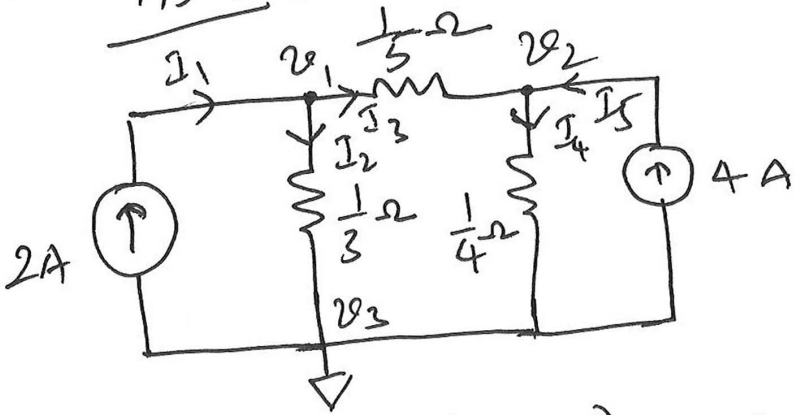
# TUTORIAL SHEET 03 SOLUTIONS



1. Fig. 1:




$v_3 = 0$

$I_1 + I_2 + I_3$

$I_3 + I_5 = I_4$

$$\frac{v_1}{Y_3} + \frac{(v_1 - v_2)}{(Y_5)} = 2 \quad -(1)$$

$$\frac{v_2}{Y_4} + \frac{v_2 - v_1}{Y_5} = 4 \quad -(2)$$

$$\begin{bmatrix} 8 & -5 \\ -5 & 9 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$v_1 = 0.8085$ V

$v_2 = 0.8936$ V

Current through 5 Ω resistor = −0.4255 A

Current through 3 Ω resistor = 2.4258 A

Current through 4 Ω resistor = 3.5744 A

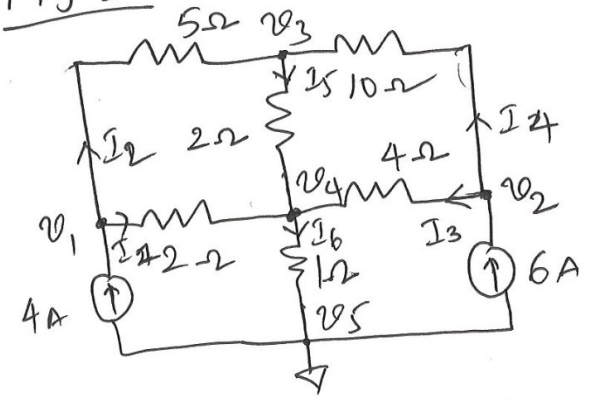
Voltage across 3 Ω resistor = 0.8085 V

Voltage across 4 Ω resistor = 0.8936 V

Voltage across 1/5 Ω resistor = (0.8085 − 0.8936)

= −0.0851 V

Fig. 2




$\left(\frac{1}{2}+\frac{1}{5}\right)v_1 - \frac{1}{5}v_3 - \frac{1}{2}v_4 = 4$ KCL at node $v_1$

$\left(\frac{1}{4}+\frac{1}{10}\right)v_2 - \frac{1}{10}v_3 - \frac{1}{4}v_4 = 6$ KCL at node $v_2$

$-\frac{1}{5}v_1 - \frac{1}{10}v_2 \left(\frac{1}{5}+\frac{1}{2}+\frac{1}{10}\right)v_3 - \frac{1}{2}v_4 = 0$ KCL at node $v_3$

