BENNETT UNIVERSITY
TIMES OF INDIA GROUP

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Name of Student -- Enrolment No. --------------------

Department / School -----------------------------

BENNETT UNIVERSITY, GREATER NOIDA

End Term Examination, Fall SEMESTER 2019-20

COURSE CODE: **EPHY203L** MAX. DURATION: **3 HOURS**

COURSE NAME: **ELECTRODYNAMICS** MAX. MARKS: **40**

Q1) Ampere's law of magnetostatics is given as $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$.

(i) Explain why it is not correct in electrodynamics.

(ii) Obtain Maxwell's correction by using equation of continuity.

- **4 Marks**

Q2)

(i) Write down electric and magnetic fields of a plane monochromatic wave travelling in the z-direction and polarized in the x-direction. How are their amplitudes related?

(ii) Consider EM waves at *normal incidence* between two linear media. Their *incident*, *reflected*, and *transmitted* amplitudes are related by $\tilde{E}_{0R} = \frac{1-\beta}{1+\beta}\tilde{E}_{0I}$, $\tilde{E}_{0T} = \frac{2}{1+\beta}\tilde{E}_{0I}$, where $\beta = \frac{\mu_1 n_2}{\mu_2 n1}$. Obtain expressions for *reflection* (R) and *transmission* (T) *coefficients* and show that they add to 1. (You may assume $\mu_1 \approx \mu_2$.)

(iii) Calculate R and T for air to glass interface.

- **6 Marks**

Q3) The electric field components of a TE$_{mn}$ mode of a rectangular wave guide of dimensions a and b ($a \geq b$) are given as $E_x = -\frac{i\omega}{\left(\frac{\omega}{c}\right)^2 - k^2}\left(\frac{n\pi}{b}\right) B_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{i(kz-\omega t)}$,

$E_y = \frac{i\omega}{\left(\frac{\omega}{c}\right)^2 - k^2}\left(\frac{m\pi}{a}\right) B_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{i(kz-\omega t)}$, and $E_z = 0$,

where $k = \sqrt{\left(\frac{\omega}{c}\right)^2 - \pi^2\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]}$.

(i) Which TE mode has lowest cutoff frequency?

(ii) Find the magnetic field components of this mode (with lowest cutoff frequency).

P. T. O.

(iii) If the dimensions of the wave guide are 5.1×3.6 cm, what is its lowest cutoff frequency?

(iv) If the driving frequency is 0.5×10^{10} Hz, what TE modes will propagate in this wave guide?

\- **6 Marks**

Q4)

(i) Convert Maxwell's equations into "potential" formalism. You will get two equations in terms of scalar potential, vector potential, charge density, and current density.

(ii) Simplify them for the cases of (a) Coulomb gauge and (b) Lorenz gauge.
[You will get inhomogeneous wave equations in the Lorenz gauge]

\- **8 Marks**

Q5) The transformation rules for electric and magnetic fields are given as $\bar{E}_x = E_x$, $\bar{E}_y = \gamma(E_y - vB_z)$, $\bar{E}_z = \gamma(E_z + vB_y)$, $\bar{B}_x = B_x$, $\bar{B}_y = \gamma(B_y + \frac{v}{c^2}E_z)$, $\bar{B}_z = \gamma(B_z - \frac{v}{c^2}E_y)$.

(i) Show that $\vec{E} \cdot \vec{B}$ is relativistically invariant.

(ii) Find the electric field of a point charge moving with uniform velocity along x-axis.

\- **8 Marks**

Q6) The scalar and vector potentials of an oscillating magnetic dipole are given as $V = 0$ and $\vec{A}(r,\theta,t) = -\frac{\mu_0 m_0 \omega}{4\pi c}\left(\frac{\sin\theta}{r}\right)\sin\left[\omega\left(t - \frac{r}{c}\right)\right]\hat{\phi}$.

(i) Find the corresponding electric and magnetic fields. Use *approximation 3*: $r \gg \frac{c}{\omega}$.

(ii) Find the intensity and total power radiated by this dipole.

(iii) What is the intensity of the radiation along the axis of the dipole?

\- **8 Marks**

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

(2) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$

(5) $\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$

(6) $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

(7) $\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$

(8) $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$

Second Derivatives

(9) $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

(10) $\nabla \times (\nabla f) = 0$

(11) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem : $\int_{\mathbf{a}}^{\mathbf{b}} (\nabla f) \cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem : $\int (\nabla \cdot \mathbf{A})\, d\tau = \oint \mathbf{A} \cdot d\mathbf{a}$

Curl Theorem : $\int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$; $\quad d\tau = dx\,dy\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$; $\quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient : $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Curl : $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}}$$
$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$; $\quad d\tau = s\,ds\,d\phi\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$$