Toutorial - 4. Samya Jain 40 F ①

Que-1. $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$

$a \geq 1, b > 1$

on comparing

$a = 3, b = 2, f(n) = n^2$

Now, $c = \log_b a = \log_2 3 = 1584$

$n^c = n^{1584} < n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = \theta(n^2)$

Que-2 $T(n) = 4T(n/2) + n^2$

$a \geq 1, b > 1$

$a = 4, b = 2, f(n) = n^2$

$\therefore c = \log_2 4 \Rightarrow 2$

$\therefore n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \theta(n^2 \log n)$

Que-3. $T(n) = T(n/2) + 2^n$

$a = 1, b = 2, f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$\therefore T(n) = \theta(2^n)$

Que-4. $T(n) = 2^n T(n/2) + n^n$

here, Master's theorem can't be applied as 'a' must be constant.

Que-5. $T(n) = 16T(n/4) + n$

$a = 16, b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 4^2 = 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = \theta(n^2)$

Que-6. $T(n) = 2T(n/2) + n \log n$

$a = 2, b = 2$

$f(n) = n \log n \Rightarrow c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

Since, $n \log n > n$

$\therefore f(n) > n^c$

$\therefore T(n) = \theta(n \log n)$

Que-7 $T(n) = 2T(n/2) + n/\log n$

$a = 2, b = 2, f(n) = n/\log n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

Since, $n/\log n < n$

$\therefore T(n) = \theta(n)$

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Que-8. $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, b = 4, f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$\therefore n^c = n^{0.5}$

Since, $n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \theta(n^{0.51})$

Que-9. $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5, b = 2$

$\because$ Acc. to Master's Theorem

$a \geq 1$, but here a is 0.5

So, M.T. cannot be applied.

Que-10. $T(n) = 16T(n/4) + n!$

$a = 16, b = 4, f(n) = n!$

$\therefore c = \log_b a \Rightarrow \log_4 16 = 2$

Now, $n^c = n^2$

as $n! > n^2$

$\therefore T(n) = \theta(n!)$

Que-11. $4T(n/2) + \log n$

$a = 4, b = 2, f(n) = \log n$

$c = \log_b a = \log_2 4 \Rightarrow 2$

$\therefore n^c = n^2$ and $f(n) = \log n$

$\because \log n < n^2$

$T(n) = \theta(n^2)$

Que-12. $T(n) = \text{sqrt}(n)\, T(n/2) + \log n$

here, MT can not be applied as a must be constant.

Que-13. $T(n) = 3T(n/2) + n$

$a = 3, b = 2, f(n) = n$

$c = \log_b a = \log_2 3 \Rightarrow 1.5849.$

$\therefore n < n^{1.5489} \Rightarrow f(n) < n^c$

$T(n) = \theta(n^{1.5489})$

Que-14. $T(n) = 3T(n/3) + \text{sqrt}(n)$

$a = 3, b = 3, c = \log_3 3 = 1$

$\therefore n^c = n^1 = n$

as $\text{sqrt}(n) < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n)$

Que-15. $T(n) = 4T(n/2) + cn$

$a = 4, b = 2$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$cn < n^2$ (for any constant)

$\therefore T(n) = \theta(n^2)$

Que-16. $T(n) = 3T(n/4) + n\log n$

$a=3, b=4, f(n) = n\log n$

$c = \log_b a \rightarrow \log_4 3 = 0.792$

$n^c = n^{0.792}$

$\therefore n^{0.792} < n\log n$

$\therefore T(n) = \theta(n\log n)$

Que-17. $T(n) = 3T(n/3) + n/2$

$a=3, \; b=3$

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$\therefore n^c = n^1 = n$

as $n/2 < n$

$\therefore T(n) = \theta(n)$

Que18.- $T(n) = 6T(n/3) + n^2\log n$

$a=6, \; b=3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

as $n^{1.6309} < n^2\log n$

$\therefore T(n) = \theta(n^2\log n)$

Que-19. $T(n) = 4T(n/2) + n\log n$

$a=4, b=2, f(n) = n/\log n$

$c = \log_2 4 = 2$

$n^c = n^2 > n/\log n$

$T(n) = \theta(n^2)$

Que-20. $T(n) = 64T(n/8) - n^2\log n$

$a=64, \; b=8$

MT can't be applied here as $f(n)$ is -ve.

Que-21. $T(n) = 7T(n/3) + n^2$

$a=7, \; b=3, \; f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712} < n^2$

$\therefore T(n) = \theta(n^2)$

Que-22. $T(n) = T(n/2) + n(2-\cos n)$

$a=1, \; b=2$

$c = \log_b a = \log_2 1 = 0$

$\therefore n^c = n^0 = 1$

$n(2-\cos n) > n^c$

$\therefore T(n) = \theta(n(2-\cos n))$