$-\frac{1}{2}v_1 - \frac{1}{4}v_2 - \frac{1}{2}v_3 + v_4\left(\frac{1}{2}+\frac{1}{2}+1+\frac{1}{4}\right) = 0$ KCL at node $v_4$

$$0.7v_1 + 0.0v_2 - 0.2v_3 - 0.5v_4 = 4$$
$$0.35v_2 - 0.1v_3 - 0.25v_4 = 6$$
$$-0.2v_1 - 0.10v_2 + 0.8v_3 - 0.50v_4 = 0$$
$$-0.5v_1 - 0.25v_2 - 0.5v_3 + 2.25v_4 = 0$$

$$\begin{bmatrix} 0.7 & 0 & -0.2 & -0.5 \\ 0 & 0.35 & -0.1 & -0.25 \\ -0.2 & -0.10 & +0.8 & -0.5 \\ -0.5 & -0.25 & -0.5 & 2.25 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 0 \\ 0 \end{bmatrix}$$

(3)

$v_1 = 16.8571\ V$ $\quad v_2 = 28.2857\ V$

$v_3 = 14\ V$ $\quad v_4 = 10\ V$

$I_1 = \frac{1}{2}(v_1 - v_4) = 3.43\ A$

$I_2 = \frac{1}{5}(v_1 - v_3) = 0.57\ A$

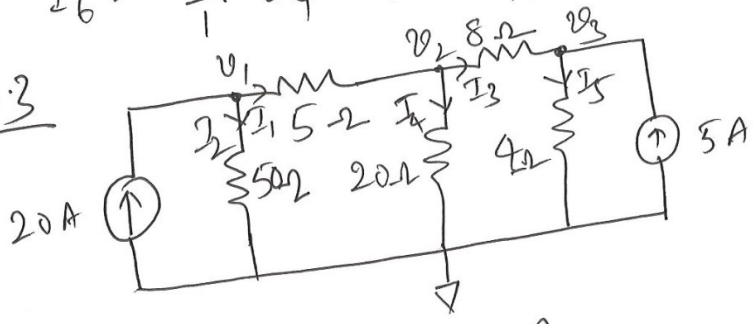
$I_3 = \frac{1}{4}(v_2 - v_4) = 4.57\ A$

$I_4 = \frac{1}{2}(v_2 - v_3) = 1.43\ A$

$I_5 = \frac{1}{2}(v_3 - v_4) = 2\ A$

$I_6 = \frac{1}{1} \cdot v_4 = 10\ A$

Fig 3




At node $v_1$ $\quad I_1 + I_2 = 20\ A$

$$\left(\frac{1}{5} + \frac{1}{50}\right)v_1 - \frac{1}{5}v_2 = 20 \quad -(1)$$

At node $v_2$ $\quad I_1 = I_3 + I_4$

$$-\frac{1}{5}v_1 + \left(\frac{1}{5} + \frac{1}{8} + \frac{1}{20}\right)v_2 - \frac{1}{8}v_3 = 0 \quad (2)$$

(4)

At node $v_3$

$$I_3 + 5 = I_4$$

$$-\frac{1}{8}v_2 + \left(\frac{1}{4}+\frac{1}{8}\right)v_3 = 5 \quad -(3)$$

$$\begin{bmatrix} 0.22 & -0.2 & 0 \\ -0.2 & 0.375 & -0.125 \\ 0 & -0.125 & 0.375 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 5 \end{bmatrix}$$

$\mathbf{v_1}$= 210 V, $\mathbf{v_2}$= 131 V, $\mathbf{v_3}$= 57 V

$$I_1 = \frac{v_1 - v_2}{5} = \frac{210-131}{5} = 15.8 \text{ A}$$

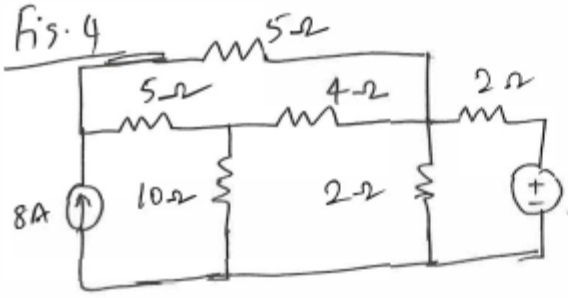
$$I_2 = \frac{v_1}{50} = \frac{210}{50} = 4.2 \text{ A}$$

