Name of Student:................ Enrollment Number:................

Department:...... Batch:.......

BENNETT UNIVERSITY,
GREATER NOIDA
Mid Term Examination, Even Semester 2019-20

COURSE CODE : EMAT102L MAX. TIME: 1 Hour
COURSE NAME : Linear Algebra and Ordinary Differential Equations MAX. MARKS: 30

Instructions: There are 7 questions in this question booklet. All questions are mandatory. All questions must be answered in this question booklet itself in the space provided. Rough sheets can be taken if required but will **NOT** be collected and evaluated.

1) State TRUE or FALSE: [5]

(a) The vector $\begin{bmatrix} -4 \\ 11 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$. (T/F)

(b) Let S be a set of m vectors in $\mathbb{R}^n$, then S is linearly dependent if $m > n$. (T/F)

(c) $S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 \leq 1 \right\}$ is a subspace of $\mathbb{R}^2$. (T/F)

(d) $S = \left\{ \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$. (T/F)

(e) Let A be any $n \times n$ matrix, then $W = \{X \in \mathbb{R}^n : AX \neq 0\}$ is a subspace of $\mathbb{R}^n$. (T/F)

2) Fill in the blanks:

(a) The set $S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 = 1 \right\}$ is not a subspace of $\mathbb{R}^2$, because ....
.... [1]

(b) Let S and T be two subspaces of $\mathbb{R}^4$ defined by

$$S = \left\{ \begin{bmatrix} s \\ t \\ 0 \\ 0 \end{bmatrix} : s, t \in \mathbb{R} \right\} \quad \text{and} \quad T = \left\{ \begin{bmatrix} 0 \\ a \\ b \\ 0 \end{bmatrix} : a, b \in \mathbb{R} \right\}. \text{ Then}$$

$\dim(S) =$ .... [1]

$\dim(T) =$ .... [1]

$\dim(S \cap T) =$ .... [1]

(c) If $T : \mathcal{P}_2(\mathbb{R}) \to \mathcal{P}_2(\mathbb{R})$ is a linear transformation and

$$T(1) = 1 + x, \quad T(x) = 2 + x^2 \quad T(x^2) = x - 3x^2.$$

Then $T(-3 + x - x^2) =$ .... [1]

3) Figure 0.1 represents the traffic entering and leaving a roundabout road junction. Construct a system of equations that describes the flow of traffic along the various branches. What is the minimum flow possible along the branch BC? What are other flows at that time? (Units are vehicles per hour.) [4]

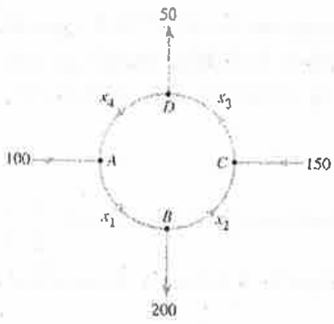

Figure 0.1: Roundabout road junction

Answer:

4) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y \\ x-y \\ z \end{bmatrix}$. Show that T is a linear transformation, and find the dimension of null space and a basis for its range space. [2+1+1]

Answer:

5) Let $A = \begin{bmatrix} 0 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Then [1+1+3+1]

(a) find the characteristic polynomial of A.

(b) find the eigenvalues of A.

(c) find the eigenvectors corresponding to each eigenvalue.

(d) Is A diagonalizable? Justify your answer.

Answer:

6) Apply the Gram-Schmidt process to the following set to find a set of orthogonal vectors : [4]

$$S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\} \quad \text{in } \mathbb{R}^3.$$

Answer:

7) Find all the values of $a, b, c \in \mathbb{R}$ for which the matrix $A = \begin{bmatrix} a & c \\ 0 & b \end{bmatrix}$ is not diagonalizable. Justify your answer. [2]

Answer: