MBA 547 Case Report, Homework 5

Topic: Exchange Rates

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Exchange Rates

Executive Summary

Over the period 2018 to 2022, this paper analyses the behavior of two main currency exchange rates - EUR/USD (Euro to US Dollar) and JPY/USD (Japanese Yen to US Dollar). The study uses several statistical techniques to investigate the links between these currencies and how world events affected their changes. We evaluated the dynamics between EUR/USD and JPY/USD by means of descriptive statistics, autocorrelation testing, cointegration analysis, VEC & VAR modeling, and volatility modeling (ARCH/GARCH) using data derived from Yahoo Finance.

With a standard deviation of 0.06, the EUR/USD exchange rate averaged 1.14 in the descriptive study; it varied between 0.96 and 1.25 across the time. The average rate for JPY/USD was 0.01, with a relatively low standard deviation of 0.001, so showing a consistent exchange rate. These figures draw attention to the variations in volatility between the two currencies.

Further testing showed no appreciable autocorrelation in both EUR/USD (DW = 2.0617, p-value = 0.861) and JPY/USD (DW = 2.0529, p-value = 0.8234), therefore past values had no appreciable bearing on future movements. The Johansen cointegration test revealed that the test statistics (0.86 for $r \leq 1$ and 7.59 for $r = 0$) were below the threshold values, therefore indicating no strong long-term link between the exchange rates. This shows that there is not a consistent, long-term equilibrium between the two currencies.

With little effect from lagged values of EUR/USD and JPY/USD on each other, the VECM analysis found poor short-term connections. Having alpha1 = 0.03895 and beta1 = 0.95695, the GARCH model for EUR/USD showed that there was significant volatility clustering. This means that times of high volatility are likely to be followed by more times of high volatility. This finding suggests that the exchange rate stays volatile over time. In the same way, the GARCH model for JPY/USD showed alpha1 = 0.08687 and beta1 = 0.89265, which confirmed that this currency pair also stays volatile. Since EUR/USD and

JPY/USD do not have strong long-term cointegration, our research suggests that traders use VAR models to make short-term predictions. Also, because volatility stays the same over time, risk management techniques that use GARCH models are suggested, especially when market volatility is high. More research should be done on macroeconomic issues to get a better idea of why currencies change value.

Introduction

Using data from reliable sources such as Yahoo Finance, this paper concentrates on the daily exchange rates of two important currency pairs: EUR/USD and JPY/USD, spanning 2018 to 2022. Analyzing the dynamic behavior of these pairs of currencies and investigating their associated over-time changes is the aim.

We used numerous analytical approaches to get at this. Using the Durbin-Watson test, which aids in the identification of whether the exchange rate data exhibits a pattern of correlation over time, we first tested for autocorrelation. We then looked for a long-term link between the two currencies using cointegration tests. We investigated short-term relationships using impulse response functions to investigate how shocks in one currency impact the other. We also constructed Vector Autoregressive Models (VAR) and Vector Error Correction Models (VECM) to probe the dynamics and link between the exchange rates. To assess the volatility and changes in the exchange rates at last, we applied ARCH/GARCH models. By means of this method, we hope to clarify the behavior of these currency pairs and the elements influencing their movements in the world market.

Data

Here, we look at the trend and fit of two exchange rates - EUR/USD and JPY/USD. We have used historical data from Yahoo Finance between January 2018 and January 2023. First, let us consider the descriptive statistics and then take a look at the trends of these two exchange rates over time.

Descriptive Statistics for Exchange Rates

Statistic	N	Mean	St. Dev.	Min	Max
Mean_EURUSD	1	1.14		1.14	1.14
SD_EURUSD	1	0.06		0.06	0.06
Min_EURUSD	1	0.96		0.96	0.96
Max_EURUSD	1	1.25		1.25	1.25
Mean_JPYUSD	1	0.01		0.01	0.01
SD_JPYUSD	1	0.001		0.001	0.001
Min_JPYUSD	1	0.01		0.01	0.01
Max_JPYUSD	1	0.01		0.01	0.01

The mean for the EUR/USD exchange rate is 1.14, meaning that on average, the value of EUR with respect to the USD is 1.14. The standard deviation is 0.06, showing moderate variation around this mean, with a minimum value of 0.96 and a maximum of 1.25. Such a range suggests that over the observation period, there has been some volatility in the EUR/USD exchange rate; the changes are relatively modest considering the mean value.

For the exchange rate for JPY/USD, it is 0.01. That is, on average, 1 USD is about 0.01 JPY. An extremely small standard deviation in this case, of 0.001, suggests that the value of JPY/USD has had very little fluctuation. As a matter of fact, the minimum and maximum values are the same at 0.01, which suggests that during the observation period, the exchange rate for JPY/USD was almost completely stable.

Graphical Illustration

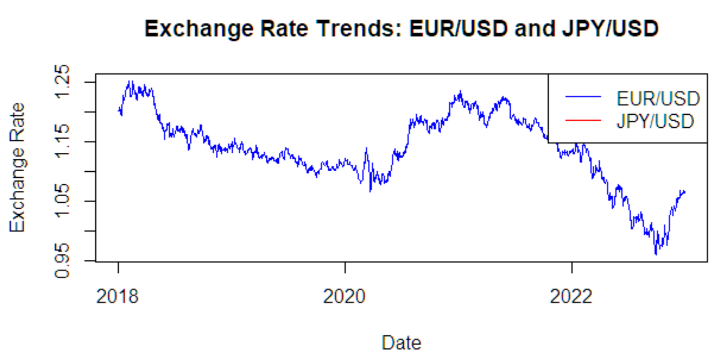

Fig 1.1

We can see in Fig. 1.1 how the Euro (EUR/USD) and the Japanese Yen (JPY/USD) changed value from 2018 to 2022. The Euro was pretty steady at first, but it dropped sharply in 2022. This was probably because of problems in Europe's economy and the US Dollar getting stronger as interest rates rose. The Yen went down in value in the same way, especially in the later years when Japan's low interest rates and events around the world made the Dollar more appealing.

These changes show how things happening around the world, like inflation and political unrest, affect the value of currencies. The big drops in 2022 stand out because they show how uncertain and linked the world's economies can be.

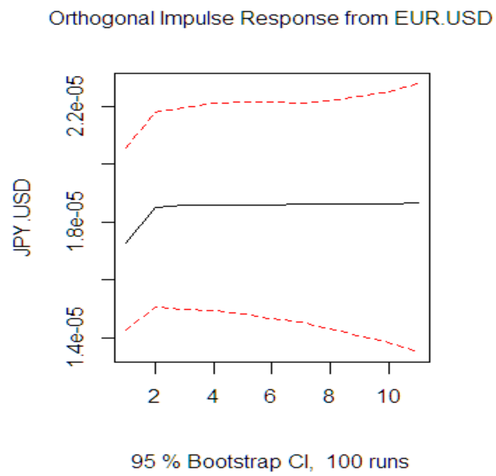

Fig 1.2

A quick change in the EUR/USD exchange rate (Euro to US Dollar) has an effect on the JPY/USD exchange rate (Japanese Yen to US Dollar) over time, as shown in Fig. 1.2. The change is shown by the black line, and the red dashed lines show the spread of possible outcomes that are 95% sure to happen. At first, the change has a clear effect on the Yen. However, this effect stops changing over time. The red lines show that it's not clear how strong the effect might be, but the reaction is generally stable within a certain range.

Analysis

1. Test for Autocorrelation – Durbin Watson test

```
# Test for autocorrelation
> dwtest(EURUSD ~ lag(EURUSD, 1), data = exchange_rates)  # For EUR/USD
autocorrelation

 Durbin-Watson test

data:  EURUSD ~ lag(EURUSD, 1)
DW = 2.0617, p-value = 0.8617
alternative hypothesis: true autocorrelation is greater than 0

> dwtest(JPYUSD ~ lag(JPYUSD, 1), data = exchange_rates)  # For JPY/USD
autocorrelation

 Durbin-Watson test

data:  JPYUSD ~ lag(JPYUSD, 1)
DW = 2.0529, p-value = 0.8234
alternative hypothesis: true autocorrelation is greater than 0
```

The Durbin-Watson (DW) test results for both EUR/USD and JPY/USD exchange rates show values close to 2. Specifically, the DW statistic for EUR/USD is 2.0617 and for JPY/USD, it is 2.0529, both of which indicate no significant autocorrelation in the data. The p-values for both tests are 0.8617 for EUR/USD and 0.8234 for JPY/USD, which are well above the common threshold of 0.05. This means that we fail to reject the null hypothesis, suggesting there is no significant autocorrelation in either currency pair's time series data. The past values of the exchange rates do not appear to influence future values in a predictable way.

