CS-1102 Problem Set 1

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1-1a Constructing a Regular Heptadecagon

Heptadecagon Geogebra Link: `https://www.geogebra.org/m/hrcntksc`

A regular heptadecagon (17-sided polygon) can be constructed using compass and straightedge by applying Gauss's trigonometric identity for $\cos(2\pi/17)$. This method avoids directly measuring angles and instead relies on constructing 17 equally spaced vertices along a circle.

The construction begins by drawing the circumscribed circle of the heptadecagon. Next, auxiliary circles are drawn that intersect the main circle using perpendicular lines. These intersections help establish points spaced at correct angular distances. Angle bisectors are then used to refine these points further.

To locate the vertices, perpendicular bisectors are drawn in parallel. The intersections of these bisectors with the primary circle determine the precise locations of the heptadecagon's vertices. Each vertex is successively marked using a circle centered at a known vertex with a radius equal to the side length of the heptadecagon. This iterative process continues until all 17 vertices are marked.

1-1b Constructing a Square of Equal Area

Operational Semantics

1. Divide the Heptadecagon into 17 Triangles: - Since the heptadecagon is cyclic, its center coincides with that of its circumscribed circle. - Draw segments from the center to each vertex, forming 17 congruent isosceles triangles.

2. Determine the Area of the Heptadecagon: - The area of one such triangle is given by:

$$A_{\text{triangle}} = \frac{1}{2} \times a \times h$$

where h is the triangle's height. - The total area of the heptadecagon is:

$$A_{\text{heptadecagon}} = 17 \times \frac{1}{2} \times a \times h$$

3. Find the Side Length of the Equivalent Square: - The square's area must be equal to that of the heptadecagon, so its side length s satisfies:

$$s^2 = 17 \times \frac{1}{2} \times a \times h$$

- Using geometric mean properties and compass constructions, construct a length equal to $s = \sqrt{A_{\text{heptadecagon}}}$.

4. Construct the Square: - Using the computed length s, construct a square by drawing perpendicular segments of this length, ensuring a right-angled quadrilateral with equal sides.

Denotational Semantics

Denotational semantics provides a mathematical description of the construction:

- Constructing the Heptadecagon:

$$\text{Heptadecagon}(a) \to \text{A regular heptadecagon with side length } a$$

- Calculating the Area:

$$\text{Area}_{\text{heptadecagon}}(a) = 17 \times \frac{1}{2} \times a \times h$$

- Defining the Equivalent Square:

$$\text{Square}(s) \text{ such that } s^2 = \text{Area}_{\text{heptadecagon}}$$

Axiomatic Semantics

The correctness of the construction is based on the following axioms:

1. Axiom 1: Area of Heptadecagon

$$A_{\text{heptadecagon}} = 17 \times \frac{1}{2} \times a \times h$$

2. Axiom 2: Area of One Triangle

$$A_{\text{triangle}} = \frac{1}{2} \times a \times h$$

3. Axiom 3: Parallel Line Construction - Given a line l and a point P outside l: 1. Choose a point A on l. 2. Construct perpendicular bisectors using a compass. 3. Use these to find equally spaced division points.

4. Axiom 4: Square Root Computation

$$s = \sqrt{17 \times \frac{1}{2} \times a \times h}$$

- This is achieved geometrically using mean proportion properties.

5. Axiom 5: Square Construction - A square is uniquely defined by constructing four perpendicular equal-length segments.