Physics formulas

1) $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\theta$
2) $\vec{a} \times \vec{b} = |\vec{a}| \cdot |\vec{b}| \sin\theta \, \hat{n}$
3) $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$
4) $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$
5) STP =

$$[\vec{a}\vec{b}\vec{c}] = a \cdot (b \times c) = b \cdot (c \times a) = c \cdot (a \times b)$$

6) VTP =

$$\vec{a} \times (\vec{b} \times \vec{c}) = b(a \cdot c) - c(a \cdot b)$$

7) Infinitesimal displacement vector

$$dl = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

8) $$\bar{A}_i = \sum_{j=1}^{3} \underbrace{R_{ij}}_{\text{Rotation matrix}} A_j$$

9) $$dT = \left(\frac{\partial T}{\partial x}\right) dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz$$

or

$$\underbrace{(\vec{\nabla} T)}_{\text{Gradient}} \cdot \underbrace{(d\vec{l})}_{\text{Infi. dis. vec.}}$$

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- Del Operator

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

Operation →

$\vec{\nabla} T$ = Gradient

$\vec{\nabla} \cdot A$ = Divergence → Scalar

$\vec{\nabla} \times A$ = Curl

→ Curl of a gradient is zero.

Laplacian of a Scalar →

$$\nabla^2 \cdot T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

$$= \vec{\nabla} \cdot (\vec{\nabla} T)$$

→ line Integrals →

$$\int_a^b v \cdot dl$$

if line or shape is closed then

$$\oint v \cdot dl$$

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→ Surface Integrals →

$$\int_S \vec{v} \cdot \vec{da}$$

$da_1 = dx\,dy$
$da_2 = dy\,dz$ } Either one is Constant
$da_3 = dz\,dx$

Closed surface:-

$$\oint_S \vec{v} \cdot d\vec{a}$$

→ Volume Integrals →

$$\int_V T \cdot d\tau$$

$d\tau$ = Infitesimal volume element

T = Any Scalar

$d\tau = dx\,dy\,dz$

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→ Fundamental theorem –

→ Gradient

$$\int_a^b (\nabla f)\cdot dl = f(b) - f(a)$$

or

$$\oint (\nabla f)\cdot dl = 0$$

→ Divergence

$$\int (\nabla\cdot A)\cdot d\tau = \oint A\cdot da$$

→ Curl

$$\int (\nabla\times A)\cdot da = \oint A\cdot dl$$

↑
Stokes theorem

→ Curvilinear Coordinates →

1) Spherical polar → (r, θ, ϕ)

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

$$A = A_r \hat{r} + A_\theta \hat{\theta} + A_\phi \hat{\phi}$$

$$\hat{r} = \sin\theta \cos\phi \,\hat{x} + \sin\theta \sin\phi \,\hat{y} + \cos\theta \,\hat{z}$$
$$\theta = \cos\theta \cos\phi \,\hat{x} + \cos\theta \sin\phi \,\hat{y} - \sin\theta \,\hat{z}$$
$$\hat{\phi} = -\sin\phi \,\hat{x} + \cos\phi \,\hat{y}$$

$$\vec{r} = r\sin\theta \cos\phi \,\hat{x} + r\sin\theta \sin\phi \,\hat{y} + r\cos\theta \,\hat{z}$$

$$\hat{r} = \frac{\frac{\partial \vec{r}}{\partial r}}{\left|\frac{\partial \vec{r}}{\partial r}\right|}, \quad \hat{\theta} = \frac{\frac{\partial \vec{r}}{\partial \theta}}{\left|\frac{\partial \vec{r}}{\partial \theta}\right|}, \quad \hat{\phi} = \frac{\frac{\partial \vec{r}}{\partial \phi}}{\left|\frac{\partial \vec{r}}{\partial \phi}\right|}$$

$$dl_r = dr$$
$$dl_\theta = r\,d\theta$$
$$dl_\phi = r\sin\theta\, d\phi$$

→ Infinitesimal displacement →

$$d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\, d\phi\,\hat{\phi}$$

→ Inf. Volume →

$$d\tau = dl_r\, dl_\theta\, dl_\phi$$
$$= r^2 \sin\theta\, dr\, d\theta\, d\phi$$

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→ Cylindrical co-ordinates →

$x = s\cos\phi$
$y = s\sin\phi$
$z = z$

$\hat{s} = \cos\phi\,\hat{x} + \sin\phi\,\hat{y}$
$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$
$z = \hat{z}$

→ Inf. dis. →
$dl_s = ds$,
$dl_\phi = s\,d\phi$
$dl_z = dz$

$dl = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}$

$d\tau = s\,ds\,d\phi\,dz$

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Electrostatics

$$F = \frac{kQq}{r^2}\hat{r} \qquad k = \frac{1}{4\pi\epsilon_0}$$

$$F = Q.E$$

→ Continuous Charge distribution

$$E(r) = k\int \frac{1}{r^2}\hat{r}\, dq$$

$$dq \to \lambda\, dl' \to \sigma\, da' \to \rho\, d\tau'$$

→ for line charge →

$$E(r) = k\int \frac{\lambda(r')}{r^2}\hat{r}\, dl'$$

→ for surface charge →

$$E(r) = k\int \frac{\sigma(r')}{r^2}\hat{r}\, da'$$

→ for vol. charge →

$$E(r) = k\int \frac{\rho(r')}{r^2}\hat{r}\, d\tau'$$

Gauss Law -

→ for any closed surface →

$$\oint E \cdot da = \frac{1}{\epsilon_0} Q_{enc.}$$

$$= \int_V (\nabla \cdot E)\, d\tau$$

$$Q_{enc.} = \int_V \underbrace{\rho}_{\text{charge density}}\, d\tau$$

or

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

→ Potential →

$$V(r) \equiv -\int_O^r E \cdot dl$$

$$V(b) - V(a) = -\int_O^b E \cdot dl + \int_O^a E \cdot dl$$

$$= -\int_O^b E \cdot dl - \int_a^O E \cdot dl$$

$$= -\int_a^b E \cdot dl$$

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$$E = -\nabla V$$

$$\nabla \times E = 0 \quad (\text{Curl of } E)$$

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0} \quad (\text{divergence of } E)$$

or

$$\nabla^2 V = \frac{-\rho}{\varepsilon_0}$$

$$V(a) - V(b) = \frac{W}{Q}$$

$$W = \frac{\varepsilon_0}{2} \int E^2 \, d\tau$$

→ <u>Conductors</u> →

→ $E = 0$ inside a conductor
→ $\rho = 0$ inside a conductor
→ Conductor is equipotential
→ E is ⊥lar to surface just outside a conductor

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→ Capacitor

2 Conductors with charge $+Q$,
$-Q$

$$V = V_+ - V_- = -\int \vec{E} \cdot d\vec{l}$$

$$\rightarrow C = \frac{Q}{V}$$

↓

Capacitance

$$\vec{E} = \frac{Q}{\epsilon_0 A} \Rightarrow V = \frac{Q}{A\epsilon_0} d$$

$$\Rightarrow C = \frac{Q}{V} = \frac{A\epsilon_0}{d}$$

[d = sep. betⁿ plates
A = area of plates]

Work done up a capacitor to Q:-

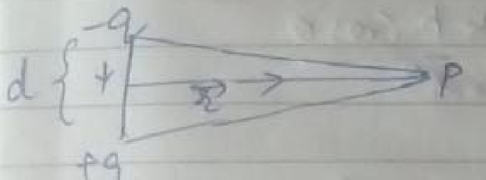
→ at some point the charge on capacitor plate $= +q$

→ potetial difference = $\frac{q}{c}$

$$W = \int_0^Q \frac{q}{c}\, dq$$

$$= \frac{Q^2}{2C} = \frac{1}{2}CV^2 \quad [Q = CV]$$

Electric Dipole →



$$V(\vec{r}) = \frac{k\, q\, d\cos\theta}{r^2}$$ → [potential due to a point]

→ Physical charges with equal and opposite charges $\pm q$
dipole moment

$$\vec{p} = q\vec{d}$$ → ('-'ve to '+'ve)

→ valid for , $r \gg d$

$$\vec{E} = -\vec{\nabla} V$$

$$V_{dip}(r,\theta) = \frac{k p \cdot \hat{r}}{r^2}$$

$$= \frac{k p \cos\theta}{r^2}$$

To get Electric field

$$E_r = -\frac{\partial V}{\partial r} = \frac{2 k p \cos\theta}{r^3}$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{k p \sin\theta}{r^3}$$

$$E_\phi = \frac{-1}{r\sin\theta}\frac{\partial V}{\partial \phi} = 0$$

$$\vec{E}_{dip}(r,\theta) = \frac{k p}{r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

Subject

→ Electric field due to dipole :→

$$\vec{E}_{dip.}(\vec{r}) = \frac{k}{r^3}\left[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}\right]$$

$[r << d]$

$$= \frac{k}{r^3}\left[2p\cos\theta\,\hat{r} + p\sin\theta\,\hat{\theta}\right]$$

$$\left[\begin{aligned}\vec{p} &= \left((\vec{p}\cdot\hat{r})\hat{r} + (\vec{p}\cdot\hat{\theta})\hat{\theta}\right) \\ &= p\cos\theta\,\hat{r} - p\sin\theta\,\hat{\theta}\end{aligned}\right]$$

$$\vec{E}_{dip.(\vec{r})} = \frac{kp}{r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$

→ Electric field due to a dipole $\alpha \frac{1}{r^3}$

→ Electric field due to a monopole $\alpha \frac{1}{r^2}$

→ Dielectric :→

→ In dielectric material, electrons cannot move about freely. They can move a bit in ~~election~~ atoms or molecules.

