BENNETT UNIVERSITY
TIMES OF INDIA GROUP

EMAT101L

Engineering Calculus

Mid Semester Examination

Total marks: 30 Time: 90 minutes

Each question carries 2 marks.

---

1. If p and q are positive real number, then the series $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)^p}{n^q}$ convergent for

   (a) $p < q - 1$

   (b) $p < q + 1$

   (c) $p \geq q - 1$

   (d) $p > q + 1$

2. Let

$$f(x) = \begin{cases} ax + 2b & \text{if } x \leq 0, \\ x^2 + 3a - b & \text{if } 0 < x \leq 2, \\ 3x - 5 & \text{if } x > 2. \end{cases}$$

   If the function f is continuous at every x, then find the values of a and b.

   (a) $a = b = -\frac{3}{2}$

   (b) $a = b = \frac{3}{2}$

   (c) $a = b = \frac{1}{2}$

   (d) $a = b = -\frac{1}{2}$

3. Interval of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{5^n}{n} (10x - 20)^n$ is

   (a) $\left(\frac{99}{50}, \frac{101}{50}\right)$

(b) $\left(\frac{99}{50}, \frac{101}{50}\right]$

(c) $\left[\frac{99}{50}, \frac{101}{50}\right)$

(d) $\left[\frac{99}{50}, \frac{101}{50}\right]$

4. Let $x_n = (-1)^n \frac{5n}{3n-2} \sin^3 n$, for all $n \in \mathbb{N}$. Then

   (i) $\{x_n\}_{n=1}^{\infty}$ is a bounded sequence.

   (ii) $\{x_n\}_{n=1}^{\infty}$ is an unbounded sequence.

   (iii) Every subsequence of $\{x_n\}_{n=1}^{\infty}$ is divergent.

   (iv) $\{x_n\}_{n=1}^{\infty}$ has a convergent subsequence.

   Choose the CORRECT option.

   (a) (i) and (iv) are correct.

   (b) Only (i) is correct.

   (c) Only (iv) is correct.

   (d) (ii) and (iv) are correct.

5. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two functions which are discontinuous at $x = 0$. Then the product function $fg : \mathbb{R} \to \mathbb{R}$

   (i) can be continuous at $x = 0$.

   (ii) is always discontinuous at $x = 0$.

   (iii) can be discontinuous at $x = 0$.

   Choose the CORRECT option.

   (a) (i) and (iii) are correct.

   (b) Only (ii) is correct.

   (c) Only (iii) is correct.

6. $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$ is

   (a) 0

   (b) does not exist

   (c) 1

   (d) -1

7. Let $A=\left\{-1,-\frac{1}{2},-\frac{1}{3},-\frac{1}{4},\cdots\right\}$ and $B=\{x\in\mathbb{Q}: x^2-2\leq 0\}$, where $\mathbb{Q}$ is the set of rational numbers. Consider the following four statements.

   (i) $\max(A)$ and $\min(B)$ do not exit.

   (ii) $\min(A)$ and $\max(B)$ do not exist.

   (iii) $\inf(A)=0$ and $\sup(B)=\sqrt{2}$

   (iv) $\sup(A)=0$ and $\inf(B)=-\sqrt{2}$

   Then which among the above statement(s) is/are CORRECT.

   (a) only (i) (b) (i) and (iv) (c) (i),(ii) and (iii) (d) only (ii)

8. Let $a_n=\frac{1}{n}$ and $b_n=-2^n$. Now consider the following four statements.

   (i) The sequences $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are both monotone sequences.

   (ii) The sequences $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are both bounded below.

   (iii) The sequence $\{a_n\}_{n=1}^{\infty}$ converges but the sequence $\{b_n\}_{n=1}^{\infty}$ diverges.

   (iv) The sequences $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are both convergent.

   Then which among the above statement(s) is/are CORRECT.

   (a) only (ii) (b) (i) and (iii) (c) (i) and (ii) (d) (i), (ii) and (iv)

9. Choose the INCORRECT option.

   (a) $\sum_{n=1}^{\infty}\frac{(-3)^n}{n!}$ is absolutely convergent.

   (b) $\sum_{n=1}^{\infty}\frac{2^n\cdot 3^n}{n^n}$ is convergent.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}-\sqrt{n}}$ is divergent.

(d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{2^n+5^n}{7^n}\right)$ is conditionally convergent.

10. If the Taylor's series expansion of the function $f(x) = \dfrac{1}{1-x}$ at $x = 2$ is $\displaystyle\sum_{n=0}^{\infty} a_n (x-2)^n$, then find the value of a_4.

(a) -1 (b) 1 (c) $\dfrac{1}{4!}$ (d) $-\dfrac{1}{4!}$

11. Let $f(x) = 3x^4 + 4x^3 - 12x^2 + 5$. Now choose the CORRECT option.

(a) 0 and -2 are the only stationary points of $f(x)$.

(b) The maximum value of $f(x)$ is 4.

(c) -2 is the global minimum of $f(x)$.

(d) 0 is a point of local minima for $f(x)$.

12. Let

$$f(x) = \begin{cases} x^2 & \text{for } x \geq 0, \\ -x^2 & \text{for } x < 0. \end{cases}$$

Then which one of the following statement is TRUE?

(a) $f(x)$ is discontinuous at $x = 0$.

(b) $f(x)$ is continuous but not differentiable at $x = 0$.

