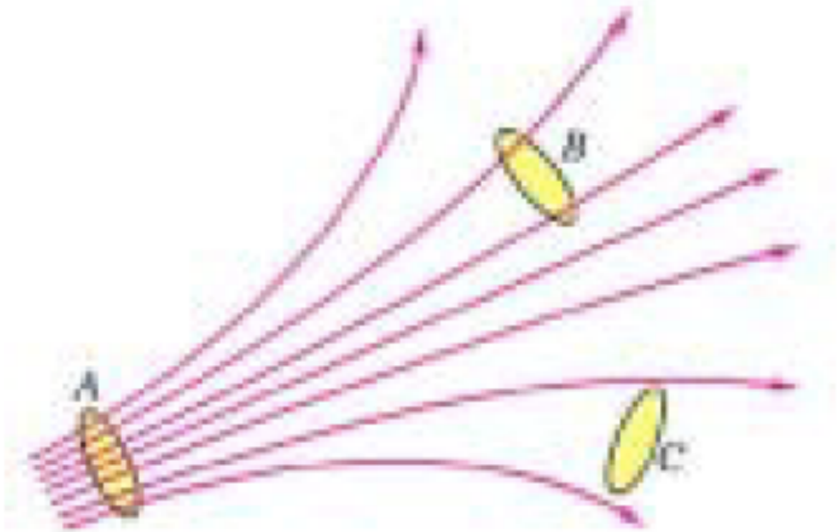
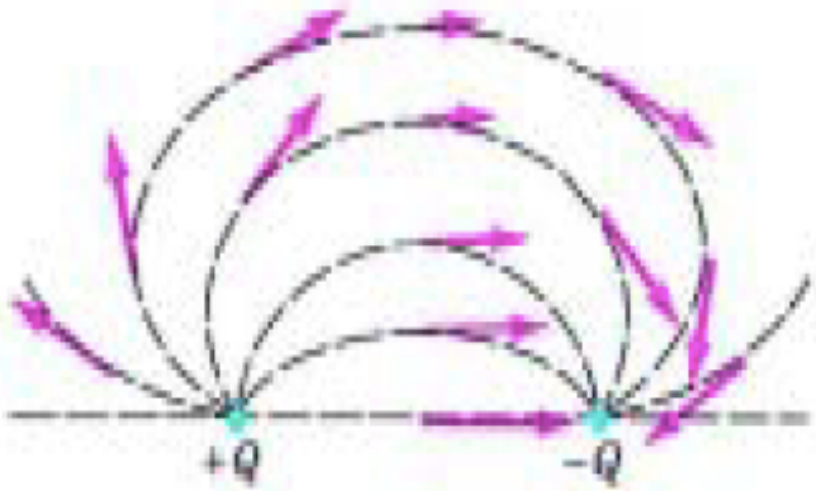
Lecture - 11

Field lines

How to determine the field strength from the field lines?

The lines are crowed together when the field is strong and spread apart where the field is weaker. The field strength is proportional to the density of the lines.





(a)Symmetry

(b)Near field

(c)Far field

(d)Null point

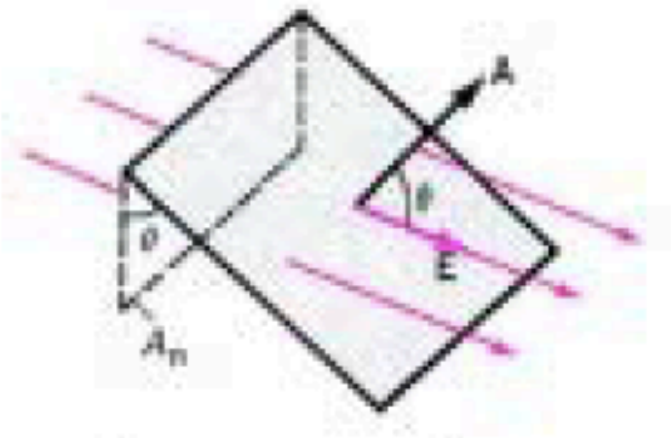
(e)Number of lines

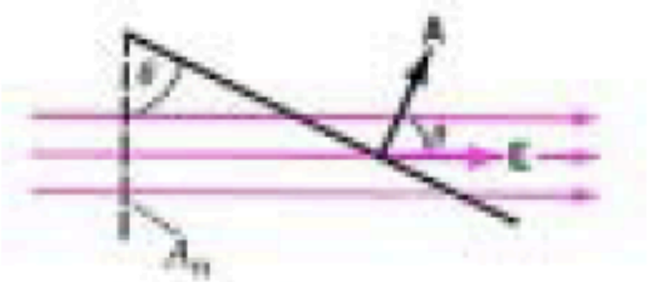
Be consistent: If your charge is doubled, number of lines need to be doubled

