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Multifractal properties of the Indian financial market

Sunil Kumar [a,b], Nivedita Deo [a,*]

[a] Department of Physics & Astrophysics, University of Delhi, Delhi-110007, India
[b] Department of Physics, Ramjas College, University of Delhi, Delhi-110007, India

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ABSTRACT

We investigate the multifractal properties of the logarithmic returns of the Indian financial indices (BSE & NSE) by applying the multifractal detrended fluctuation analysis. The results are compared with that of the US S&P 500 index. Numerically we find that qth-order generalized Hurst exponents $h(q)$ and $\tau(q)$ change with the moments q. The nonlinear dependence of these scaling exponents and the singularity spectrum $f(\alpha)$ show that the returns possess multifractality. By comparing the MF-DFA results of the original series to those for the shuffled series, we find that the multifractality is due to the contributions of long-range correlations as well as the broad probability density function. The financial markets studied here are compared with the Binomial Multifractal Model (BMFM) and have a smaller multifractal strength than the BMFM.

1. Introduction

The financial markets exhibit very complex dynamics and in recent years have been the focus of some physicists' attempts to apply statistical mechanics to economic problems. The financial markets are open systems in which many subunits interact nonlinearly in the presence of feedback [1] and many records of the non-stationary time series have been investigated [2–4]. The simplest multifractal analysis is based on the standard partition function multifractal formalism, which has been developed for the multifractal characterization of normalized, stationary measurements [5–7]. Unfortunately, this standard formalism does not give correct results for non-stationary time series that are affected by trends or that cannot be normalized. Thus, in the early 1990s an improved multifractal formalism was developed, the wavelet transform modulus maxima method (WTMM) [8], which is based on wavelet analysis and involves tracing the maxima lines in the continuous wavelet transform over all scales. The other method, the multifractal detrended fluctuation analysis (MF-DFA) [9,21–24], is based on the identification of scaling of the qth-order moments depending on the signal length and is a generalization of the standard DFA using only the second moment $q = 2$. The MF-DFA does not require the modulus maxima procedure in contrast to the WTMM method, and hence does not require more effort in programming and computing than the conventional DFA. On the other hand, often experimental data are affected by non-stationarity trends, which have to be well distinguished from the intrinsic fluctuations of the system in order to find the correct scaling behavior of the fluctuations. In addition very often we do not know the reasons for the underlying trends in collected data

* Corresponding author. Tel.: +91 11 22456389; fax: +91 11 27667061.
E-mail address: ndeo@physics.du.ac.in (N. Deo).

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and even worse we do not know the scales of the underlying trends. Usually the available record data is small. For the reliable detection of correlations, it is essential to distinguish trends from the fluctuations intrinsic in the data. Hurst's rescaled-range analysis [10] and other non-detrending methods work well if the records are long and do not involve trends. But if trends are present in the data, they might give wrong results. Detrended fluctuation analysis (DFA) is a well-established method for determining the scaling behavior of noisy data in the presence of trends without knowing their origin and shape [11–15]. By comparing the multifractal-DFA results for the original time series with those of the shuffled time series one can distinguish two different kinds of multifractality: (i) Multifractality due to a broad probability density function for the values of the time series, this multifractality cannot be removed by shuffling the series. (ii) Multifractality due to different long-range correlations for small and large fluctuations, here the corresponding shuffled series will exhibit non-multifractal scaling, since all long-range correlations are destroyed by the shuffling procedure. If both kinds of multifractality are present in a given time series than the shuffled series will show weaker multifractality than the original one [9].

The Hurst exponent, proposed by Hurst [10] provides a measure for long-term memory and fractality of a time-series. Since it is robust with few assumptions about the underlying system, it has broad applicability for time-series analysis and gives information about the stochastic phenomena behind the fluctuations present in the series. $H = 0.5$ indicates a random series, $H > 0.5$ and $H < 0.5$ corresponds to persistent and anti-persistent series respectively.

This paper is organized as follows: Section 2, describes the MF-DFA procedure. In Section 3, we apply the MF-DFA method to the financial markets (BSE, NSE, S&P 500 indices) and study the multifractal properties and long-range correlations present in the financial time series. The series generated from the BMFM is compared with the BSE, NSE and S&P 500 indices. Finally we conclude in Section 4.

2. Method

We have used the multifractal-DFA (MF-DFA) procedure of Ref. [9] which consists of five steps. The first three steps are essentially identical to the conventional-DFA procedure [11,16,17]. We define the normalized logarithmic returns as $g_t = \frac{\log P(t+1) - \log P(t)}{\sigma}$ of length N, where $P(t)$ denotes the daily closing prices of the index and σ is the standard deviation of logarithmic returns.

Step 1: Calculate the "profile",

$$Y(i) \equiv \sum_{k=1}^{i} [g_k - \langle g \rangle], \quad i = 1, \ldots, N, \tag{1}$$

where N is the length of series and $\langle g \rangle$ is the mean of g_t.

Step 2: Divide the profile $Y(i)$ into $N_s \equiv \mathrm{int}(N/s)$ non-overlapping segments of equal length s. Since the length of the series is often not a multiple of the considered time scale s, a short part of the series remains, the same procedure is repeated starting from the opposite end. Thereby, $2N_s$ segments are obtained altogether.