$$I_3 = \frac{v_2 - v_3}{8} = \frac{131-57}{8} = 9.25 \text{ A}$$

$$I_4 = \frac{v_2}{20} = \frac{131}{20} = 6.55 \text{ A}$$

$$I_5 = \frac{v_3}{4} = \frac{57}{4} = 14.25 \text{ A}$$

Fig. 4

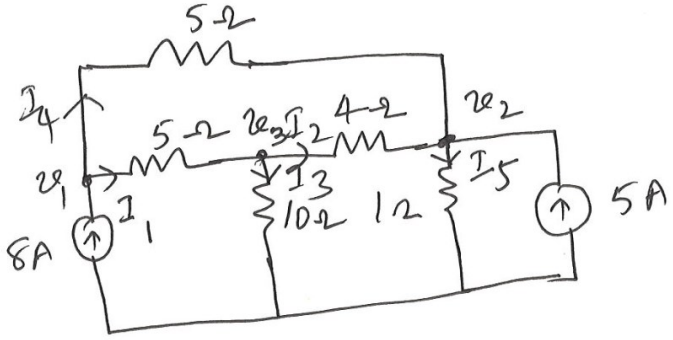


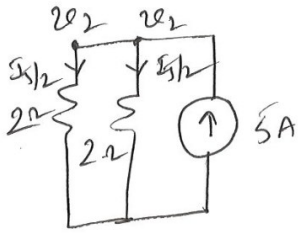

Using Source transformation

$I_{s1} = \frac{10}{2} = 5A$

$2\Omega // 2\Omega = 1\Omega$

(5)




$$\left(\frac{1}{5}+\frac{1}{5}\right)v_1 - \frac{1}{5}v_2 - \frac{1}{5}v_3 = 8 \quad -①$$

$$-\frac{1}{5}v_1 \quad \left(\frac{1}{1}+\frac{1}{4}+\frac{1}{5}\right)v_2 - \frac{1}{4}v_3 = 5 \quad -②$$

$$-\frac{1}{5}v_1 \quad -\frac{1}{4}v_2 + \left(\frac{1}{5}+\frac{1}{10}+\frac{1}{4}\right)v_3 = 0 \quad -③$$

$$\begin{bmatrix} 0.4 & -0.2 & -0.2 \\ -0.2 & 1.45 & -0.25 \\ -0.2 & -0.25 & 0.55 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 0 \end{bmatrix}$$

$v_1 = 34.43\ V \quad v_2 = 11.24\ V \quad v_3 = 17.63\ V$

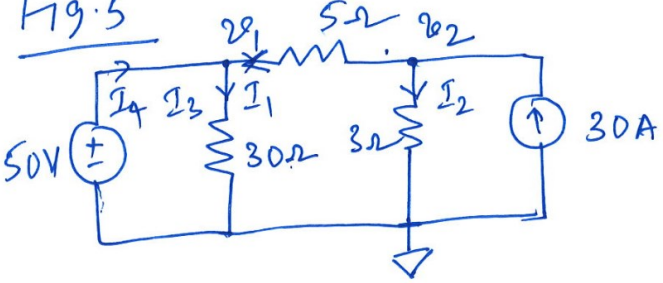
$I_1 = \frac{v_1 - v_3}{5} = 3.36A \quad I_2 = \frac{v_3 - v_2}{4} = 1.5975\ A$

$I_3 = \frac{v_3}{10} = 1.763A \quad I_4 = \frac{v_1 - v_2}{5} = 4.838\ A$

$I_5 = \frac{v_2}{2\Omega} = \frac{11.24}{2}A = 5.62A$

$\frac{I_5}{2} = \underline{5.62\ A}$ (Current through 2Ω resistor)

Fig·5




At node $v_1$, $v_1 = 50$ V.

At node $v_2$, $I_1 + I_2 = 30$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{3} = 30$$

$v_1 = 50$ $\quad \left(\frac{1}{5} + \frac{1}{3}\right) v_2 = 40$

$$v_2 = 75 \text{ V}$$

$$I_2 = \frac{75}{3} = 25 \text{ A}$$

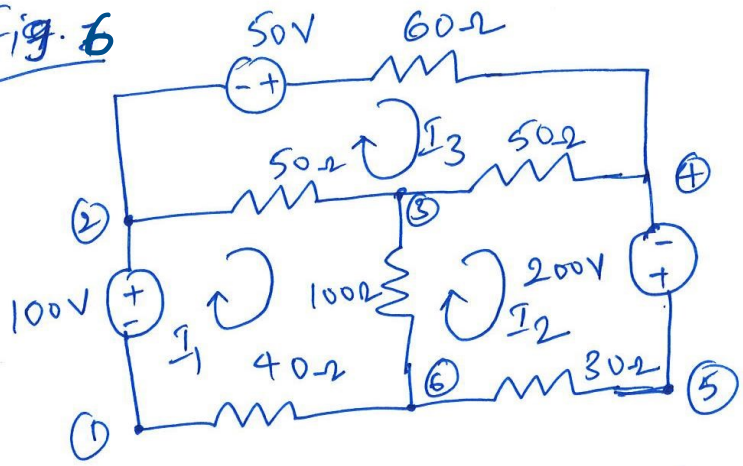
$$I_1 = \frac{75 - 50}{5} = 5 \text{ A}$$

$$I_3 = \frac{v_1}{30} = \frac{50}{30} = 1.67 \text{ A}$$

$I_1 + I_4 = I_3 \Rightarrow I_4 = I_3 - I_1 = \underline{3.33 \text{ A}}$

2/

Fig. 6




Applying KVL to mesh 1-2-3-6-1

$$40I_1 + 50(I_1 - I_3) + 100(I_1 - I_2) = 100$$

$$190I_1 - 100I_2 - 50I_3 = 100 \quad -(1)$$

Applying KVL to mesh 6-3-4-5-6

$$30I_2 + 100(I_2 - I_1) + 50(I_2 - I_3) = 200$$

$$-100I_1 + 180I_2 - 50I_3 = 200 \quad -(2)$$

Applying KVL to mesh 2-4-3-2

$$50(I_3 - I_1) + 60I_3 + 50(I_3 - I_2) = 50$$

(9)

$$-50I_1 - 50I_2 + 160I_3 = 50 \quad -(3)$$

$$\begin{bmatrix} 190 & -100 & -50 \\ -100 & 180 & -50 \\ -50 & -50 & 160 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 50 \end{bmatrix}$$

$I_1 = 2.873\ A \quad I_2 = 3.33\ A \quad I_3 = 2.252A$

$I_{40\Omega} = 2.873A \quad V_{40\Omega} = 114.92\ V$

$I_{100\Omega} = I_1 - I_2 = -0.46\ A \quad V_{100\Omega} = -46\ V$

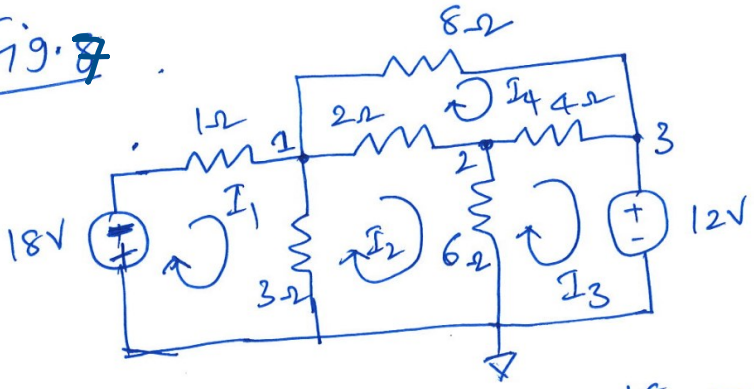
$I_{60\Omega} = 2.252\ A \quad V_{60\Omega} = 135.12\ V$