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Teacher's Singnatura ....

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→ If movements of e^- is restricted = Insulators (small dielec. cons.)

→ If charge particles can move a little bit = Dielectrics (larger dielec. cons.)

→ If material is made up of non-polar molec. or neutral atoms → in an electric field (ext.) → tiny dipoles are induced. pointing in same dirn of electric field.

→ If material is made up of polar molec. ⇒ permanent dipoles in system. → external electric field exerts a torque on them → aligns them along the dirn of the electric field.

→ Net effect is a lot of tiny dipoles pointing along the dirn of the field.

⇒ Polarisation

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Subject

→ Polarisation ($\vec{P}$) :→

$\vec{P}$ = Dipole moment per unit volume.

→ Field due to a polarised object :→

for a single dipole $\vec{p}$ →

$$V(\vec{r}) = k \frac{\vec{p}\cdot\hat{\mathscr{r}}}{\mathscr{r}^2}$$

dipole moment :- $\vec{p} = \vec{P}.d\tau'$ for each infinitesimal volume element $d\tau'$

→ The total potential →

$$V = k \int_V \frac{\vec{P}(\vec{r})\cdot\hat{\mathscr{r}}}{\mathscr{r}^2}\, d\tau'$$

⇒ $\boxed{\begin{aligned}\sigma_b &= \vec{P}\cdot\hat{n} \\ \rho_b &= -\vec{\nabla}\cdot\vec{P}\end{aligned}}$

Bound surface density

Bound charge

Bound volume charge density

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Teacher's Singnature

Subject

$\rightarrow V(\vec{r}) = k \oint_S \frac{\sigma_b}{\mathscr{r}} da' + k \int_V \frac{\rho_b}{\mathscr{r}} d\tau'$

→ Potential of a system with surface charge density $P \cos\theta$ and volume charge density $= 0$ or $\vec{P}$ is uniform along z axis.

$$V(r,\theta) = \begin{cases} \dfrac{P r \cos\theta}{3\epsilon_0} & \text{for } r \leq R \\[2ex] \dfrac{P R^3 \cos\theta}{3\epsilon_0 r^2} & \text{for } r \geq R \end{cases}$$

as we know,

$$z = r\cos\theta$$

for $r < R$,

$$E = -\vec{\nabla} V = \frac{P}{3\epsilon_0}\hat{z} = \frac{\vec{P}}{3\epsilon_0}$$

potential outside the sphere →

$$V(r,\theta) = \frac{P R^3 \cos\theta}{3\epsilon_0 r^2} = \frac{k\,\vec{p}\cdot\hat{r}}{r^2}$$

where $\left[\vec{p} = \frac{4}{3}\pi R^3 \vec{P}\right]$

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Teacher's Signature

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Subject

→ Gauss law in presence of dielectric →

Polarisation → accumulation of bound charges.

$\downarrow$

$\rho_b = -\vec{\nabla}\cdot\vec{P}$

$\sigma_b = \vec{P}\cdot\hat{n}$

→ These bound charges give rise to electric field.

→ Total field = Field due to bound chrges
+
Field due to free chrges.

$\Rightarrow \rho = \rho_b + \rho_f$

$\left[\text{using Gauss law} \Rightarrow \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}\right]$

$\Rightarrow \varepsilon_0(\vec{\nabla}\cdot\vec{E}) = \rho = -\vec{\nabla}\cdot\vec{P} + \rho_f$

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$\Rightarrow \vec{\nabla}\cdot(\epsilon_0\vec{E}+\vec{P}) = \rho_f$

$\Rightarrow \rho_f = \vec{\nabla}\cdot\vec{D}$

$\Rightarrow \underline{\vec{D} = \epsilon_0\vec{E}+\vec{P} \equiv \text{Electric Displacement}}$

Hence,

$$\boxed{\oint \vec{D}\cdot d\vec{a} = \int \rho_f \, d\tau = (Q_f)_{enc}}$$

Gauss law for dielectrics

total free charge enclosed

→ <u>Linear Dielectrics</u> :→

→ A class of dielectrics for which the induced polarisation is proportional to the electric field or $\vec{P} \propto \vec{E}$

$$\vec{P} = \epsilon_0 \boxed{\chi_e} \vec{E} \quad [\vec{E} \text{ is not strong}]$$

Dimensionless

Electric suspectibility of medium

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Subject

$\vec{E}$ = total electric field.