Step 3: Calculate the local trend for each of the $2N_s$ segments by a least-square fit of the time-series. Then determine the variance

$$F^2(s, \nu) \equiv \frac{1}{s} \sum_{i=1}^{s} \{Y[(\nu - 1)s + i] - y_\nu(i)\}^2 \tag{2}$$

for each segment ν, $\nu = 1, \ldots, N_s$ and

$$F^2(s, \nu) \equiv \frac{1}{s} \sum_{i=1}^{s} \{Y[N - (\nu - N_s)s + i] - y_\nu(i)\}^2 \tag{3}$$

for $\nu = N_s + 1, \ldots, 2N_s$. Here, $y_\nu(i)$ is the fitting polynomial in segment ν.

Step 4: Average over all segments to obtain the qth-order fluctuation function,

$$F_q(s) \equiv \left\{ \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} [F^2(s, \nu)]^{q/2} \right\}^{1/q} \tag{4}$$

here, the variable q can take any real value except zero [9].

Step 5: Determine the scaling behavior of the fluctuation functions by analyzing log–log plot of $F_q(s)$ versus s for each value of q. If the time series g_t are long-range power-law correlated, $F_q(s)$ increases for large value of s, as a power law

$$F_q(s) \sim s^{h(q)}. \tag{5}$$

The family of scaling exponents $h(q)$ can be obtained by observing the slope of the log–log plot of $F_q(s)$ versus s. $h(q)$ is the generalization of the Hurst exponent $H(\equiv h(2))$. The monofractal time series are characterized by a single exponent over all time scales i.e. $h(q)$ is independent of q, whereas for multifractal time series, $h(q)$ is varies with q. The $h(q)$ obtained from the MF-DFA is directly related to the classical multifractal scaling exponent [9] by

$$\tau(q) = qh(q) - 1. \tag{6}$$

Using the spectrum of generalized Hurst exponents $h(q)$, one can calculate the singularity strength α and the singularity spectrum $f(\alpha)$ by using

$$\alpha = h(q) + qh'(q) \quad \text{and} \quad f(\alpha) = q[\alpha - h(q)] + 1. \tag{7}$$

A Hölder exponent denotes monofractality, while in the multifractal case, the different parts of the structure are characterized by different values of α, leading to the existence of the spectrum $f(\alpha)$.

3. Multifractal analysis of the financial markets

We have analyzed the multifractal properties of the Indian financial markets (BSE and NSE) and the US market (S&P 500 index). We have carried out the multifractal detrended fluctuation analysis of the BSE (Bombay Stock Exchange) index for the period July, 1997 to December, 2007; NSE (National Stock Exchange) index for the period August, 2002 to December, 2007 and the US's S&P 500 index for the period July, 1997 to December, 2007 [18].

The daily closing prices and normalized log-returns of BSE, NSE, and S&P 500 indices are shown in Fig. 1(a) and (b), Fig. 2(a) and (b), and Fig. 3(a) and (b) respectively. To see the presence of long-range correlations in the time series, we have calculated the Hurst exponents using the DFA method. The time-dependent Hurst exponents [19] $H(t)$ for the window size of 25 days are shown in Figs. 1(c), 2(c), and 3(c) for BSE, NSE, and S&P 500 indices respectively. Here we have observed that for most of the time Hurst exponents are greater than 0.5 for BSE and NSE indices. This shows that BSE and NSE indices are not random, but on average they indicate the persistence behavior. Hurst exponents tend to fluctuate around 0.5 for the S&P 500 index as shown in Fig. 3(c). The Hurst exponent is useful for predicting future price changes and can distinguish emerging markets from mature markets [26,27]. Therefore, we find that the Indian markets (BSE and NSE) is an emerging market while the US market (S&P 500) is a mature market. Refs. [30,29,31] study other aspects of the Indian and US financial markets.

