

例 1

求函数 $y = \ln(x^2 + 1)$ 的导数

$$y' = \frac{1}{x^2 + 1} \cdot 2x = \frac{2x}{x^2 + 1}$$

例 2 求函数 $y = \ln(x^2 + 1)$ 的二阶导数

$$y' = \frac{2x}{x^2 + 1}$$

$$y'' = \frac{(x^2 + 1) \cdot 2 - 2x \cdot 2x}{(x^2 + 1)^2} = \frac{2 - 2x^2}{(x^2 + 1)^2}$$

例 3 求函数 $y = \ln(x^2 + 1)$ 的三阶导数

$$y'' = \frac{2 - 2x^2}{(x^2 + 1)^2}$$

$$y''' = \frac{(x^2 + 1)^2 \cdot (-4x) - (2 - 2x^2) \cdot 2(x^2 + 1) \cdot 2x}{(x^2 + 1)^4} = \frac{-4x(x^2 + 1) - 4x(2 - 2x^2)}{(x^2 + 1)^3} = \frac{-4x^3 - 4x - 8x + 8x^3}{(x^2 + 1)^3} = \frac{4x^3 - 12x}{(x^2 + 1)^3}$$

例 4

$$y = \ln(x^2 + 1)$$

$$y' = \frac{2x}{x^2 + 1}$$

$$y'' = \frac{2 - 2x^2}{(x^2 + 1)^2}$$

$$y''' = \frac{4x^3 - 12x}{(x^2 + 1)^3}$$

$$\frac{y'''}{y'} = \frac{4x^3 - 12x}{(x^2 + 1)^3} \cdot \frac{x^2 + 1}{2x} = \frac{2x^2 - 6}{(x^2 + 1)^2}$$

$$y'' = \frac{2 - 2x^2}{(x^2 + 1)^2}$$

$$y''' = \frac{4x^3 - 12x}{(x^2 + 1)^3}$$

$$y'' = \frac{2 - 2x^2}{(x^2 + 1)^2}$$

$$\frac{y'''}{y'} = \frac{4x^3 - 12x}{(x^2 + 1)^3} \cdot \frac{x^2 + 1}{2x} = \frac{2x^2 - 6}{(x^2 + 1)^2}$$

$$y'' = \frac{2 - 2x^2}{(x^2 + 1)^2}$$