$I_{50\Omega} = I_3 - I_2 = -1.081A \quad V_{50\Omega} = -54.05\ V$

$I_{50\Omega} = I_3 - I_1 = -0.621A \quad V_{50\Omega} = -31.05\ V$

$I_{30\Omega} = 3.333A \quad V_{30\Omega} = 100\ V$

Fig. 8




Mesh 1: $4I_1 - 3I_2 = -18 \quad -(1)$

Mesh 2: $-8I_1 + 11I_2 - 6I_3 - 2I_4 = 0 \quad -(2)$

Mesh 3: $-6I_2 + 10I_3 - 4I_4 = -12 \quad (3)$

Mesh 4:

$$-2I_2 - 4I_3 + 14I_4 = 0 \quad -④$$

$$\begin{bmatrix} 4 & -3 & 0 & 0 \\ -3 & 11 & -6 & -2 \\ 0 & -6 & 10 & -4 \\ 0 & -2 & -4 & 14 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -18 \\ 0 \\ -12 \\ 0 \end{bmatrix}$$

$I_1 = -9.182\ A \quad I_2 = -6.243\ A$

$I_3 = -5.987\ A \quad I_4 = -2.602\ A$

$I_{1\Omega} = -9.182\ A \quad V_{1\Omega} = -9.182\ V$

$V_{3\Omega} = I_1 - I_2 = -2.939\ A \quad V_{3\Omega} = -8.817\ V$

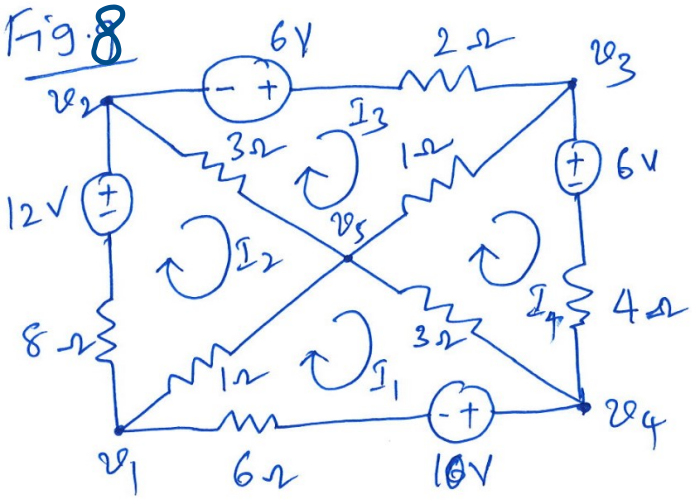
$I_{6\Omega} = I_2 - I_3 = -0.256\ A \quad V_{6\Omega} = -1.536\ V$

$I_{8\Omega} = -2.602\ A \quad V_{8\Omega} = -20.816\ V$

$I_{2\Omega} = I_4 - I_2 = +3.641\ A, \quad V_{2\Omega} = +9.884\ V$

$I_{4\Omega} = I_4 - I_3 = 3.385\ A \quad V_{4\Omega} = 13.54\ V$

Fig. 8



(11)

Mesh 1: $10I_1 - I_2 - 3I_4 = -10$

Mesh 2: $-I_1 + 12I_2 - 3I_3 = 12$

Mesh 3: $-3I_2 + 6I_3 - I_4 = 6$

Mesh 4: $-3I_1 - I_3 + 8I_4 = -6$

$$\begin{bmatrix} 10 & -1 & 0 & -3 \\ -1 & 12 & -3 & 0 \\ 0 & -3 & 6 & -1 \\ -3 & 0 & -1 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \\ 6 \\ -6 \end{bmatrix}$$

