EPHY105L-3 4.10.21

Transformation of vectors

For rotation about an arbitrary axis in 3-D

$$\begin{pmatrix} \bar{A}_x \\ \bar{A}_y \\ \bar{A}_z \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{zy} \\ R_{zx} & R_{zy} & R_{zz} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

In compact notation,

$$\bar{A}_i = \sum_{j=1}^{3} R_{ij} A_j$$

$i = 1 \equiv x$

$i = 2 \equiv y$

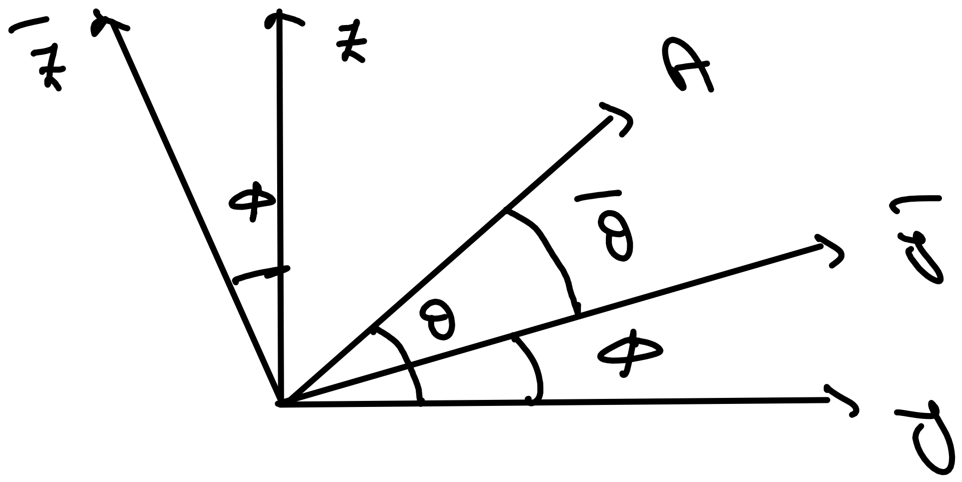
$i = 3 \equiv z$

For rotation along x-axis by an angle ϕ:

$R_{xx} = 1, \quad R_{xy} = 0, \quad R_{xz} = 0$

$R_{yx} = 0, \quad R_{yy} = \cos\phi, \quad R_{yz} = \sin\phi$

$R_{zx} = 0, \quad R_{zy} = -\sin\phi, \quad R_{zz} = \cos\phi$



$$x = \bar{x}$$

$$\begin{cases} \bar{A}_x = A_x \\ \bar{A}_y = A_y \cos\phi + A_z \sin\phi \\ \bar{A}_z = -A_y \sin\phi + A_z \cos\phi \end{cases}$$

$$\begin{pmatrix} \bar{A}_x \\ \bar{A}_y \\ \bar{A}_z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

$$\bar{A}_y = A\cos\bar{\theta} = A\cos(\theta - \phi)$$

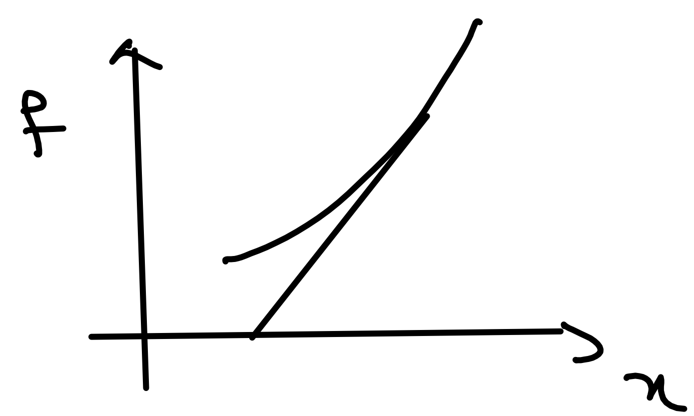
$$= A(\cos\theta\cos\phi + \sin\theta\sin\phi)$$

$$= A_y\cos\phi + A_z\sin\phi$$

Gradient

Derivative of a fn. tells us how fast the fn varies with respective coordinate

$$\underline{\underline{df}} = \left(\frac{df}{dx}\right) dx$$



↳ Slope of the graph.

Say, $T = T(x, y, z)$

$$\Rightarrow dT = \left(\frac{\partial T}{\partial x}\right) dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz$$

↳ tells us how T varies when we alter all 3 variables by infinitesimal amounts dx, dy, dz.

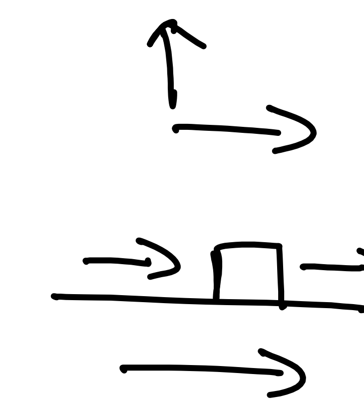
Rewrite dT as:

$$dT = \left(\hat{x}\,\frac{\partial T}{\partial x} + \hat{y}\,\frac{\partial T}{\partial y} + \hat{z}\,\frac{\partial T}{\partial z}\right)\cdot\left(\hat{x}\,dx + \hat{y}\,dy + \hat{z}\,dz\right)$$

$$= \underbrace{\left(\vec{\nabla} T\right)}_{\text{Gradient of } T\text{.}}\cdot\left(d\vec{l}\right)$$

We can write,

$$dT = |\vec{\nabla} T|\,|d\vec{l}|\cos\theta \leftarrow$$



Then for fixed $|d\vec{l}|$, dT is maximum when $\theta = 0$ or $\cos\theta = 1$

→ The magnitude $|\vec{\nabla} T|$ gives the slope along this maximal direction.

→ $\vec{\nabla} T$ is directed along the direction of maximum increase of T.

Link to the Recording:

https://bennettu.sharepoint.com/sites/EPHY105L-Odd2021/Shared%20Documents/General/Recordings/Meeting%20in%20_General_-20211004_154403-Meeting%20Recording.mp4?web=1