2. Test for Cointegration – Johansen Procedure

	Test Statistic	10pct	5pct	1pct
r <= 1	0.86	7.52	9.24	12.97
r = 0	7.59	17.85	19.96	24.6

Table 1.1

We did a Johansen Cointegration test on the EUR/USD and JPY/USD exchange rates. The results show that neither of the two hypotheses for cointegration ($r \leq 1$ and $r = 0$) is a strong indicator of a long-term relationship. There is a test statistic for $r \leq 1$ that is less than the important numbers at 10%, 5%, and 1% (7.52, 9.24, and 12.97, respectively). It is 0.86. Also, the test result of 7.59 for $r = 0$ is also less than the critical values of 17.85, 19.96, and 24. The results show that the two exchange rates are not significantly cointegrated, which means they do not have a long-term equilibrum relationship.

	EUR.USD.l1	JPY.USD.l1	constant
EUR.USD.l1	1	1	1
JPY.USD.l1	-64.218478	-155.4659164	-2251.97
constant	-0.561195	-0.02010427	19.21818

Table 1.2

This table shows how the EUR/USD and JPY/USD exchange rates have changed over time. The numbers in the first row, EUR.USD.l1, are set to 1 so that they can be used as a standard.

The large negative coefficient (-64.218478) in the second row, JPY.USD.l1, shows that changes in JPY/USD are strongly and negatively linked to changes in EUR/USD over the long term. In other words, when JPY/USD goes up, EUR/USD usually goes down a lot.

With a big positive value (19.21818), the constant term shows an adjustment that needs to be made to keep this relationship in balance. Over time, the table shows how the two exchange rates are linked and what changes need to be made to keep them balanced.

	EUR.USD.l1	**JPY.USD.l1**	**constant**
EUR.USD.d	-0.008563	0.000211529	1.02655E-18
JPY.USD.d	3.44027E-06	3.95937E-06	9.73282E-21

Table 1.3

In Table 1.3, we can see how long-term links affect changes in the EUR/USD and JPY/USD rates over the short term. For EUR/USD, past EUR/USD prices have a small negative effect. For JPY/USD, however, they almost never have an effect. For JPY/USD, the past prices of both EUR/USD and JPY/USD don't matter much. Rates don't change much in the short run because of long-term relationships.

3. VEC Model

Component	**Details**
rlm	12 (mlm list)
beta	3 (numeric)
Log Likelihood	16426.751
Cointegration Rank	1

Table 1.4

We used the Vector Error Correction Model (VECM) to look at the link between the EUR/USD and JPY/USD exchange rates. This table shows the results. The model sees one long-term link between these two currencies, which is shown by a cointegration rank of 1. In other words, changes in EUR/USD and JPY/USD tend to happen at the same time over time. If they move in different directions, they will return to a balanced state.

The log-likelihood number of 16426.751 is a great sign that the model fits the data very well. The model does a good job of capturing the patterns in the data if the log-likelihood number is high. In the long run, the beta values show how the two currencies are linked to each other. This number tells you how the other currency will react if one changes.

4. VAR Model

```
Estimation results for equation EURUSD:
=======================================
EURUSD = EURUSD.l1 + JPYUSD.l1 + EURUSD.l2 + JPYUSD.l2 + const
```

Variable	Estimate	Std. Error	t value	Pr(>|t|)	Significance
EUR.USD.l1	1.031482	0.030383	33.949	<2e-16 ***	Highly significant
JPY.USD.l1	2.581125	3.630256	0.711	0.4772	Not significant
EUR.USD.l2	-0.03983	0.030351	-1.312	0.1896	Not significant
JPY.USD.l2	-2.06411	3.634866	-0.568	0.5702	Not significant
const	0.004801	0.002687	1.787	0.0742 .	Weakly significant

```
Residual standard error: 0.005049 on 1297 degrees of freedom
Multiple R-Squared: 0.993,    Adjusted R-squared: 0.993
F-statistic: 4.597e+04 on 4 and 1297 DF,  p-value: < 2.2e-16
```

Table 1.5

Table 1.5 shows how the present and past values of EUR/USD and JPY/USD are linked, along with the times between them. With a coefficient of 1.0315, the first lag of EUR/USD (EUR.USD.l1) is very important. This means that past values of EUR/USD have a big effect on the current exchange rate. This means that EUR/USD tends to stay the same over time and that its past behavior is a good indicator of its present number.

The first lag of JPY/USD, on the other hand (JPY.USD.l1), does not have a big effect on EUR/USD, as shown by its high p-value (0.4772). In the same way, the second lags for EUR/USD (EUR.USD.l2) and JPY/USD (JPY.USD.l2) are not important either, with p-values of 0.1896 and 0.5702, respectively. This means that numbers from before the first lag don't help us figure out what the EUR/USD exchange rate will be.