$I_1 = -1.1754\,A \quad I_2 = 1.2687\,A \quad I_3 = 1.4664\,A$

$I_4 = -1.0075\,A$

$I_{6\Omega} = -1.1754\,A \quad V_{6\Omega} = -7.0524\,V$

$I_{8\Omega} = 1.2687\,A \quad V_{8\Omega} = 10.1496\,V$

$I_{2\Omega} = 1.4664\,A \quad V_{2\Omega} = 2.9328\,V$

$I_{4\Omega} = -1.0075\,A \quad V_{4\Omega} = -4.03\,V$

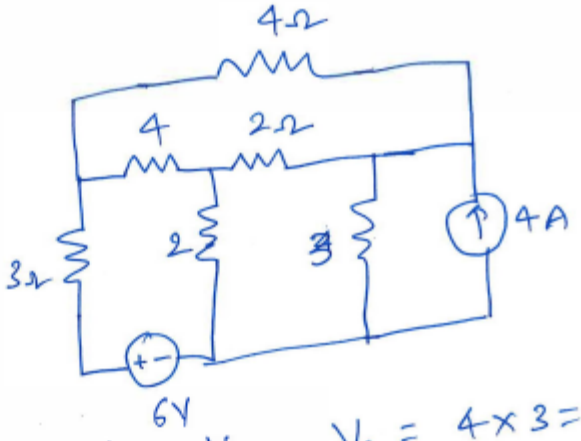
$I_{1\Omega} = I_1 - I_2 = -2.4441\,A \quad V_{1\Omega} = -2.4441\,V$

$I_{1\Omega} = I_3 - I_4 = 2.4739\,A \quad V_{1\Omega} = 2.4739\,V$

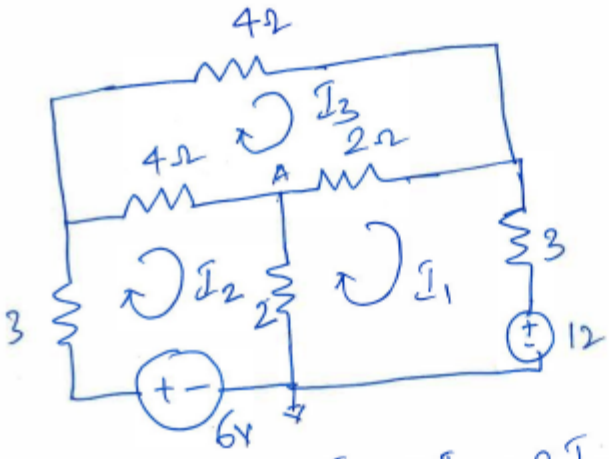
$I_{3\Omega} = I_2 - I_3 = -0.1977 \quad V_{3\Omega} = -0.5931\,V$

$I_{3\Omega} = I_1 - I_4 = -0.1679 \quad V_{3\Omega} = -0.5037\,V$

Fig. 9




Using Source transformation, $V_s = 4 \times 3 = 12V$




In mesh 1 : $7I_1 - 2I_2 - 2I_3 = -12$ — (1)

Mesh 2 : $-2I_1 + 9I_2 - 4I_3 = 6$ — (2)

Mesh 3 : $-2I_1 - 4I_2 + 10I_3 = 0$ — (3)

$$\begin{bmatrix} 7 & -2 & -2 \\ -2 & 9 & -4 \\ -2 & -4 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 6 \\ 0 \end{bmatrix}$$

$I_1 = -1.7561\ A \quad I_2 = 0.1463A \quad I_3 = -0.2927A$ (13)

