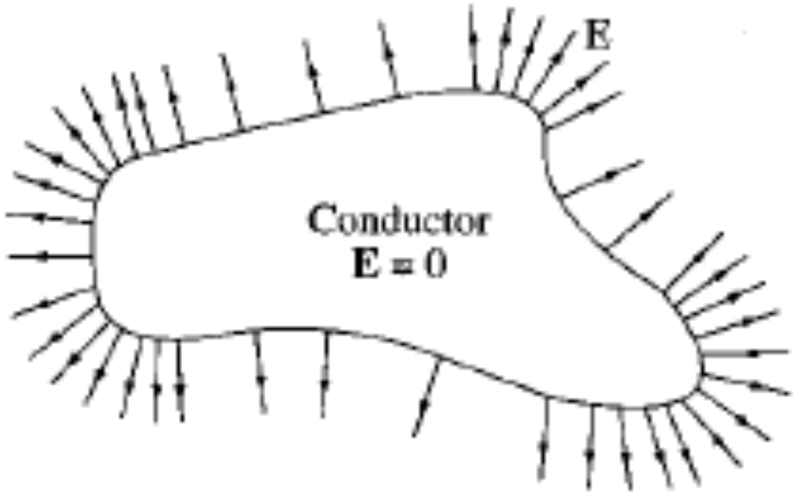
Lecture - 17

$\mathbf{E} = 0$ inside a conductor



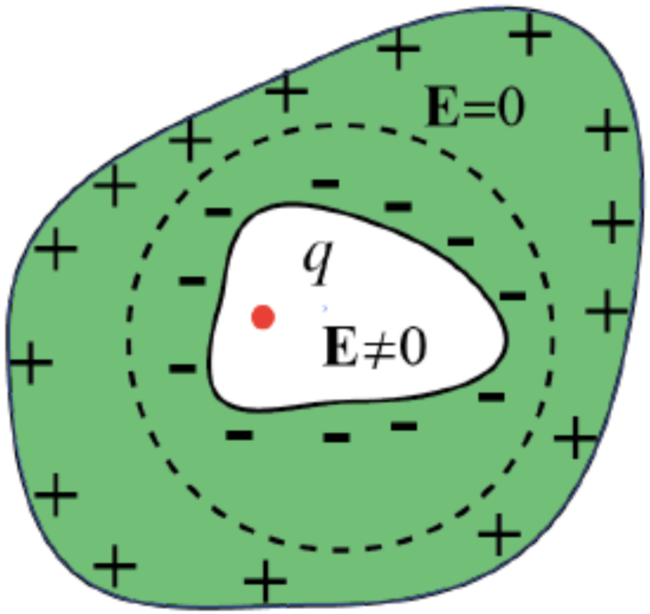
$\rho = 0$ inside a conductor

Any net charge resides on the surface

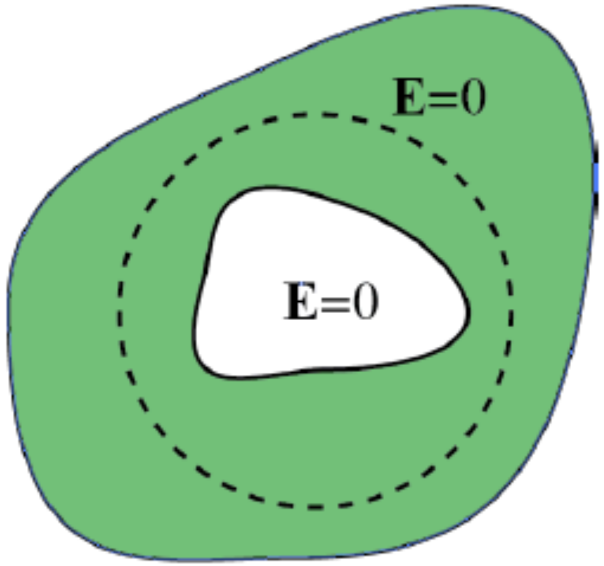
A conductor is an equipotential

$\mathbf{E}$ is perpendicular to the surface, just outside a conductor.