We have observed a change in the Hurst exponent by moving the time window of 25 days near the 9/11 crash. A large dip in the value of the Hurst exponent for BSE index can be seen in Fig. 4 as compare to S&P 500 index. This observation may not be statistically significant. In Fig. 5, we have compared the Gaussian cumulative distribution function with the cumulative distribution function of BSE, NSE, and S&P 500 indices respectively, the clear departure from the Gaussian normality can be seen here. The MF-DFA for BSE index (original and shuffled) and Binomial Multifractal Model (BMFM) [5,9] is shown in Fig. 6(a), (b), (c), and (d) respectively. In Fig. 6(b), $h(q = 2)$ is the well known Hurst exponent H and is found to be 0.56728 for the BSE index. From Fig. 6(b), (c), and (d) we find that the scaling exponents $h(q)$ and $\tau(q)$ show nonlinear dependence which proves that the BSE index exhibit multifractal behavior. For a signal with a multifractal structure the shape of $f(\alpha)$ resembles an inverted parabola. The strength of multifractality ($\Delta\alpha$) can be defined as $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$; the bigger the $\Delta\alpha$, the richer the multifractality. We find $\Delta\alpha = 0.4415$ for the BSE index (original). To find the type of multifractality present in the time series, we have randomly shuffled the series of log-returns for BSE, NSE, and S&P 500 indices respectively [20]. The dependence of $h_{\text{shuf}}(q)$ on q and the value of $\Delta\alpha = 0.399$ for the shuffled series indicates weaker multifractality as compared to the original series for the BSE index. This shows that the BSE index exhibits multifractality due to the contribution of the broad probability density function and long-range correlations. Using the MF-DFA for the short series, we find the generalized Hurst exponents: $h_{\text{orig}}(-10) = 0.7763$, $h_{\text{orig}}(+10) = 0.4689$; $h_{\text{shuf}}(-10) = 0.5568$, $h_{\text{shuf}}(+10) = 0.2875$ for the BSE index. The values of $h(-10)$ for the original and shuffled series are larger than $h(+10)$. We always expect this difference for the short series [9]. We focus on the parts with small fluctuations from the negative values of q described by the larger scaling exponent; whereas for positive values of q, we focus on the parts with large fluctuations usually described by smaller scaling exponent. We have also compared the MF-DFA results of the NSE index (original and shuffled) with BMFM. All these results are shown in Fig. 7(a), (b), (c), and (d). In Fig. 7(b) $h(q = 2)$ is found to be 0.42845. From Fig. 7(d), we find the multifractal strength $\Delta\alpha = 0.5584$ for the original and $\Delta\alpha = 0.4729$ for the shuffled series of the NSE index. We find the generalized Hurst exponents: $h_{\text{orig}}(-10) = 0.6673$, $h_{\text{orig}}(+10) = 0.2633$; $h_{\text{shuf}}(-10) = 0.5472$, $h_{\text{shuf}}(+10) = 0.2234$ for the NSE index. These results show that the NSE index also exhibits both type of multifractality. We see that the multifractal strength for the BSE index is slightly less than that for the NSE index, this shows that the BSE index has a slightly less multifractal strength as compared to the NSE index.The MF-DFA results for S&P 500 index are shown in Fig. 8(a), (b), (c), and (d). We find the generalized Hurst exponents: $h_{\text{orig}}(-10) = 0.8798$, $h_{\text{orig}}(+10) = 0.4814$; $h_{\text{shuf}}(-10) = 0.4857$ and $h_{\text{shuf}}(+10) = 0.2755$ for the S&P 500 index. The multifractal strength for S & P 500 index is found to be 0.5479 for the original series and 0.3280 for the shuffled series. These results also show that both type of multifractality are present in the S&P 500 index.

We have compared the MF-DFA results for the financial indices (BSE, NSE, and S & P 500 indices) with the series obtained by the Binomial Multifractal Model (BMFM). The binomial multifractal series x_k is generated by using the procedure given in the Ref. [5,9] for $n_{\max} = 12$ with $a = 0.6$. The multifractal strengths for the BMFM series corresponding to BSE, NSE, and S&P 500 indices are 0.7619, 0.7755 and 0.7576 respectively. By comparing the MF-DFA results for the binomial multifractal series with the results for the financial time series, we find that the financial markets have less multifractal strength as compared

