## EMAT101L

## **Engineering Calculus**

## **Tutorial Sheet 8**

## (Applications of Integrals)

1. Find the area of the region bounded by the given curves.

(a) 
$$f(x) = 2x^2 + 5x - 3$$
 and  $g(x) = x^2 + 4x - 1$   
Ans.  $\frac{9}{2}$ 

(b) 
$$f(x) = \sin x$$
 and  $g(x) = \cos x$  from  $x = 0$  to  $x = \frac{\pi}{4}$   
Ans.  $\sqrt{2} - 1$ 

(c) 
$$x = 2y^2$$
 and  $x + y = 1$   
Ans.  $\frac{9}{8}$ 

(d) 
$$y = 2x, y = 5x \text{ and } x = 3$$
  
Ans.  $\frac{27}{2}$ 

(e) 
$$y = x^2$$
 and  $x = y^2$   
Ans.  $\frac{1}{3}$ 

2. Use the Disk/Washer to find the volume of the solid of revolution formed by rotating the region about each of the given axes. Region bounded by:  $y = \sqrt{x}, y = 0$  and x = 1. Rotated about

(a) the x-axis Ans. 
$$\frac{\pi}{2}$$
 unit<sup>3</sup>

(b) 
$$y = 1$$
 Ans.  $\frac{\pi}{6}$  unit<sup>3</sup>

(c) the y-axis Ans. 
$$\frac{\pi}{5}$$
 unit<sup>3</sup>

(d) 
$$x = 1$$
 Ans.  $\frac{8}{15}\pi$  unit<sup>3</sup>

3. Find the volume of the following solids of revolution using the Shell method.

Region bounded 
$$y = \sqrt{x}, y = 0$$
 and  $x = 1$  and rotated about  $x = 3$ .

Hint: 
$$r(x) = 3 - x$$
 and  $h(x) = \sqrt{x}$ , Ans.  $\frac{16\pi}{5}$  unit<sup>3</sup>.

4. Find the volume of the solid of revolution where  $y = \sin x$  on  $[0, \pi/2]$  is revolved around the x-axis.

Hint: Use disk method, Ans. 
$$\frac{\pi^2}{4}$$
 unit<sup>3</sup>

5. Find the volume of the solid created when the area between  $f(x) = x^2 + 1$  and g(x) = x on [0,1] is rotated about x-axis.

Hint: Use the Washer method, Ans. 
$$\frac{23}{15}\pi$$
 unit<sup>3</sup>

6. Find the volume of the solid created when the area contained by  $f(x) = x^2$  and  $g(x) = x^3$  is revolved around the x-axis.

Hint: Use the Washer method, Ans. 
$$\frac{2}{35}\pi$$
 unit<sup>3</sup>

7. Using the Shell method, find the volume of the region enclosed by  $y = \sqrt{x}, x = 1$  and x = 4 when revolved about y-axis.

Ans. 
$$2\pi \int_{1}^{4} x \sqrt{x} \ dx = \frac{124}{5} \pi \text{ unit}^{3}$$