Department of Physics, Bennett University

EPHY105L (I Semester 2021-2022)

Tutorial Set-1

1. **Three vertices of a triangle are located at A(6,-1,2), B(-2,3,-4) and C(-3,1,5). Find**
   (a) $\vec{r}_{\rm AB}$ **and** $\vec{r}_{\rm AC}$
   (b) The angle $\theta_{\rm BAC}$ **at vertex A .**

2. **Find the area of a parallelogram determined by the vectors** $\vec{a} = \hat{x} + 3\hat{y}$ **and** $\vec{b} = \hat{x} - 3\hat{y}$**.**

3. **Find the volume of a parallelopiped generated by the vectors** $\vec{u} = \hat{x} + 3\hat{y}$**,** $\vec{v} = \hat{x} - 3\hat{y}$ **and** $\vec{w} = -\hat{x} - \hat{y} - \hat{z}$**.**

4. **Find the vector normal to the plane that contains the points P(1,0,0), Q(1,2,3) and R(2,2,2).**

5. **Find the gradient (**$\vec{\nabla}\phi$**) of the following scalar functions at a point** P **with Cartesian coordinates (2,-1,2):**

$$(a) f(x,\ y,\ z) = x^2 + y^2 + z^2 - 9$$
$$(b) g(x,\ y,\ z) = x^2 + y^2 - z - 3$$

   Using gradients obtain the angle between the surfaces given by $f(x,\ y,\ z) = 0$ **and** $g(x,\ y,\ z) = 0$ **at the point** P**.**

6. **Obtain the maximum directional derivative of the scalar function** $f(x,\ y,\ z) = x^2yz^3$ **at a point with coordinates (2,1,-1).**

7. **Calculate divergence (**$\nabla.\vec{F}$**) of the following vector functions:**

$$(a)\vec{F}_1 = \hat{x}x - \hat{y}y$$
$$(b)\vec{F}_2 = \hat{z}z$$
$$(c)\vec{F}_3 = \alpha\vec{r} = \alpha(\hat{x}x + \hat{y}y + \hat{z}z)$$
$$(d)\vec{F}_4 = \beta\frac{\hat{r}}{r^2} = \beta\frac{\vec{r}}{r^3} = \beta\frac{(\hat{x}x + \hat{y}y + \hat{z}z)}{(x^2 + y^2 + z^2)^{3/2}} \quad \text{for } r \neq 0$$

8. **Calculate curl (**$\nabla \times \vec{F}$**) of the following vector functions:**

$$(a)\vec{F}_1 = \hat{x}\alpha y$$
$$(b)\vec{F}_2 = \hat{x}\alpha x + \hat{y}\beta y^2$$
$$(c)\vec{F}_3 = \hat{x}x^2 + \hat{y}3xz^2 - \hat{z}2xz$$

9. **Consider the scalar function given by** $f(x,\ y,\ z) = \alpha xy^2$**.**
   (a) Calculate the gradient of the function f**.**
   (b) Obtain the curl of the gradient of the function and show that it is zero. [*Note that the curl of the gradient of a function is always zero. Thus if we find a vector function whose curl is zero, then the vector function can always be represented by the gradient of a scalar function.*]

10. Consider a vector function given by $\vec{G} = \hat{x}x^2 + \hat{y}3xz^2 - \hat{z}2xz$.
(a) Calculate the curl of the vector function $\vec{G}$.
(b) If $\nabla \times \vec{G} = \vec{A}$ then show that $\nabla.\vec{A} = 0$. [*Note that the divergence of a vector function is always zero. Thus if we find a vector function whose divergence is zero, then we can always represent the vector function as the curl of another vector function.*]

11. Find $\vec{\nabla}.(\vec{\nabla} \times (\vec{\nabla} f))$ for $f(x, y, z) = x^3 + y^2 + z$.