Electric Flux

The electric flux Φ_E through this surface is defined as

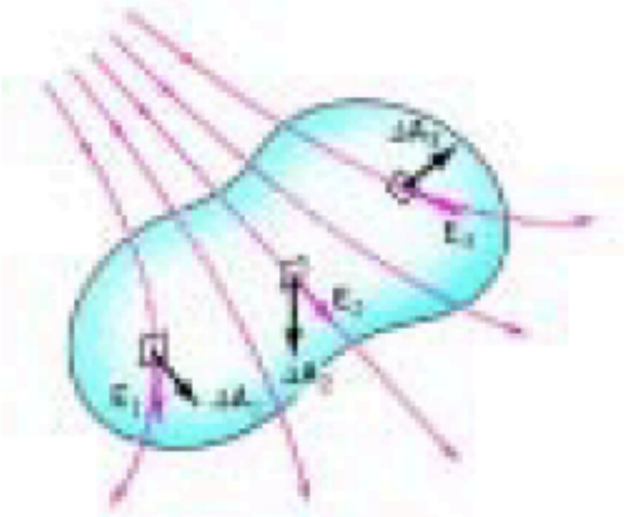
$$\Phi_E = EA\cos\theta$$
$$= \mathbf{E}\cdot\mathbf{A}$$

