TSP–NNG Intersection Proof Pseudocode

TSP Tour

Notation:

- Index set for all possible NNG configurations. Note that $\big|S_{NNG}(n)\big| = (n-3)^n$.

$$S_{NNG}(n) := \Big\{(\sigma_0, \ldots, \sigma_{n-1}) : \sigma_i \in \{0, \ldots, n-1\} \setminus \{j \bmod n : j = i-1, i, i+1\}\Big\}.$$

- Edge swap index set. Note that $|T(n)| = 3^n - 1$.

$$T(n) = \big\{(t_1, \ldots, t_n) : t_i \in \{-1, 0, 1\}\big\} \setminus \{(0, \ldots, 0)\}$$

Algorithm 1: TSP tour vs. NNG intersection tester for graphs on n vertices

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1 **procedure** TOURNNG (n);
   Input   : integer $n \geq 3$
   Output: TRUE if a Hamilton cycle on the complement of the 1-NNG (of a
              complete weighted graph on n vertices) can always be shortened via
              edge exchanges with the 1-NNG; FALSE otherwise
2 **for** $(\sigma_0, \ldots, \sigma_{n-1})$ **in** $S_{NNG}(n)$ **do**
3   foundEdgeSwap $\leftarrow$ FALSE;
4   **for** t **in** $T(n)$ **do**
5           $F \leftarrow \big(\{v_0, \ldots, v_{n-1}\}, \{(v_0, v_1), \ldots, (v_{n-2}, v_{n-1}), (v_{n-1}, v_0)\}\big)$;
6   **for** t_i **in** t **do**
7   **if** $t_i = -1$ **or** $t_i = 1$ **then**
8   **try:**
9                       $F \leftarrow F \setminus \big\{(v_i, v_{(i+t_i) \bmod n})\big\}$;
10   **catch:**
11   **pass**
12   **end**
13                  $F \leftarrow F \cup \big\{(v_i, v_{\sigma_i})\big\}$;
14   **end**
15   **end**
16   **if** isCycle(F) **then**
17   foundEdgeSwap $\leftarrow$ TRUE;
18   **break**
19   **end**
20   **end**
21   **if not** foundEdgeSwap **then**
22   **return** FALSE;
23   **end**
24 **end**
25 **return** TRUE;

TSP Path

Notation:

- Index set for all possible NNG configurations.

$$S_{NNG}(n) := \Big\{ (\sigma_0, \ldots, \sigma_{n-1}) :$$
$$\sigma_0 \in \{2, \ldots, n-1\}$$
$$\wedge \ \sigma_i \in \{0, \ldots, n-1\} \setminus \{i-1, i, i+1\}, i = 1, \ldots, n-2$$
$$\wedge \ \sigma_{n-1} \in \{0, \ldots, n-3\} \Big\}$$

- Edge swap index set.

$$T(n) = \Big\{ (t_1, \ldots, t_n) :$$
$$t_0 \in \{0, 1\}$$
$$\wedge \ t_i \in \{-1, 0, 1\}, i = 1, \ldots, n-2$$
$$\wedge \ t_{n-1} \in \{-1, 0\} \Big\} \setminus \{(0, \ldots, 0)\}$$

Algorithm 2: TSP path vs. NNG intersection tester for graphs on n vertices
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1 procedure PATHNNG (n);

   Input : integer $n \geq 3$

   Output: TRUE if a Hamilton path on the complement of the 1-NNG (of a complete weighted graph on n vertices) can always be shortened via edge exchanges with the 1-NNG; FALSE otherwise

2 **for** $(\sigma_0, \ldots, \sigma_{n-1})$ **in** $S_{NNG}(n)$ **do**

3   foundEdgeSwap $\leftarrow$ FALSE;

4   **for** t **in** $T(n)$ **do**

5 $F \leftarrow \big(\{v_0, \ldots, v_{n-1}\}, \{(v_0, v_1), \ldots, (v_{n-2}, v_{n-1})\}\big)$;

6   **for** t_i **in** t **do**

7   **if** $t_i = -1$ **or** $t_i = 1$ **then**

8   **try:**

9 $F \leftarrow F \setminus \big\{(v_i, v_{i+t_i})\big\}$;

10   **catch:**

11   **pass**

12   **end**

13 $F \leftarrow F \cup \big\{(v_i, v_{\sigma_i})\big\}$;

14   **end**

15   **end**

16   **if** isPath(F) **then**

17   foundEdgeSwap $\leftarrow$ TRUE;

18   **break**

19   **end**

20   **end**

21   **if not** foundEdgeSwap **then**

22   **return** FALSE;

23   **end**

24 **end**

25 **return** TRUE;

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