

Enrollment No.: _____

Name: _____

Department/School: _____

Mid-Semester Examination, Even Semester 2022-23

Course Code: EMAT102L

Maximum Time Duration: 1 hour

Course Name: Linear Algebra and ODEs

Maximum Marks: 15

GENERAL INSTRUCTIONS:

1. Do not write anything on the question paper except name, enrollment number and department/school.
2. Carrying mobile phone, smart watch and any other non-permissible materials in the examination hall is an act of UFM.

1. Let $S = \{(x, y, z) \in \mathbb{R}^3 : x + y = 0 \text{ or } y - z = 0\}$. Then check whether S forms a subspace of \mathbb{R}^3 with respect to the usual addition and scalar multiplication operations over \mathbb{R} . [2 marks]

$(1, 0, 0) \in S$

2. Find all the values of a, b and c such that the matrix A (whose all entries are real) is in reduced row echelon form (RREF) [2 marks]

$$A = \begin{pmatrix} 0 & a & 0 & 2 & 0 \\ 0 & 0 & b & c & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \Rightarrow \quad a < b = 1$$

$\sqrt{c} = 1R$

3. Find the values of λ and μ such that the following system of linear equations have an infinite number of solutions. [2 marks]

$$x + y + z = 6, \quad x + 2y + 3z = 10, \quad x + 2y + \lambda z = \mu$$

4. Determine whether the subset $\{(1, 1, 1, 1), (1, -1, 1, -1), (1, 1, -1, -1)\}$ of the vector space \mathbb{R}^4 are linearly dependent or linearly independent [2 marks]

5. Find the range space and the rank of the linear transformation [2 marks]

$$T : \mathbb{R}^3 \rightarrow \mathbb{R}^2 \text{ given by } T(x, y, z) = (x + y, 0).$$

6. Find a basis and the dimension of the vector space of all 2×2 skew symmetric matrices [1 marks]

7. Find the null space of the linear transformation [1 marks]

$$T : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \text{ given by } T(x, y) = (x - y, 2x - 2y).$$

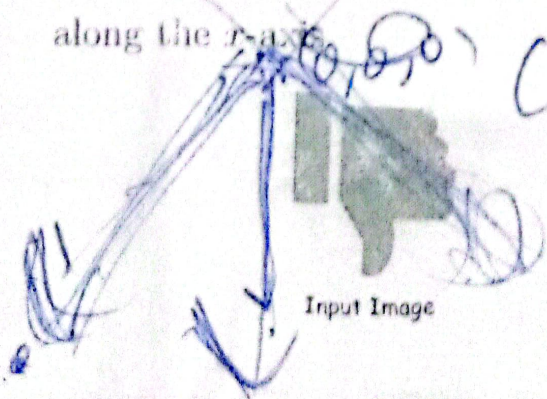
8. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 4 & 5 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 6 & 9 & 12 \end{pmatrix}$. Find the elementary matrix E such that $EA = B$. [1 mark]

$$\text{Row } 3 \leftarrow 0$$

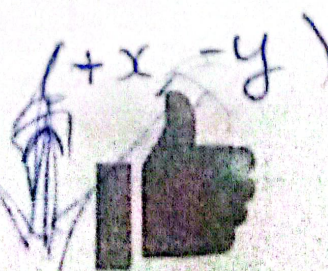
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

9. Let $S = \{(1, 2, 1), (1, 0, 1)\}$ be a set of vectors of the vector space \mathbb{R}^3 . Determine whether the vector $(2, 2, 2)$ belongs to $\text{Span}(S)$ or not. Justify your answer. [1 mark]

10. Find the linear transformation T from \mathbb{R}^2 to \mathbb{R}^2 which flips the below image vertically along the x -axis. [1 mark]



Input Image



Output Image

$$+x, -y$$