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$
$$= \epsilon_0 \vec{E} + \epsilon_0 \chi_e \vec{E}$$

$$\boxed{\vec{D} = \epsilon_0 (1+\chi_e)\vec{E}}$$

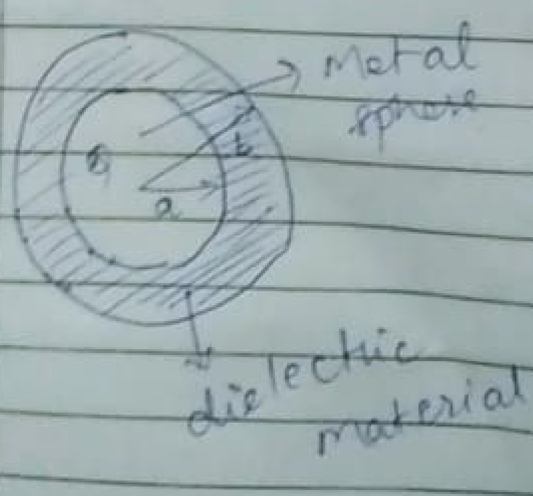
or

$$\boxed{\vec{D} = \epsilon \vec{E}} \quad \underbrace{[\epsilon = \epsilon_0 (1+\chi_e)]}_{\text{permitivity of the material}}$$

$\boxed{\epsilon_r = 1+\chi_e}$ → Relative permitivity or Dielectric constant

$\Rightarrow$ ~~E~~ $\epsilon_r = \dfrac{\epsilon}{\epsilon_0}$

Potential at centre of the ~~hollow~~ sphere



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यहाँ पे $\varepsilon = \varepsilon_0 \cdot \varepsilon_r$

$$\vec{E} = \begin{cases} \dfrac{Q}{4\pi\varepsilon r^2}\hat{r} & a < r < b \\[2ex] \dfrac{KQ}{r^2}\hat{r} & r > b \end{cases}$$

यहाँ पे सिर्फ ε_0

for $r < a$ $\vec{E} = \vec{P} = \vec{D} = 0$

potetial at centre of surface

$$V = \frac{Q}{4\pi}\left(\frac{1}{\varepsilon_0 b} + \frac{1}{\varepsilon a} - \frac{1}{\varepsilon b}\right)$$

Polarisation →

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

$$= \frac{\varepsilon_0 \chi_e Q}{4\pi\varepsilon r^2}\hat{r}$$ (in the dielectric)

Bound charges →

$$\rho_b = 0$$

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Subject

$\sigma_b = \vec{P}\cdot\hat{n}$ → $\dfrac{\varepsilon_0 \chi_e Q}{4\pi\varepsilon\cdot b^2}$ (at outer surface)

→ $-\dfrac{\varepsilon_0 \chi_e Q}{4\pi\varepsilon\cdot a^2}$ (at inner surface)

→ $\hat{n}$ points outwards wrt dielectrics

$\Rightarrow \hat{n} = \hat{r}$ at $r = b$

$\hat{n} = -\hat{r}$ at $r = a$

→ for a linear dielectric →

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$
$$\vec{D} = \varepsilon \vec{E}$$

$\vec{\nabla}\times\vec{D}$ and $\vec{\nabla}\times\vec{P} \neq 0$ in general.

→ But if entire space is filled with a homogenius leniar dielectric.

$$\vec{\nabla}\times\vec{D} = 0$$
and
$$\vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\vec{D} = \varepsilon_0 \vec{E}_{vac}$$ $\left[\varepsilon_0 = \dfrac{\varepsilon}{?}\right]$

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Teacher's Singnature

$$\vec{n} = \frac{\vec{E}_{vac}}{E_r}$$

↳ The factor by which the field is reduced in presence of homogeneous linear dielectric.

Magnetostatics

→ A magnetic field lines form closed loops.

Stationary charges → Cons. Electric field → Electro-statics.

Steady currents → Cons. Magnetic field → Magneto-statics

→ Force on a point charge Q :→

$$F_{elec} = QE$$
$$F_{mag.} = Q(V \times B)$$ → Lorentz force Law.

Subject

Total force →

$$F = F_{ele.} + F_{mag.}$$

$$= Q(E + V \times B)$$

→ Magnetic force do not work.

