# Econometric measures of connectedness and systemic risk in the finance and insurance sectors

## Abstract

1. Econometric measures: Quantitative measures used in econometric analysis to study the relationship between economic variables.
2. Principal components analysis: A statistical technique used to reduce the dimensionality of a dataset by transforming the variables into a new set of uncorrelated variables called principal components.
3. Granger-causality networks: A method used to analyze the causal relationship between time series variables, where one variable is said to Granger-cause another if it provides useful information in predicting the future values of the other variable.
4. Systemic risk: The risk of a widespread disruption or collapse of a financial system, where the failure of one institution can have a domino effect on other interconnected institutions.
5. Hedge funds: Investment funds that pool capital from accredited individuals or institutional investors and use various strategies to generate high returns.
6. Banks: Financial institutions that accept deposits from the public and provide loans and other financial services.
7. Broker/dealers: Financial firms that facilitate the buying and selling of securities on behalf of clients.
8. Insurance companies: Institutions that provide insurance coverage and risk management services to individuals and businesses.


## Mathematical Formulas

### PCAS

Defining the total risk of the system as $\Omega \equiv \sum_{k=1}^{N} \lambda_k$ and the risk associated with the first $n$ principal components as $\omega_n \equiv \sum_{k=1}^{n} \lambda_k$, we compare the ratio of the two (i.e., the Cumulative Risk Fraction) to the prespecified critical threshold level $H$ to capture periods of increased interconnectedness:

$$\frac{\omega_n}{\Omega} \equiv h_n \geq H. \tag{6}$$

When the system is highly interconnected, a small number $n$ of $N$ principal components can explain most of the volatility in the system, hence, $h_n$ will exceed the threshold $H$. By examining the time variation in the magnitudes of $h_n$, we are able to detect increasing correlation among institutions, i.e., increased linkages and integration as well as similarities in risk exposures, which can contribute to systemic risk.

The contribution $\text{PCAS}_{i,n}$ of institution $i$ to the risk of the system – conditional on a strong common component across the returns of all financial institutions ($h_n \geq H$) – is a univariate measure of connectedness for each company $i$, i.e.:

$$\text{PCAS}_{i,n} = \frac{1}{2} \frac{\sigma_i^2}{\sigma_S^2} \frac{\partial \sigma_S^2}{\partial \sigma_i^2} \bigg|_{h_n \geq H}. \tag{7}$$

It is easy to show that this measure also corresponds to the exposure of institution $i$ to the total risk of the system, measured as the weighted average of the square of the factor loadings of the single institution $i$ to the first $n$ principal components, where the weights are simply the eigenvalues. In fact:

$$\text{PCAS}_{i,n} = \frac{1}{2} \frac{\sigma_i^2}{\sigma_S^2} \frac{\partial \sigma_S^2}{\partial \sigma_i^2} \bigg|_{h_n \geq H} = \sum_{k=1}^{n} \frac{\sigma_i^2}{\sigma_S^2} L_{ik}^2 \lambda_k \bigg|_{h_n \geq H}. \tag{8}$$

Intuitively, since we are focusing on endogenous risk, this is both the contribution and the exposure of the $i$-th institution to the overall risk of the system given a strong common component across the returns of all institutions.

# Linear Granger Causality

values of $i$ alone. The mathematical formulation of this test is based on linear regressions of $R_{t+1}^i$ on $R_t^i$ and $R_t^j$.

Specifically, let $R_t^i$ and $R_t^j$ be two stationary time series, and for simplicity assume they have zero mean. We can represent their linear inter-relationships with the following model:

$$R_{t+1}^i = a^i R_t^i + b^{ij} R_t^j + e_{t+1}^i,$$

$$R_{t+1}^j = a^j R_t^j + b^{ji} R_t^i + e_{t+1}^j, \tag{9}$$

where $e_{t+1}^i$ and $e_{t+1}^j$ are two uncorrelated white noise processes, and $a^i, a^j, b^{ij}, b^{ji}$ are coefficients of the model.

We consider a Generalized AutoRegressive Conditional Heteroskedasticity (GARCH)(1,1) baseline model of returns:

$$R_t^i = \mu_i + \sigma_{it} \epsilon_t^i, \quad \epsilon_t^i \sim \text{WN}(0, 1),$$

$$\sigma_{it}^2 = \omega_i + \alpha_i (R_{t-1}^i - \mu_i)^2 + \beta_i \sigma_{it-1}^2 \tag{10}$$

1. *Degree of Granger causality*. Denote by the *degree of Granger causality* (DGC) the fraction of statistically significant Granger-causality relationships among all $N(N-1)$ pairs of $N$ financial institutions:

$$\text{DGC} \equiv \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j \neq i} (j \to i). \tag{14}$$

   The risk of a systemic event is high when DGC exceeds a threshold $K$ which is well above normal sampling variation as determined by our Monte Carlo simulation procedure (see Appendix B).
2. *Number of connections*. To assess the systemic importance of single institutions, we define the following simple counting measures, where $S$ represents the system:

$$\#\text{Out} : (j \to S)\big|_{\text{DGC} \geq K} = \frac{1}{N-1} \sum_{i \neq j} (j \to i)\big|_{\text{DGC} \geq K},$$

$$\#\text{In} : (S \to j)\big|_{\text{DGC} \geq K} = \frac{1}{N-1} \sum_{i \neq j} (i \to j)\big|_{\text{DGC} \geq K},$$

$$\#\text{In+Out} : (j \longleftrightarrow S)\big|_{\text{DGC} \geq K} = \frac{1}{2(N-1)} \sum_{i \neq j} (i \to j) + (j \to i)\big|_{\text{DGC} \geq K}. \tag{15}$$

   *#Out* measures the number of financial institutions that are significantly Granger-caused by institution $j$, *#In* measures the number of financial institutions that significantly Granger-cause institution $j$, and *#In+Out* is the sum of these two measures.
3. *Sector-conditional connections*. Sector-conditional connections are similar to (15), but they condition on the

type of financial institution. Given $M$ types (four in our case: banks, broker/dealers, insurers, and hedge funds), indexed by $\alpha,\beta = 1,\ldots,M$, we have the following three measures:

$$\#\text{Out-to-Other} : \left( (j|\alpha) \rightarrow \sum_{\beta \neq \alpha} (S|\beta) \right) \Bigg|_{\text{DGC} \geq K} = \frac{1}{(M-1)N/M} \sum_{\beta \neq \alpha} \sum_{i \neq j} ((j|\alpha) \rightarrow (i|\beta)) \Bigg|_{\text{DGC} \geq K}, \tag{16}$$

$$\#\text{In-from-Other} : \left( \sum_{\beta \neq \alpha} (S|\beta) \rightarrow (j|\alpha) \right) \Bigg|_{\text{DGC} \geq K} = \frac{1}{(M-1)N/M} \sum_{\beta \neq \alpha} \sum_{i \neq j} ((i|\beta) \rightarrow (j|\alpha)) \Bigg|_{\text{DGC} \geq K}, \tag{17}$$

$$\#\text{In} + \text{Out} - \text{Other} : \left( (j|\alpha) \longleftrightarrow \sum_{\beta \neq \alpha} (S|\beta) \right) \Bigg|_{\text{DGC} \geq K} = \frac{\sum_{\beta \neq \alpha} \sum_{i \neq j} ((i|\beta) \rightarrow (j|\alpha)) + ((j|\alpha) \rightarrow (i|\beta)) \big|_{\text{DGC} \geq K}}{2(M-1)N/M}, \tag{18}$$

4. *Closeness.* Closeness measures the shortest path between a financial institution and all other institutions reachable from it, averaged across all other financial institutions. To construct this measure, we first define $j$ as weakly causally $C$-connected to $i$ if there exists a causality path of length $C$ between $i$ and $j$, i.e., there exists a sequence of nodes $k_1,\ldots,k_{C-1}$ such that:

$$(j \rightarrow k_1) \times (k_1 \rightarrow k_2) \cdots \times (k_{C-1} \rightarrow i) \equiv (j \xrightarrow{C} i) = 1. \tag{19}$$

Denote by $C_{ji}$ the length of the shortest $C$-connection between $j$ to $i$:

$$C_{ji} \equiv \min_{C} \{ C \in [1,N-1] : (j \xrightarrow{C} i) = 1 \}, \tag{20}$$

where we set $C_{ji} = N-1$ if $(j \xrightarrow{C} i) = 0$ for all $C \in [1,N-1]$. The closeness measure for institution $j$ is then defined as

$$C_{jS}|_{\text{DGC} \geq K} = \frac{1}{N-1} \sum_{i \neq j} C_{ji} (j \xrightarrow{C} i)|_{\text{DGC} \geq K}. \tag{21}$$

5. *Eigenvector centrality*. The *eigenvector centrality* measures the importance of a financial institution in a network by assigning relative scores to financial institutions based on how connected they are to the rest of the network. First, define the adjacency matrix $A$ as the matrix with elements:

$$[A]_{ji} = (j \rightarrow i). \tag{22}$$

The eigenvector centrality measure is the eigenvector $v$ of the adjacency matrix associated with eigenvalue

1. Granger non-causality from $Z_{h,t}$ to $Z_{b,t}$ ($Z_{h,t} \nRightarrow Z_{b,t}$):
   Decompose the joint probability:

$$P(Z_{h,t},Z_{b,t}|Z_{h,t-1},Z_{b,t-1}) = P(Z_{h,t}|Z_{b,t},Z_{h,t-1},Z_{b,t-1}) \times P(Z_{b,t}|Z_{h,t-1},Z_{b,t-1}). \quad (28)$$

   If $Z_{h,t} \nRightarrow Z_{b,t}$, the last term becomes

$$P(Z_{b,t}|Z_{h,t-1},Z_{b,t-1}) = P(Z_{b,t}|Z_{b,t-1}). \quad (29)$$

2. Granger non-causality from $Z_{b,t}$ to $Z_{h,t}$ ($Z_{b,t} \nRightarrow Z_{h,t}$):
   This requires that if $Z_{b,t} \nRightarrow Z_{h,t}$, then

$$P(Z_{h,t}|Z_{h,t-1},Z_{b,t-1}) = P(Z_{h,t}|Z_{h,t-1}). \quad (30)$$

## Interpretation of Data Tables

### Table 1:

Summary statistics for monthly returns of individual hedge funds, broker/dealers, banks, and insurers for the full sample: January

1994 to December 2008, and five time periods: 1994–1996, 1996–1998, 1999–2001, 2002–2004, and 2006–2008. The annualized mean, annualized

standard deviation, minimum, maximum, median, skewness, kurtosis, and first-order autocorrelation are reported.

### Table 2:

Summary statistics for PCAS measures. Mean, minimum, and maximum values for PCAS 1, PCAS 1–10, and PCAS 1–20. These measures are based on the

monthly returns of individual hedge funds, broker/dealers, banks, and insurers for the five time periods: 1994–1996, 1996–1998, 1999–2001, 2002–2004,

and 2006–2008. Cumulative Risk Fraction (i.e., eigenvalues) is calculated for PC 1, PC 1–10, and PC 1–20 for all five time periods.

### Table 3:

Summary statistics of asset-weighted autocorrelations and linear Granger-causality relationships (at the 5% level of statistical significance) among the

monthly returns of the largest 25 banks, broker/dealers, insurers, and hedge funds (as determined by average AUM for hedge funds and average market

capitalization for broker/dealers, insurers, and banks during the time period considered) for five sample periods: 1994–1996, 1996–1998, 1999–2001, 2002–2004, and 2006–2008. The normalized number of connections, and the total number of connections for all financial institutions, hedge funds,

broker/dealers, banks, and insurers are calculated for each sample including autoregressive terms and filtering out heteroskedasticity with a

GARCH(1,1) model.

## Table 4:

P-values of nonlinear Granger-causality likelihood ratio tests. P-values of nonlinear Granger-causality likelihood ratio tests for the monthly residual returns indexes (from a market-model regression against S&P 500 returns) of Banks, Brokers, Insurers, and Hedge funds for two subsamples: January 1994 to December 2000, and January 2001 to December 2008. Statistics that are significant at the 5% level are shown in bold.

## Table 5:

Predictive power of PCAS measures. Regression coefficients, t-statistics,

p-values, and Kendall (1938) t rank-correlation coefficients for regressions of Max%Loss on PCAS 1, PCAS 1–10, and PCAS 1–20. The maximum

percentage loss (Max%Loss) for a financial institution is the dollar amount of the maximum cumulative decline in market capitalization or fund size for each financial institution during July 2007–December 2008 divided by the market capitalization or total fund size of the institution at the end of June 2007.

## Table 6:

Predictive power of Granger-causality-network measures. Regression coefficients, t-statistics, p-values, and Kendall (1938) t rank-correlation

coefficients for regressions of Max%Loss on Granger-causality-network measures. The maximum percentage loss (Max%Loss) for a financial institution is the dollar amount of the maximum cumulative decline in market capitalization or fund size for each financial institution during July 2007–December 2008 divided by the market capitalization or total fund size of the institution at the end of June 2007.

## Table 7:

Out-of-sample analysis. Parameter estimates of a multivariate rank regression of Max% Loss for each financial institution during July 2007–December 2008 on PCAS 1, leverage, size, first-order autocorrelation, and Granger-causality-network measures. The maximum percentage loss (Max% Loss) for a financial institution is the dollar amount of the maximum cumulative decline in market capitalization or fund size for each financial institution during July 2007–December 2008 divided by the market capitalization or total fund size of the institution at the end of June 2007. PCAS 1, leverage, size, first-order autocorrelation, and Granger-causality-network measures are calculated over October 2002–September 2005 and July 2004–June 2007.

## Conclusion

1. The financial system has become more complex and interconnected over the past two decades due to financial innovation and deregulation, leading to greater interdependence among different sectors of the finance and insurance industries .
2. The paper proposes econometric measures of connectedness based on principal components analysis and Granger-causality networks to quantify this interdependence .
3. These measures are capable of identifying periods of market dislocation and distress, and have promising out-of-sample characteristics .
4. The empirical results suggest that the banking and insurance sectors may be more important sources of connectedness than other parts, which aligns with the anecdotal evidence from the recent financial crisis .
5. The illiquidity of bank and insurance assets, coupled with their vulnerability to rapid and large losses, makes these sectors natural repositories for systemic risk .

6. The paper emphasizes the importance of developing methods for measuring, monitoring, and anticipating financial crises to mitigate their disruptive consequences .
7. By using a broad array of tools to gauge the topology of the financial network, there is a better chance of identifying potential risks and vulnerabilities.