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***Abstract*—Write your abstract text here.**

I. Waveport Boundary Condition

The following is not necessarily written using the proper prose as mentioned in lecture. This is just a derivation, I will re-word this later.

Starting with (3.3.34) in Ch. 9 of our textbook

$$\mathbf{E}(u,v,w) = \mathbf{E}^{inc}(u,v,w) + \mathbf{E}^{ref}(u,v,w). \tag{1}$$

Substituting in the $\mathbf{e}_{10}$ definition in (9.3.35) yields the following

$$\mathbf{E}(u,v,w) = \hat{v}E_0 \sin\frac{\pi u}{a} e^{-j\beta w} + \hat{v}RE_0 \sin\frac{\pi u}{a} e^{j\beta w}. \tag{2}$$

where R is the reflection coefficient and a is the width of the waveguide.

This can be cleanly written in vector notation as

$$\begin{aligned}\mathbf{E}(u,v,w) &= \begin{bmatrix} 0 \\ E_0 \sin\frac{\pi u}{a}(e^{-j\beta w} + Re^{j\beta w}) \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ c(u)(e^{-j\beta w} + Re^{j\beta w}) \\ 0 \end{bmatrix}. \end{aligned} \tag{3}$$

With

$$\mathbf{E}^{inc}(u,v,w) = \begin{bmatrix} 0 \\ c(u)e^{-j\beta w} \\ 0 \end{bmatrix}. \tag{4}$$

The waveport source boundary condition as found in (9.3.37) is

$$\hat{n} \times (\nabla \times \mathbf{E}) + j\beta\hat{n} \times (\hat{n} \times \mathbf{E}) = -2j\beta\mathbf{E}^{inc}. \tag{5}$$

Substituting the above field definitions into 5 yields the following

$$\begin{aligned}&\hat{n} \times \left(\nabla \times \begin{bmatrix} 0 \\ c(u)(e^{-j\beta w} + Re^{j\beta w}) \\ 0 \end{bmatrix}\right) \\ &\quad + j\beta\hat{n} \times \left(\hat{n} \times \begin{bmatrix} 0 \\ c(u)(e^{-j\beta w} + Re^{j\beta w}) \\ 0 \end{bmatrix}\right) \\ &\quad = -2j\beta \begin{bmatrix} 0 \\ c(u)e^{-j\beta w} \\ 0 \end{bmatrix}. \end{aligned} \tag{6}$$

Assuming $\hat{u}, \hat{v}, \hat{w}$ form an orthogonal, right-handed set of basis vectors with $\hat{n} = \hat{w}$ as it is normal to the waveport surface in the direction of propagation 6 becomes

$$\begin{aligned}&\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} \partial_u \\ \partial_v \\ \partial_w \end{bmatrix} \times \begin{bmatrix} 0 \\ c(u)(e^{-j\beta w} + Re^{j\beta w}) \\ 0 \end{bmatrix}\right) \\ &\quad + j\beta \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ c(u)(e^{-j\beta w} + Re^{j\beta w}) \\ 0 \end{bmatrix}\right) \\ &\quad = -2j\beta \begin{bmatrix} 0 \\ c(u)e^{-j\beta w} \\ 0 \end{bmatrix}. \end{aligned} \tag{7}$$

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Evaluating the first set of cross products results in

$$\begin{aligned}&\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -\partial_w(c(u)(e^{-j\beta w} + Re^{j\beta w})) \\ 0 \\ \partial_u(c(u)(e^{-j\beta w} + Re^{j\beta w})) \end{bmatrix} \\ &\quad + j\beta \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -c(u)(e^{-j\beta w} + Re^{j\beta w}) \\ 0 \\ 0 \end{bmatrix} \\ &\quad = -2j\beta \begin{bmatrix} 0 \\ c(u)e^{-j\beta w} \\ 0 \end{bmatrix}. \end{aligned} \tag{8}$$

Evaluating the second set of cross products equals

$$\begin{aligned}&\begin{bmatrix} 0 \\ -\partial_w(c(u)(e^{-j\beta w} + Re^{j\beta w})) \\ 0 \end{bmatrix} \\ &\quad + j\beta \begin{bmatrix} 0 \\ -c(u)(e^{-j\beta w} + Re^{j\beta w}) \\ 0 \end{bmatrix} \\ &\quad = -2j\beta \begin{bmatrix} 0 \\ c(u)e^{-j\beta w} \\ 0 \end{bmatrix}. \end{aligned} \tag{9}$$

Resulting in the following expression for the $\hat{v}$ direction

$$\begin{aligned}&-\partial_w(c(u)(e^{-j\beta w} + Re^{j\beta w})) - \\ &\quad j\beta c(u)(e^{-j\beta w} + Re^{j\beta w}) \\ &\quad = -2j\beta c(u)e^{-j\beta w}. \end{aligned} \tag{10}$$

Which is equal to

$$\partial_w E_v + j\beta E_v = 2j\beta E_v^{inc}. \tag{11}$$