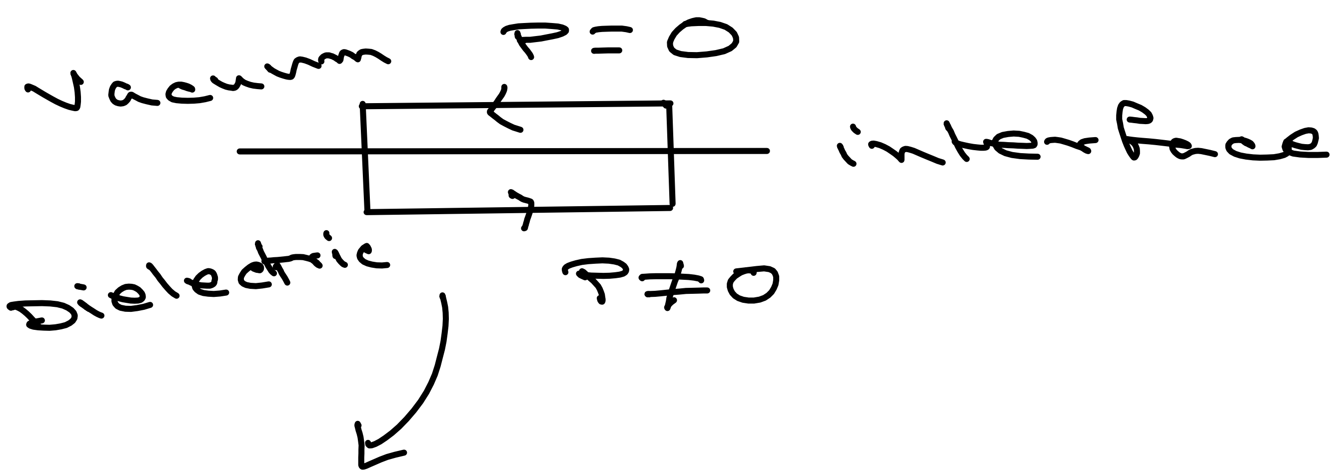
For a linear dielectric,

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$\vec{D} = \epsilon \vec{E} \quad , \quad \epsilon = \underbrace{\epsilon_r}_{\text{Dielectric const.}} \epsilon_0$$

(*) Does this mean $\vec{\nabla} \times \vec{P} = 0$?



→ In order for $\vec{\nabla} \times \vec{P} = 0$ we should have $\oint \vec{P} \cdot d\vec{l} = 0$

Around this loop

$$\oint \vec{P} \cdot d\vec{l} \neq 0 \implies \vec{\nabla} \times \vec{P} \neq 0$$

→ If the space is completely filled with dielectrics, $\vec{\nabla} \cdot \vec{D} = \rho_f$ & $\vec{\nabla} \times \vec{D} = 0$

$\Rightarrow \vec{D}$ can be obtained from free charge density

$$\vec{D} = \epsilon_0 \vec{E}_{vac.}$$

↳ the field free charge will produce in absence of any dielectric.

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

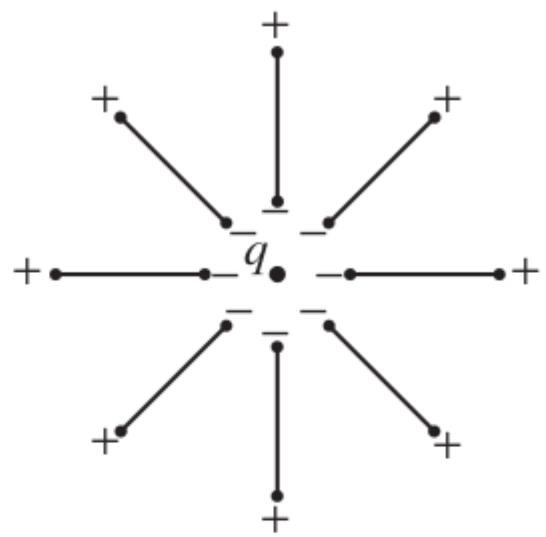
$$\vec{E} = \frac{1}{\epsilon} \vec{D}$$

$$= \frac{1}{\epsilon_r} \vec{E}_{vac.}$$

↳ the factor by which the field is reduced in presence of homogeneous linear dielectric.

