BENNETT UNIVERSITY

DO NOT WRITE ANYTHING ON QUESTION PAPER EXCEPT YOUR NAME, DEPARTMENT AND ENROLMENT No.

Name of Student -- Enrolment No. ------------------

Department / School ------------------------------

BENNETT UNIVERSITY, GREATER NOIDA

End Term Examination, Fall SEMESTER 2019-20

COURSE CODE: **ECSE209L** MAX. DURATION: **2 Hours**

COURSE NAME: **Discrete Mathematical Structures** MAX. MARKS: **40**

Note: Attempt all the questions. All the questions are compulsory.

Q.1 Determine whether each of the following functions is an injection and/or a surjection:

(a) $f: Z^+ \to Z^+, defined\ by\ f(x) = x^2 + 2$ **(1+1=2 Marks)**

(b) $f: R \to R, defined\ by\ f(x) = -4x^2 + 12x - 9$

Q.2 (a) Consider the set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, compute the corresponding fuzzy set for the following membership function:

$$\mu_A(x) = \left(\frac{1}{1 + 10x}\right)^2$$

Also, represent the membership function graphically. **(2 Marks)**

(b) Determine whether the following sets are equal (**Justify your answer**): **(1 Mark)**

$A = \{x : x\ is\ a\ letter\ in\ the\ word\ REAP\}\ and\ B = \{x : x\ is\ a\ letter\ in\ the\ word\ PAPER\}$

(c) Let $A\ and\ B$ be two sets with $A'\ and\ B'$ as their complements respectively, then the set $(A - B) \cup (B - A) \cup (A \cap B)$ is equal to: **(1 Mark)**

(i) $A \cup B$

(ii) $A' \uplus B'$

(iii) $A \cap B$

(iv) $A' \cap B'$

Q.3 Translate the following statements into their symbolic representations (**using quantifiers, variables and predicate symbols**): **(2 Marks)**

(a) There is a student who can speak Tamil and who knows C++.

(b) Given any integer whose square is odd, that integer is itself odd.

Q.4 (a) Determine the truth value of $[P \rightarrow ((Q \wedge (\neg R)) \vee S)] \wedge [(\neg T) \leftrightarrow (S \wedge R)]$, where P, Q, R *and* S are all *true* while T is *false*. **(2 Marks)**

(b) Show that $[(P \vee Q) \vee ((Q \vee (\neg R)) \wedge (P \vee R))] \Leftrightarrow \neg[(\neg P) \wedge (\neg Q)]$ using laws of equivalence. **(2 Marks)**

Q.5 Of 30 personal computers (PCs) owned by faculty members in a certain university department, 20 do not have A drives, 8 have 19-inch monitors, 25 are running Windows XP, 20 have at least 2 of these properties and 6 have all three. Considering the stated scenario, compute the following: **(2 Marks)**

(a) Number of PCs that have at least one of these properties.

(b) Number PCs that have exactly one property.

Q.6 Comment on the truth value of the following statements: **(3 Marks)**

(a) If A_1, A_2 *and* A_3 are pairwise disjoint sets with $|A_1| = 6, |A_2| = 8,$ *and* $|A_3| = 5$, then $|A_1 \cup A_2 \cup A_3| = 6 \times 8 \times 5 = 240$.

(b) There are 8 ways of choosing a chairperson for a committee consisting of 5 men and 3 women.

(c) The number of different ways of answering 10 **True/False** type questions is 2^{10}.

Q.7 Discuss the following statement using the concept of pigeonhole principle: **(1 Mark)**

"In a group of 49 people, there must be 6 who have their birthdays in the same month."

Q.8 (a) Determine the number of edges in a graph with 6 vertices, 2 of degree 4 and 4 of degree 2. Draw two such graphs. **(3 Marks)**

(b) Show that the following graphs are isomorphic: **(2 Marks)**

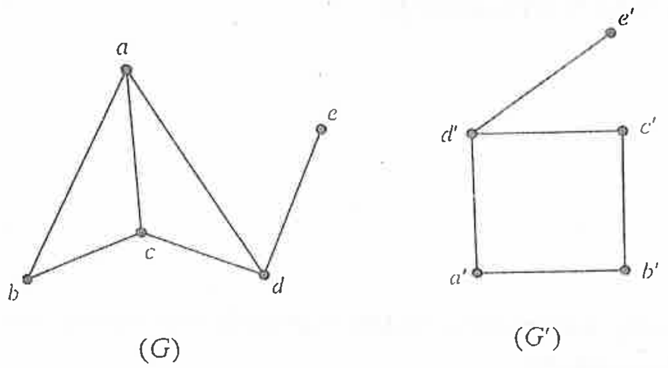

Fig. 1: Graphs G and G'

(c) Determine the minimum spanning tree for the following graph using Prim's algorithm:

(2 Marks)

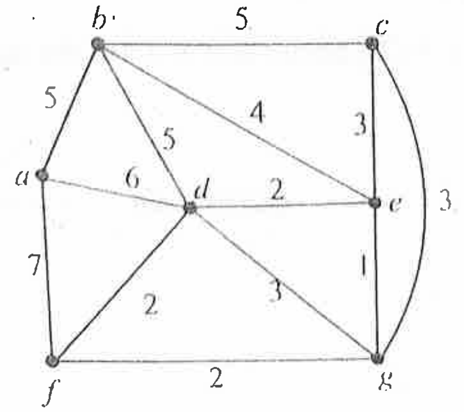

Fig. 2: Weighted Graph G_2

Q.9 Draw a graph corresponding to the map shown below and find a colouring that requires the least number of colours (vertex colouring). Calculate the chromatic number of the graph.

(3 Marks)

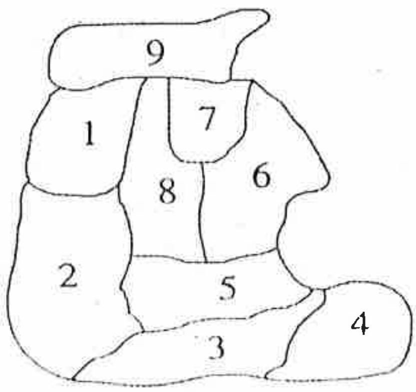

Fig. 3: A map representing important locations of a city

Q.10 (a) Compute the number of ways in which 6 boys and 6 girls can be arranged in a row under the following conditions: **(2 Marks)**

(i) All boys are to be seated together and all girls are to be seated together.

(ii) Boys and girls take their seats alternately.

(b) Determine the number of ways in which 20 students out of a class of 32 can be chosen to attend class on a late thursday afternoon if: **(2 Marks)**

(i) Paul refuses to go to the class.

(ii) Jim and Michelle insist on going to the class.

Q.11 (a) The set $L = \{1, 2, 3, 4, 5, 6, 12\}$ of factors of 12 under divisibility forms a lattice. Prove by Hasse Diagram. **(2 Marks)**

(b) Determine whether the group $(G, +_6)$ is a cyclic group where $G = \{0, 1, 2, 3, 4, 5\}$. If yes, point out the generators. **(3 Marks)**

(c) Show that $S = \{a + b\sqrt{2}\}$ $where\ a, b \in Z$ for the operations $+$, $\times$ is an integral domain but not a field. **(3 Marks)**

---GOOD LUCK---