$I_{3\Omega} = -1.756\ A \quad V_{3\Omega} = -5.268\ V$

$I_{2\Omega} = I_1 - I_2 = -1.9023\ A \quad V_{2\Omega} = -3.8046V$

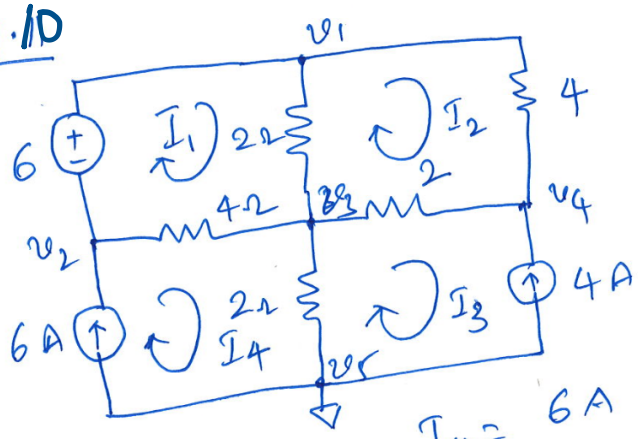
$I_{2\Omega} = I_1 - I_3 = -1.4634\ A \quad V_{2\Omega} = -2.9268\ V$

$I_{3\Omega} = I_2 = 0.1463A \quad V_{3\Omega} = 0.4389\ V$

$I_{4\Omega} = I_2 - I_3 = 0.439\ A \quad V_{4\Omega} = 1.756\ V$

$I_{4\Omega} = I_3 = -0.2927\ A \quad V_{4\Omega} = -1.1708V$

Fig. 10




$I_3 = -4A \quad I_4 = 6A$

In mesh I: $6I_1 - 2I_2 - 4I_4 = 6$

$6I_1 - 2I_2 = 30 \quad -(1)$

In mesh II: $-2I_1 + 8I_2 - 2I_3 = 0$

$-2I_1 + 8I_2 = -8 \quad -(2)$

Solving, $I_1 = 5.09\ A \quad I_2 = 6.273\ A$

(14)

$I_{2\Omega} = I_1 = 5.09A \quad V_{2\Omega} = 10.18V$

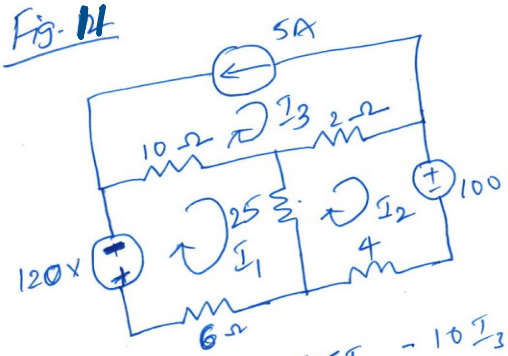
$I_{4\Omega} = I_1 - I_4 = -0.91A \quad V_{4\Omega} = -3.64V$

$I_{4\Omega} = I_2 = 0.273A \quad V_{4\Omega} = 1.092V$

$I_{2\Omega} = I_2 - I_3 = 4.273A \quad V_{2\Omega} = 8.546V$

$I_{2\Omega} = I_4 - I_3 = 10A \quad V_{2\Omega} = 20V$

Fig. 14




$I_3 = -5A$

Mesh 1: $41I_1 - 25I_2 - 10I_3 = -120$ — (1)

Mesh 2: $-25I_1 + 31I_2 - 2I_3 = +100$ — (2)

Simplifying $41I_1 - 25I_2 = -170$

$-25I_1 + 31I_2 = 90$

Solving $I_1 = -4.675\ A \quad I_2 = -0.867\ A$

$I_{10\Omega} = I_3 - I_1 = -0.325A \quad V_{10\Omega} = -3.25V$

$I_{2\Omega} = I_2 - I_2 = -4.132A \quad V_{2\Omega} = -8.266V$

$I_{4\Omega} = I_2 = -0.867A \quad V_{4\Omega} = -3.468V$

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(15)

$I_{25\Omega} = I_1 - I_2 = -3.808A \quad V_{25\Omega} = 95.2V$

$I_{6\Omega} = I_1 = -4.675A \quad V_{6\Omega} = 28.05V$