Disentangling shock diffusion on complex networks: identification through graph planarity

Abstract Summary:

The paper uses the concept of network theory and statistical modelling. It tries to analyse distress propagation from a given epicentre to the entire network in interconnected dynamical systems using multivariate time series data. The problems are threefold: one, behaviour across nodes can be interdependent making it difficult to disentangle cause and effects; two, all nodes might react to similar exogenous shocks; three, all nodes may possess very similar characteristics leading to correlated behaviour.The first step involves estimating a Vector Autoregression (VAR) model using observational data. The VAR estimation allows us to capture dependence across many different time series and hence, allows us to create a dependency network. But, it retains all possible signals of co-movements of all pairs of time series.The second step introduces two network filtering techniques: Planar Maximally Filtered Graphs (PMFG) and Partially Correlation Planar Graphs (PCPG). These filters are based on planar graphs, which are graphs that can be embedded in the plane without edge crossings. With the help of the filtered networks, we can retain all the informative edges, that is, the ones that represent most important co-movements.

Key Words:

Reflection Problem: refers to a situation where network traffic or data packets are inadvertently or intentionally reflected back to the source, creating a loop or feedback loop. This can lead to various issues in network communication, including congestion, performance degradation, and potential security concerns
Cascading Failure:refers to a situation where a failure or disruption in one part of the system triggers a sequence of additional failures or disruptions in other interconnected components.
Business cycle:also known as the economic cycle, refers to the recurring pattern of economic expansion and contraction, typically characterised by periods of growth (boom) and contraction (recession).
MST: Minimum Spanning Tree is a concept in graph theory and network analysis. It is a subset of the edges of a connected, undirected graph that connects all the vertices together with the minimum possible total edge weight.

Mathematical Models:

1.Vector Autoregressive Model:The vector autoregressive model (VAR(p)) model represents evolution of N-dimensional time series xt as a function of their lagged values of the order $1 \leq ... \leq p$ along with cross-dependence and a vector of error terms

- **Matrix form of VAR:**

$$\boldsymbol{x}_t = A_1\boldsymbol{x}_{t-1} + \ldots + A_p\boldsymbol{x}_{t-p} + \boldsymbol{u}_t,$$

where xt = (x1t, ... , xNt) denote N-dimensional time series, Ak denotes (N × N) coefficient matrix with elements ak ij where k corresponds to the lag order, i, j are row and column index respectively. ut is the vector of error terms.

- **Wold Moving Average Form:**

$$\boldsymbol{x}_t = \Phi_0\boldsymbol{u}_t + \Phi_1\boldsymbol{u}_{t-1} + \Phi_2\boldsymbol{u}_{t-2} + \ldots$$

Phi_s are matrices that can be calculated recursively :

$$\Phi_s = \sum_{j=1}^{s} \Phi_{s-j}A_j \text{ for } s = 1, 2, \ldots$$

- **Orthogonalization of shocks:**

Shocks ut are orthogonalized using the Cholesky decomposition of the covariance matrix:
u=P'P
Where P is a lower triangular matrix.

2.**Structural VAR:**
models put additional restrictions in the form of A and B matrix on the reduced form VAR model described above to uniquely identify the propagation of shocks:

$$A\boldsymbol{x}_t = A_1\boldsymbol{x}_{t-1} + \cdots + A_p\boldsymbol{x}_{t-p} + B\boldsymbol{\varepsilon}_t.$$

PMFG Construction:
PMFG retains loops and cliques of three and four nodes in the network. It uses Pearson correlation (Cij) as the adjacency matrix for the complete graph. Pearson correlation is calculated using the following formula:

$$(C_{ij} = [E(x_i.x_j) - E(x_i)E(x_j)]/\sigma_i\sigma_j,$$

Where:
Cij is the Pearson correlation between variables
Xi and xj .E(xi) and E(xj) are the means of variables xi and xj.σi and σj are the standard deviations of variables xi and xj. The adjacency matrix of the PMFG graph is symmetric by construction.

3.Partial Correlation Calculation:
The partial correlation between two variables, i and j, is calculated using the following formula:

$$PC_{ij|k} = \frac{C_{ij} - C_{ik}C_{jk}}{\sqrt{1 - C_{ik}^2}\sqrt{1 - C_{jk}^2}}.$$

Where:PCij∣k is the partial correlation between variables i and j while controlling for variable k.Cik and Cjk are the correlations of variables i and j with variable k.
The PCPG is constructed by calculating partial correlations to measure the influence of one variable on another while controlling for all other variables. This results in a directed, weighted, and planar graph that retains asymmetric influences.

4.numerical optimization of the log-likelihood function:
The parameters of the SVAR model are estimated by minimizing the negative of the log-likelihood function, denoted as log L(B). The negative log-likelihood function is defined as follows:

$$log\ \mathscr{L}(B) = -\frac{NT}{2}\ln(2\pi) - \frac{T}{2}\ln|B|^2 - \frac{T}{2}tr(B^{-1^\top}B^{-1}\tilde{\Sigma}_u),$$

where N and T denote the number and length of time series, tr(.) denotes trace of matrix, ˜ u is the estimated reduced form covariance matrix of the error term.The likelihood function is nonlinear, and the parameter space is high-dimensional, as the number of free parameters in the N-dimensional VAR framework is O(N^2).

