Logistic Regression

ML Kaggle SMP

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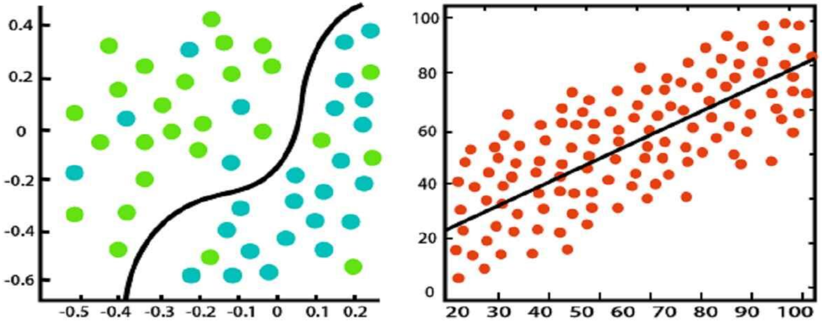
Classification

In machine learning, classification refers to a predictive modeling problem where a class label is predicted for a given example of input data.

Examples of classification problems include: Given an example, classify if it is spam or not. Given a handwritten character, classify it as one of the known characters.



Classification Regression

Hypothesis Representation

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \cdots + \theta_n x_n$$

Linear Regression Hypothesis

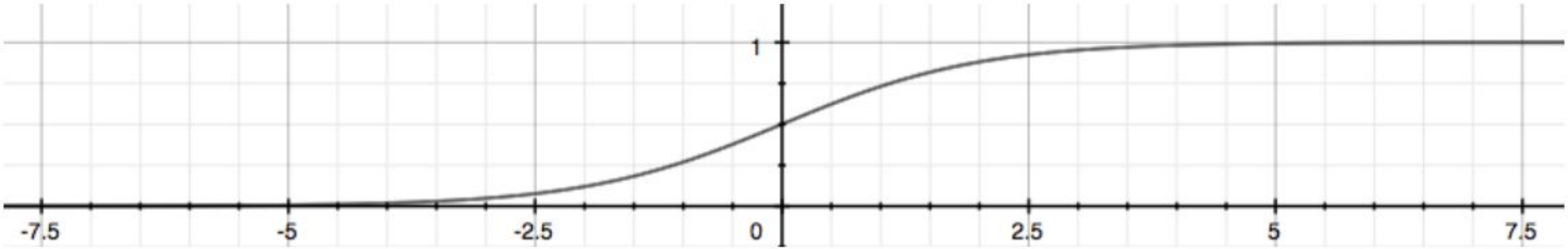
$$h_\theta(x) = g(\theta^T x)$$

$$z = \theta^T x$$

$$g(z) = \frac{1}{1 + e^{-z}}$$

Logistic Regression hypothesis

$$h_\theta(x) = P(y=1|x;\theta) = 1 - P(y=0|x;\theta)$$
$$P(y=0|x;\theta) + P(y=1|x;\theta) = 1$$

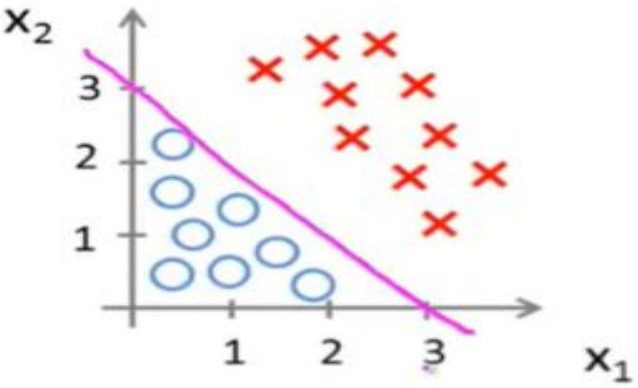


Decision Boundary

The decision boundary is the line that separates the area where y = 0 and where y = 1. It is created by our hypothesis function.