(c) $f(x)$ is differentiable but its first derivative is not continuous at $x = 0$.

(d) $f(x)$ is differentiable but its first derivative is not differentiable at $x = 0$.

13. Let $f(x) = \begin{cases} 0 & \text{if } x \in \mathbb{Q}, \\ 1 & \text{if } x \notin \mathbb{Q} \end{cases}$ $g(x) = \begin{cases} 0 & \text{if } 0 \leq x < 1, \\ 1 & \text{if } 1 \leq x < 2 \end{cases}$ and $h(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0, \\ 0 & \text{otherwise.} \end{cases}$

Choose the INCORRECT option.

(a) f is Riemann integrable in $[0,1]$.

(b) g is Riemann integrable in $[0,1]$.

(c) h is Riemann integrable in $[0, 1]$.

(d) g is Riemann integrable in $[1, 2]$.

14. Find the value of $\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^n$.

(a) e (b) e^2 (c) 0 (d) 1

15. Let f be a continuous function over the closed interval $[0, 3]$ and let $f(0) = 1$. If the derivative of f is the step function as shown in Figure 1, then which among the following figures represents f CORRECTLY.

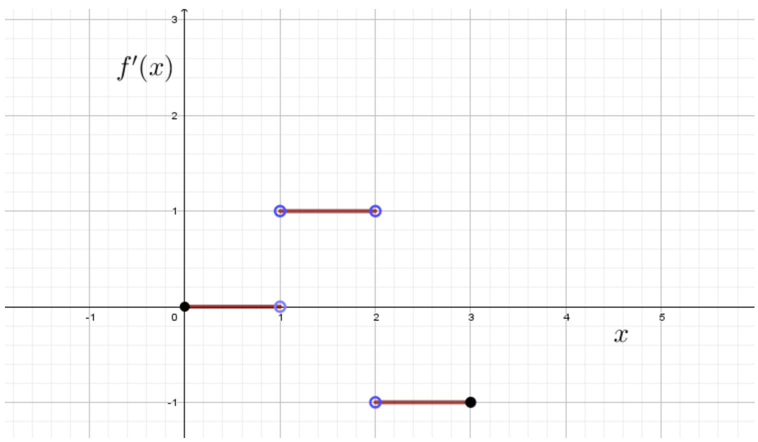
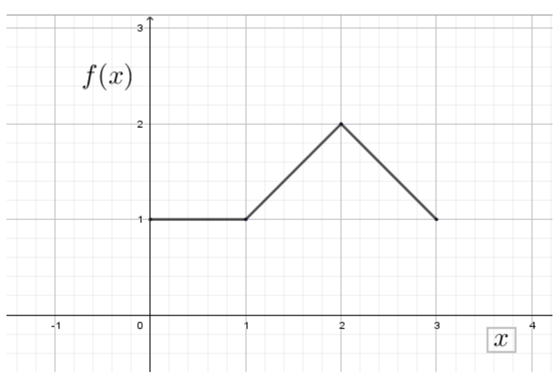
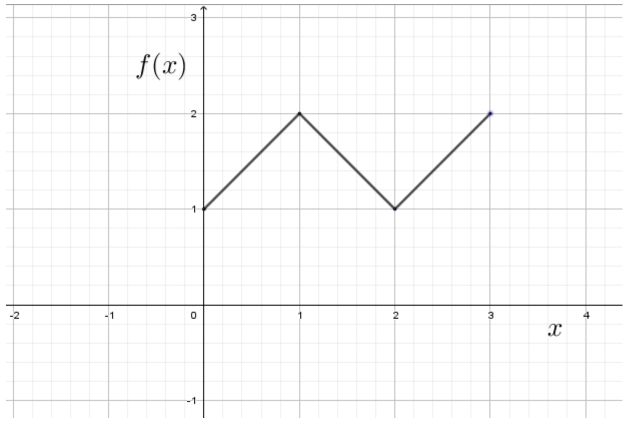


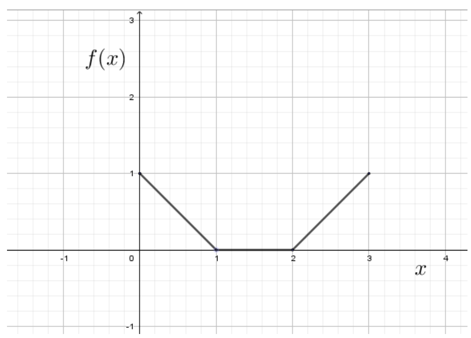
Figure 1



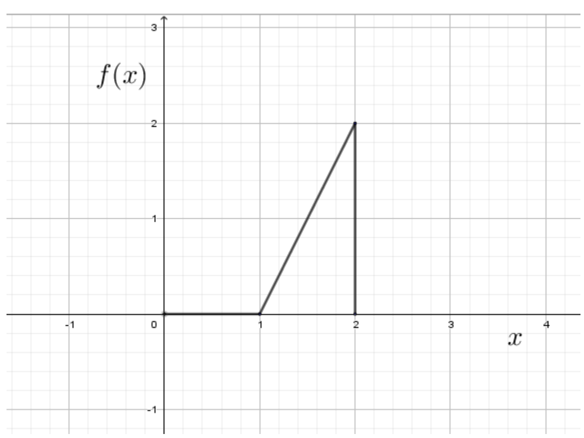
(a)



(b)



(c)



(d)