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POSSESSION OF MOBILE IN EXAMINATION IN UFM PRACTICE

Name of Student -- Enrolment No. ------------------

Department ----------------------------

BENNETT UNIVERSITY, GREATER NOIDA

Supplementary, July 2019

COURSE CODE: **EPHY203L** MAX. DURATION: **THREE HOURS**

COURSE NAME: **Electrodynamics**

COURSE CREDIT: **3-1-0** MAX. MARKS: **100**

Note:

- This question paper contains **FOUR** questions.
- All the questions are compulsory.
- Marks of each question are indicated next to it.
- Rough work must be carried out at the back of the answer script.
- Do not **derive** an expression **unless explicitly asked** in the question. When the question is "**Write** an expression for ....", the derivation of the same is **NOT** required.
- Please write precisely and neatly. Please make clear diagram wherever required.
- Use of calculator is allowed.

1.

a) The time-averaged potential of a neutral hydrogen atom is given by **5 Marks**

$$\Phi = \frac{q}{4\pi\epsilon_0}\frac{e^{-\alpha r}}{r}\left(1+\frac{\alpha r}{2}\right),$$

where q is the magnitude of electronic charge and $\alpha^{-1} = \frac{a_0}{2}$, a_0 being Bohr radius. Find the distribution of charge that will give this potential.

b) A thick spherical shell (inner and outer radius a and b, respectively) carries charge density as **4 Marks**

$$\rho(r) = \frac{k}{r^2} \quad (a \leq r \leq b).$$

Find the electric field within the shell. Also find the total charge enclosed within the shell.

c) A rectangular pipe running parallel to the z-axis (from ∞ to $-\infty$), has three grounded metal sides, at $x = 0, x = a,$ and $y = 0$. The forth side at $y = b$, is maintained at a constant potential V_0. Find the potential inside the pipe. **8 Marks**

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d) If the current density in the wire is inversely proportional to the distance from the axis of the wire, find the total current in the wire. Also, find the magnetic field inside and outside the wire. **4 Marks**

e) Are the following vectors representing electrostatic field? **4 Marks**

I. $\vec{E} = k(xy\,\hat{x} + 2yz\,\hat{y} + 3xz\,\hat{z})$,

II. $\vec{E} = k[y^2\,\hat{x} + (2xy + z^2)\hat{y} + 2yz\,\hat{z}]$.

2 a) Using Maxwell's equations in vacuum, show that $\vec{E}$ and $\vec{B}$ fields satisfy equation for wave travelling with speed of light $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$. **5 Marks**

(You may need the identity: $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} . \vec{A}) - \nabla^2 \vec{A}$)

b) Show that $\vec{E}$ and $\vec{B}$ waves travelling in vacuum are transverse waves. **4 Marks**

c) Reflected and transmitted electric field at the interface of nonconducting medium are given as **6 Marks**

$$\tilde{E}_{0_R} = \frac{\alpha - \beta}{\alpha + \beta}\tilde{E}_{0_I} \text{ and } \tilde{E}_{0_T} = \frac{2}{\alpha + \beta}\tilde{E}_{0_I},$$

where $\alpha \equiv \frac{\cos\theta_T}{\cos\theta_I}$ and $\beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2}$. Here symbols represent their usual meaning. Obtain the expression for angle (known as *Brewster's* angle), at which reflected wave is completely extinguished.

d) If the wave incident on the glass ($n_2 = 1.5$) from air ($n_1 = 1$), obtain the *Brewster's* angle. **4 Marks**

e) Silver is an excellent conductor ($\sigma \gg \epsilon\omega$), but it is expensive. Suppose you have been asked to design a microwave experiment to be operated at frequency $\omega = 2\pi \times 10^{10} s^{-1}$. How thick would you make the silver coatings? You may use $\sigma \approx 4 \times 10^7\ \Omega^{-1}m^{-1}$ and $\epsilon \approx \epsilon_0 = 8.85 \times 10^{-12}\frac{C^2}{Nm^2}$ and $\mu \approx \mu_0 = 4\pi \times 10^{-7}$. **6 Marks**

(you may need the following information: The real and imaginary parts of complex wave number are given as

$$k_r = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[1 + \sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2}\right]^{\frac{1}{2}} \text{ and } k_i = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[-1 + \sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2}\right]^{\frac{1}{2}},$$

where symbols represent their usual meaning.)