With a p-value of 0.0742, the constant term is weakly significant, which means it has a small but noticeable effect. This could mean that the base amount of EUR/USD has changed slightly, but it's not the main reason.

Looking at model fit, an R-squared value of 0.993 means that the model can explain 99.3% of the changes in EUR/USD. This is a very high amount of explanatory power. Even though the number of variables was taken into account, the adjusted R-squared of 0.993 shows that the model still fits the data well. The F-statistic of 45,970 and the p-value of less than 2.2e-16 show that the model as a whole is very significant and that the relationships between the factors are meaningful from a statistical point of view.

```
Estimation results for equation JPYUSD:
=======================================
JPYUSD = EURUSD.l1 + JPYUSD.l1 + EURUSD.l2 + JPYUSD.l2 + const
```

Variable	Estimate	Std. Error	t value	Pr(>|t|)	Significance
EUR.USD.l1	0.000349	0.000255	1.372	0.170	Not significant
JPY.USD.l1	0.9692	0.03042	31.864	<2e-16 ***	Highly significant
EUR.USD.l2	-0.00034	0.000254	-1.345	0.179	Not significant
JPY.USD.l2	0.02992	0.03046	0.982	0.326	Not significant
const	-2E-06	2.25E-05	-0.089	0.929	Not significant

```
Residual standard error: 4.23e-05 on 1297 degrees of freedom
Multiple R-Squared: 0.9964,   Adjusted R-squared: 0.9964
F-statistic: 8.916e+04 on 4 and 1297 DF,  p-value: < 2.2e-16
```

Table 1.6

In Table 1.6, we can see how the past rates of EUR/USD and JPY/USD have affected the current JPY/USD exchange rate. With a score of 0.9692 and a p-value of less than 2e-16, the first lag of JPY/USD (JPY.USD.l1) is very important. This means that the values of JPY/USD in the past have a big and good effect on the values of JPY/USD now. This is common for time series data with a lot of autocorrelation.

The first lag of EUR/USD, on the other hand (EUR.USD.l1), does not have a big impact on JPY/USD, as shown by its p^0.170 number. With p-values of 0.179 and 0.326, the second lags of both EUR/USD (EUR.USD.l2) and JPY/USD (JPY.USD.l2) are also not significant. This means that the changes of EUR/USD in the past and the second lag of JPY/USD have little to no effect on the value of JPY/USD right now.

With a p-value of 0.929, the constant term is also not significant. This means that there is not a big change or move in the JPY/USD exchange rate in this model.

For example, an R-squared number of 0.9964 means that the model explains 99.64% of the changes in the JPY/USD exchange rate. This is a very good fit. Even when the number of variables is taken into account, the adjusted R-squared value of 0.9964 shows that the model is stable and does not fit too well. The overall model is highly statistically significant, as shown by the F-statistic of 89,160 and the p-value of less than 2.2e-16. It accurately shows how the factors are related.

Covariance matrix of residuals

	EUR.USD	**JPY.USD**
EUR.USD	2.55E-05	8.72E-08
JPY.USD	8.72E-08	1.79E-09

Table 1.7

There are errors (residuals) in the model for both the EUR/USD and JPY/USD exchange rates. Table 1.7 shows how these mistakes are connected. From left to right, the numbers show how much the mistakes change for each exchange rate on their own. For instance, EUR/USD changes more (2.55E-05) than JPY/USD changes (1.79E-09).

The numbers that are not on the diagonal (8.72E-08) show how the two exchange rate mistakes are linked. In other words, there is a weak connection between the strange changes in EUR/USD and JPY/USD.

Correlation matrix of residuals

	EUR.USD	**JPY.USD**
EUR.USD	1	0.4085
JPY.U	0.4085	1

SD		

Table 1.8

In Table 1.8, we see how the model's mistakes (or moves that cannot be explained) for the EUR/USD and JPY/USD exchange rates are connected. Since the diagonal numbers are all 1, it makes sense that each exchange rate is perfectly linked to the others.

An error of 0.4085 means that there is a moderately good relationship between the EUR/USD and JPY/USD rates. In other words, if the model's error for EUR/USD goes up, it's likely that the error for JPY/USD will go up too, though the link is not very strong.

