DO NOT WRITE ANYTHING ON QUESTION PAPER EXCEPT YOUR NAME, DEPARTMENT AND ENROLMENT No.

POSSESSION OF MOBILE, SMART WATCH ETC, IN EXAMINATION IS A UFM PRACTICE

Name of Student --- Enrolment No. ------------------

Department /School ----------------------------

BENNETT UNIVERSITY, GREATER NOIDA

Mid-Term Examination, SPRING SEMESTER 2018-19

COURSE CODE: **EPHY108L** MAX. DURATION: **ONE HOUR**

COURSE NAME: **Mechanics**

COURSE CREDIT: **3** MAX. MARKS: **25**

Note

- All questions are mandatory
- Rough work must be carried out at the back of the answer script

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Answer the following questions (1(i) to 1(v)) in brief. The answers should be no longer than 2-3 lines.

1(i). The potential energy due to a force field ($\mathbf{F}(\mathbf{r})$) is given by:

$$V(x, y, z) = xe^{-x^3-2y^2+5x-z} + 7y^4 - 190x^3 + z$$

A particle moves in xy plane in a circle of radius 5 m centred at origin, such that it makes one complete turn and returns back to its starting position. What is the work done by the force $\mathbf{F}(\mathbf{r})$. Justify your answer. (1 Mark)

1(ii). For a square of side a find three mutually perpendicular axes for which the moment of inertia tensor is diagonal. Indicate your answer by a simple schematic figure. (1 Mark)

1(iii). A particle moves in the region $1\text{m} \leq x \leq 8\text{m}$. A conservative force whose potential energy varies as shown in figure 1 below, acts on the particle. Total energy of the particle at x = 1 m is 7 J. What is the maximum kinetic energy that the particle can attain? Justify your answer. (1 Mark)

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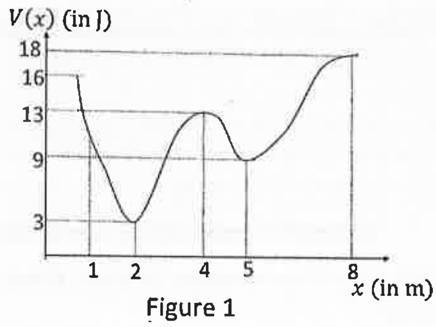

Figure 1

1(iv). Consider two vectors $\mathbf{A} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\mathbf{B} = -\hat{\imath} + 7\hat{\jmath} + 3\hat{k}$. What is the angle between them? (1 Mark)

1(v). Find the force that gives rise to the potential energy $V(x, y, z) = A(x^2 + y^2 - z^2)$. (1 Mark)

2(i). A uniform square plate of mass M, length a and negligible thickness lies in the xy plane. Find the moment of inertia tensor of the plate about x, y and z axes, whose orientation and location are shown in figure 2 below. (7 Marks)

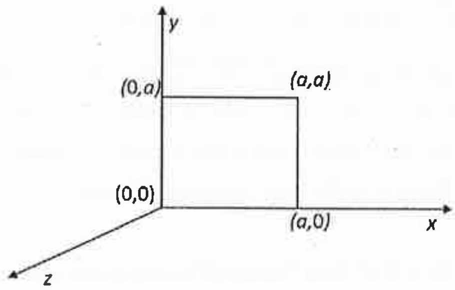

Figure 2

2(ii). Suppose instead of the square plate we have a different rigid body whose moment of inertia tensor about the chosen axes is given by

$$I = \begin{pmatrix} I_0 & -\frac{1}{2}I_0 & 0 \\ -\frac{1}{2}I_0 & I_0 & 0 \\ 0 & 0 & 2I_0 \end{pmatrix}$$

Where I_0 is a constant. The body rotates with a constant angular velocity, $\boldsymbol{\omega} = \omega_0 \hat{\imath}$, about the *x* axis. Find its angular momentum, $\mathbf{L} = I\boldsymbol{\omega}$. Write the final answer in vector form ($\mathbf{L} = L_x\hat{\imath} + L_y\hat{\jmath} + L_z\hat{k}$). (2 Marks)

2(iii). There are two external torques ($\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_2$) acting on the rigid body of the previous problem. If $\boldsymbol{\tau}_1 = 5\hat{\imath} - 6\hat{\jmath} - 2\hat{k}$ what is $\boldsymbol{\tau}_2$? (1 Mark)

3. Consider the force field $\mathbf{F} = xy(\hat{\imath} + \hat{\jmath} + \hat{k})$. What is the work done by this force in going around a closed path which is a square in the *xy* plane. Coordinates of the vertices of the square are (0,0,0), (a, 0, 0), (a, a, 0), and (0, a, 0). Instead of directly doing the line integral, find the answer by using the Stokes' theorem. The path is traversed in a counter clockwise manner. (6 Marks)

4 A particle is moving in a 2D plane such that its polar coordinates are $r(t) = e^{-t}$ and $\theta(t) = \frac{1}{2}t^2$. Find the acceleration in polar coordinates. When does the net force acting on the particle becomes radial in direction. (4 Marks)

End of Question Paper

Useful relations (symbols have their usual meanings):

Vector multiplication:

A.B = AB cosθ, $|\mathbf{A} \times \mathbf{B}| = AB$ sinθ

$\mathbf{A.B} = A_xB_x + A_yB_y + A_zB_z$

$$\mathbf{A \, X \, B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Plane polar coordinates:

$x = r\cos\theta, y = r\sin\theta, \hat{\mathbf{r}} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}, \hat{\boldsymbol{\theta}} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$

Kinematics in polar coordinates:

$\mathbf{r} = r\hat{\mathbf{r}}, \mathbf{v} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}, \boldsymbol{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\boldsymbol{\theta}}$

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Gradient of a scalar function: $\nabla V = \hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}$

Conservative Force: $\mathbf{F} = -\nabla V$

Curl of a vector: $\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

Stokes' Theorem: $\oint \mathbf{F}.d\mathbf{l} = \int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$

Moment of Inertia tensor:

$$I = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}$$

$I_{xx} = \int (y^2 + z^2) dm,\ I_{yy} = \int (x^2 + z^2) dm,\ I_{zz} = \int (x^2 + y^2) dm$

$I_{xy} = -\int xy\, dm,\ I_{xz} = -\int xz\, dm,\ I_{yz} = -\int yz\, dm,\ I_{yx} = I_{xy},\ I_{zx} = I_{xz},\ I_{zy} = I_{yz}$

Rotational motion:

$\mathbf{L} = \mathbf{r} \times \mathbf{P},\ \boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \frac{d\mathbf{L}}{dt}$