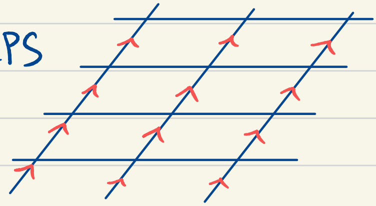
MPS

imp

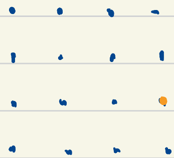
MPS

PEPS



1. Notation for generic quantum lattice system

$l : 1 \cdots \mathcal{L}$



$\mathcal{L}$ sites

spin-s :

$\{2s+1\}$

local state space l:

$|\sigma_l\rangle_l \in \{|1\rangle_l, |2\rangle_l, \dots, |2s+1\rangle_l\}$.

local dim

$d = 2s+1$

$|\sigma_l\rangle \in \mathcal{H}$

Full physical system ($\mathcal{L}$-sites)

$$|\sigma_\mathcal{L}\rangle \otimes |\sigma_{\mathcal{L}-1}\rangle \otimes \cdots \otimes |\sigma_2\rangle \otimes |\sigma_1\rangle$$

$$:= |\sigma_\mathcal{L} \cdots \sigma_2 \sigma_1\rangle := |\vec{\sigma}\rangle_\mathcal{L} \in \mathcal{H}^{\otimes \mathcal{L}}$$

General quantum states :

wave-function

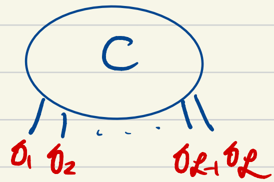
$$|\Psi\rangle_\mathcal{L} = \sum_{\sigma_1 \cdots \sigma_\mathcal{L}} |\sigma_1, \dots, \sigma_\mathcal{L}\rangle C^{\sigma_1 \cdots \sigma_\mathcal{L}} := |\vec{\sigma}\rangle C^{\vec{\sigma}}$$

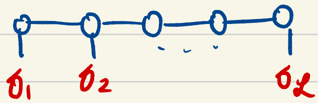
Dimension of full Hilbert space $\mathcal{H}^{\otimes \mathcal{L}}$: $(2S+1)^{\mathcal{L}} = d^{\mathcal{L}}$

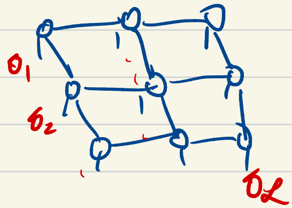
↳ Specifying quantum state $|\psi\rangle_{\mathcal{L}}$: specify $C^{\vec{\sigma}}$

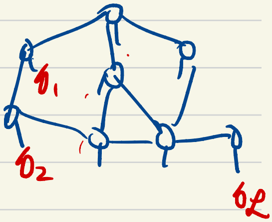
$\Rightarrow$ specify $d^{\mathcal{L}}$ complex #.

$C^{\vec{\sigma}}$ is a tensor of rank $\mathcal{L}$







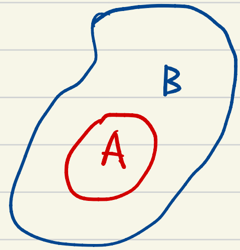


2. Entanglement & area law.

Quantum system in pure state $|\psi\rangle$,

density matrix $\hat{\rho} = |\psi\rangle\langle\psi|$. Divide

2 parts, A and B,



$\mathcal{H}_A = \text{span}\{|\vec{\sigma}_A\rangle\}$, $\mathcal{H}_B = \text{span}\{|\vec{\sigma}_B\rangle\}$

Reduced density matrix (RDM):

RDM for A: $\hat{\rho}_A = \text{Tr}_B\, \hat{\rho}$ and RDM for B: $\hat{\rho}_B = \text{Tr}_A\, \hat{\rho}$

EE: $S_{A/B} = -\text{Tr}\, \hat{\rho}_A \log_2 \hat{\rho}_A = -\sum_\alpha w_\alpha \log_2 w_\alpha$

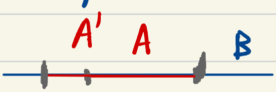
$\uparrow$ e-vals of $\hat{\rho}_A$

Fact: for Hamiltonian that has local interaction,

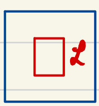
the ground state EE is govern by area law:

$S := S_{A/B} \sim$ (area of boundary A) $= \partial_A$

1D: $\sim$ const



2D: $\sim \mathcal{L}$

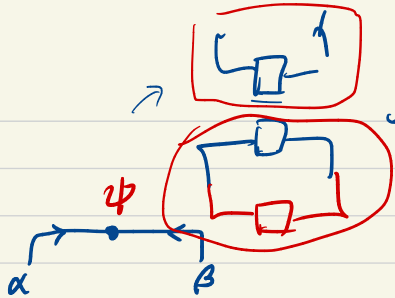


⋮

3D: $\sim \mathcal{L}^2$

1D critical: const $+ \log \mathcal{L}$

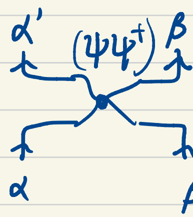
Pure state in $A \cup B$:



$$|\psi\rangle = |\beta\rangle_B |\alpha\rangle_A \psi^{\alpha\beta}$$

$$\langle\psi| = \underbrace{\psi^\dagger_{\beta'\alpha'}}_{\left(\psi^{\alpha'\beta'}\right)^*} {}_A\langle\alpha'| {}_B\langle\beta'|$$

$$|\psi\rangle \underbrace{\langle\psi|\psi\rangle}$$
$$\hat{\rho}|\psi\rangle$$



Density matrix: $\hat{\rho} = |\psi\rangle\langle\psi|$

$$\Downarrow$$

$$|\beta\rangle_B |\alpha\rangle_A \psi^{\alpha\beta} \psi^\dagger_{\beta'\alpha'} {}_A\langle\alpha'| {}_B\langle\beta'|$$

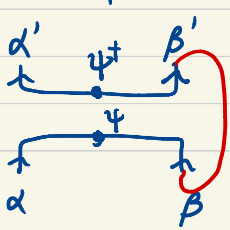
RDM A:

$$\hat{\rho}_A = \mathrm{Tr}_B |\psi\rangle\langle\psi| = \sum_{\bar{\beta}} {}_B\langle\bar{\beta}|\beta\rangle_B |\alpha\rangle_A \psi^{\alpha\beta} \psi^\dagger_{\beta'\alpha'} {}_A\langle\alpha'| {}_B\langle\beta'|\bar{\beta}\rangle$$

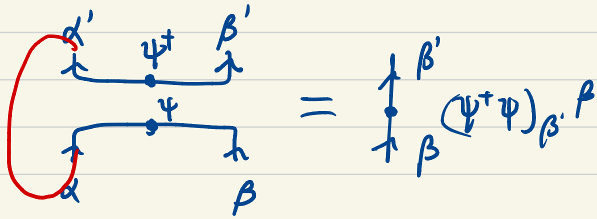
$$= |\alpha\rangle_A \underline{(\hat{\rho}_A)^\alpha{}_{\alpha'}} {}_A\langle\alpha'|$$

$$(\hat{\rho}_A)^\alpha{}_{\alpha'} = \sum_{\bar{\beta}} \underbrace{{}_B\langle\bar{\beta}|\beta\rangle_B}_{\delta_{\bar{\beta}\beta}} \psi^{\alpha\beta} \psi^\dagger_{\beta'\alpha'} \underbrace{{}_B\langle\beta'|\bar{\beta}\rangle}_{\delta_{\beta'\bar{\beta}}}$$

$$= \psi^{\alpha\bar{\beta}} \psi^\dagger_{\bar{\beta}\alpha'} = (\psi\psi^\dagger)^\alpha{}_{\alpha'}$$

$(\rho_A)^\alpha{}_{\alpha'} =$  $=$  $\psi^{\alpha\beta} \psi^\dagger{}_{\beta\alpha'} = (\psi\psi^\dagger)^\alpha{}_{\alpha'}$

RDM B: ...



$\mathrm{tr}(\hat{\rho}_A) = 1$

$\hat{\rho}_A : (\psi\psi^\dagger)^{\alpha}{}_{\alpha'} = \delta^{\alpha}{}_{\alpha'} \underline{W_\alpha}$ svd values

$\sum_\alpha W_\alpha = 1$ bond-dim

$\alpha = 1, \cdots, \underline{D}$

EE

$$S = -\sum_{\alpha=1}^{D} W_\alpha \log_2 W_\alpha$$

$D = 2$

$W = 0.5$

$\underline{S = w\log w + (1-w)\log(1-w)}$

$W_\alpha = \frac{1}{D}$

$$S = -\sum_{\alpha=1}^{D} \frac{1}{D}\log_2\frac{1}{D} = \frac{D}{D}\log_2 D = \log_2 D$$

$S \leq \log_2 D$

$\Rightarrow 2^S = D$

$\Rightarrow \underline{2^S \leq D}$

<u>Area law</u>

<u>1D gappend</u> : $S \sim \text{const} \Rightarrow D \sim 2^{\text{const}} \sim \text{const}$

indep. system size ☺

1D critical : $S \sim \text{const} + \log L \Rightarrow D \sim 2^{\text{const}+\log L} \sim$ power law in L

<u>2D gapped</u>: $S \sim L \Rightarrow D \sim 2^{L}$

<u>3D gapped</u>: $S \sim L^2 \Rightarrow D \sim 2^{L^2}$

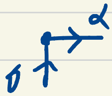
1D: For gapped and gapless system $\times$ exponential scaling