5. ARCH/GARCH Model

Performing ARCH Test on EUR/USD

```
# Perform ARCH test on EUR/USD returns
> eur_usd_arch_test <- ArchTest(eur_usd_returns, lags = 1, demean = TRUE)
> print(eur_usd_arch_test)

        ARCH LM-test; Null hypothesis: no ARCH effects

data:  eur_usd_returns
Chi-squared = 7.7403, df = 1, p-value = 0.0054
```

The test statistic Chi-squared = 7.7403 with 1 degree of freedom. The p-value is 0.0054, which is less than the commonly used threshold of 0.05. This indicates that we reject the null hypothesis and conclude that there are significant ARCH effects present in the EUR/USD returns. In other words, the volatility of the EUR/USD exchange rate is time-varying and exhibits volatility clustering.

Performing ARCH Test on JPY/USD

```
# Perform ARCH test on JPY/USD returns
> jpy_usd_arch_test <- ArchTest(jpy_usd_returns, lags = 1, demean = TRUE)
> print(jpy_usd_arch_test)

        ARCH LM-test; Null hypothesis: no ARCH effects

data:  jpy_usd_returns
Chi-squared = 15.117, df = 1, p-value = 0.000101
```

The Chi-squared = 15.117 with 1 degree of freedom, and the p-value is 0.000101, which is much smaller than the typical significance level of 0.05. This means we reject the null hypothesis that there are no ARCH effects in the data.

Fitting an ARCH(1) Model for EUR/USD

```
> summary(eur_usd_arch)

Call:
garch(x = eur_usd_returns, order = c(0, 1))

Model:
GARCH(0,1)

Residuals:
    Min      1Q  Median      3Q     Max 
-6.0996 -0.6047  0.0000  0.5589  3.7019 

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)    
a0 1.773e-05   7.816e-07   22.682  < 2e-16 ***
a1 1.476e-01   3.462e-02    4.262 2.02e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Diagnostic Tests:
	Jarque Bera Test

data:  Residuals
X-squared = 226.98, df = 2, p-value < 2.2e-16


	Box-Ljung test

data:  Squared.Residuals
X-squared = 0.60176, df = 1, p-value = 0.4379
```

The results of the ARCH(1) model on EUR/USD shows volatility of the exchange rate. The intercept term, estimated at 1.773e-05, is highly significant with a t-value of 22.682 and a p-value less than 2e-16. This suggests that the model’s baseline volatility is statistically important. In other words, there is a small but significant level of volatility in the EUR/USD exchange rate even when past volatility is not taken into account.

The ARCH(1) coefficient is estimated at 0.1476. This coefficient is also highly significant with a t-value of 4.262 and a p-value of 2.02e-05. This indicates that past fluctuations in the EUR/USD exchange rate play a key role in predicting current volatility.

The residuals from the model, which represent the difference between the actual and predicted values, range from -6.0996 to 3.7019, with the median close to zero. This suggests that the model captures both positive and negative shocks in the exchange rate, effectively explaining the fluctuations in EUR/USD.

Further diagnostic tests support the validity of the model. The Jarque-Bera test, which checks for the normality of residuals, produces a p-value less than 2.2e-16, indicating that the residuals are not normally distributed. This is expected in financial time series data, where returns often exhibit skewness and excess kurtosis. Meanwhile, the Box-Ljung test for autocorrelation of squared residuals shows a p-value of 0.4379, meaning that there is no significant autocorrelation in the squared residuals. This suggests that the model has captured the volatility clustering well, and there are no further patterns in the residuals that need to be modeled.

Fitting an ARCH(1) Model for JPY/USD

```
> summary(jpy_usd_arch)

Call:
garch(x = jpy_usd_returns, order = c(0, 1))

Model:
GARCH(0,1)

Residuals:
     Min       1Q   Median       3Q      Max 
-4.48780 -0.56585 -0.06013  0.45512  8.40350 

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)    
a0 1.913e-05   4.786e-07   39.960   <2e-16 ***
a1 2.579e-01   2.893e-02    8.915   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Diagnostic Tests:
	Jarque Bera Test

data:  Residuals
X-squared = 2348.9, df = 2, p-value < 2.2e-16


	Box-Ljung test

data:  Squared.Residuals
X-squared = 0.76394, df = 1, p-value = 0.3821
```

The results from the ARCH(1) model on the JPY/USD highlights volatility dynamics of the Japanese Yen exchange rate relative to the US Dollar. The coefficient for the intercept term, a0 = 1.913e-05, is highly significant with a t-value of 39.960 and a p-value less than 2e-16. This suggests that there is a small but statistically significant baseline level of volatility in the JPY/USD exchange rate, even when past volatility is not considered.

The coefficient for the ARCH term, a1 = 0.2579, indicates that past volatility has a significant influence on current volatility. With a t-value of 8.915 and a p-value of less than 2e-16, this result demonstrates that recent fluctuations in the JPY/USD exchange rate are important for forecasting future volatility.

