EPHY105L - 9 1.11.21

The Electric field

Electric charge is quantised. Charge of any system can be written as $q = ne$

$e \simeq 1.602 \times 10^{-19}\,C$ $\equiv$ charge carried by one electron.

$n \equiv$ A multiple, "almost always" an integer (exception: quarks)

Electrostatics $\equiv$ Study of electric charges which are stationary.

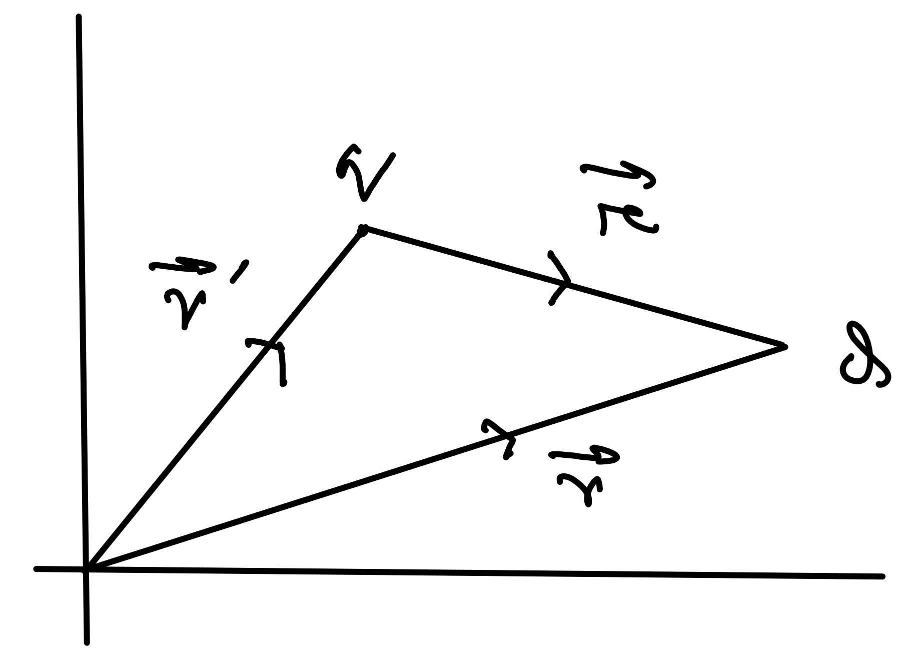
Coulomb's law

Force on a test q due to a single

point charge q at rest and distance 'r' away:

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q\,\mathcal{J}}{r^2}\,\hat{r}$$

$$\vec{r} = \vec{r} - \vec{r}\,'$$



ϵ_0 = Permittivity of free space. In SI unit, $\epsilon_0 = 8.85 \times 10^{-12}\,\frac{C^2}{N.m^2}$

Electric Field

Say, we have several such point charges $q_1, q_2, \ldots, q_n$ at distances r_1, r_2, r_3, ..., r_n away from $\mathcal{J}$.

Then the total force on $\mathcal{J}$:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n$$

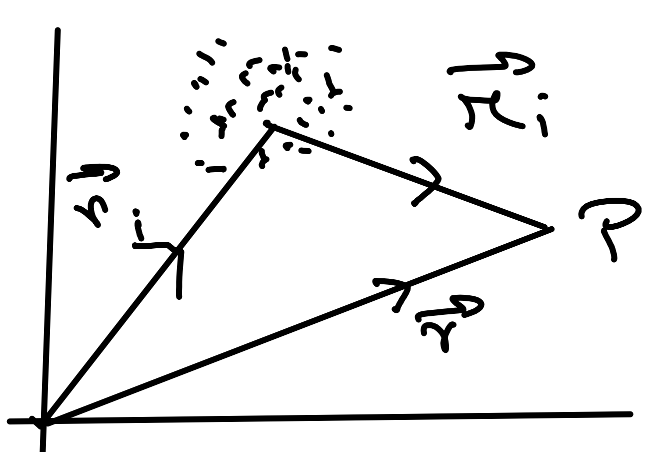
$$= \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 Q}{r_1^2}\hat{r}_1 + \frac{q_2 Q}{r_2^2}\hat{r}_2 + \cdots + \frac{q_n Q}{r_n^2}\hat{r}_n\right)$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \cdots + \frac{q_n}{r_n^2}\hat{r}_n\right)$$

