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***Abstract*—Write your abstract text here.**

I. Waveport Boundary Condition

The following is not necessarily written using the proper prose as mentioned in lecture. This is just a derivation, I will re-word this later.

Starting with (3.3.34) in Ch. 9 of our textbook

$$\mathbf{E}(u,v,w) = \mathbf{E}^{inc}(u,v,w) + \mathbf{E}^{ref}(u,v,w). \tag{1}$$

Substituting in the $\mathbf{e}_{10}$ definition in (9.3.35) yields the following

$$\mathbf{E}(u,v,w) = \hat{v}E_0 \sin\frac{\pi u}{a} + \hat{v}RE_0 \sin\frac{\pi u}{a}. \tag{2}$$

where R is the reflection coefficient and a is the width of the waveguide.

This can be cleanly written in vector notation as

$$\mathbf{E}(u,v,w) = \begin{bmatrix} 0 \\ E_0 \sin\frac{\pi u}{a}(1+R) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ c(u)(1+R) \\ 0 \end{bmatrix}. \tag{3}$$

With

$$\mathbf{E}^{inc}(u,v,w) = \begin{bmatrix} 0 \\ c(u) \\ 0 \end{bmatrix}. \tag{4}$$

The waveport source boundary condition as found in (9.3.37) is

$$\hat{n} \times (\nabla \times \mathbf{E}) + j\beta\hat{n} \times (\hat{n} \times \mathbf{E}) = -2j\beta\mathbf{E}^{inc}. \tag{5}$$

Substituting the above field definitions into 5 yields the following

$$\begin{aligned} &\hat{n} \times \left(\nabla \times \begin{bmatrix} 0 \\ c(u)(1+R) \\ 0 \end{bmatrix}\right) \\ &\qquad + j\beta\hat{n} \times \left(\hat{n} \times \begin{bmatrix} 0 \\ c(u)(1+R) \\ 0 \end{bmatrix}\right) \\ &\qquad\qquad = -2j\beta \begin{bmatrix} 0 \\ c(u) \\ 0 \end{bmatrix}. \end{aligned} \tag{6}$$

Assuming $\hat{u}, \hat{v}, \hat{w}$ form an orthogonal, right-handed set of basis vectors with $\hat{n} = \hat{w}$ as it is normal to the waveport surface in the direction of propagation 6 becomes

$$\begin{aligned} &\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} \partial_x \\ \partial_y \\ \partial_z \end{bmatrix} \times \begin{bmatrix} 0 \\ c(u)(1+R) \\ 0 \end{bmatrix}\right) \\ &\qquad + j\beta \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ c(u)(1+R) \\ 0 \end{bmatrix}\right) \\ &\qquad\qquad = -2j\beta \begin{bmatrix} 0 \\ c(u) \\ 0 \end{bmatrix}. \end{aligned} \tag{7}$$

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Evaluating the first set of cross products results in

$$\begin{aligned} &\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ \partial_u(c(u)(1+R)) \end{bmatrix} \\ &\qquad + j\beta \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -c(u)(1+R) \\ 0 \\ 0 \end{bmatrix} \\ &\qquad\qquad = -2j\beta \begin{bmatrix} 0 \\ c(u) \\ 0 \end{bmatrix}. \end{aligned} \tag{8}$$

this creates a problem as the spatial derivative drops