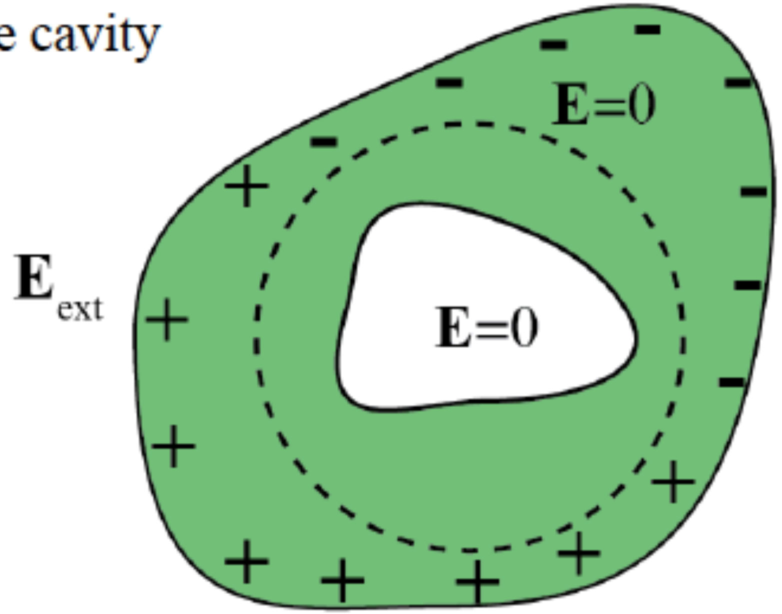
Induced Charges



Charge inside the cavity



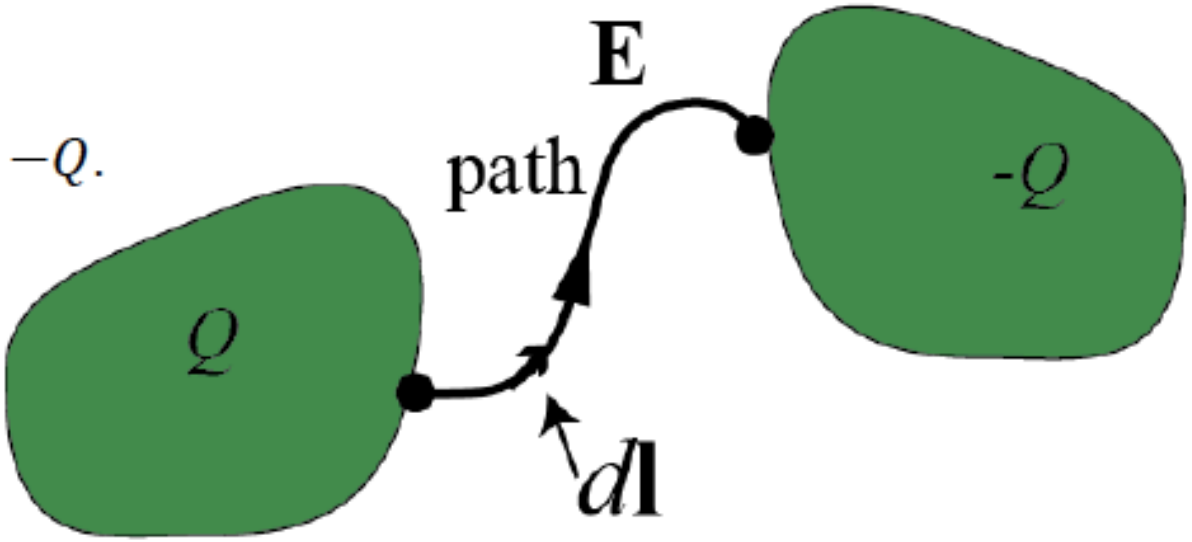
No charge inside the cavity



No charge inside the cavity,
Conductor in an external field

Capacitor:

Two conductors with charge Q and $-Q$.



What is the potential difference between them?