Numerical Optimization Algorithms:

- Nelder-Mead: A simplex-based method that is robust but can be slow. It shows tendency of inappropriate termination [38] even before reaching a local minima.
- Broyden-Fletcher-Goldfarb-Shanno (BFGS): A quasi-Newton method that performs well for smooth convex functions.

For each run, 30 iterations of the optimization algorithm are performed. The starting values for each iteration are the estimated parameters from the previous iteration. Beyond 30 iterations, the improvement in likelihood is negligible.
Results from the optimization runs show that the BFGS method outperforms Nelder-Mead in terms of achieving lower negative log-likelihood values. However, there is a trade-off, as BFGS is non-robust and can get stuck in local optima if the starting point is not chosen carefully.

Interpretation of table1:

Table 1 shows the information about the data used for gdp and stock return volatility return analysis. The dataset shows that from where the data has been taken. Frequency tells us about how frequent /often the data has been recorded. GDP has been reported quarterly (1st quarter of 1980 to 4th quarter of 2018) whereas Stock return volatility has been reported monthly(January 2000 to December2018). The column countries list the countries which are in G-20 often considered as major economies worldwide.

Shock spillover on business cycle network:
GDP data contain both trend and cyclical components. So we decompose the data into two parts.For decomposition we use Hodrick–Prescott filter.The focus is on extracting the cyclical component. We are using the cycle network to analyse the propagation of economic shock. The estimated impulse response function allow us to analyse shock propagation using both the filtration methods.The shock spillover effects are more prominent in developed countries and less so in developing countries like India, Indonesia, and South Korea. However, the impact is more significant for developing countries geographically close to the USA, such as Mexico and Argentina.

Shock spillover on stock index network:
The major focus is to analyse the volatility shock spillover in the global financial market by studying the stock market of G-20 countries.To assess volatility, latent volatility series is extracted using a GARCH(1,1) model. GARCH models are commonly used to estimate latent volatility in financial return series.

$$r_t = \sigma_t \epsilon_t,$$
$$\sigma_t^2 = c + \sum_{i=1}^{p} \alpha_i r_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2,$$

where rt is the return series and σt is latent volatility series which is unobservable in real data
The PCPG-SVAR estimation shows direct spillover to other developed countries, including the UK, France, and Germany. There is also some influence on countries like Argentina, Japan, and China. After four time-points (equivalent to 4 months), the shock permeates and becomes muted in the affected countries. countries like Indian South Korea, South Africa, Turkey were barely affected and show very muted response overall.

Comparison between PCPG-SVAR and PMFG-SVAR

The methods are evaluated in three dimensions- applicability, parsimony and uniqueness.
Applicability: Applicability refers to the suitability of the algorithms for general networks. The empirical results demonstrate that PMFG provides sufficient restrictions for SVAR model identification only when the number of nodes (N) is either very small (N ≤ 2) or relatively large (N ≥ 11). In contrast, PCPG offers sufficient restrictions for VAR identification for all integer values of N greater than or equal to 4. Thus, PCPG-SVAR is generally more applicable and less restrictive.
Parsimony: In this context, parsimony means providing an effective level of explanation with fewer estimated parameters. PCPG-SVAR imposes more restrictions (N(N − 1)/2 − 3(N − 2)) with fewer free parameters (3(N−2)), while PMFG-SVAR imposes fewer restrictions (N2−2×3(N−2)) with twice the number of free parameters (2 × 3(N − 2)). As a result, PCPG-SVAR is more parsimonious, which is advantageous in VAR framework applications, particularly in overcoming the curse of dimensionality.
Uniqueness: Uniqueness implies that the estimation method should lead to unique and interpretable results. Empirical exercises indicate that PMFG-SVAR estimation results in a flat likelihood surface with multiple local minima. In contrast, numerical results demonstrate that PCPG-SVAR provides a smooth likelihood surface with enough curvature for effective optimization.
PCPG, as a directed network, captures asymmetric interactions between nodes. Thus, PCPG-SVAR strikes a balance between MST and PMFG by retaining crucial network properties and providing sufficient degrees of freedom for SVAR model estimation.

Conclusion:

The application of network theory often relies on credible models to understand how shocks at the micro-level impact the macro-level network response. The article introduces an algorithm that combines network filtering techniques with econometric estimation of dynamic models to capture this relationship.e an algorithm by combining tools from network filtering, along with econometric estimation of dynamic models to capture the same link between micro-level shock and macro-level repercussions has been provided.The article proposes a Structural Vector Autoregressive (SVAR) model with identification criteria derived from the topological properties of co-movement networks of dynamic variables.The algorithm is not limited to economic and financial networks; it can be applied to any multivariate time series data with a non-trivial correlation structure. It is particularly suited for high-frequency financial data, such as estimating volatility spillover within a stock market.