$$\vec{F} = Q\vec{E}$$

↳ electric field

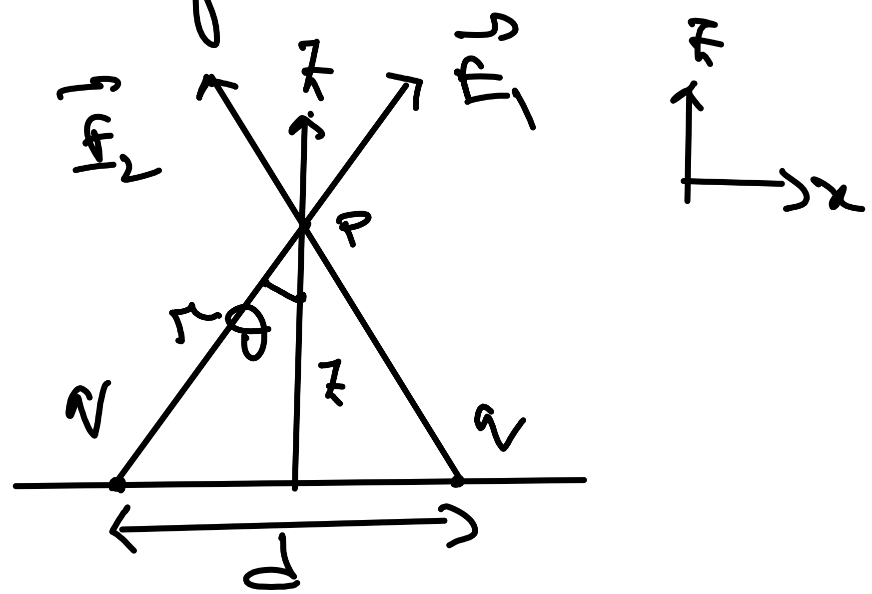
where, $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$



↳ Electric field of the source charges.

Physically, $E(\vec{r})$ is force per unit charge.

Ex



$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$\hat{z}$ x-components cancel and z-components add up.

$$E_z = 2 \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \cos\theta$$

$$r = \left[z^2 + \left(\frac{d}{2}\right)^2 \right]^{1/2}$$

$$\cos\theta = \frac{z}{r}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2qz}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}} \hat{z}$$

Far away from charge, $z \gg d$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2q}{z^2} \hat{z}$$

<u>Continuous charge distribution</u>

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{r}}{r^2} dq$$

* charge spread over a line

$dq = \lambda \, dl'$ ↳ small line element of source

λ ↳ charge per unit length

* charge spread over a surface

$dq = \sigma \, da'$ ↳ infinitesimal surface

σ ↳ charge per unit surface

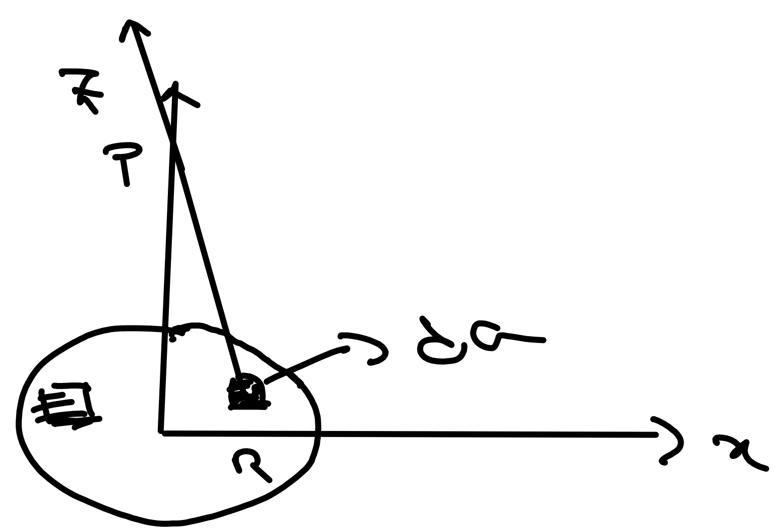
(1) charge spread over a volume

$$dq = \rho \, d\tau$$

$d\tau$ ↳ infinitesimal vol.

ρ ↳ charge density.

Ex



(2) uniform surface charge distn. (σ)

$$da = r\,dr\,d\phi$$

$$= 2\pi r\,dr$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int_0^R \frac{(2\pi r\,dr)\,\sigma}{(z^2 + r^2)^{3/2}}\, z \,\hat{z}$$

$$= \frac{1}{4\pi\epsilon_0} 2\pi\sigma z \left(\frac{1}{z} - \frac{1}{\sqrt{R^2 + z^2}} \right) \hat{z}$$