***POSSESSION OF MOBILES IN EXAM IS UFM PRACTICE***

**Name of student** ____________________ **Enrolment No.**____ ______

**BENNETT UNIVERSITY, GREATER NOIDA**

**B.TECH/ TEST – End Term Examination: FALL SEMESTER A.Y. 2018-2019**

| | | |
|---|---|---|
| COURSE CODE | **EPHY105L/EPHY103L** | MAX. TIME: **2 hours** |
| COURSE NAME | : **Electromagnetics** | |
| COURSE CREDIT: | **3** | MAX . MARKS: **50** |

**ALL QUESTIONS ARE COMPULSORY**

1. Give brief answers with appropriate reasons to the following questions: **(8x2=16)**
   a) A positive charge of 1 μC is placed at the center of a cavity formed inside a *spherical conducting shell* having an inner radius 0.2 m and an outer radius 1 m. What is the charge density on the inner surface of the sphere?
   b) A sphere of radius $R$ carries a charge density given by $\rho(r) = \rho_0(1 - r^2/R^2)$. What is the value of $\nabla.\vec{\boldsymbol{E}}$ at a point at a distance $R/2$ from the center?
   c) A dielectric sphere of radius $R$ and dielectric constant $K$ carries a polarization given by $\vec{\boldsymbol{P}} = P_0\vec{\boldsymbol{r}}$ where $P_0$ is a constant. Obtain the bound surface and volume charge densities.
   d) Determine whether the vector function $\vec{\boldsymbol{G}} = x^2\hat{\mathbf{x}} + 3xz^2\hat{\mathbf{y}} + 2xz\hat{\mathbf{z}}$ can represent a magnetic field.
   e) A circular current loop having a radius $R$ and carrying a current $I$ is placed at the center of a sphere of radius $2R$. What is the net magnetic flux passing through the sphere?
   f) A cylinder of circular cross section of radius $R$ and length $L$ is uniformly magnetized with magnetization $\vec{\boldsymbol{M}} = M_0\hat{\mathbf{z}}$ parallel to the axis of the cylinder. Obtain the corresponding bound surface current densities on the cylindrical and plane end surfaces.
   g) Verify whether the following two vector potentials $\vec{\boldsymbol{A}}_1$ and $\vec{\boldsymbol{A}}_2 = \vec{\boldsymbol{A}}_1 + (y\hat{\mathbf{x}} + x\hat{\mathbf{y}})$ correspond to the same magnetic field.
   h) An infinitely long straight solenoid with circular cross section of radius $R$ has a cylindrical rod of radius $R/2$ placed coaxially with the solenoid. If the magnetic susceptibility of the rod is $\chi_m$ what is the ratio of $\vec{\boldsymbol{H}}$ in the rod to $\vec{\boldsymbol{H}}$ in the air gap?

2. A point charge $Q$ is placed at the center of a sphere which has free space in the region $0 < r < R_1$ and a linear, homogeneous dielectric with susceptibility $\chi_e$ for $R_1 < r < R_2$ and free space for $r > R_2$.
   a) Using Gauss's law find the electric field in all regions. **(4)**
   b) Obtain all the bound surface and volume charge densities in the dielectric. **(3)**
   c) What is the value of $\nabla.\vec{\boldsymbol{D}}$ at a point $r_0$ with $R_1 < r_0 < R_2$? **(1)**

**P.T.O**

3. A infinitely long straight cylindrical wire of radius $R$ made of a material with magnetic susceptibility $\chi_m$ carries a current $I$ which is uniformly distributed across its cross section.
   a) Using Ampere's law, obtain the magnetic field $\vec{B}$ in the regions $r < R$ and $r > R$. **(4)**
   b) Obtain the surface bound currents on the wire. **(3)**
   c) What is the value of $\nabla.\vec{B}$ at a distance $R/2$ from the axis of the cylinder? **(1)**

4. Consider an infinitely long hollow solenoid of radius $R$ having $n$ turns per unit length. The current in the solenoid is varied with time and given by $I(t) = I_0 \sin \omega t$. Assuming that the induced electric field is along $\hat{\boldsymbol{\phi}}$ direction,
   a) Obtain the induced electric field $\vec{E}$ both within and outside the solenoid. **(3)**
   b) What will be the values of $\nabla.\vec{E}$ and $\nabla \times \vec{E}$ inside and outside the solenoid? **(3)**

5. A parallel plate capacitor with circular plates with radius $R$ and free space between the capacitor plates is being charged with a time dependent current given by $I(t)$.
   a) Obtain the displacement current density flowing between the plates of the capacitor. **(3)**
   b) Show that the total displacement current is equal to the conduction current $I(t)$. **(2)**

   Given that the displacement current density is $\vec{J}_D = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$ where symbols have their usual meaning.

6. An electromagnetic wave propagating in free space (velocity of the wave is 3 x $10^8$ m/s) is described by the following expression for the electric field ($y$ is measured in meters):
$$\vec{E} = E_0 \hat{\boldsymbol{x}} \cos[2\pi(10^6 y - vt)]$$
   a) What are the values of frequency and wavelength of the wave? **(2)**
   b) What is the direction of propagation of the wave? **(2)**
   c) Given that the corresponding magnetic field of the wave is $\vec{B} = \vec{B}_0 \cos[2\pi(10^6 y - vt)]$, using Maxwell's equations obtain the magnitude and direction of $\vec{B}_0$. **(3)**

**Some useful formulas**

- In Cartesian coordinates:

$$\nabla f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}$$

$$\nabla.\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\nabla \times \vec{F} = \left[\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right]\hat{\imath} + \left[\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right]\hat{\jmath} + \left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right]\hat{k}$$

- In spherical polar coordinates:

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\nabla.\vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$$

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- $\epsilon_0 = 8.85 \times 10^{-12}$ $C^2/N\text{-}m^2$

- Maxwell's equations:

$$\nabla.\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla.\vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial \vec{E}}{\partial t}$$

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