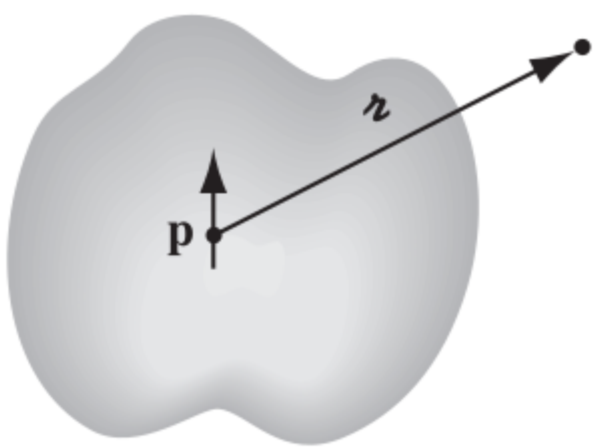
Polarization ($\vec{P}$)

$\vec{P} \equiv$ Dipole moment per unit volume

Field due to a Polarized object



* We have a polarized material. All the dipoles are pointing in the same direction.

For a single dipole $\vec{p}$,

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p}\cdot\hat{r}}{r^2}$$

$\vec{r} \equiv$ vector from the dipole to the reference point.

Dipole moment: $\vec{p} = \underline{\vec{P}\, d\tau'}$ for each infinitesimal volume element $d\tau'$.

→ The total potential,

$$V = \frac{1}{4\pi\epsilon_0} \int_V \frac{\vec{P}(\vec{r}\,') \cdot \hat{\mathscr{r}}}{\mathscr{r}^2} \, d\tau'$$

$$= \frac{1}{4\pi\epsilon_0} \int_V \vec{P} \cdot \vec{\nabla}'\left(\frac{1}{\mathscr{r}}\right) d\tau'$$

$$\vec{\nabla}'\left(\frac{1}{\mathscr{r}}\right) = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\Rightarrow V = \frac{1}{4\pi\epsilon_0} \left[\int_V \vec{\nabla}' \cdot \left(\frac{\vec{P}}{\mathscr{r}}\right) d\tau' - \int_V \frac{1}{\mathscr{r}} (\vec{\nabla} \cdot \vec{P}) \, d\tau' \right]$$

Using divergence theorem,

$$V = \underbrace{\frac{1}{4\pi\epsilon_0} \oint_S \frac{1}{r} \vec{P} \cdot d\vec{a}'}_{\text{Potential due to a surface charge distribution}} - \underbrace{\frac{1}{4\pi\epsilon_0} \int_V \frac{1}{r} (\vec{\nabla}' \cdot \vec{P})\, d\tau'}_{\text{Potential due to a volume charge distribution}}$$

charge density: $\underline{\sigma_b \equiv \vec{P} \cdot \hat{n}}$

$\underline{\rho_b \equiv -\vec{\nabla}' \cdot \vec{P}}$

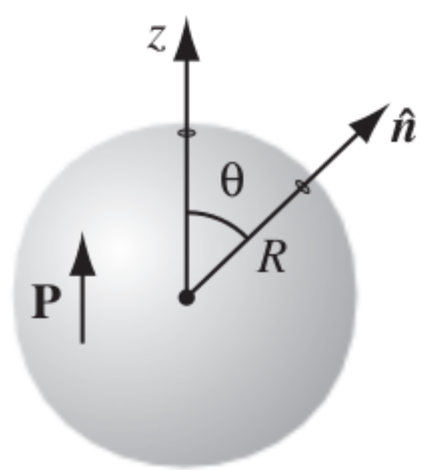
Hence,

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \oint_S \frac{\sigma_b}{r}\, da' + \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho_b}{r}\, d\tau'$$

→ Potential of a polarised object is the same as that produced by a volume and a surface charge densities.

These are called <u>bound</u> <u>charges</u>.