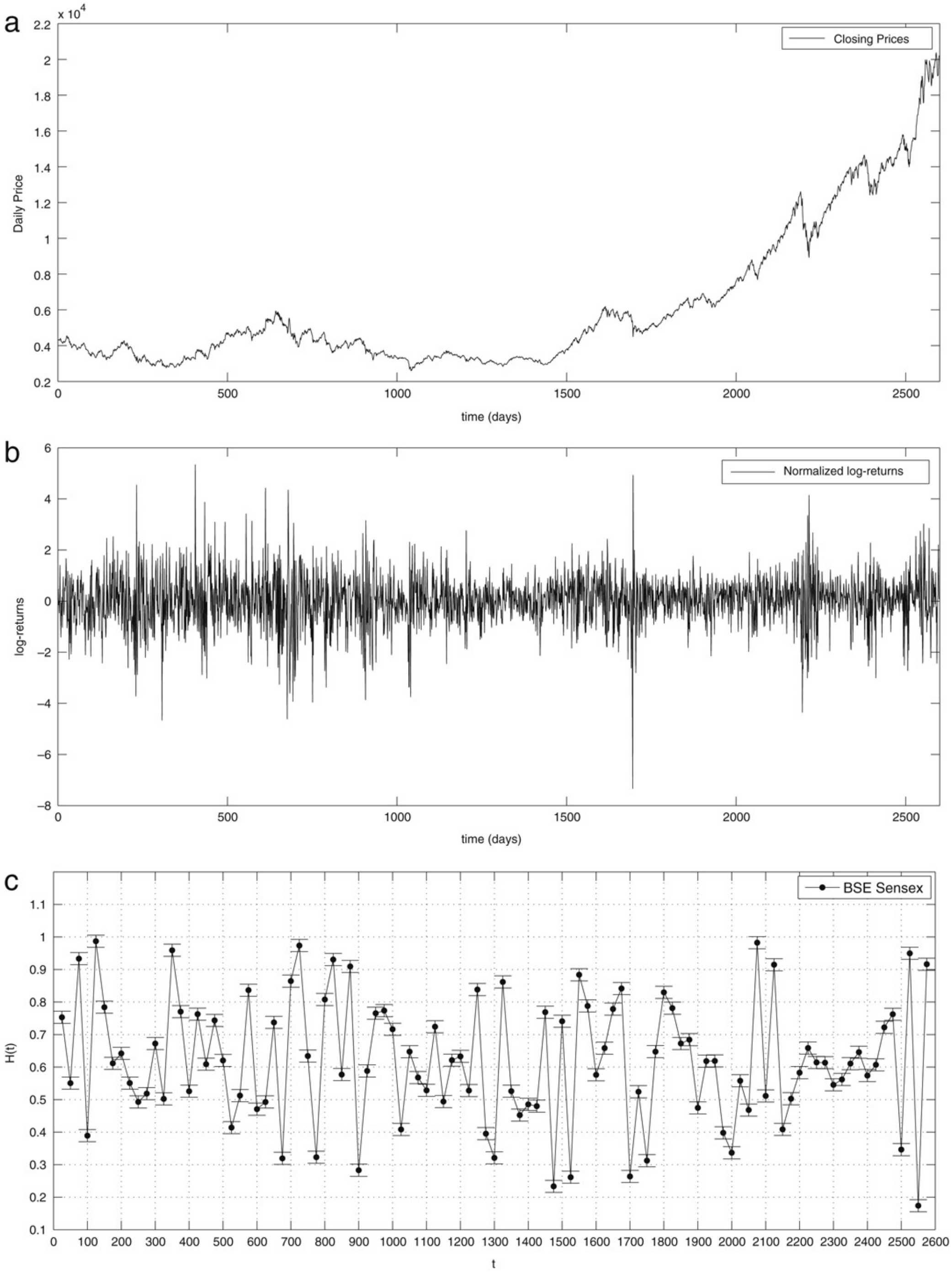

Fig. 1. (a) BSE price index for the period July, 1997 to December, 2007 (b) Corresponding log-returns (c) The Hurst exponents as a function of t for BSE index for the time window of 25 days with errorbar ± 0.0187.

to the Binomial multifractal series. To find a realistic multifractal model which will fit these results of the financial markets is left for a future work.

We plot the multifractal spectrum in Fig. 9(a) and (b). The parameters multifractal strength ($\Delta\alpha$), Hurst exponent, and volatility of the BSE and S&P 500 indices for two year windows for the period (1997–2007) are given in Table 1. If we focus on a two year window, we do not observe any relation among the parameters but if we take the average of 10 years data we find that the higher value of the Hurst exponent leads to the higher average volatility and higher average multifractal strength.

