POSSESSION OF MOBILES IN EXAM IS UFM PRACTICE

Name of Student: .... **Enrollment No.** ....

Department: ....

BENNETT UNIVERSITY, GREATER NOIDA
SUPPLEMENTARY EXAMINATION, DECEMBER 2019

COURSE CODE : EMAT101L — MAX. DURATION: 2 Hours
COURSE NAME: Engineering Calculus
COURSE CREDIT: 3-1-0 — MAX. MARKS: 100

Instructions:

- All questions are mandatory.
- Space for rough work is provided at the end.
- Please write down the solution in the given space.

1. Find the limit of the following sequences: [5+5]

(a) $a_n = \frac{1}{n}\sin^2 n + 3\left(1 + \frac{2}{n}\right)^n \ \forall\, n \in \mathbb{N}$.
Answer:

(b) $a_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(n+n)^2} \ \forall\, n \in \mathbb{N}$.
Answer:

2. Determine which of the following series converges/diverges. [5+5]

(a) $\sum_{n=1}^{\infty} \frac{n!}{10^n}$ (b) $\sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$

Answer:

3. Find the critical points and their nature of the function $f(x, y) = 2(x-4)^2 + 3(y-7)^2$. [5]

Answer:

4. Find all the critical points of the function $f(x, y) = \sin x \sin y$, for all $-2 \leq x, y \leq 2$. [5]

Answer:

5. Calculate $L(P_n, f)$ and $U(P_n, f)$ where $f(x) = x^2$, and $P_n = \{0, \frac{1}{n}, \frac{2}{n}, ..., \frac{n-1}{n}, \frac{n}{n}\}$. Then show that f is Riemann integrable on $[0, 1]$. [8]
Answer:

6. Do any TWO of the following: [$2 \times 6 = 12$]

(a) Determine if f' is continuous at 0 for the following function $f(x) = \begin{cases} x^2 \cos \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$.

(b) Show that $\lim_{x \to 0} \sin\left(\frac{1}{x^2}\right)$ does not exist.

(c) Evaluate $\int_0^{\pi/2} \sqrt{\tan x}\, dx$ using Beta and Gamma integral.

Answer:

7. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^4} & \text{if} \quad (x,y) \neq (0,0), \\ 0 & \text{if} \quad (x,y) = (0,0). \end{cases}$$

Examine

(a) Continuity of f at $(0,0)$. [3]

(b) Existence of the directional derivatives $D_u f$ at $(0,0)$ along each unit vector u. [3]

(c) Differentiability of f at $(0,0)$. [4]

8. Evaluate the following integral:

(a) $\displaystyle\iint_R x^2 dA$ where R is the region bounded by $y = x^2, y = x + 2$. [5]

(b) $\int_0^{\ln 3} \int_1^{\ln 4} e^{3x+2y} dy\, dx$. [5]

Answer:

9. Check whether the following statements are true/false. Justify your answer. $[3 \times 5 = 15]$

(a) Every continuous function is differentiable function.

Answer:

(b) If $|f|$ is Riemann integrable then f is Riemann integrable.

Answer:

(c) $|\cos 4x| - |\cos 4y| > 4|x - y|$ for every value $x, y \in \mathbb{R}$.

Answer:

(d) Every monotonic sequence is convergent.

Answer:

(e) The series $\sum_{n=1}^{\infty} n^{\frac{1}{n}}$ converges.

Answer:

(f) $\int_1^{\infty} \frac{dx}{x^{\frac{7}{2}}}$ converges.
Answer:

10. Fill in the blanks. [$2 \times 5 = 10$]

(a) $\bigcap_{n \in \mathbb{N}} \left(-\frac{1}{n}, \frac{1}{n}\right) = \ldots\ldots\ldots\ldots\ldots\ldots$

(b) Directional derivative of $f(x) = 2x^2 + 3xy$ at $(0, 2)$ in the direction of $(1, 0)$ is ....

(c) Limit superior and limit inferior of $a_n = \begin{cases} \frac{1}{2^n}, & \text{if } n \text{ is odd} \\ \frac{1}{3^n}, & \text{if } n \text{ is even} \end{cases}$. are ....and .... respectively.

(d) Supremum and Infimum of the set $S = \{a_n : n \in \mathbb{N}\}$ where $a_n = \begin{cases} \frac{1}{2^n}, & \text{if } n \text{ is odd} \\ \frac{1}{3^n}, & \text{if } n \text{ is even} \end{cases}$. are ....and .... respectively.

(e) All the critical points of $f(x, y) = 2 + 2x + 2y - x^2 - y^2$ are...

11. Give an example of a function which is discontinuous at every point of $\mathbb{R}$ but modulus of the function is continuous in $\mathbb{R}$. [5]
Answer:

SPACE FOR ROUGH WORK

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