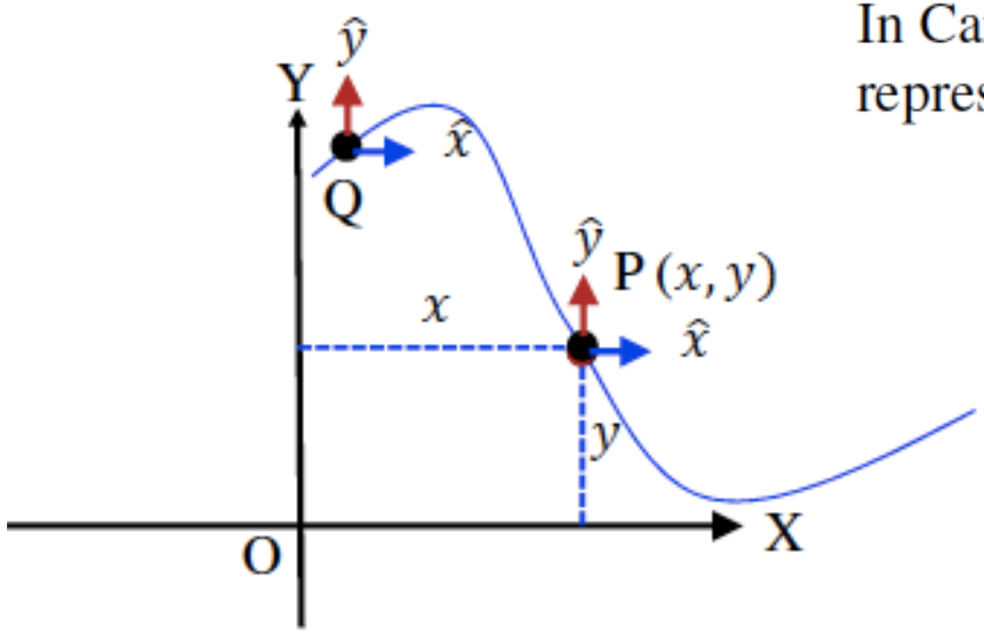
# Lecture - 8

# Coordinate Systems




In Cartesian coordinate position $P$ is represented by $(x, y)$.

$$\overrightarrow{OP} = \vec{r} = x\,\hat{x} + y\,\hat{y}$$

**Note:**

- $\hat{x}$ and $\hat{y}$ are unit vectors **pointing the increasing direction** of $x$ and $y$.