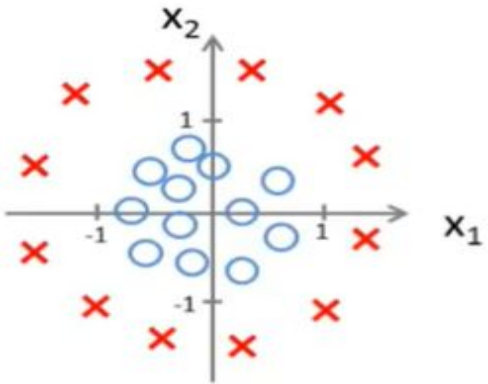
$$h_\theta(x) \geq 0.5 \rightarrow y = 1$$
$$h_\theta(x) < 0.5 \rightarrow y = 0$$
$$g(z) \geq 0.5$$
$$when\ z \geq 0$$
$$h_\theta(x) = g(\theta^T x) \geq 0.5$$
$$when\ \theta^T x \geq 0$$

Decision Boundary



$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

Predict "$y = 1$" if $-3 + x_1 + x_2 \geq 0$

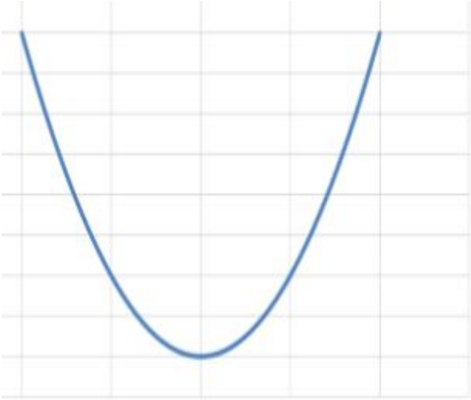


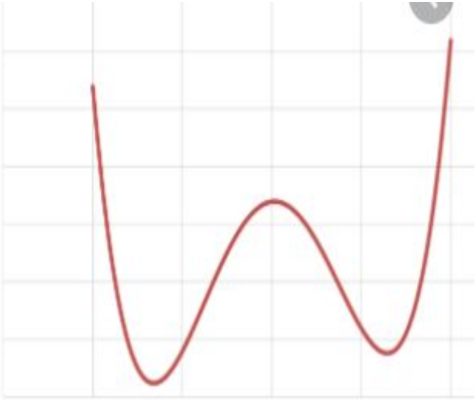
$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2)$$

Predict "$y = 1$" if $-1 + x_1^2 + x_2^2 \geq 0$

Cost Function

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \frac{1}{2} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$



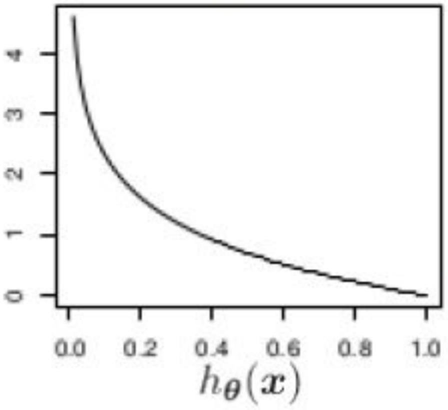


Convex Non Convex

Cost Function

$$\text{cost}\,(h_{\boldsymbol{\theta}}(\boldsymbol{x}),\; y) = \begin{cases} -\log(h_{\boldsymbol{\theta}}(\boldsymbol{x})) & \text{if } \; y = 1 \\ -\log(1 - h_{\boldsymbol{\theta}}(\boldsymbol{x})) & \text{if } \; y = 0 \end{cases}$$

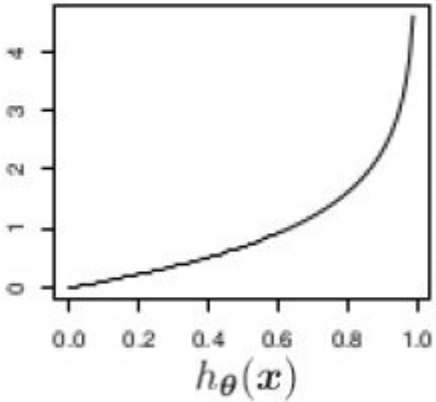
if $\; y = 1$



if $h_{\boldsymbol{\theta}}(\boldsymbol{x}) = 1$
then cost $= 0$

if $h_{\boldsymbol{\theta}}(\boldsymbol{x}) \to 0$
then cost $\to \infty$
predicted
$\text{prob}(y = 1 \mid \boldsymbol{x}; \boldsymbol{\theta}) = 0$
but $y = 1$

if $\; y = 0$



if $h_{\boldsymbol{\theta}}(\boldsymbol{x}) = 0$
then cost $= 0$

if $h_{\boldsymbol{\theta}}(\boldsymbol{x}) \to 1$
then cost $\to \infty$
predicted
$\text{prob}(y = 0 \mid \boldsymbol{x}; \boldsymbol{\theta}) = 0$
but $y = 0$

Cost Function

Compressed Logistic Cost Function:

$$\mathrm{Cost}(h_\theta(x), y) = -y \ \log(h_\theta(x)) - (1 - y) \log(1 - h_\theta(x))$$

$$J(\theta) = -\frac{1}{m} \sum_{i=1}^{m} [y^{(i)} \log(h_\theta(x^{(i)})) + (1 - y^{(i)}) \log(1 - h_\theta(x^{(i)}))]$$

Gradient Descent

Want $\min_\theta J(\theta)$:

Repeat $\{$

$$\theta_j := \theta_j - \alpha \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(simultaneously update all θ_j)

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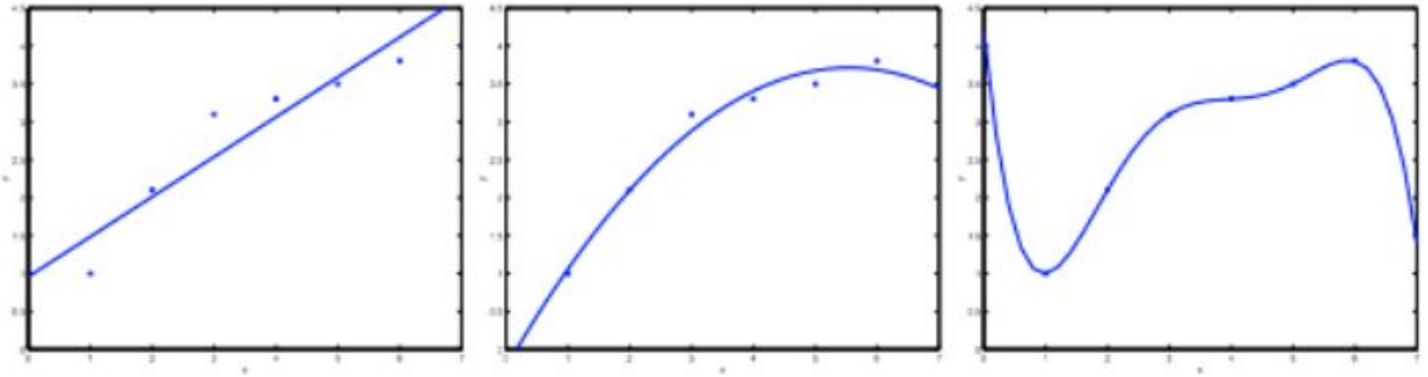
Underfitting and Overfitting



Underfitting, or high bias, is when the form of our hypothesis function h maps poorly to the trend of the data. It is usually caused by a function that is too simple or uses too few features. At the other extreme, overfitting, or high variance, is caused by a hypothesis function that fits the available data but does not generalize well to predict new data. It is usually caused by a complicated function that creates a lot of unnecessary curves and angles unrelated to the data.

Solution:

This terminology is applied to both linear and logistic regression. There are two main options to address the issue of overfitting:

1) Reduce the number of features:

Manually select which features to keep.

2) Regularization

Keep all the features, but reduce the magnitude of parameters..

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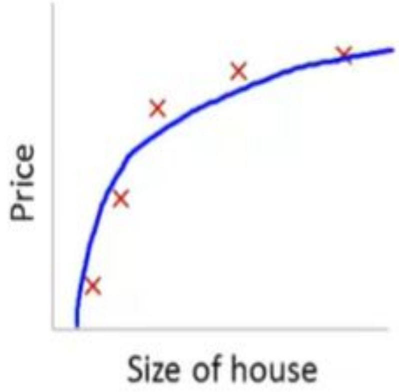
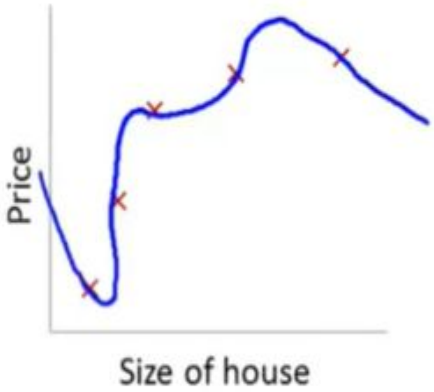
Regularization





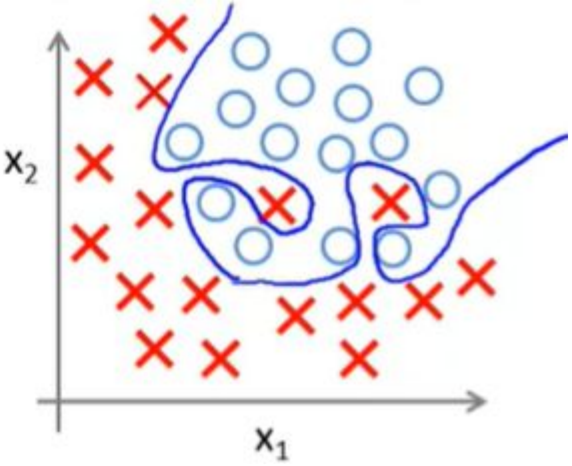
$$min_\theta \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^{n} \theta_j^2$$

The λ, or lambda, is the regularization parameter. It determines how much the costs of our theta parameters are inflated.

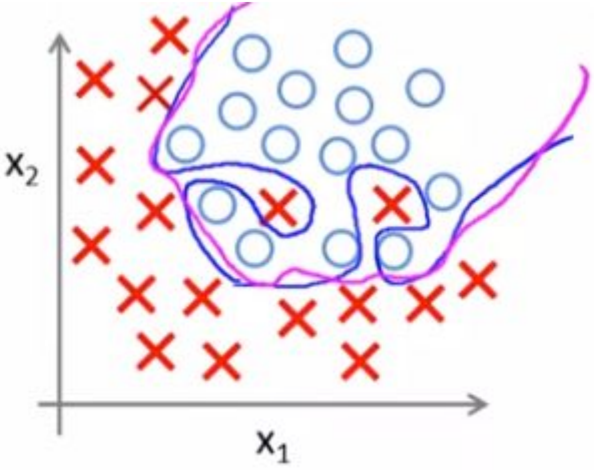
This smoothens the output of hypothesis function to reduce overfitting. However If lambda is chosen to be too large, it may smooth out the function too much and cause underfitting.

Regularisation in Logistic Regression

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m}[y^{(i)}\ \log(h_\theta(x^{(i)})) + (1-y^{(i)})\ \log(1-h_\theta(x^{(i)}))] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$$



Overfitted



Regularized