Topological isomorphisms of human brain and financial market networks

Abstract Summary:

The paper analyzes the similarities and differences between the human brain and the financial market.

Functional MRI data were acquired from 18 volunteers, and regional time series were estimated by averaging voxel series in each of the 90 anatomical regions of the brain.
1000 daily closing prices for 116 stocks from the NYSE were obtained. 90 stocks at random were sampled so that both the networks would contain 90 nodes. To construct a simpler financial model, 90 time series simulating the evolution of 90 stocks were generated per the Black-Scholes model.
A political network based on correlations in voting patterns of 90 random senators out of 100 was also constructed.

In each of the three graphs, a node corresponded to a different brain region, a senator, or a stock. The edges between nodes represent statistical associations, e.g., correlations, between the nodes.

Then, the network graphs are analyzed using minimum spanning trees, connection density, and topology of a connection density to determine the similarities and contrasts between them.

Keywords and Definitions:

1) *Black-Scholes Model:* This mathematical equation estimates the theoretical value of derivatives based on other investment instruments, considering the impact of time and other risk factors. It assumes the price follows a geometric Brownian motion with constant drift and volatility.

$$Y_i(t) = Y_0 \exp\left(\frac{\mu - \sigma^2}{2} + \sigma B(t)\right)$$

where $Y_i(t)$ is the price of the i-th stock at time t, Y(0) = 30, μ = 0.0006 is the drift rate, σ = 0.024 is the volatility of the stock's returns, and B(t) follows Brownian motion.

2) *Correlation coefficient:* It represents how one quantity varies with respect to the other. It ranges from -1 to 1. A positive correlation means the two move in the same direction, while a negative value represents the inverse. Values of 1 and -1 indicate perfect positive and negative correlation, respectively.

$$\rho_{i,j} = \frac{< S_i S_j > - < S_i >< S_j >}{\sqrt{\left(< S_i^2 > - < S_i >^2\right)\left(< S_j^2 > - < S_j >^2\right)}}$$

where $S_i(t) = \ln[Y_i(t + \Delta t)] - \ln[Y_i(t)]$, where $Y_i(t)$ is the price of the ith stock on day t.

3) *Network Measures*:

a) Degree of a node: The degree of a node represents the number of edges connecting it to other nodes.
b) Assortavity: The tendency for nodes to connect to other nodes with similar/dissimilar properties within a network.
c) Clustering Coefficient (C_i): of a node is defined as the ratio of the number of triangular connections between the node's nearest neighbors to the maximal possible number of such triangular motifs.

$$C(G) = \frac{1}{N} \sum_{i \in G} C_i$$

d) Path length (L_{ij}): The path length L_{ij} between a pair of nodes i and j is defined as the minimum number of edges that need to be traversed to get from i to j. Global efficiency (E(G)) is defined as the average inverse

$$E(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in G} \frac{1}{L_{ij}}$$

path length.

e) Cost Efficiency CE(G): It is defined as the global efficiency of a network minus its (arbitrary) topological cost or connection density.

$$CE(G) = (E(G) - \kappa).$$

f) Small Worldness: It is a property of a network with high clustering but low characteristic path length, compared to the clustering and path length of a random graph with the same number of nodes and edges

$$\sigma(G) = \frac{C/C_R}{L/L_R}$$

and the same degree distribution.

g) Modularity, Q(G): It measures the structure of a graph, measuring the density of connections within a module or community.

$$Q(G) = \frac{1}{2m} \sum_{i \neq j} (\mathcal{A}_{ij} = P_{ij}) \delta(M_i, M_j)$$

4) *Minimum Spanning Trees:* A minimum-spanning tree is a subset of the edges of a connected graph that connects all the vertices with the most negligible possible total weight of the edges. It is a way of finding the most economical way to connect a set of vertices. It may not be unique. The main advantage of MSTs over more complex network analyses is that the trees are guaranteed to have a fixed number of nodes and edges, without any disconnected islands.

Interpretation of Graphs:

1) Figure 1: The figure has two MSTs drawn in it. It is a comparison of the modular structure of financial and brain-functional networks. We observe that stocks tend to group on branches according to the industrial sector. Similarly, brain regions corresponding to the same functional modules are concentrated together.

2) Figure 2: It shows the growth of financial and brain networks as the cost is increased. As the graph grows, new edges are added to the small clusters already present in both cases, but this property is slightly diminished in brain networks.

3) Figure 3: The plots show the evolution of various terms defined above under "network measures" with increasing cost in the financial network, brain-functional network, political network, and model based on the Black-Scholes model.

4) Figure 5: It is a comparison of the degree distribution, hierarchy, and robustness to random and targeted attacks for the four series same as in Figure 3. Both financial and brain functional networks have a fat-tailed degree distribution are hierarchical, displaying a negative correlation between degree and clustering coefficient over all nodes in each network. They also have similar responses to attacks i.e., robustness.

5) Figure 6: It depicts the network measures at cost 21.2% (maximizing cost efficiency in brain functional network). The financial network is more clustered, modular, and efficient but less robust to targeted attacks than the brain functional network.

Conclusion:

The financial and brain networks have a lot of common structural properties that distinguish them from random graphs. It highlights the potential to solve methodological problems by direct analogy across them.
However, the two are not identical. The financial markets are less robust than the brain networks to targeted attacks. It may be possible that the human brain is less smart than the market but is also less prone to disintegration while removing nodes.
Many exciting questions are similar in the two fields. Insights gained in one may even potentially be applied directly to the other. This approach can prove to be helpful in understanding financial and brain functions.