POSSESSION OF MOBILES IN EXAM IS UFM PRACTICE

Name of Student: .... Enrollment No. ....
Department: ....

BENNETT UNIVERSITY, GREATER NOIDA
Supplementary Examination, July 2019

COURSE CODE : EMAT101L	MAX. DURATION: 2 Hours
COURSE NAME: Engineering Calculus	
COURSE CREDIT: 3-1-0	MAX. MARKS: 100

Instructions:

- All questions are mandatory.

1. Show that
   (a) $\lim_{x\to 0} \sin(\frac{1}{x^2})$ does not exist. [6]
   (b) $\lim_{n\to\infty} \frac{1}{n} = 0$. [6]
2. Show that $\lim_{n\to\infty} \frac{\cos n}{n} = 0$. [8]
3. Check the convergence of the infinite series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$. [8]
4. Find all $x \in \mathbb{R}$ for which the series $\sum_{n=1}^{\infty} \frac{(-1)^n(x-1)^n}{2^n n^2}$ converges. [8]
5. Show that there is a root of the equation $x^3 - x - 1 = 0$ between 1 and 2. [8]
6. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^2} & if \quad (x,y) \neq (0,0), \\ 0 & if \ (x,y) = (0,0). \end{cases}$$

   Examine
   (a) Continuity of f at $(0,0)$ [4]
   (b) Existence of partial derivatives f_x and f_y at $(0,0)$ [4]
   (c) Existence of the directional derivatives $D_u f$ at $(0,0)$ along each unit vector u. [4]
   (d) Differentiability of f at $(0,0)$ [4]
7. Check whether $\lim_{(x,y)\to(0,0)} f(x,y)$ exists or not where [6]

$$f(x,y) = \left(1 + \sqrt{x^2+y^2}, x \sin\frac{1}{y}, \frac{e^x \sin y}{y}\right).$$

8. Evaluate the following integral [6]

$$\int_0^{\ln 2} \int_1^{\ln 5} e^{2x} e^{2y} \, dy \, dx.$$

9. Find the critical points and their nature of the function $f(x,y) = 4(x - y)^2$. [8]

10. Use the transformation $x + 2y = u$ and $x - y = v$ to evaluate the integral [10]

$$\int_0^{2/3} \int_y^{2-2y} e^{(y-x)}(x+2y)\,dx\,dy.$$

11. Let $R = \{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 1 \text{ and } x^2 \leq y \leq x\}$. Find the area of the region $\mathbb{R}$. [10]