Ex. If a free charge 'q' is embedded in a large dielectric,

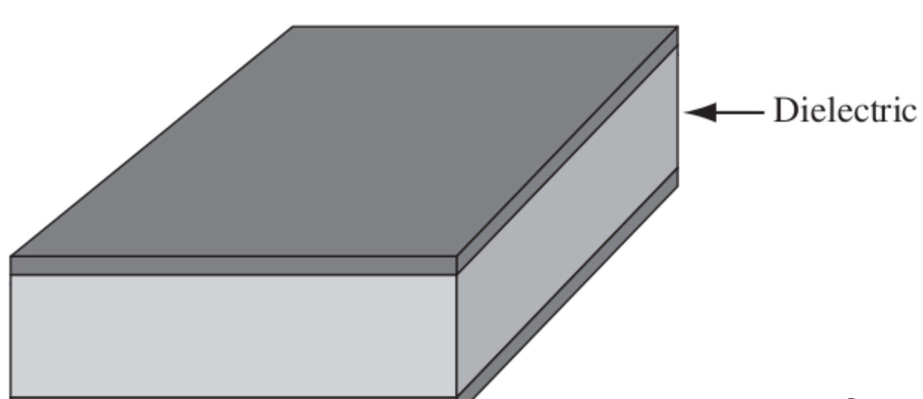
$$\vec{E} = \frac{1}{4\pi\epsilon} \frac{q}{r^2} \hat{r}$$



→ The free charge is shielded from all sides due to the polarisation.

⇒ This shielding effect reduces the electric field.

(*) This property can be used to enhance capacitance of a system.



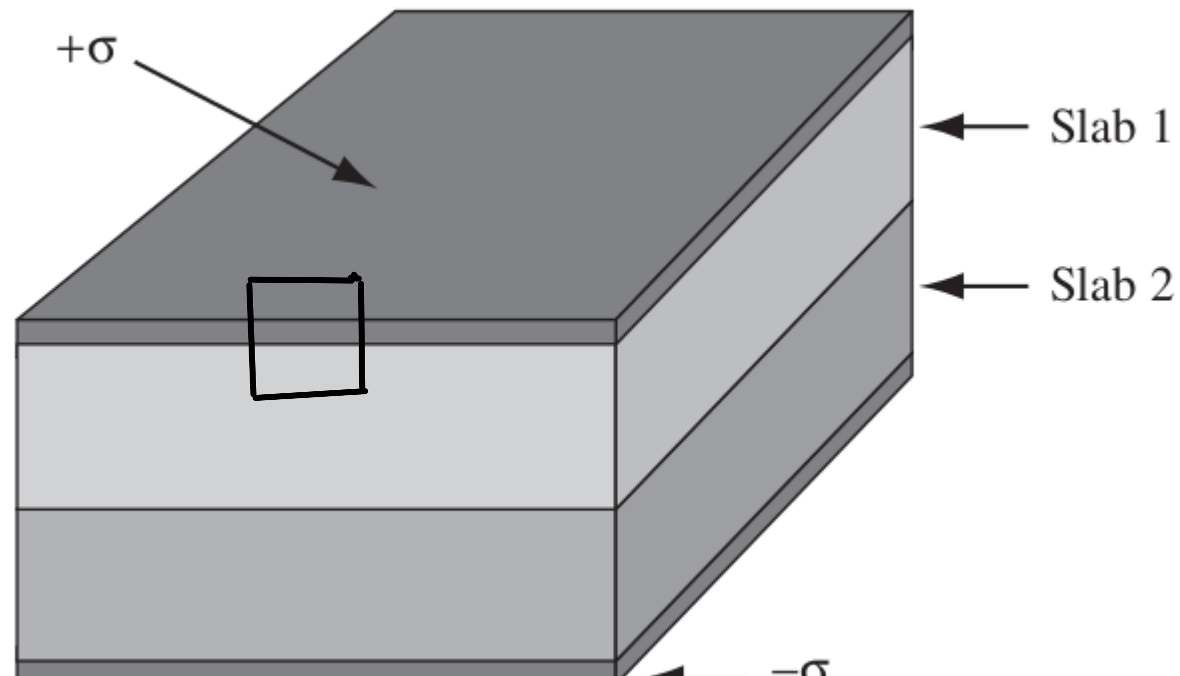
→ A parallel plate Capacitor filled with dielectric material

→ The dielectric reduces $\vec{E}$ and ΔV (potential diff.) by a factor $\frac{1}{\epsilon_r}$.

