e_i \in \{-1, 0, 1\}\big\} \setminus \{(0, \ldots, 0)\}$$

Algorithm 1: TSP tour vs. NNG intersection tester for weighted graphs

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procedure TOURNNG (n);
Input : integer $n \geq 3$
Output: TRUE if a Hamiltonian cycle on the complement of the NNG (of an undirected weighted graph on n vertices) can always be shortened via edge exchanges with a representative of the NNG; FALSE otherwise
for $(\sigma_0, \ldots, \sigma_{n-1})$ **in** $S_{NNG}(n)$ **do**
    foundEdgeSwap $\leftarrow$ FALSE;
    for $(e_0, \ldots, e_{n-1})$ **in** $\mathcal{E}(n)$ **do**
        $F \leftarrow \big(\{v_0, \ldots, v_{n-1}\}, \big\{(v_0, v_1), \ldots, (v_{n-2}, v_{n-1}), (v_{n-1}, v_0)\big\}\big)$;
        for i **in** $\{0, \ldots, n-1\}$ **do**
            if $(v_i, v_{(i+e_i) \bmod n}) \in F$ **then**
                $F \leftarrow F \setminus \big\{(v_i, v_{(i+e_i) \bmod n})\big\}$;
                $F \leftarrow F \cup \big\{(v_i, v_{\sigma_i})\big\}$;
            end
        end
        if isCycle(F) **then**
            foundEdgeSwap $\leftarrow$ TRUE;
            break
        end
    end
    if not foundEdgeSwap **then**
        return FALSE;
    end
end
return TRUE;

TSP Path

Notation:

- Index set for all possible NNG configurations.

$$
\begin{aligned}
S_{NNG}(n) := \Big\{(\sigma_0, \ldots, \sigma_{n-1}) :\ & \sigma_0 \in \{2, \ldots, n-1\} \\
& \wedge\ \sigma_i \in \{0, \ldots, n-1\} \setminus \{i-1, i, i+1\}, i = 1, \ldots, n-2 \\
& \wedge\ \sigma_{n-1} \in \{0, \ldots, n-3\}\Big\}
\end{aligned}
$$

- Edge swap index set.

$$
\begin{aligned}
\mathcal{E}(n) = \Big\{(e_0, \ldots, e_{n-1}) :\ & e_0 \in \{0, 1\} \\
& \wedge\ e_i \in \{-1, 0, 1\}, i = 1, \ldots, n-2 \\
& \wedge\ e_{n-1} \in \{-1, 0\}\Big\} \setminus \{(0, \ldots, 0)\}
\end{aligned}
$$

Algorithm 2: TSP path vs. NNG intersection tester for graphs on n vertices

procedure PATHNNG (n);
Input : integer $n \geq 3$
Output: TRUE if a Hamiltonian path on the complement of the NNG (of an undirected weighted graph on n vertices) can always be shortened via edge exchanges with a representative of the NNG; FALSE otherwise
for $(\sigma_0, \ldots, \sigma_{n-1})$ **in** $S_{NNG}(n)$ **do**
    foundEdgeSwap $\leftarrow$ FALSE;
    for $(e_0, \ldots, e_{n-1})$ **in** $\mathcal{E}(n)$ **do**
        $F \leftarrow \big(\{v_0, \ldots, v_{n-1}\}, \big\{(v_0, v_1), \ldots, (v_{n-2}, v_{n-1})\big\}\big)$;
        for i **in** $\{0, \ldots, n-1\}$ **do**
            if $(v_i, v_{(i+e_i) \bmod n}) \in F$ **then**
                $F \leftarrow F \setminus \big\{(v_i, v_{(i+e_i) \bmod n})\big\}$;
                $F \leftarrow F \cup \big\{(v_i, v_{\sigma_i})\big\}$;
            end
        end
        if isPath(F) **then**
            foundEdgeSwap $\leftarrow$ TRUE;
            break
        end
    end
    if not foundEdgeSwap **then**
        return FALSE;
    end
end
return TRUE;