Name of Student: .... Enrollment No. ....
Department: ....

BENNETT UNIVERSITY, GREATER NOIDA
Supplementary Examination, July 2019

COURSE CODE : EMAT102L MAX. DURATION: 2 hrs.
COURSE NAME: Linear Algebra and Ordinary Differential Equations

COURSE CREDIT: 3-1-0 MAX. MARKS: 100

Instructions:

- There are **ten** questions in this question paper and all questions are mandatory.
- Rough work must be carried out at the back of the answer script.
- Calculators are not allowed.

1. For what values of $a, b \in \mathbb{R}$, the following system of equations [7]

$$x + y + 2z = 3, \ ay + z = 4, \ (b-2)z = 0.$$

has (i) no solution (ii) a unique solution and (iii) infinitely many solutions.

2. Find the **rank** and **row reduced echelon form** of the following matrix [5]

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 2 \\ 1 & 0 & 1 & 0 & 5 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix}$$

3. The following statements are true/false. Justify your answer(Do any six parts). [3× 6=18]

   (a) If the eigenvalues of a 3×3 matrix A are 1, $2i$, then $traceA = 3$, $detA = -2$.

   (b) The vectors $(1, 1, i)$ and $(0, i, 1)$ in $\mathbb{C}^3(\mathbb{R})$ are not orthogonal.

   (c) Eigenvectors corresponding to eigenvalues of a symmetric matrix are not orthogonal.

   (d) The set of functions x^2, $3x + 1$, $3x^2 + 6x + 2$ are linearly independent.

   (e) $T : \mathbb{R} \to \mathbb{R}$ defined as $T(x) = x + 2$ is a linear transformation.

   (f) The set $S = \{[x, \ y, \ z]^t \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 0\}$ is a subspace of $\mathbb{R}^3$.

   (g) The set $W = \{[x, \ y, \ z]^t \in \mathbb{R}^3 : ax + by + cz = 0\}$ is not a subspace of $\mathbb{R}^3$.

4. Attempt all parts. [$5 \times 4 = 20$]

   (a) Find a basis and dimension for the subspace $W = \{p(x) \in \mathcal{P}_3(\mathbb{R}) : p(-1) = p(1) = 0\}$.

   (b) Find a matrix/linear transformation whose null space consists of all multiples of $(1, 2, -5, 1)$.

(c) Find the determinant of the following matrix over $\mathbb{Z}_5$

$$M = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

(d) Apply Gram-Schmidt process to the set $\{[1,\ 1,\ 0]^t, [0,\ 0,\ 1]^t, [2,\ 1, 1]^t\}$ to obtain an orthonormal set in $\mathbb{R}^3$.

5. Test whether the differential equation $(x+y)^2dx - (y^2 - 2xy - x^2)dy = 0$ is exact or not and hence solve it. [6]

6. Discuss the existence and uniqueness of the solution for the IVP [6]

$$\frac{dy}{dx} = y^2 + x^2,\ y(0) = 0,\ |x| \leq 1,\ |y| \leq 1.$$

7. (a) Show that $y = (A/x) + B$ is a solution of $\dfrac{d^2y}{dx^2} + (2/x)\dfrac{dy}{dx} = 0$. [4]

   (b) Check whether $y_1(x) = e^x$ and $y_2(x) = xe^x$ are linearly independent solutions of the differential equation $y'' - 2y' + y = 0$, $x \in \mathbb{R}$ or not? [4]

   (c) Without solving, determine the Wronskian of two solutions for the following differential equation. [4]

$$t^2y'' - 2ty' - t^8y = 0,\ t \in (0, \infty).$$

8. Find the another linearly independent solution using the method of reduction of order. Also write the general solution. [6]

$$9y'' - 12y' + 4y = 0,\ y_1 = e^{\frac{2x}{3}}.$$

9. Solve the differential equation $y'' + y = \sin x$. [6]

10. (a) Find the Laplace transform of $e^{-t}\sin 3t$. [4]

   (b) Find the inverse Laplace transform of $\dfrac{1}{s(s+1)}$. [4]

   (c) Solve the following differential equation using Laplace transforms [6]

$$y' + 4y = e^t,\ y(0) = 2.$$