→ Magnetic force can change direction but cannot change speed with which a particles moves

→ Currents. →

$$I = \frac{dq}{dt} = \frac{\lambda dl}{dt} = \lambda v$$

$\frac{dl}{dt} \to v$

$\lambda \to$ linear charge density

$$F_{mag.} = (V \times B) Q = \int (V \times B)\, dq$$

$$= \int (V \times B) \lambda\, dl$$

↓

$$F_{mag.} = \int (I \times B)\, dl.$$

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Teacher's Singnature ....

→ for surface current :-

$$K = \frac{dI}{dl_\perp} = \sigma v$$

$$F_{mag} = \int (v \times B)\,\sigma\, da = \int (K \times B)\, da$$

→ for volume current :-

$$J = \frac{dI}{da_\perp} = \rho v$$

$$F_{mag} = \int (J \times B)\, d\tau = \int (v \times B)\,\rho\, d\tau$$

→ For a closed surface

$$I = \oint_S J \cdot da = \int_V (\nabla \cdot J)\, d\tau$$

$$-\frac{d}{dt}\int_V \rho\, d\tau = -\int \frac{d\rho}{dt}\, d\tau$$

$$\Rightarrow \nabla \cdot J = -\frac{d\rho}{dt} \longrightarrow \text{The Continuity Eqn}$$

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Teacher's Signature

Subject

The Biot - Savart's law →

→ for steady current →

$$B(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{I \times \hat{\mathfrak{r}}}{\mathfrak{r}^2} = \frac{\mu_0 I}{4\pi}\int \frac{dI' \times \hat{\mathfrak{r}}}{\mathfrak{r}^2}$$

$$\mu_0 = 4\pi \times 10^{-7}\ N/A^2$$

$N/Amp.m$ or T → unit

→ for surface current →

$$B(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{K(r') \times \hat{\mathfrak{r}}}{\mathfrak{r}^2}\, da'$$

→ for volume current →

$$B(r) = \frac{\mu_0}{4\pi}\int \frac{J(r') \times \hat{\mathfrak{r}}}{\mathfrak{r}^2}\, d\tau'$$

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Subject

Net magnetic flux →

$$\oint \vec{B}.d\vec{a} = 0$$

$$\Rightarrow \int_V (\vec{\nabla}.\vec{B})\, d\tau = 0$$

$$\Rightarrow \boxed{\vec{\nabla}.\vec{B} = 0}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

$$\oint \vec{B}.d\vec{l} = \mu_0 . I_{enc.}$$

→ Ampere's law

→ Magnetic vector potential $(\vec{A})$ →

$$\vec{\nabla}.\vec{B} = 0$$

or

$$\vec{\nabla}.(\vec{\nabla}\times\vec{A}) = 0 \quad \left[\vec{B} = \vec{\nabla}\times\vec{A}\right]$$

→ A is not unique, we can always add any scalar function to $\vec{A}$ that won't change the actual $\vec{B}$.

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Subject

$$\vec{B} = \vec{\nabla} \times \vec{A}$$
$$\vec{A'} = \vec{A} + \vec{\nabla}\lambda \rightarrow \text{scalar fn.}$$

$$\Rightarrow \vec{B} = \vec{\nabla} \times \vec{A'}$$

→ Ampere's law :→

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu_0 \vec{J}$$
$$\Rightarrow \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 A = \mu_0 \vec{J}$$

If $\vec{\nabla} \cdot \vec{A}$ is 0 then

$$\vec{\nabla}^2 \cdot \vec{A} = -\mu_0 \vec{J}$$

→ Magnetisation of matter :→

$\vec{M}$ = mag. dipole moment per unit vol.

Bound Currents. →

Bound volume current ⇒

$$\vec{J_b} = \vec{\nabla} \times \vec{M}$$

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Subject

Bound surface current →

$$\vec{K}_b = \vec{M} \times \hat{n}$$

→ $\vec{H} = \dfrac{\vec{B}}{\mu_0} - \vec{M}$ [$\vec{H}$ is like $\vec{D}$ → electro]

$$\vec{\nabla} \times \vec{H} = \vec{J}_f$$

$$\oint \vec{H} \cdot d\vec{l} = I_{f\,enc}$$

→ Linear medium →

$$\vec{M} = \chi_m \vec{H}$$

χ_m → mag. suspectibility

$$\vec{B} = \mu_0 (1 + \chi_m) \vec{H}$$

$$= \mu \vec{H}$$

$$\mu = \mu_0 (1 + \chi_m)$$

μ → permeability of metarial

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