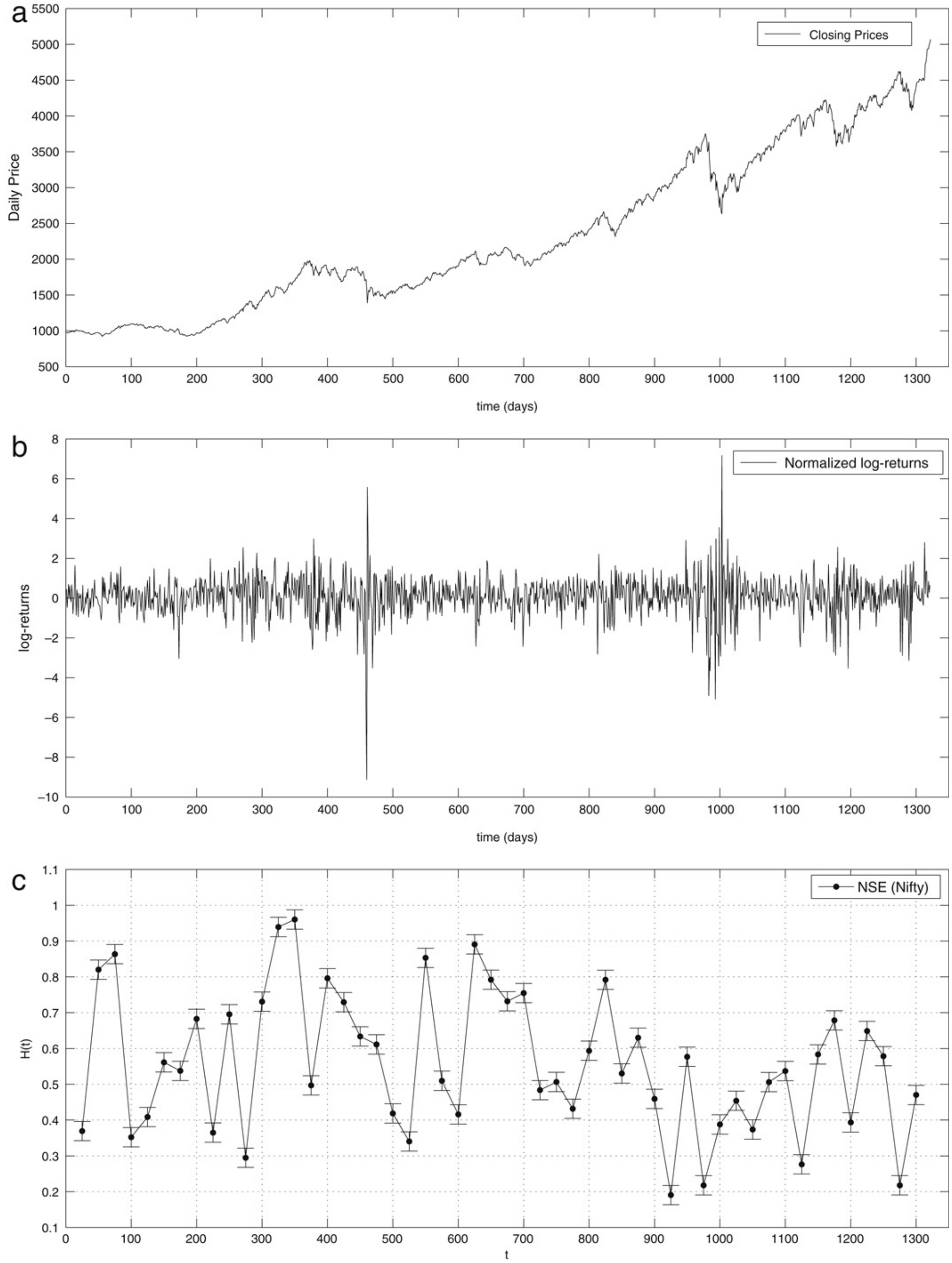

Fig. 2. (a) NSE nifty index daily price index for the period from August, 2002 to December, 2007 (b) Corresponding log-returns (c) The Hurst exponents as a function of t for NSE nifty index for the window size of 25 days with errorbar ± 0.0270.

The value $f(\alpha) = 0$ is not reached in the case of the BSE, NSE and S&P 500 indices studied. This may be a characteristic feature of financial markets, as $f(\alpha) = 0$ is reached for other systems such as river runoff records [28].

We have carried out the cumulative time series analysis to compare the result reported in Fig. 11 of Ref. [25]. We study the cumulant Q(t) for the daily prices of the three different indices BSE, NSE and S&P 500 in Fig. 10. We find that the Q(t) shows a nonlinear behavior for the BSE and NSE indices and a linear behavior for the S&P 500 index for the period which we have studied. In Refs. [32,33] linear slopes are also found for NYSE index between January 1966 and December 1979. The initial slope of the BSE curve exactly matches with that of Ref. [25]. Here the data for the last two and half years considered was absent in their data bank and deviations are seen.