The residuals from the model, which represent the difference between actual and predicted values, range from -4.4878 to 8.4035, with the median close to -0.06013. This distribution of residuals indicates that the model captures both large positive and negative shocks in the exchange rate, while the residuals are fairly spread out.

The Jarque-Bera test for the normality of residuals reveals a p-value of less than 2.2e-16, meaning that the residuals are not normally distributed. This is common in financial time series data, as exchange rate returns often exhibit skewness and excess kurtosis. Meanwhile, the Box-Ljung test for autocorrelation of squared residuals shows a p-value of 0.3821, indicating that there is no significant autocorrelation in the squared residuals. This suggests that the ARCH model has effectively captured the volatility clustering, and there are no remaining patterns in the squared residuals.

GARCH FIT

```
> summary(eur_usd_garch_fit)
   Length     Class      Mode 
        1 uGARCHfit        S4 


print(eur_usd_garch_coef)
           mu         omega        alpha1         beta1         shape 
-9.831828e-05  1.021638e-07  3.895109e-02  9.569521e-01  8.990736e+00 


> summary(jpy_usd_garch_fit)
   Length     Class      Mode 
        1 uGARCHfit        S4 


print(jpy_usd_garch_coef)
           mu         omega        alpha1         beta1         shape 
-1.655131e-04  5.564645e-07  8.687444e-02  8.926501e-01  5.702161e+00 
```

The results from the GARCH model for EUR/USD and JPY/USD reveal insights about the volatility characteristics of these currency pairs. For EUR/USD, the mean return (mu) is almost zero at -9.83e-05, which suggests that there was very little average return over the analyzed period. The baseline level of volatility, as indicated by omega, is very low at 1.02e-07, suggesting that, in the absence of other factors, there is minimal volatility. However, the model does account for past volatility in predicting future volatility, as evidenced by the alpha1 coefficient of 0.03895, which indicates a moderate effect from previous volatility. The beta1 coefficient, which is quite large at 0.95695, implies that shocks to the exchange rate have a long-lasting effect on its volatility, meaning that once volatility increases, it tends to remain high for a while.

Similarly, for JPY/USD, the mean return (mu) is also very close to zero at -1.655e-04, which, like EUR/USD, suggests little average return over the time period studied. The omega value is even smaller at 5.56e-07, pointing to a baseline level of volatility that is slightly lower than that of EUR/USD. The alpha1 coefficient for JPY/USD is 0.08687, which indicates a slightly stronger reliance on past volatility compared to EUR/USD. The beta1 coefficient is 0.89265, which again suggests that volatility shocks have a lasting impact on the exchange rate's future volatility. Finally, the shape parameter of 5.70 indicates that the distribution of volatility for JPY/USD is slightly less heavy-tailed than that of EUR/USD, meaning that extreme volatility events, while still significant, occur less frequently.

Graphical Illustration

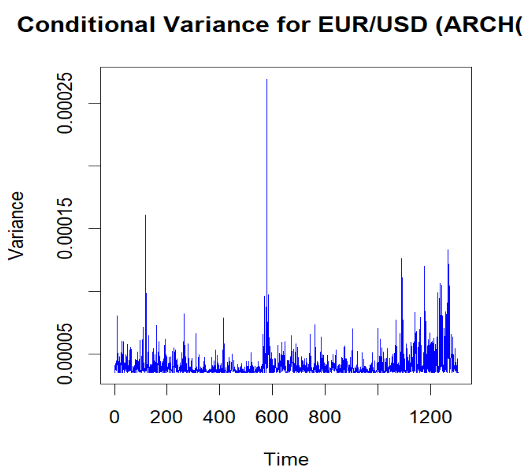

Fig 1.3

Fig 1.3 shows the conditional variance of EUR/USD exchange rates over time, based on the ARCH(1) model. The vertical spikes represent periods of higher volatility, while the smaller fluctuations indicate lower volatility. We can see that the variance is relatively stable for most of the time, but there are significant spikes in certain periods, which suggest that there were instances of high volatility in the EUR/USD exchange rate.

