# TSP–NNG Intersection Proof Pseudocode

**TSP Tour**

Notation:

- Index set for all possible NNG configurations. Note that $\left|S_{NNG}(n)\right| = (n-3)^n$.

$$S_{NNG}(n) := \Big\{(\sigma_0, \ldots, \sigma_{n-1}) : \sigma_i \in \{0, \ldots, n-1\} \setminus \{j \mod n : j = i-1, i, i+1\}\Big\}.$$

- Edge swap index set. Note that $|\mathcal{E}(n)| = 3^n - 1$.

$$\mathcal{E}(n) = \Big\{(e_0, \ldots, e_{n-1}) : t_i \in \{-1, 0, 1\}\Big\} \setminus \{(0, \ldots, 0)\}$$

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**Algorithm 1:** TSP tour vs. NNG intersection tester for weighted graphs

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 1 procedure TOURNNG (n);
   Input   : integer n ≥ 3
   Output: TRUE if a Hamiltonian cycle on the complement of the NNG (of
            an undirected weighted graph on n vertices) can always be
            shortened via edge exchanges with a representative of the NNG;
            FALSE otherwise
 2 for (σ₀, …, σₙ₋₁) in S_NNG(n) do
 3       foundEdgeSwap ← FALSE;
 4       for (e₀, …, eₙ₋₁) in ℰ(n) do
 5             F ← ({v₀, …, vₙ₋₁}, {(v₀, v₁), …, (vₙ₋₂, vₙ₋₁), (vₙ₋₁, v₀)});
 6             for i in {0, …, n − 1} do
 7                   if (vᵢ, v₍ᵢ₊ₑᵢ₎ mod n) ∈ F then
 8                         F ← F \ {(vᵢ, v₍ᵢ₊ₑᵢ₎ mod n)};
 9                         F ← F ∪ {(vᵢ, v_σᵢ)};
10                   end
11             end
12             if isCycle(F) then
13                   foundEdgeSwap ← TRUE;
14                   break
15             end
16       end
17       if not foundEdgeSwap then
18             return FALSE;
19       end
20 end
21 return TRUE;
```

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**TSP Path**

Notation:

- Index set for all possible NNG configurations.

$$S_{NNG}(n) := \Big\{ (\sigma_0, \ldots, \sigma_{n-1}) : \ \sigma_0 \in \{2, \ldots, n-1\}$$
$$\wedge \ \sigma_i \in \{0, \ldots, n-1\} \setminus \{i-1, i, i+1\}, i = 1, \ldots, n-2$$
$$\wedge \ \sigma_{n-1} \in \{0, \ldots, n-3\} \Big\}$$

- Edge swap index set.

$$\mathcal{E}(n) = \Big\{ (t_1, \ldots, t_n) : \ t_0 \in \{0, 1\}$$
$$\wedge \ t_i \in \{-1, 0, 1\}, i = 1, \ldots, n-2$$
$$\wedge \ t_{n-1} \in \{-1, 0\} \Big\} \setminus \{(0, \ldots, 0)\}$$

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**Algorithm 2:** TSP path vs. NNG intersection tester for graphs on $n$ vertices

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1 **procedure** PATHNNG $(n)$;
   **Input** : integer $n \geq 3$
   **Output:** TRUE if a Hamiltonian path on the complement of the NNG (of
               an undirected weighted graph on $n$ vertices) can always be
               shortened via edge exchanges with a representative of the NNG;
               FALSE otherwise
2 **for** $(\sigma_0, \ldots, \sigma_{n-1})$ **in** $S_{NNG}(n)$ **do**
3      foundEdgeSwap $\leftarrow$ FALSE;
4      **for** $(e_0, \ldots, e_{n-1})$ **in** $\mathcal{E}(n)$ **do**
5           $F \leftarrow \big(\{v_0, \ldots, v_{n-1}\}, \{(v_0, v_1), \ldots, (v_{n-2}, v_{n-1})\}\big)$;
6           **for** $i$ **in** $\{0, \ldots, n-1\}$ **do**
7                **if** $(v_i, v_{(i+e_i) \bmod n}) \in F$ **then**
8                    $F \leftarrow F \setminus \big\{(v_i, v_{(i+e_i) \bmod n})\big\}$;
9                    $F \leftarrow F \cup \big\{(v_i, v_{\sigma_i})\big\}$;
10                **end**
11           **end**
12           **if** isPath$(F)$ **then**
13                foundEdgeSwap $\leftarrow$ TRUE;
14                **break**
15           **end**
16      **end**
17      **if not** foundEdgeSwap **then**
18           **return** FALSE;
19      **end**
20 **end**
21 **return** TRUE;

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