<u>Ex:</u>



Electric field produced by a uniformly polarized sphere of radius 'R'.

z-axis coincides with $\vec{P}$

→ if $\vec{P}$ is uniform, $\rho_b = 0$

The surface bound charge density,

$$\sigma_b = \vec{P} \cdot \hat{n} = P\cos\theta$$

We need to find the potential of a system with surface charge density $P\cos\theta$

$$V(r,\theta) = \frac{P r \cos\theta}{3\epsilon_0} \quad \text{for } r \leq R$$

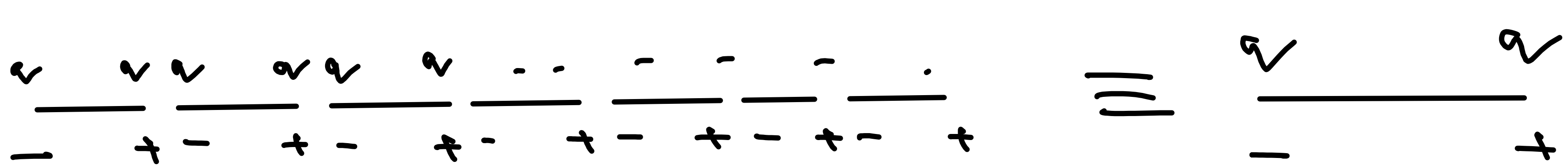
$$= \frac{P R^3 \cos\theta}{3\epsilon_0 r^2} \quad \text{for } r \geq R$$

We know, $z = r\cos\theta$

for $r < R$,

$$\vec{E} = -\vec{\nabla} V = -\frac{P}{3\epsilon_0}\hat{z} = -\frac{\vec{P}}{3\epsilon_0}$$

Physical interpretation of bound charge.

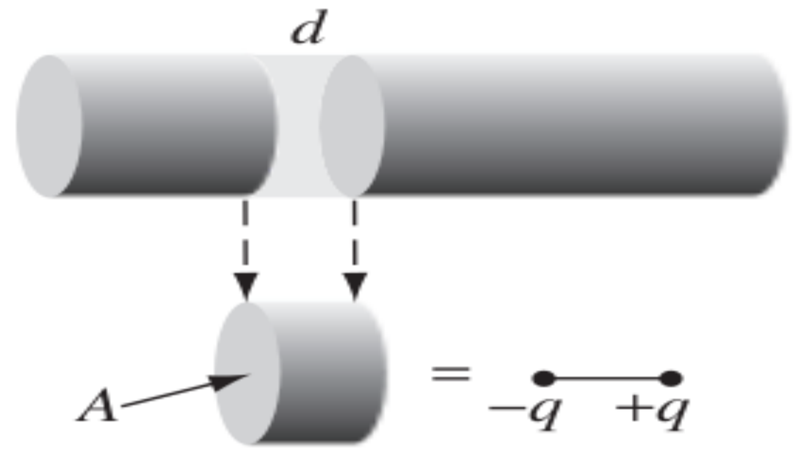


$\longrightarrow$ Bound charges.

↳ String of dipoles made up of same amount of +ve and -ve charges.

To calculate the amount of bound charges resulting from a polarization:

(+) A tube of dielectric parallel to $\vec{P}$



$\vec{P}$ $(\hat{x})$

On the small chunk, the dipole moment

$= P(Ad)$

A: Cross-section of the sides

d: length of the chunk

In terms of charges at the end, dipole moment $= qd$

$A = A_{end} \cos\theta$

→ The charges on the end are still the same, but

$$\sigma_b = \frac{q}{A_{end}} = \frac{q\cos\theta}{A} = P\cos\theta$$

Then the bound charge

$$P V = q d$$

$$\Rightarrow q = \frac{P V}{d} = \frac{P A d}{d}$$

$$\Rightarrow q = P A$$

$$\sigma_b = \frac{q}{A} = P = \vec{P} \cdot \hat{n}$$

$$= \vec{P} \cdot \hat{n}$$

The net effect of polarisation is to create a bound charge density $\boxed{\sigma_b = \vec{P} \cdot \hat{n}}$ over the surface of the material.