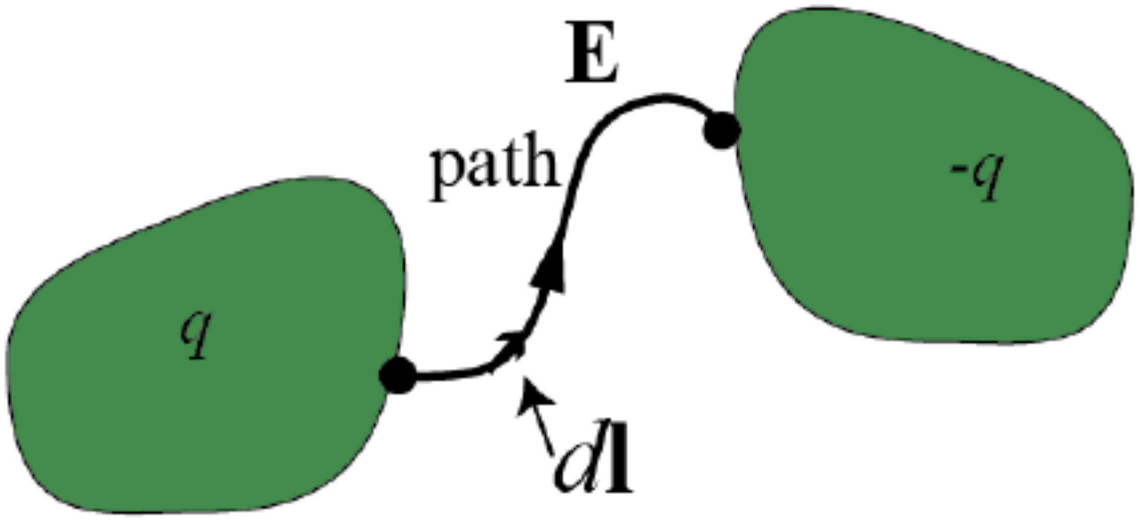
$$V = V_+ - V_- = -\int_{(-)}^{(+)} \mathbf{E}.\,d\mathbf{l}$$

Capacitance C is defined as: $C \equiv \dfrac{Q}{V}$

- Capacitance is the ability of a system to store electric charge.
- It is purely a geometric quantity.
- C is measured in farads (F), Coulomb/Volt.
- Practical units are microfarad (10^{-6}) or picofarad (10^{-12}).

Work needed to charge a Capacitor:

Two conductors with charge q and $-q$.



How much work needs to be done to increase the charge by dq

Recall $\boxed{\text{The work required to create a system of a point charge } Q\text{:} \quad W = QV(\mathbf{r})}$

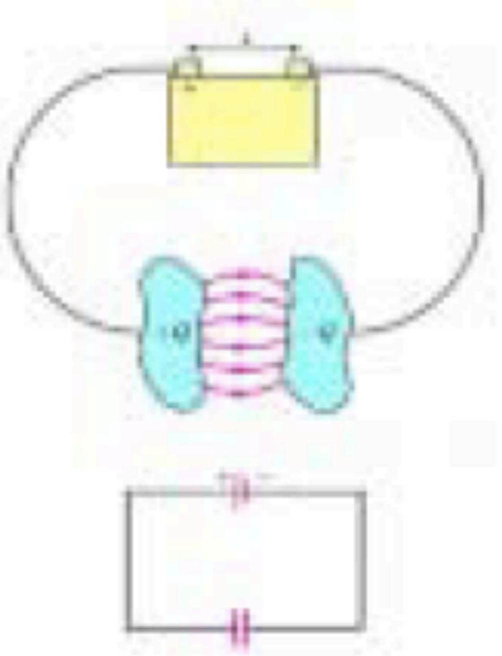
$$dW = V dq \;\; = \left(\frac{q}{C}\right) dq$$

The work necessary to go from $q = 0$ to $q = Q$ is

$$W = \int_0^Q \left(\frac{q}{C}\right) dq \;\; = \frac{1}{2}\frac{Q^2}{C} \;\; = \frac{1}{2}CV^2$$

The magnitude of the charge Q stored on either plate of a capacitor is directly proportional to the potential difference V between the plates. Therefore, we may write

$$Q = CV$$



Where C is a constant of proportionality called the capacitance of the capacitor.

The SI unit of a capacitance is the farad (F). 1Farad =1 coulomb/volt

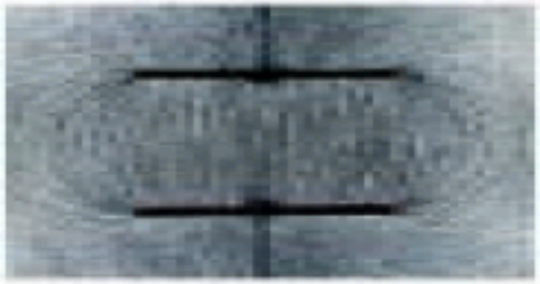
The capacitance of a capacitor depends on the ***geometry*** of the plates (their size, shape, and relative positions) and the ***medium*** (such as air, paper, or plastic) between them.

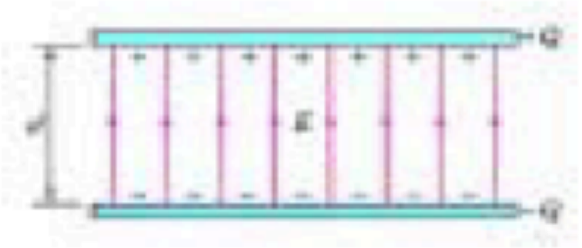
Parallel-plate capacitor

A common arrangement found in capacitors consists of two plates.

