POSSESSION OF MOBILES IN EXAM IS UFM PRACTICE

Name of Student:——————————Enrollment Number: ——————

BENNETT UNIVERSITY, GREATER NOIDA
Supplementary Examination, August 2018

COURSE CODE :	EMAT102L	MAX. TIME: 2 Hours.
COURSE NAME :	Linear Algebra and Ordinary Differential Equations	
COURSE CREDIT:	3-1-0-4	MAX. MARKS: 100

Instructions

There are **eleven** questions in this question paper and all questions are mandatory. Rough work must be carried out at the back of the answer script.

1. For what values of $\lambda \in \mathbb{R}$, the following system of equations has (i) no solution, (ii) a unique solution, and (iii) infinitely many solutions?

$$(5-\lambda)x + 4y + 2z = 4, \quad 4x + (5-\lambda)y + 2z = 4, \quad 2x + 2y + (2-\lambda)z = 2.$$

Also, find the solutions whenever they exist. [10]

2. (a) Show that $W = \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \in \mathbb{R}^4 : w - z = y - x \right\}$ is a subspace of $\mathbb{R}^4$, spanned by $[1,0,0,-1]^t, [0,1,0,1]^t$ and $[0,0,1,1]^t$. [6]

   (b) Prove that e^x and e^{3x} are linearly independent over $\mathbb{R}$. [4]

3. (a) Show that $T : \mathbb{R} \to \mathbb{R}^2$ defined by $T(x) = [0\ x]^t$ for $x \in \mathbb{R}$ is one-one but not onto. [5]

   (b) Prove that if T is a linear map from $\mathbb{R}^4$ to $\mathbb{R}^2$ such that

$$\text{null}(T) = \left\{ [x_1\ x_2\ x_3\ x_4]^t \in \mathbb{R}^4 \ :\ x_1 = 5x_2 \text{ and } x_3 = 7x_4 \right\},$$

   then T is surjective. [5]

4. Let A be a 6×6 matrix with characteristic polynomial $p(\lambda) = (\lambda+1)(\lambda-1)^2(2-\lambda)^3$.

   (a) Prove that it is not possible to find three linearly independent vectors v_1, v_2, v_3 in $\mathbb{R}^6$ such that $Av_1 = v_1,\ Av_2 = v_2$ and $Av_3 = v_3$. [5]

   (b) If A is diagonalizable, find the dimensions of the eigenspaces E_{-1}, E_1 and E_2?[5]

5. On $\mathbb{P}_2(\mathbb{R})$, consider the inner product given by $f \cdot g = \int_0^1 f(x)g(x)dx$. Apply the Gram-Schmidt process to the basis $\{1, x, x^2\}$ to produce an orthonormal basis of $\mathbb{P}_2(\mathbb{R})$. [10]

6. Find the value of constant λ such that $(2xe^y + 3y^2)\frac{dy}{dx} + (3x^2 + \lambda e^y) = 0$ is exact. Further for this value of λ, solve the equation. [6]

7. Discuss the existence and uniqueness of the solution for the IVP [6]

$$\frac{dy}{dx} = y^2 + \sin(3x + y),\ y(0) = 0,\ |x| \leq 1,\ |y| \leq 1.$$

8. (a) Show that $y = a\cos(mx + b)$ is a solution of the differential equation [4]

$$\frac{d^2y}{dx^2} + m^2y = 0.$$

(b) Check whether the following functions are linearly dependent or linearly independent. [4]

$$f(x) = \sin 2x,\ g(x) = \sin x \cos x.$$

(c) Without solving, determine the Wronskian of two solutions for the following differential equation. [4]

$$t^4y'' - 2t^3y' - t^8y = 0,\ t \in (0, \infty).$$

9. Given that $y = e^x$ is a solution of [6]

$$xy'' - (2x - 1)y' + (x - 1)y = 0,$$

find the another linearly independent solution using the method of reduction of order. Also write the general solution.

10. Find the general solution of the following differential equation using the method of variation of parameters. [6]

$$y'' + y = \sin x$$

11. (a) Find the Laplace transform of $te^{-4t}\sin 3t$. [4]

(b) Find the inverse Laplace transform of $\frac{1}{s^2(s+2)}$. [4]

(c) Solve the following system of differential equations using Laplace transforms [6]

$$\frac{dx}{dt} - 6x + 3y = 8e^t,$$

$$\frac{dy}{dt} - 2x - y = 4e^t$$

with the initial conditions $x(0) = -1,\ y(0) = 0$.