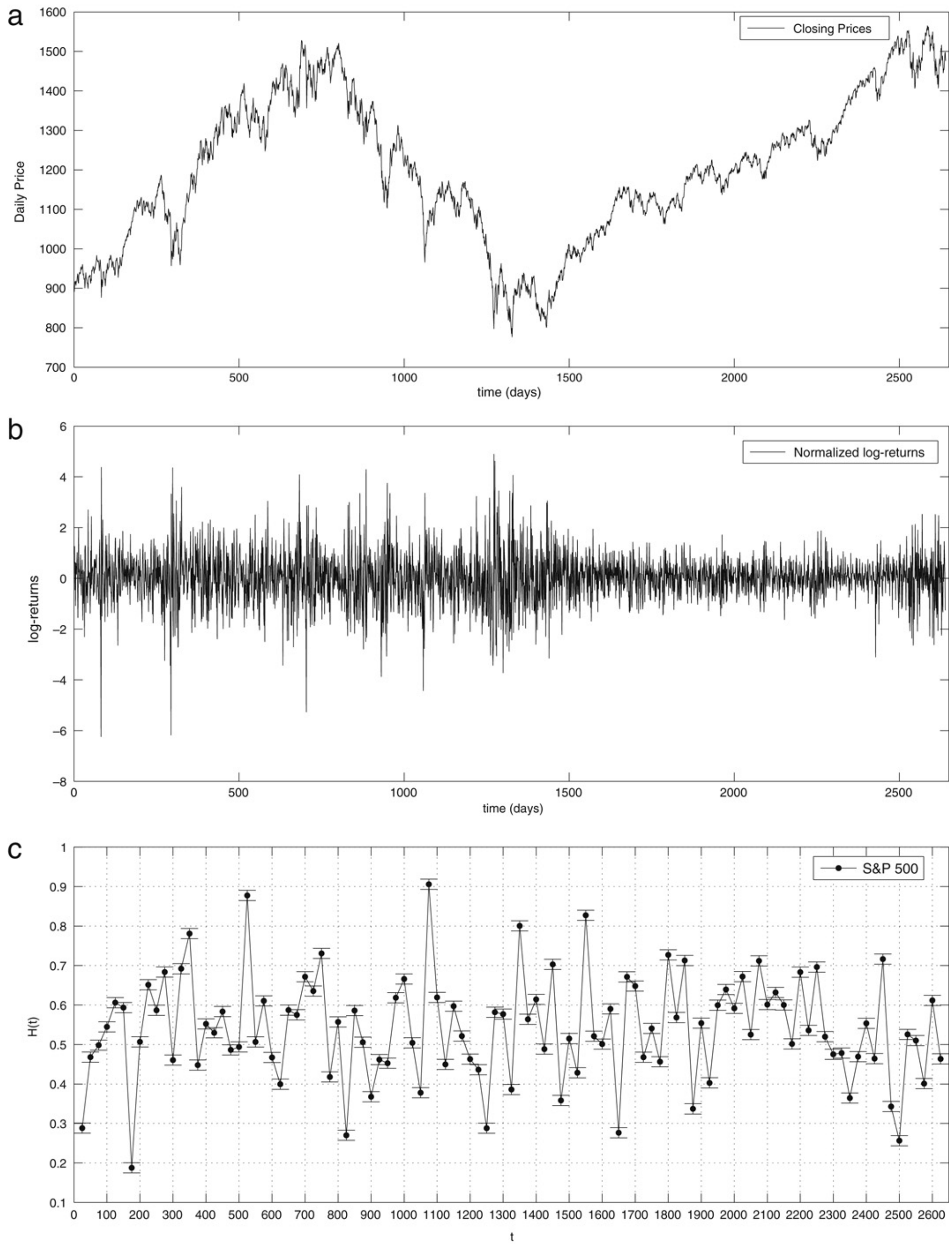

Fig. 3. (a) S&P 500 index for the period July, 1997 to December, 2007 and (b) Corresponding log-returns (c) The generalized Hurst exponents as a function of t for S&P 500 index for the time window of 25 days with errorbar ± 0.0129.

4. Conclusion

The MF-DFA method allows a reliable multifractal characterization of the Indian financial markets (BSE and NSE indices) and the US S&P 500 index. We find that the multifractal scaling exponents $h(q)$ and $\tau(q)$ have a nonlinear dependence on the moment q. On the basis of the nonlinearity of these scaling exponents and singularity spectrum $f(\alpha)$, we prove that the Indian and US markets both exhibit multifractality. By analyzing the results for the original and shuffled time series, we find that the multifractality in these financial markets is due to long-range correlations and broad probability density function. The above results are compared with the series generated by the BMFM. We find that the multifractal strengths for the financial markets are smaller than the BMFM.

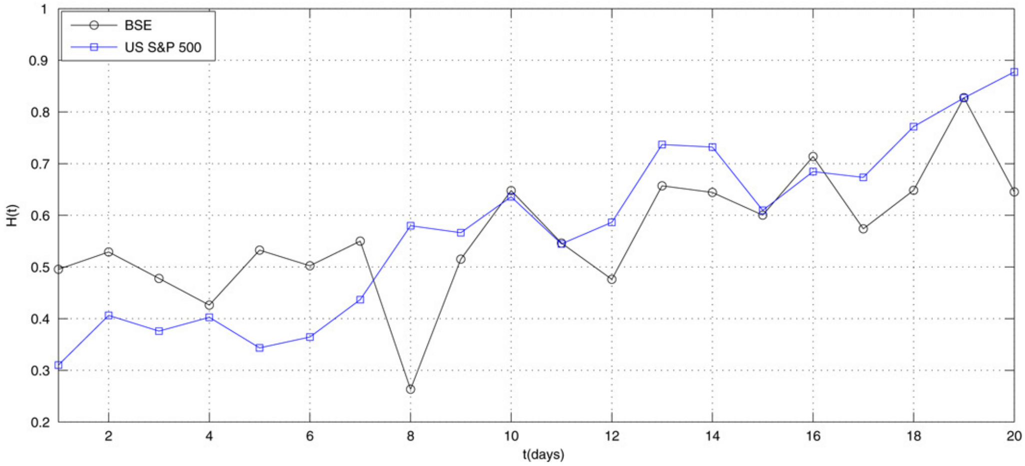

Fig. 4. The change of the Hurst exponent by moving window size of 25 days near the 9/11 crash for BSE (circle) and S&P 500 (square). The fall in the value of the Hurst exponent can be seen here for BSE.

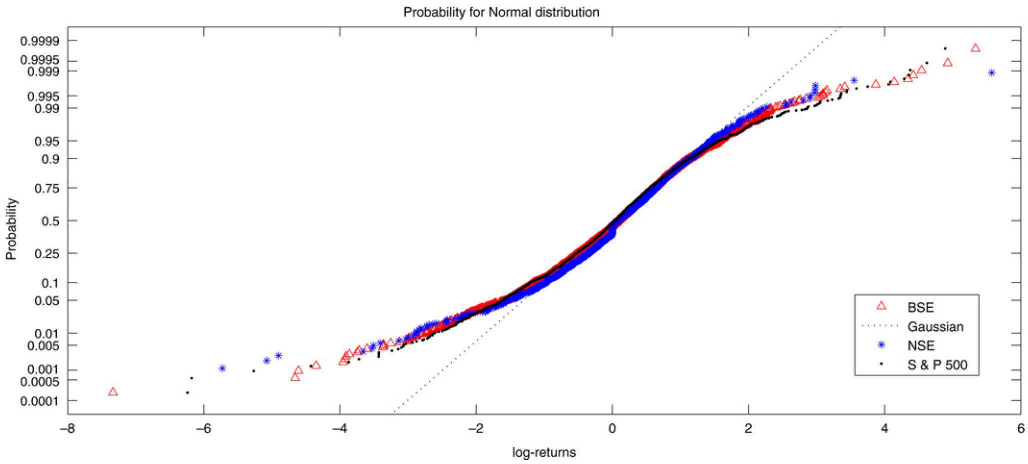

Fig. 5. Cumulative distribution functions of normalized log-returns of BSE (Δ—red), NSE ($*$—blue) and S&P 500 ($\cdot$) indices against the Gaussian cumulative distribution (dashed line).

