POSSESSION OF MOBILES IN EXAM IS UFM PRACTICE

Name of Student: .... **Enrollment No.** ....
Department: ....

BENNETT UNIVERSITY, GREATER NOIDA
End Term Examination, SPRING SEMESTER 2018-19

COURSE CODE : EMAT102L MAX. DURATION: 2 hrs.
COURSE NAME: Linear Algebra and Ordinary Differential Equations

COURSE CREDIT: 3-1-0 MAX. MARKS: 50

Instructions:

- There are **eight** questions in this question paper and all questions are mandatory.
- Rough work must be carried out at the back of the answer script.
- Calculators are not allowed.

1. The following statements are true/false. Justify your answer.(Do any **four**) [2× 4=8]
   (a) $W = \{A \in M_{n\times n}(\mathbb{R}) : A \text{ is non-singular}\}$ is a subspace of $M_{n\times n}(\mathbb{R})$.
   (b) If the eigenvalues of a 3×3 matrix A are $2, i$, then $traceA = 3$, $detA = -2$.
   (c) Let $T : M_{3\times 4}(\mathbb{R}) \to M_{2\times 3}(\mathbb{R})$ be a linear transformation which is onto, then dimension of nullspace of T is 4.
   (d) The vectors $(2, 1, 0, 1)$ and $(-1, 2, i, 1)$ in $\mathbb{C}^4(\mathbb{R})$ are orthogonal.
   (e) If f, g both are continuous functions on $[0, 1]$, then
   $$\int_0^1 f(x)g(x)dx \geq \left(\int_0^1 |f(x)|^2 dx\right)^{\frac{1}{2}} \left(\int_0^1 |g(x)|^2 dx\right)^{\frac{1}{2}}.$$

2. Find an orthogonal basis for the subspace $W = \{p(x) \in \mathcal{P}_3(\mathbb{R}) : p(0) = p(1) = 0\}$, where the inner product is given by $\langle p, q\rangle = \int_{-1}^{1} p(x)q(x)dx$. [5]

3. Solve the differential equation $a\left(\frac{dy}{dx}\right) + by = ke^{-\lambda x}$, where a, b and k are positive constants and λ is a nonnegative constant. Also, show that [5]
   (a) if $\lambda = 0$, then every solution approaches to k/b as $x \to \infty$.
   (b) if $\lambda > 0$, every solution approaches to 0 as $x \to \infty$.

4. Attempt any **four** parts. $[2 \times 4 = 8]$

(a) Find the value of c for which the following differential equation is exact.

$$(4xe^{2y} + 3y)dx + (cx^2e^{2y} + 3x)dy = 0.$$

(b) Let y_1 and y_2 be any two linearly independent solutions of $y'' + a(x)y = 0, x \in (a, b)$ where $a(x)$ is continuous on (a, b). Find $W(y_1, y_2)$.

(c) If the two roots of a cubic auxiliary equation with real coefficients are $m_1 = 0,\ m_2 = 5+i$, then what is the corresponding homogeneous differential equation?

(d) Find the inverse Laplace transform of $\frac{1}{s(s+5)}$.

(e) Check whether the function $f(x, y) = \cos x + y^2$ satisfies Lipschitz condition or not in the region $R : |x| \leq 1, |y| \leq 1$.

5. (a) Find the general solution of $\frac{d^4y}{dx^4} - a^4y = 0$. [3]

(b) Find a matrix whose null space consists of all multiples of $(2, 3, 4, 1)$. [3]

OR

Let $y_1(x)$ and $y_2(x)$ be two solutions of $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + (\sec x)y = 0$ with Wronskian $W(x)$. If $y_1(0) = 1$, $y_1'(0) = 0$ and $W\left(\frac{1}{2}\right) = \frac{1}{3}$, then find $y_2'(0)$?

(c) Find the first three approximations using Picard's iterative method. [3]

$$\frac{dy}{dx} = xy,\ y(0) = 1.$$

6. If $y_1 = x^a$ is a solution of $x^2y'' - (2a-1)xy' + a^2y = 0,\ (x > 0, a \neq 0)$, then find the second linearly independent solution using the method of reduction of order. Hence find the general solution. [5]

7. Solve the differential equation $y'' - 4y = \sin x + e^{-2x}$. [5]

8. Let $x(t)$ be the solution of the initial value problem [5]

$$\frac{d^2x}{dt^2} + x = 6\cos 2t + t^2e^{2t},\ y(0) = 3,\ y'(0) = 1.$$

Let the Laplace transform of $x(t)$ be $X(s)$. Then find the value of $X(1)$.