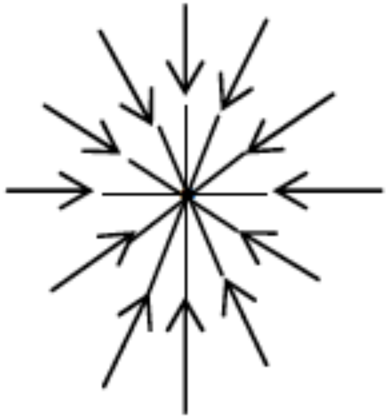




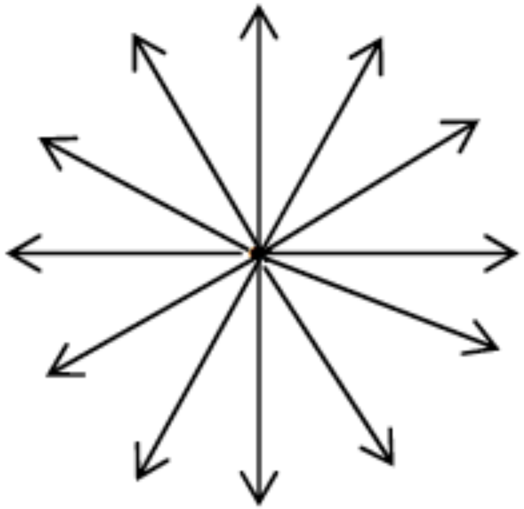
For a nonuniform electric field

$$\Phi_E = \int \mathbf{E}\cdot\hat{\mathbf{n}}\,da$$





$q_s = -e$
inward



$q_s = +e$
outward

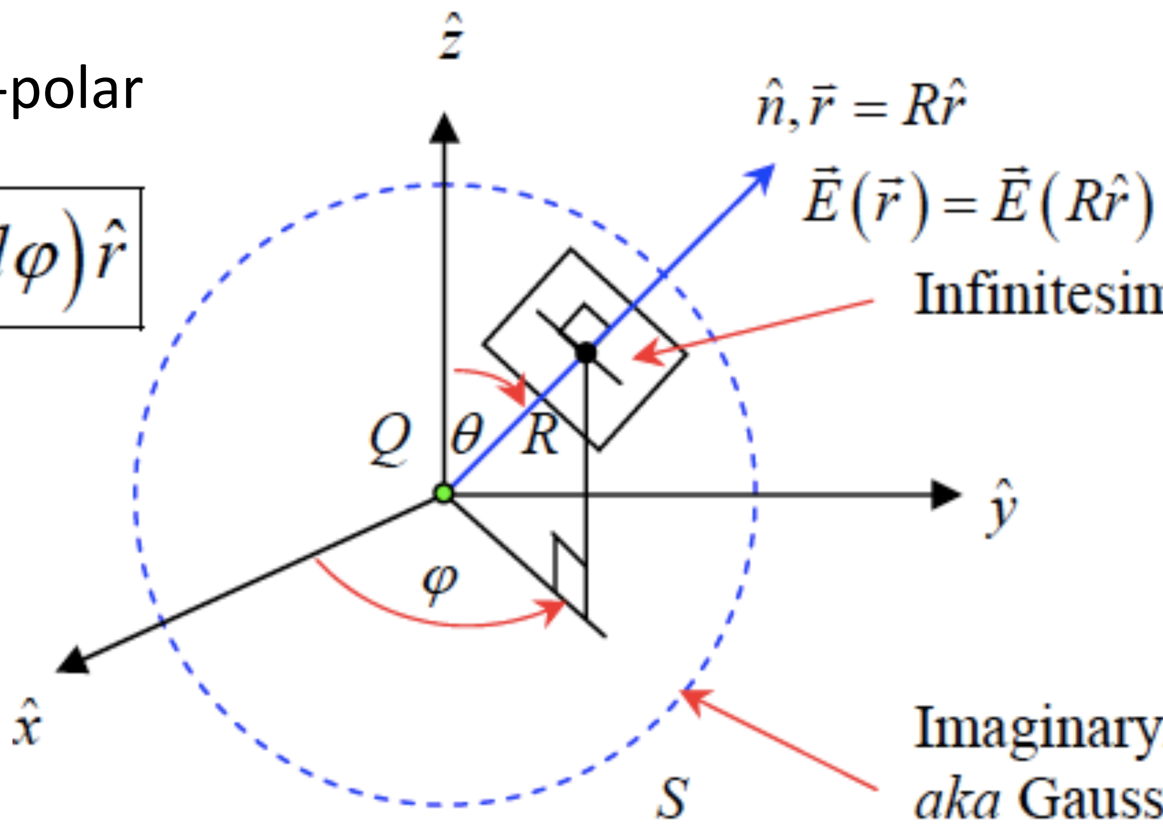
- Flux leaving a closed surface is positive, whereas flux entering a closed surface is negative.

- The net flux through the surface is zero if the number of lines that enter the surface is equal to the number that leave.

Gauss's law

Elemental area in Spherical-polar

$$\boxed{d\vec{A} = dA\hat{r} = \left(r^2 \sin\theta d\theta d\varphi\right)\hat{r}}$$



The total flux through this closed Gaussian surface is

$$\Phi_E = \oint_S \vec{E}(\vec{r})\bullet d\vec{A} = \frac{Q}{4\pi\varepsilon_o}\int_S\left(\frac{1}{\cancel{r^2}}\hat{r}\right)\bullet\underbrace{\left(\cancel{r^2}\sin\theta d\theta d\varphi\hat{r}\right)}_{=d\vec{A}}$$

$$\text{Thus: } \Phi_E = \frac{Q}{4\pi\varepsilon_o}\int_{\theta=o}^{\theta=\pi}\int_{\varphi=o}^{\varphi=2\pi}\sin\theta\, d\theta\, d\varphi \underbrace{(\hat{r}\bullet\hat{r})}_{=1} = \frac{\cancel{2}\pi Q}{\underbrace{\cancel{4}\pi}_{2}\,\varepsilon_o}\int_{\theta=o}^{\theta=\pi}\sin\theta\, d\theta$$

$$= \frac{\cancel{2}Q}{\cancel{2}\varepsilon_o} = \frac{Q}{\varepsilon_o}$$

<u>Gauss' Law (in Integral Form)</u>: $\boxed{\Phi_E = \oint_s \vec{E}(\vec{r})\bullet d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}}$

Electric flux through closed surface S = (electric charge enclosed by surface S)/ ε_o

The circle on the integral sign indicates that the Gaussian surface must be enclosed.

Can we prove the above statement for arbitrary closed shape?

volume charge density $\rho(\vec{r}')$, then: $Q_{encl} = \int_v \rho(\vec{r}')d\tau'$

$$\Phi_E = \oint_S \vec{E}(\vec{r})\cdot d\vec{A} = \int_v \overbrace{\left(\vec{\nabla}\cdot\vec{E}(\vec{r})\right)} d\tau' = \frac{Q_{encl}}{\varepsilon_o} = \frac{1}{\varepsilon_o}\int_v \overbrace{\rho(\vec{r})} d\tau'$$

This relation holds for *any* volume v $\Rightarrow$ the *integrands* of $\int_v (\quad) d\tau'$ *must* be equal

$\therefore$ Gauss' Law (in Differential Form): $\boxed{\vec{\nabla}\cdot\vec{E}(\vec{r}) = \rho(\vec{r}) / \varepsilon_o}$