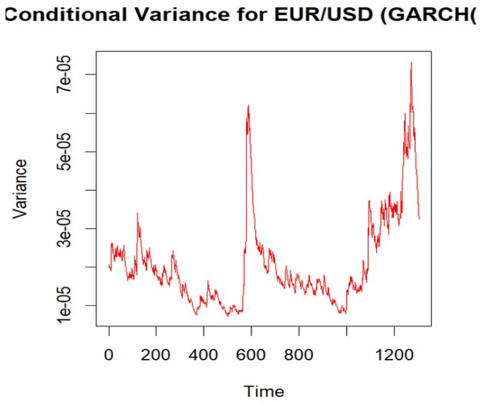

Fig 1.4

Fig 1.4 highlights the conditional variance of the EUR/USD exchange rate, based on the GARCH(1,1) model. The red lines represent periods of volatility, and just like the ARCH model, we can observe significant spikes in certain areas. These spikes indicate moments of high volatility, where the exchange rate's variability was particularly large. The GARCH model captures this more clearly, showing how volatility persists over time and highlighting the clustering effect where volatility tends to follow periods of high or low volatility.

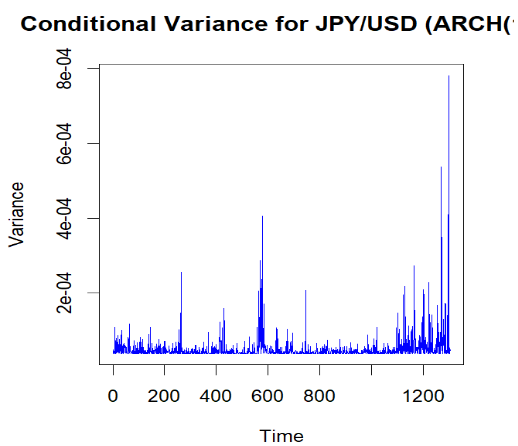

Fig 1.5

Fig 1.5 illustrates the conditional variance of the JPY/USD exchange rate based on the ARCH(1) model. The blue lines indicate periods of volatility, with noticeable spikes showing times when the exchange rate was more volatile. Similar to the EUR/USD graph, the large peaks represent moments of high market turbulence, suggesting that the volatility of JPY/USD can experience sudden increases.

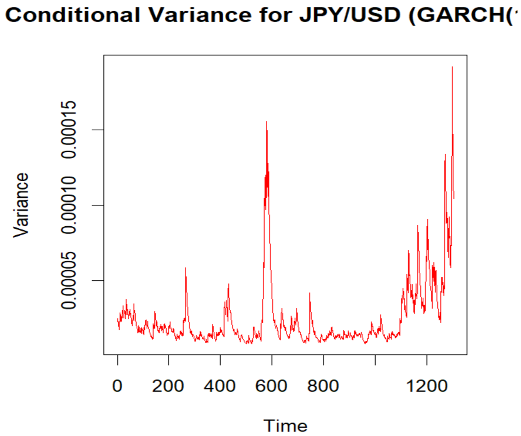

Fig 1.6

Fig 1.6 shows the conditional variance of the JPY/USD exchange rate using a GARCH(1,1) model. The red lines represent the volatility over time, highlighting moments of heightened fluctuations in the exchange rate. Similar to the ARCH model, the graph shows periods where the variance is high, especially during sharp spikes, which indicate times of increased market uncertainty or volatility.

Conclusion and Recommendation

The analysis of EUR/USD and JPY/USD exchange rates from 2018 to 2022 provided several key insights. The autocorrelation tests showed a Durbin-Watson statistic of 2.0617 for EUR/USD (p-value = 0.8617) and 2.0529 for JPY/USD (p-value = 0.8234), suggesting that there is no significant autocorrelation in the exchange rates. The Johansen cointegration test showed a test statistic of 0.86 for $r \leq 1$ and 7.59 for $r = 0$, both below the critical values, indicating that there is no long-term cointegration between the EUR/USD and JPY/USD rates.

In the VECM analysis, with a cointegration rank of 1, the model identified a weak short-term relationship, where the lagged values of EUR/USD and JPY/USD had minimal impact on each other. The ARCH(1) model for EUR/USD highlighted significant volatility with a0 = 1.773e-05 ($p < 2e-16$) and a1 = 0.1476 ($p < 2.02e-05$), indicating persistence in volatility. Similarly, for JPY/USD, the ARCH(1) model showed significant coefficients with a0 = 1.913e-05 ($p < 2e-16$) and a1 = 0.2579 ($p < 2e-16$), reflecting high volatility persistence. The GARCH models confirmed the persistent nature of volatility in both currency pairs.

Based on these results, we recommend focusing on short-term exchange rate predictions using VAR models, given that the long-term relationships are not significant. Traders should also incorporate volatility modeling, particularly using GARCH models, to better manage risk during high-volatility periods. Future research should consider integrating macroeconomic factors to further enhance the understanding of exchange rate dynamics.

