DO NOT WRITE ANYTHING ON QUESTION PAPER EXCEPT YOUR NAME, DEPARTMENT AND ENROLMENT No.

Name of student: .... Enrollment No. ....

Department/School:..

BENNETT UNIVERSITY, GREATER NOIDA
Mid Term Examination, FALL SEMESTER 2019-20

COURSE CODE : EMAT101L — MAX. TIME: 1 Hour

COURSE NAME: ENGINEERING CALCULUS — MAX. MARKS: 30

Note: This paper contains 6 questions and all questions are mandatory.

1. TRUE/FALSE. Give proper justifications for any **FIVE** of the following statements: [2× 5=10]

   (a) If $|f|$ is continuous, then f is continuous.

   (b) $|\sin(2x) - \sin(2y)| \leq 2|x - y|$ for all $x, y \in \mathbb{R}$.

   (c) The radius of convergence of the series $\sum_{n=0}^{\infty} n!x^n$ is 0.

   (d) If $f : \mathbb{R} \to \mathbb{R}$ is such that $\lim_{h\to 0} \frac{f(x+h) - f(x-h)}{h}$ exists in $\mathbb{R}$ for every $x \in \mathbb{R}$, then f must be differentiable on $\mathbb{R}$.

   (e) If both the series $\sum_{n=1}^{\infty} x_n$ and $\sum_{n=1}^{\infty} y_n$ of real numbers converge, then the series $\sum_{n=1}^{\infty} x_n y_n$ converges.

   (f) The function $f(x) = \begin{cases} x^2, & x \in \mathbb{Q} \\ 0, & x \notin \mathbb{Q}. \end{cases}$ is continuous at $x = 1$.

2. Examine the convergence of any **TWO** of the following : [5]

$$(a)\sum_{n=1}^{\infty} \frac{n^n}{2^{n^2}} \qquad (b)\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{1}{\sqrt{n}}\right) \qquad (c)\sum_{n=1}^{\infty} \frac{n^2}{5(2n)!}.$$

3. If $a_1 = 1$ and $a_{n+1} = 1 + \sqrt{a_n}$ $\forall$ $n \in \mathbb{N}$, then show that the sequence $\{a_n\}$ is convergent. Also determine the limit if convergent. [3]

4. Show that the function $f : [0, 1] \to \mathbb{R}$ defined as $f(x) = \begin{cases} x^2 \cos\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ is uniformly continuous on [0,1]. Also Check the differentiability of f at 0. [5]

5. Find the values of $x \in \mathbb{R}$ for which the series $\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n.4^n}$ is convergent. [4]

   OR

   Determine all $p \in \mathbb{R}$ for which the series $\sum_{n=1}^{\infty} \frac{n^2 + 2n}{(2n^4 + 1)^p}$ is convergent.

6. Let a, b, c be distinct positive real numbers and let $x_n = (a^n + b^n + c^n)^{1/n}$ $\forall$ $n \in \mathbb{N}$. Then find the limit of $\{x_n\}$. [3]