

Tutorial Sheet 9
Improper Integral, Beta and Gamma Function

1. Discuss the convergence/divergence of improper integrals of first kind

$$(a) \int_0^{\infty} e^{-x} \cos x \, dx, \quad (b) \int_1^{\infty} \frac{dx}{x^2(1+e^x)} \quad (c) \int_1^{\infty} \frac{x+1}{\sqrt{x^3}} dx$$

2. Discuss the convergence/divergence of improper integrals of second kind

$$(a) \int_1^2 \frac{\sqrt{x}}{\ln x} dx \quad (b) \int_0^1 \frac{\sin(\frac{1}{x})}{\sqrt{x}} dx \quad (c) \int_1^{\pi/2} \frac{\tan x}{x^{3/2}} dx$$

3. Discuss the convergence/divergence of improper integrals

$$(a) \int_0^{\infty} x^{\frac{-1}{2}} e^{x^2} dx \quad (b) \int_{-\infty}^{\infty} \frac{x}{x^2+1} dx \quad (c) \int_0^{\infty} \frac{1+x}{1+x^3} dx$$

4. Show that $\int_0^1 \frac{\sin x}{x^p} dx$ converges if and only if $0 < p < 2$.

5. Show the following:

$$(a) \int_0^1 \frac{x^t - 1}{\log x} dx = \log(1+t), \quad (b) \int_0^{\infty} e^{-tx} \frac{\sin x}{x} dx = \frac{\pi}{2} - \tan^{-1} t.$$

6. Using Beta and Gamma functions, evaluate the following:

$$(a) \int_0^{\infty} e^{-x^2} dx \quad (b) \int_0^{\pi/2} \sqrt{\tan x} dx \quad (c) \int_0^{\infty} x^{2/3} e^{-\sqrt{x}} dx$$

7. Find the values of the gamma function at the given points.

$$(a) \frac{5}{2} \quad (b) \frac{9}{2} \quad (c) -\frac{1}{2}$$