$\hat{x}$ is the unit vector perpendicular to $x = constant$ line

$\hat{y}$ is the unit vector perpendicular to $y = constant$ line

**Velocity** $$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(x\,\hat{x} + y\,\hat{y})$$

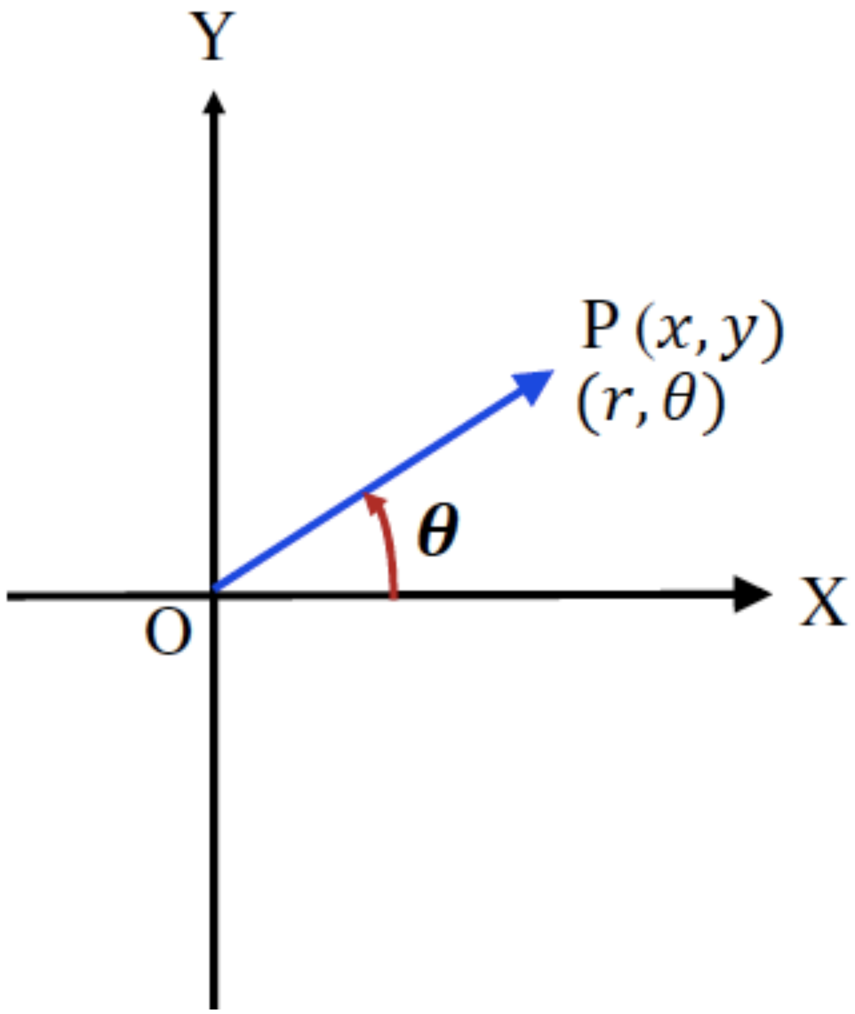
$$= \dot{x}\hat{x} + x\frac{d\hat{x}}{dt} + \dot{y}\hat{y} + y\frac{d\hat{y}}{dt}$$

$$\boxed{\text{Since, } \frac{d\hat{x}}{dt} = \frac{d\hat{y}}{dt} = 0}$$

$$\boxed{\vec{v} = \dot{x}\hat{x} + \dot{y}\hat{y}}$$

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt} = \ddot{x}\hat{x} + \ddot{y}\hat{y}}$$

# Plane-polar coordinate




Each point P $(x, y)$ on the plane can also be represented by its distance $(r)$ from the origin O and the angle $(\theta)$ OP makes with X-axis.

