POSSESSION OF MOBILES IN EXAM IS UFM PRACTICE

Name of student ____________________ **Enrolment No.**__________

BENNETT UNIVERSITY, GREATER NOIDA

B.TECH/ TEST – Supplementary Examination: SPRING SEMESTER A.Y. 2018-2019

COURSE CODE	**EPHY105L**	MAX. TIME: **2 hours**
COURSE NAME :	**Electromagnetics**	
COURSE CREDIT:	**3**	MAX . MARKS: **100**

ALL QUESTIONS ARE COMPULSORY

1. Give brief answers with appropriate reasons to the following questions: **(8x5=40)**
   a) A positive charge of 1 μC is placed at the center of a cavity formed inside a *spherical conducting shell* having an inner radius 1 m and an outer radius 2 m. What is the electric field at a distance of 0.5 m from the center?
   b) A sphere of radius R carries a charge density given by $\rho(r) = \rho_0(1 - r/R)$. What is the value of $\nabla.\vec{E}$ at a point at a distance $R/2$ from the center?
   c) A charge Q is placed at the center of a dielectric sphere of radius R and uniform dielectric constant K. Using Gauss's law obtain $\vec{E}$ within the sphere.
   d) Determine whether the vector function $\vec{G} = \frac{C}{r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$ (expressed in spherical polar coordinates) can represent a magnetic field.
   e) A cylindrical wire of radius R is carrying a current given by $I(r) = I_0\left(1 - \frac{r}{R}\right)$ (here r is the cylindrical polar coordinate) where I_0 is a constant. What is the value of $\nabla \times \vec{B}$ at a distance $2R$ from the axis?
   f) An infinitely long straight wire made of copper (with $\mu = \mu_0$) and of radius R carries a current I which is uniformly distributed across its cross section. Using Ampere's law obtain the magnetization $\vec{M}$ within the wire.
   g) Verify whether the following two vector potentials $\vec{A}_1$ and $\vec{A}_2 = \vec{A}_1 + (x\hat{x} + y\hat{y})$ correspond to the same magnetic field.
   h) An infinitely long straight solenoid with circular cross section of radius R has a cylindrical rod of radius $R/2$ placed coaxially with the solenoid. If the magnetic susceptibility of the rod is χ_m what is the ratio of $\vec{H}$ in the rod to $\vec{H}$ in the air gap?
2. A point charge Q is placed at the center of a sphere which has free space in the region $0 < r < R_1$ and a linear, homogeneous dielectric with susceptibility χ_e for $R_1 < r < R_2$ and free space for $r > R_2$.
   a) Using Gauss's law find the electric field in all regions. **(6)**
   b) What is the value of $\oint \vec{E}.d\vec{a}$ integrated over a sphere of radius $2R_2$. **(4)**
   c) Obtain the bound surface and volume charge densities in the dielectric. **(4)**

3. An infinitely long straight cylindrical tube with inner radius R_1 and outer radius R_2 made of a material with magnetic permeability μ carries a current I which is uniformly distributed across its cross section.
   a) Using Ampere's law, obtain the fields $\vec{H}$ and $\vec{B}$ in all the regions. **(8)**
   b) Obtain the surface bound current and volume bound current in the wire. **(6)**
   c) What is the value of $\nabla \times \vec{B}$ at a distance $R_1/2$ from the axis of the cylinder? **(2)**

4. A uniform magnetic field $\vec{B} = B(t)\hat{z}$ exists in the cylindrical region $r < R$ with a time varying magnetic field given by $B(t) = B_0 \sin \omega t$; the magnetic field is zero for $r > R$. Assuming that the induced electric field due to the time varying magnetic field is along $\hat{\phi}$ direction (in cylindrical coordinates),
   a) Obtain the induced electric field $\vec{E}$ at a distance $r < R$ and $r > R$. **(9)**
   b) What will be the values of $\nabla \times \vec{E}$ for $r < R$ and $r > R$? **(5)**

5. An electromagnetic wave propagating in free space (velocity of the wave is 3 x 10^8 m/s) is described by the following expression for the electric field (z is measured in meters):
$$\vec{E} = E_0\hat{y}\cos[2\pi(2\times 10^6 z + \nu t)]$$
   a) What are the values of frequency ν and wavelength λ of the wave? **(4)**
   b) What is the direction of propagation of the wave? **(4)**
   c) Given that the corresponding magnetic field of the wave is $\vec{B} = \vec{B}_0 \cos[2\pi(2\times 10^6 z + \nu t)]$, using Maxwell's equations obtain the magnitude and direction of $\vec{B}_0$. **(8)**

Some useful formulas

- $\epsilon_0 = 8.85 \times 10^{-12}$ C²/N-m²
- In Cartesian coordinates:

$$\nabla f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}$$

$$\nabla.\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\nabla\times\vec{F} = \left[\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right]\hat{\imath} + \left[\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right]\hat{\jmath} + \left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right]\hat{k}$$

- In spherical polar coordinates:

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\nabla.\vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi}$$

$$\nabla\times\vec{F} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, F_\phi) - \frac{\partial F_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial F_r}{\partial\phi} - \frac{\partial}{\partial r}(rF_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial F_r}{\partial\theta}\right]\hat{\phi}$$

- In cylindrical coordinates:

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial\phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$$

$$\nabla.\vec{F} = \frac{1}{r}\frac{\partial}{\partial r}(rF_r) + \frac{1}{r}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}$$

$$\nabla\times\vec{F} = \left[\frac{1}{r}\frac{\partial F_z}{\partial\phi} - \frac{\partial F_\phi}{\partial z}\right]\hat{r} + \left[\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right]\hat{\phi} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rF_\phi) - \frac{\partial F_r}{\partial\phi}\right]\hat{z}$$

- Maxwell's equations:

$$\nabla.\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla.\vec{B} = 0$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$