3 a) The potential $V_0(\theta) = k\ cos\theta$ is specified on the surface of hollow sphere, where θ is measured with respect to z −axis. The general solution of Laplace's equation in spherical-polar coordinate system with azimuthal symmetry is given as **8 Marks**

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(cos\theta),$$

where the Legendre polynomials are defined as

$$P_l(x) = \frac{1}{2^l l!}\frac{d^l}{dx^l}(x^2-1)^l.$$

These Legendre polynomials also satisfy orthogonality conditions:

$$\int_{-1}^{1} P_l(x)P_{l'}(x)dx$$

$$= \int_0^{\pi} P_l(cos\theta)P_{l'}(cos\theta)sin\theta\ d\theta$$

$$= \begin{cases} 0, & \text{if } l \neq l' \\ \dfrac{2}{2l+1}, & if\ l = l'. \end{cases}$$

Obtain the potential inside the sphere.

b) Let us suppose that electromagnetic waves are confined to the interior of a hollow pipe of perfect conductor. Please specify the boundary conditions at the inner wall of the guide (hollow pipe). **5 Marks**

c) The longitudinal component of magnetic field for a transverse electric wave in a rectangular wave guide of height a and width b is given as **4 Marks**

$$B_z = B_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right).$$

The wave number, k, for this wave is given as

$$k = \sqrt{\left(\frac{\omega}{c}\right)^2 - \pi^2\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]}.$$

If $a < b$, obtain the lowest cut off frequency for a guided wave.

d) Which statement is incorrect expression for $\vec{E}$ and $\vec{B}$ field at the boundary **3 Marks**

i. $E^{\perp}_{above} - E^{\perp}_{below} = \frac{\sigma}{\epsilon_0}$

ii. $E^{||}_{above} - E^{||}_{below} = \frac{\sigma}{\epsilon_0}$

iii. $B^{\perp}_{above} - B^{\perp}_{below} = 0$

iv. $B^{||}_{above} - B^{||}_{below} = \mu_0 K$

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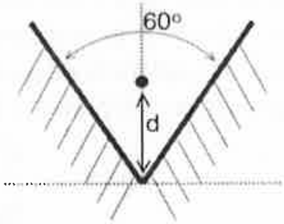
e) Two grounded semi-infinite conducting planes meet at angle of 60° between them as shown below. A charge q is placed at a point midway between these planes at a distance d from their line of intersection. Obtain the image charges (other than the real charge q) and their locations to satisfy the boundary conditions (Just draw this arrangement). **5 Marks**



4 The scalar potential of an oscillating magnetic dipole is zero, whereas the vector potential is given by

$$\vec{A}(r,\theta,t) = -\frac{\mu_0 m_0 \omega}{4\pi c}\left(\frac{\sin\theta}{r}\right)\sin\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{\phi}.$$

a) Show that these potentials satisfy Lorentz gauge $\left(\vec{\nabla}.\vec{A} + \frac{1}{c^2}\frac{\partial V}{\partial t} = 0\right)$. **5 Marks**

b) Obtain the electric and magnetic fields for in the radiation zone **10 Marks**

c) Show that these $\vec{E}$ and $\vec{B}$ fields satisfy Maxwell's equations. **5 Marks**

d) Obtain the total power radiated by the source. **5 Marks**

In spherical polar coordinate, gradient, divergence, curl, and Laplacian are:

$$\vec{\nabla} V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}.$$

$$\vec{\nabla}.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}.$$

$$\vec{\nabla}\times\vec{A} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_\phi) - \frac{\partial A_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\hat{\phi}.$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin\theta}\frac{\partial^2 V}{\partial\phi^2}.$$

....Paper Ends...