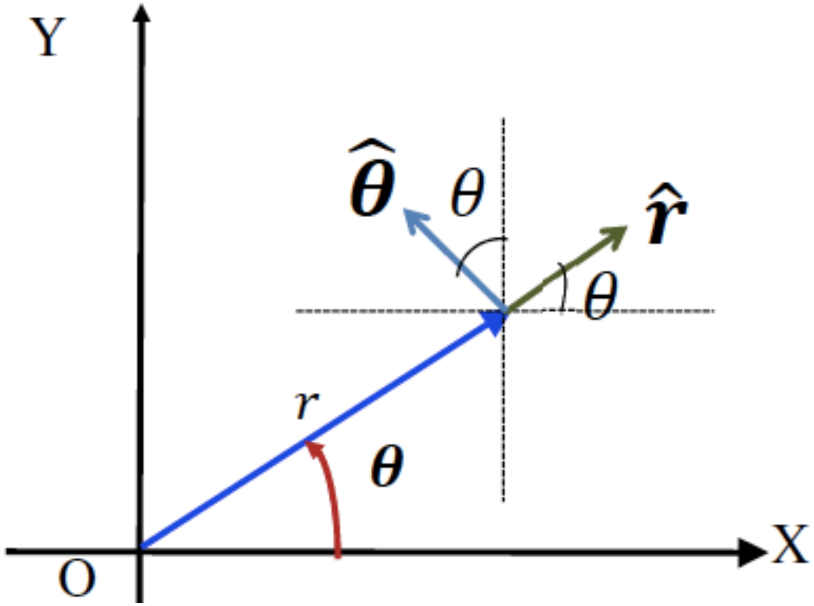
Relationship with Cartesian coordinates

$$x = r\ \cos\theta \quad \& \quad y = r\ \sin\theta$$

Thus ,

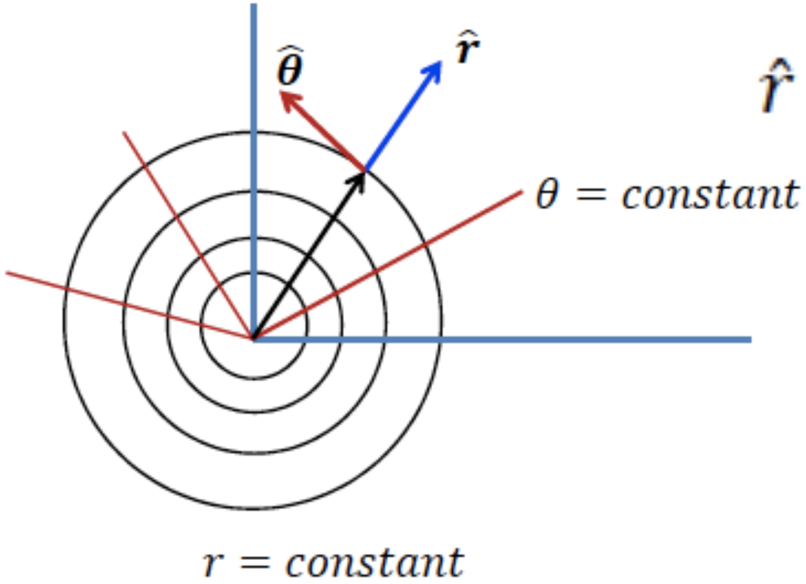
$$r = (x^2 + y^2)^{1/2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$



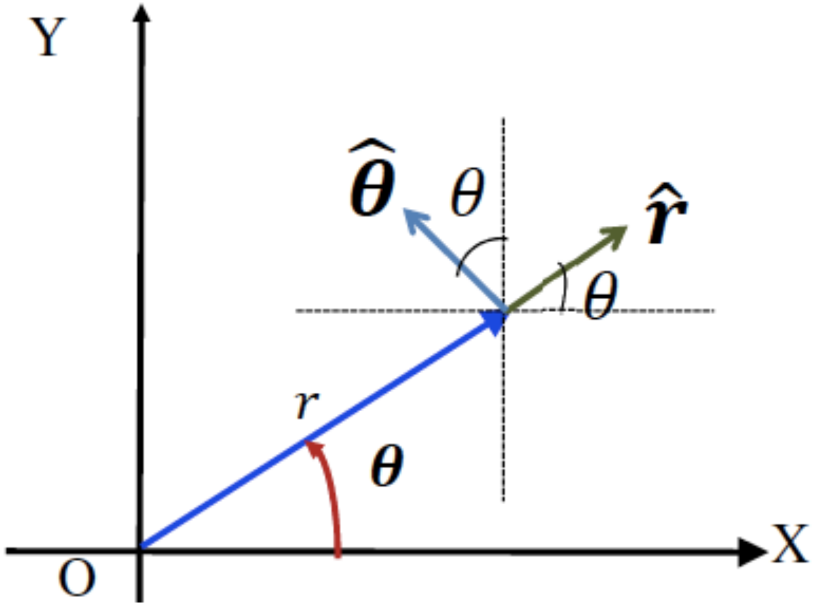

- $\hat{r}$ and $\hat{\theta}$ are unit vector along increasing direction of coordinate $r$ and $\theta$.

$\hat{r}$ and $\hat{\theta}$ are **orthogonal:** $\boldsymbol{\hat{r} \cdot \hat{\theta} = 0}$




$\hat{r}$ is the unit vector perpendicular to $r = constant$

$\hat{\theta}$ is the unit vector perpendicular to $\theta = constant$




$$\boxed{\bar{r} = r\cos\boldsymbol{\theta}\ \hat{\boldsymbol{x}} + r\sin\boldsymbol{\theta}\ \hat{\boldsymbol{y}}}$$

$$\hat{r} = \frac{\vec{r}}{|r|} \qquad \hat{r} = \frac{\partial\vec{r}}{\partial r}$$

$$\boxed{\begin{aligned}\hat{\boldsymbol{r}} &= \cos\boldsymbol{\theta}\ \hat{\boldsymbol{x}} + \sin\boldsymbol{\theta}\ \hat{\boldsymbol{y}}\\ \hat{\boldsymbol{\theta}} &= -\sin\boldsymbol{\theta}\ \hat{\boldsymbol{x}} + \cos\boldsymbol{\theta}\ \hat{\boldsymbol{y}}\end{aligned}}$$