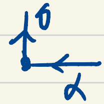
3. Tensor network diagrams

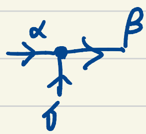
rank-0 : scalar A  $A^{\dagger} := \bar{A}$

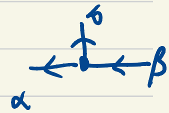
-1 : vector A^{σ} 

$A^{\dagger}_{\sigma} := \bar{A}^{\sigma}$ 

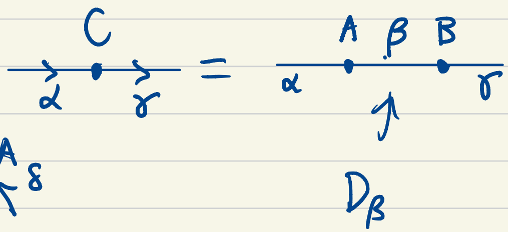
-2 : matrix $A^{\sigma}{}_{\alpha}$ 

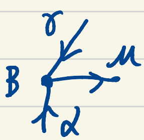
$A^{\dagger\alpha}{}_{\sigma} := \bar{A}^{\sigma}{}_{\alpha}$ 

-3 : tensor $A^{\alpha\sigma}{}_{\beta}$ 

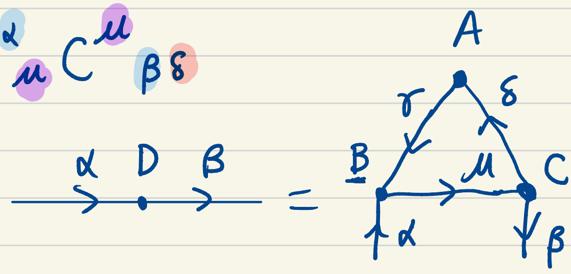
$A^{\dagger\beta}{}_{\sigma\alpha} := \bar{A}^{\alpha\sigma}{}_{\beta}$ 

$$C^{\alpha}{}_{\gamma} = \sum_{\beta=1}^{D} A^{\alpha}{}_{\beta} B^{\beta}{}_{\gamma}$$





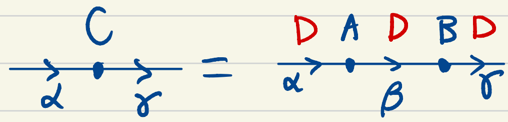
$$D^{\alpha}{}_{\beta} = A^{\delta}{}_{\gamma} B^{\gamma\alpha}{}_{\mu} C^{\mu}{}_{\beta\delta}$$



$\alpha, \gamma = \{1, \cdots, D\}$

Cost of computing contraction

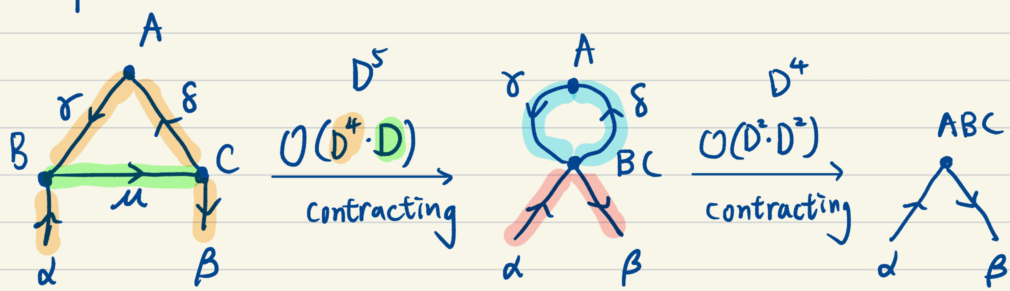
$\beta = \{1, \cdots D\}$



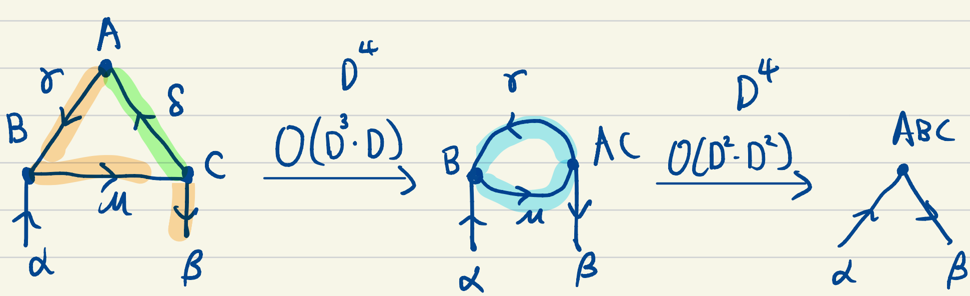
$$C^{\alpha}{}_{\gamma} = A^{\alpha}{}_{\beta} B^{\beta}{}_{\gamma}$$

Cost: $\underbrace{D^2}_{\text{open legs}} \cdot \underbrace{D}_{\text{sum}}$

Example:



Cost $\sim O(D^5)$.



Cost $\sim O(D^4)$.