Appendix

```
# Fit an ARCH(1) model for EUR/USD
> eur_usd_arch <- garch(eur_usd_returns, c(0, 1))  # ARCH(1) Model

 ***** ESTIMATION WITH ANALYTICAL GRADIENT ***** 


     I     INITIAL X(I)        D(I)

     1     1.953938e-05     1.000e+00
     2     5.000000e-02     1.000e+00

    IT   NF      F         RELDF    PRELDF    RELDX   STPPAR   D*STEP   
NPRELDF
     0    1 -6.381e+03
     1    9 -6.381e+03  7.86e-06  1.50e-05  2.4e-06  1.6e+12  2.4e-07  
1.21e+07
     2   10 -6.381e+03  1.68e-08  1.72e-08  2.4e-06  2.0e+00  2.4e-07  5.46e-
01
     3   20 -6.384e+03  4.68e-04  8.10e-04  3.8e-01  2.0e+00  6.2e-02  5.46e-
01
     4   21 -6.384e+03  5.22e-05  3.86e-05  8.3e-02  0.0e+00  2.0e-02  3.86e-
05
     5   22 -6.384e+03  1.01e-05  8.85e-06  4.6e-02  0.0e+00  1.3e-02  8.85e-
06
     6   23 -6.384e+03  2.68e-07  2.52e-07  7.6e-03  0.0e+00  2.2e-03  2.52e-
07
     7   24 -6.384e+03  1.25e-09  1.25e-09  5.9e-04  0.0e+00  1.7e-04  1.25e-
09
     8   25 -6.384e+03  8.41e-13  9.06e-13  1.1e-06  0.0e+00  3.2e-07  9.06e-
13

 ***** RELATIVE FUNCTION CONVERGENCE *****

 FUNCTION    -6.384281e+03   RELDX        1.074e-06
 FUNC. EVALS      25         GRAD. EVALS      9
 PRELDF       9.059e-13      NPRELDF      9.059e-13

     I      FINAL X(I)        D(I)          G(I)

     1    1.772912e-05     1.000e+00     9.994e+00
     2    1.475693e-01     1.000e+00     1.157e-04


# Fit an ARCH(1) model for JPY/USD
> jpy_usd_arch <- garch(jpy_usd_returns, c(0, 1))  # ARCH(1) Model

 ***** ESTIMATION WITH ANALYTICAL GRADIENT ***** 


     I     INITIAL X(I)        D(I)

     1     2.349617e-05     1.000e+00
     2     5.000000e-02     1.000e+00
```

```
    IT    NF      F          RELDF     PRELDF     RELDX    STPPAR   D*STEP
NPRELDF
     0     1 -6.271e+03
     1     8 -6.271e+03  6.55e-05  1.49e-04  1.0e-05  9.3e+11  1.0e-06
6.93e+07
     2     9 -6.271e+03  1.39e-06  1.51e-06  9.9e-06  2.0e+00  1.0e-06
3.95e+00
     3    18 -6.280e+03  1.41e-03  1.95e-03  3.6e-01  2.0e+00  5.5e-02
3.95e+00
     4    19 -6.284e+03  7.26e-04  5.21e-04  2.1e-01  3.8e-02  5.5e-02  5.22e-
04
     5    20 -6.286e+03  3.23e-04  2.41e-04  1.4e-01  0.0e+00  5.1e-02  2.41e-
04
     6    21 -6.287e+03  7.26e-05  5.78e-05  7.0e-02  0.0e+00  3.2e-02  5.78e-
05
     7    22 -6.287e+03  7.71e-06  6.82e-06  2.5e-02  0.0e+00  1.2e-02  6.82e-
06
     8    23 -6.287e+03  1.91e-07  1.89e-07  4.6e-03  0.0e+00  2.4e-03  1.89e-
07
     9    24 -6.287e+03  1.60e-09  1.98e-09  1.3e-04  0.0e+00  6.7e-05  1.98e-
09
    10    25 -6.287e+03  6.34e-11  6.33e-11  8.1e-06  0.0e+00  4.2e-06  6.33e-
11

 ***** RELATIVE FUNCTION CONVERGENCE *****

 FUNCTION    -6.286959e+03   RELDX        8.054e-06
 FUNC. EVALS      25         GRAD. EVALS      11
 PRELDF       6.333e-11      NPRELDF      6.333e-11

     I      FINAL X(I)        D(I)          G(I)

     1    1.912499e-05     1.000e+00     4.455e-01
     2    2.578546e-01     1.000e+00    -5.885e-07
```