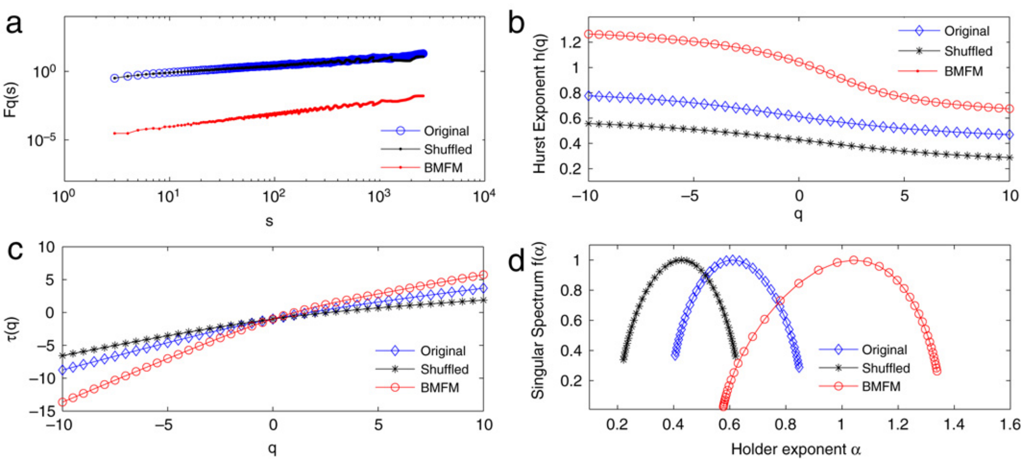

Fig. 6. (a) Log–log plot of the fluctuation function $F_{q=2}(s)$ versus s, (b) The q-dependence of generalized Hurst exponent $h(q)$ by the fit in the regime $3 < s < 2596$, (c) Multifractality shown by $\tau(q)$ versus q, and (d) Corresponding singularity spectrum $f(\alpha)$ versus α, for the BSE index (original and shuffled) and BMFM.

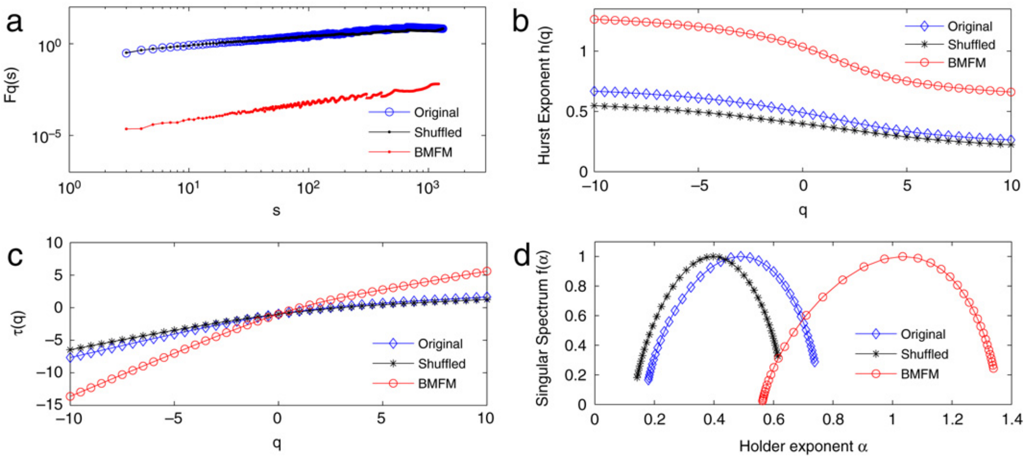

Fig. 7. (a) Log–log plot of the fluctuation function $F_{q=2}(s)$ versus s, (b) The q-dependence of generalized Hurst exponent $h(q)$ by the fits in the regime $3 < s < 1216$, (c) Multifractality shown by $\tau(q)$ versus q, and (d) Corresponding singularity spectrum $f(\alpha)$ versus α, for the NSE index (original and shuffled) and BMFM.