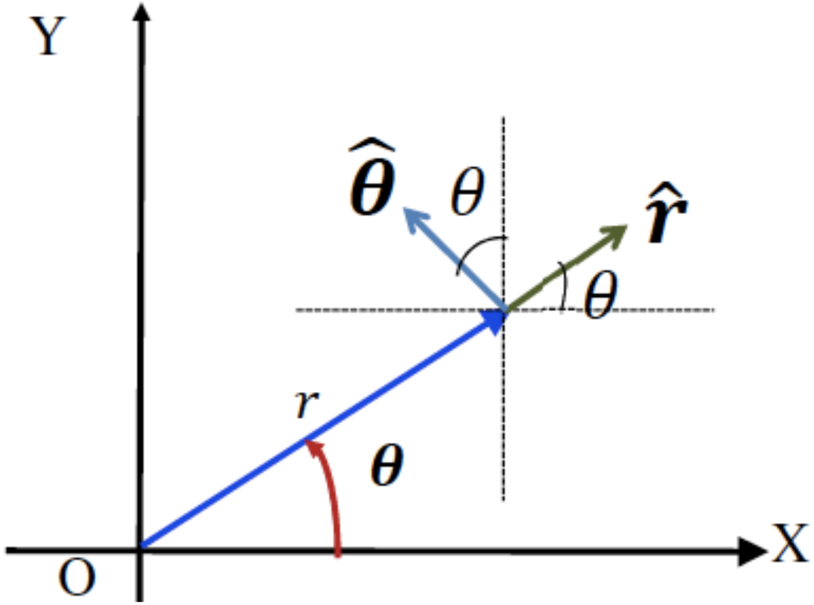
$$\hat{\theta} = \frac{\partial\hat{r}}{\partial\theta}$$

$$\vec{r} = r\ \hat{r}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(r\ \hat{r}) = \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt} = \dot{r}\hat{r} + r\frac{\partial\hat{r}}{\partial\theta}\frac{d\theta}{dt}$$

$$\boxed{\vec{\boldsymbol{v}} = \dot{\boldsymbol{r}}\hat{\boldsymbol{r}} + \boldsymbol{r}\dot{\boldsymbol{\theta}}\hat{\boldsymbol{\theta}}}$$

# Plane-polar coordinate




$$\boxed{\bar{r} = r\cos\theta\ \hat{x} + r\sin\theta\ \hat{y}}$$

$$\hat{r} = \frac{\vec{r}}{|r|} \qquad \hat{r} = \frac{\partial \vec{r}}{\partial r}$$

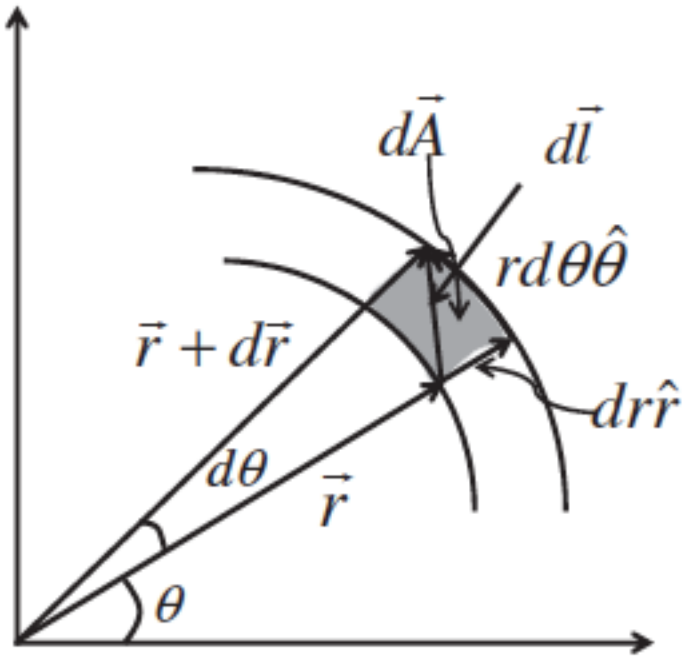
$$\boxed{\begin{aligned} \hat{r} &= \cos\theta\ \hat{x} + \sin\theta\ \hat{y} \\ \hat{\theta} &= -\sin\theta\ \hat{x} + \cos\theta\ \hat{y} \end{aligned}}$$

