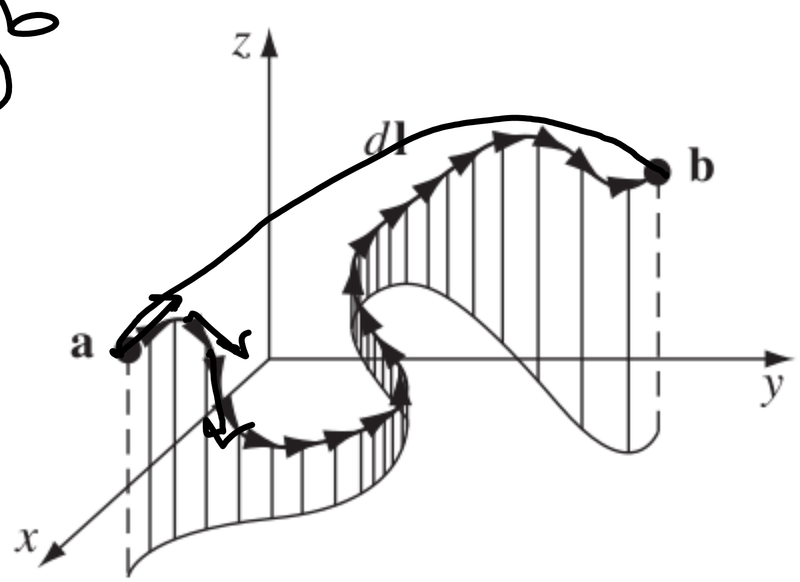
EPHY105L-5 11.10.21

Integral Calculus

Line Integral

Integrals of form : $\int_a^b \vec{v}\cdot \underline{\underline{d\vec{l}}}$ ←



→ integration to be carried out along the shown path from A to B.

(*) Closed integral : $\oint \vec{v}\cdot d\vec{l}$

$$= \int_a^b \underline{\vec{v}\cdot d\vec{l}} + \int_b^a \underline{\vec{v}\cdot d\vec{l}}$$

(*) One simple example : $W = \int \vec{F}\cdot d\vec{l}$ (= work done)

* Usually the line integral depends on the path taken, but for a class of vectors the integral only depends on the end points.

→ A force that has this property is called "conservative force."

for such vectors,

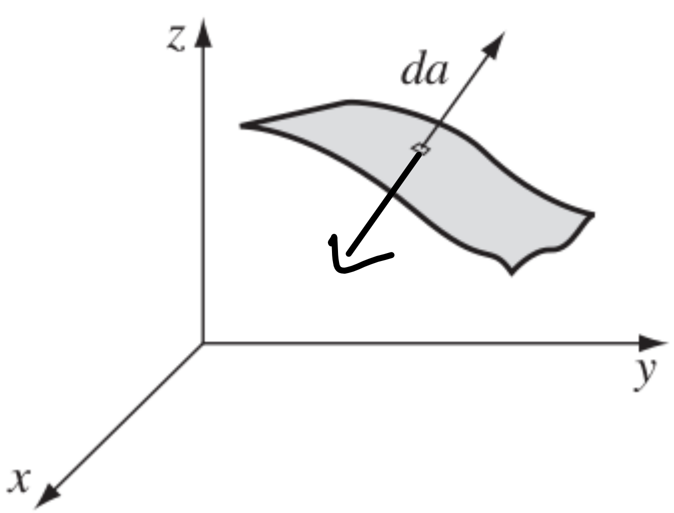
$$\oint = \int_a^b (\vec{F} \cdot d\vec{l}) + \int_b^a (\vec{F} \cdot d\vec{l})$$

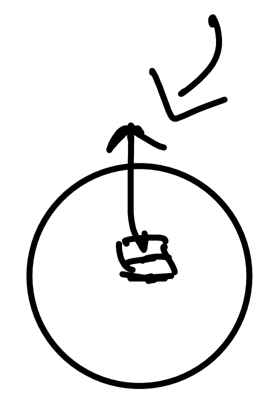
$$= \int_a^b (\vec{F} \cdot d\vec{l}) - \int_a^b (\vec{F} \cdot d\vec{l}) = 0$$

Surface Integral

Integral is of form: $\int_S \underline{\vec{v}} \cdot d\vec{a}$ ←

closed surface $\equiv \oint_S \vec{v} \cdot d\vec{a}$





$d\vec{a} \equiv$ infinitesimal surface area element

→ integral is taken over a specified surface area.

⊕ There are two directions perpendicular to any surface.

→ if surface is closed $d\vec{a}$ is always outward.

Flux: $\vec{v}$ describes flow of a fluid.
(mass per unit area)

Then $\int \vec{v} \cdot d\vec{a}$ $\Rightarrow$ total mass per unit time passing through the surface.

$\Rightarrow$ **Flux.**

Volume Integral

Integral is of the form: $\int_V T \, d\tau$

$d\tau \equiv$ infinitesimal volume element

$T \equiv$ Any scalar.

In Cartesian coordinate system
$d\tau = dx\,dy\,dz$

Ex: if $T \equiv$ density of some substance,

$\int T \, d\tau \equiv$ Total mass.

Fundamental theorem for gradients

Let us consider a line integral.

Here the vector, $\vec{v} = \vec{\nabla} T$

Then $dT = (\vec{\nabla} T) \cdot d\vec{l}$

$\rightarrow$ many infinitesimal steps taken from a to b.

Total change in 'T'

$$\int_a^b dT = \int_a^b (\vec{\nabla} T) \cdot d\vec{l} = T_b - T_a$$

(independent of path)

Corollary

(1) $\int_a^b (\vec{\nabla} T) \cdot d\vec{l}$ is independent of path

(2) $\oint (\vec{\nabla} T) \cdot d\vec{l} = 0$

Link to the recording:

https://bennettu.sharepoint.com/sites/EPHY105L-Odd2021/Shared%20Documents/Forms/AllItems.aspx?id=%2Fsites%2FEPHY105L%2DOdd2021%2FShared%20Documents%2FGeneral%2FRecordings%2FEPHY105L%20theory%20Class%2D20211011%5F154532%2DMeeting%20Recording%2Emp4&parent=%2Fsites%2FEPHY105L%2DOdd2021%2FShared%20Documents%2FGeneral%2FRecordings