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \Rightarrow V = Ed = \frac{dQ}{\varepsilon_0 A} \quad \therefore \quad C = \frac{\varepsilon_0 A}{d}$$

where ε_0 is 8.85×10^{-12} F/m.





Example 2.10 A parallel-plate capacitor with a plate separation of 1 mm has a capacitance of 1 F. What is the area of each plate?

$$A = \frac{Cd}{\varepsilon_0} = \frac{1\times10^{-3}}{8.85\times10^{-12}} = 1.13\times10^{8}\ \mathrm{m}^2$$

What is the capacitance of an isolated sphere of radius R?

Solution:

$$V = \frac{Q}{4\pi\varepsilon_0 R} \Rightarrow C = 4\pi\varepsilon_0 R$$

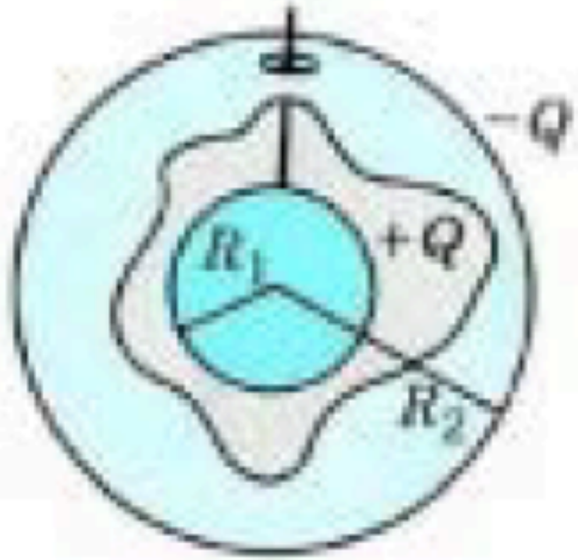
If we assume that earth is a conducting sphere of radius 6370 km, then its capacitance would be 710 uF.

A *spherical capacitor* consist of two concentric conducting spheres, as shown in the figure. The inner sphere, of radius R_1, has charge $+Q$, while the outer shell of radius R_2, has charge $-Q$. Find its capacitance.

Solution:

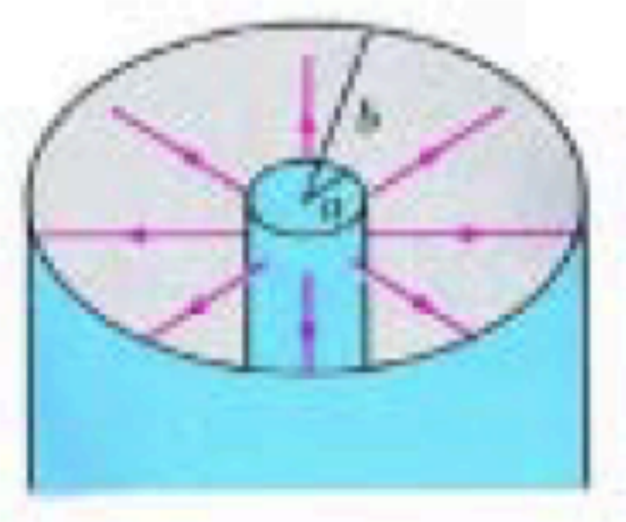
$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \Rightarrow V = -\int_{R1}^{R2} E dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

$$C = 4\pi\varepsilon_0\left(\frac{R_1 R_2}{R_2 - R_1}\right)$$



The capacitance happens to be negative quantity.

A *cylindrical capacitor* consists of a central conductor of radius a surrounded by a cylindrical shell of radius b, as shown below. Find the capacitance of a length L assuming that air is between the plates.



Solution:

$$E_r = \frac{\lambda L}{\varepsilon_0 2\pi r L} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$V_r = -\int_a^b E_r dr = -\frac{\lambda}{2\pi\varepsilon_0}\ln(\frac{b}{a})$$

$$= -\frac{Q}{2\pi\varepsilon_0 L}\ln(\frac{b}{a})$$

$$C = -\frac{2\pi\varepsilon_0 L}{\ln(b/a)}$$

- Capacitance C is defined as: $C \equiv \frac{Q}{V}$
- The work necessary to charge a capacitor upto charge Q: $W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$