$$\hat{\theta} = \frac{\partial \hat{r}}{\partial \theta}$$

$$\vec{r} = r\ \hat{r}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(r\ \hat{r}) = \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt} = \dot{r}\hat{r} + r\frac{\partial \hat{r}}{\partial \theta}\frac{d\theta}{dt}$$

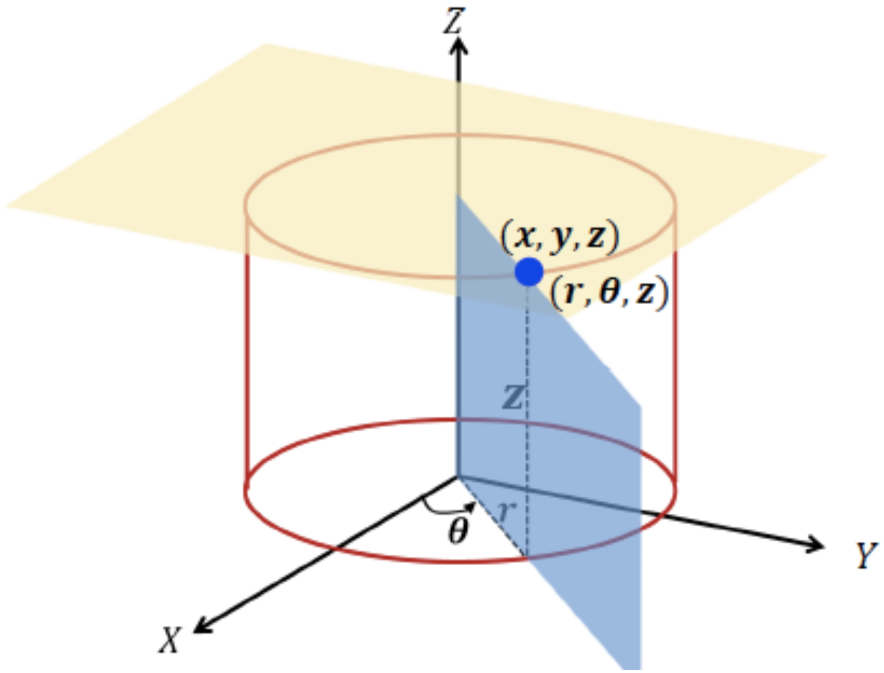
$$\boxed{\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}}$$




Line element: $d\vec{l} = dr\hat{r} + rd\theta\hat{\theta}$

Surface element: $d\vec{A} = dr\hat{r} \times rd\theta\hat{\theta} = rdrd\theta\hat{k}$

# Cylindrical coordinate System




How to specify a point $\boldsymbol{P}$ in space?
$(r, \boldsymbol{\theta}, z)$

- ✓ z is the Height from the *XY*-plane
- ✓ Coordinate of the foot of the point in XY plane.

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$

**Polar coordinate unit vectors $(\hat{r}, \hat{\theta})$ + additional unit vector in the $z$ –direction.**

**Reverse transformation**

$$r = (x^2 + y^2)^{1/2}$$
$$\theta = \tan^{-1}\frac{y}{x}$$
$$z = z$$

- ❑ $\hat{r}, \hat{\theta}$ and $\hat{z}$ are unit vectors along increasing direction of coordinates $r, \theta$ and $z$.

**Line element:**

$$\vec{dl} = dr\hat{r} + rd\theta\hat{\theta} + dz\hat{z}$$

**Surface element with fix r:**

$$\vec{dA} = rd\theta dz\hat{r}$$

**Volume element:** $dv = rdrd\theta dz$