The capacitance $\left(\frac{Q}{V}\right)$ is increased by a factor, $C = \epsilon_r C_{vac}$.

$\frac{E_x}{\hat{z}}$:

$\vec{D}, \vec{E}, \vec{P}$



$d = 2a$

We have a parallel plate capacitor with charge density $\pm\sigma$ on the plates. The space betⁿ the plates is filled with two dielectric slabs with thickness 'a'.

→ Slab 1 has dielectric const = 2

Slab 2 - - - - - = 1.5

(*) $\vec{D}$ in each slab:

$$\oint \vec{D} \cdot d\vec{a} = (Q_f)_{enc}.$$

$$\Rightarrow \quad DA = \sigma A$$

$$\Rightarrow \quad D = \sigma$$

Then, $\vec{D} = -\sigma \hat{z}$

$\vec{D} = 0$ inside metal plates.

(c) $\vec{E}$ in each slab:

$$\vec{D} = \epsilon \vec{E}$$

for slab ① $\Rightarrow |\vec{E}_1| = \dfrac{\sigma}{\epsilon_1}$

slab ② $\Rightarrow |\vec{E}_2| = \dfrac{\sigma}{\epsilon_2}$

$$\left.\begin{array}{l} \epsilon_1 = 2\epsilon_0 \\ \epsilon_2 = 1.5\epsilon_0 \end{array}\right\} \begin{array}{l} |\vec{E}_1| = \sigma / 2\epsilon_0 \\ |\vec{E}_2| = \sigma / 1.5\epsilon_0 = 2\sigma / 3\epsilon_0 \end{array}$$

* Polarisation:

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$\Rightarrow |\vec{P}| = \frac{\epsilon_0 \chi_e \sigma}{\epsilon_0 \epsilon_r}$$

$$= \frac{\chi_e}{\epsilon_r} \sigma$$

$$(\chi_e = \epsilon_r - 1)$$

$$\Rightarrow |\vec{P}| = \frac{\epsilon_r - 1}{\epsilon_r} \sigma = \left(1 - \frac{1}{\epsilon_r}\right)\sigma$$

→ for slab 1, $|\vec{P}_1| = \frac{\sigma}{2}$

$$|\vec{P}_2| = \frac{\sigma}{3}$$

* Potential difference,

$$\Delta V = -\int_b^a \vec{E}\cdot d\vec{l}$$

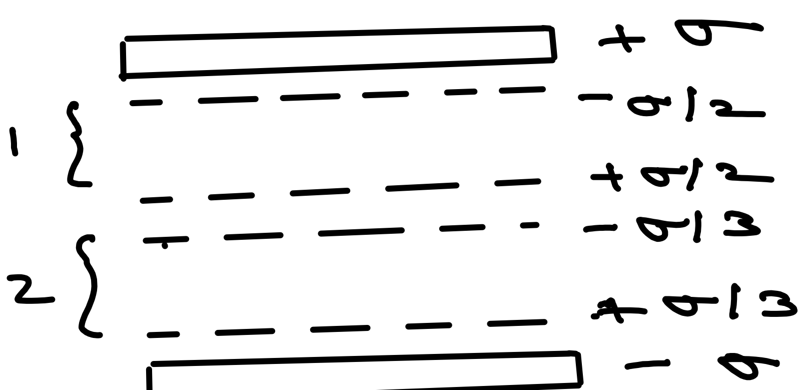
$$= E_1 a + E_2 a = \frac{\sigma a}{\epsilon_0}\left(\frac{1}{2} + \frac{2}{3}\right)$$

$$= \frac{7\sigma a}{6\epsilon_0}$$

(f) Bound charges:

Constant polarisation $\Rightarrow \rho_b = 0$

The bound surface charge density:



$$\sigma_b = -P_1 \quad (\text{top of slab 1})$$
$$= +P_1 \quad (\text{bottom of slab 1})$$
$$= -P_2 \quad (\text{top of slab 2})$$
$$= +P_2 \quad (\text{bottom of slab 2})$$

$$\sigma_b = \vec{P}\cdot\hat{n}$$
$$\vec{P}_1 = \frac{\sigma}{2}(-\hat{z})$$
$$\vec{P}_2 = \frac{\sigma}{3}(-\hat{z})$$