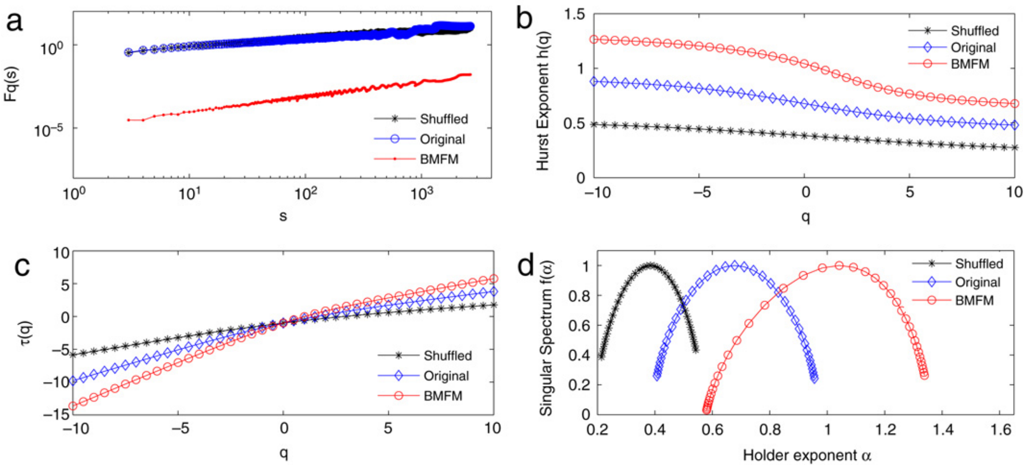

Fig. 8. (a) Log–log plot of the fluctuation function $F_{q=2}(s)$ versus s, (b) The q-dependence of generalized Hurst exponent $h(q)$ by the fit in the regime $3 < s < 2637$, (c) Multifractality shown by $\tau(q)$ versus q, and (d) Corresponding singularity spectrum $f(\alpha)$ versus α, for the S&P 500 index (original and shuffled) and BMFM.

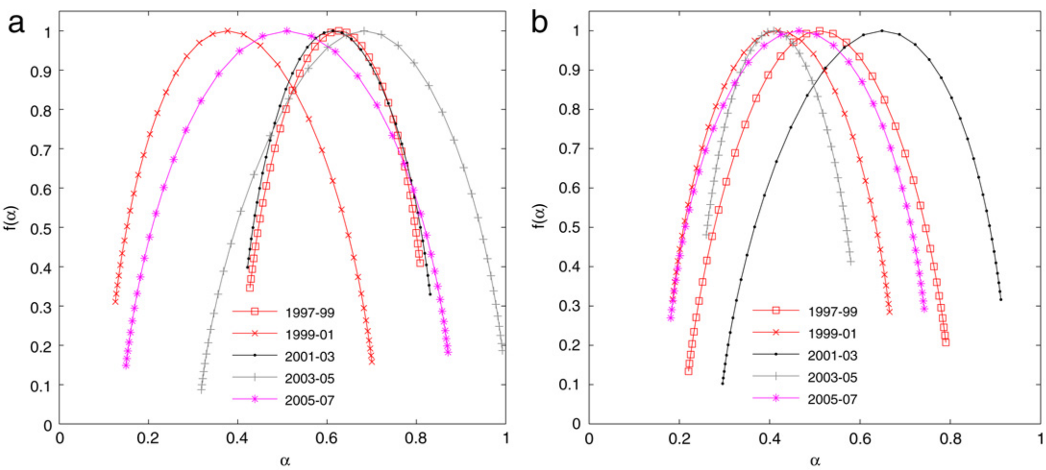

Fig. 9. The multifractal spectra for the two year windows are shown for the (a) BSE sensex, (b) S&P 500 index.

Table 1
Comparison of results for BSE and S&P 500 indices for the 10 year period.

Time window (years)	Hurst exponent		Volatility		$\Delta\alpha$	
	BSE	S&P 500	BSE	S&P 500	BSE	S&P 500
1997–1999	0.5908	0.3942	0.0141	0.0060	0.3815	0.5616
1999–2001	0.3310	0.3920	0.0146	0.0053	0.5744	0.3199
2001–2003	0.5744	0.5645	0.0091	0.0105	0.4091	0.6164
2003–2005	0.6005	0.3781	0.0098	0.0104	0.6744	0.4802
2005–2007	0.4173	0.4604	0.0115	0.0092	0.7210	0.5698
Average	0.5028	0.43784	0.01182	0.00828	0.55208	0.50958

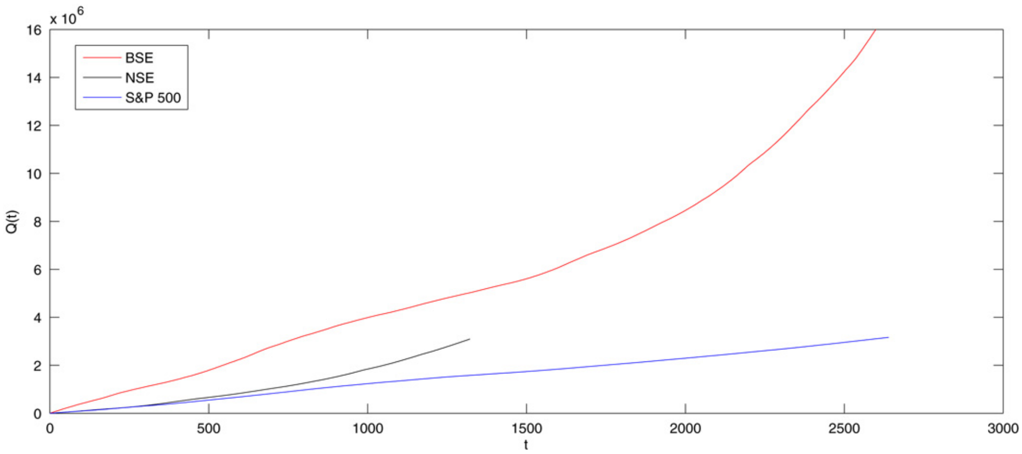

Fig. 10. Cumulative Q(t) of the time series shows a nonlinear behavior with time t for BSE and NSE indices whereas it shows a linear behavior for the S&P 500 index.

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