**BENNETT UNIVERSITY, GREATER NOIDA**
**End Term Examination, Fall SEMESTER 2019-20**

Name of Student ---------------------------------------------------- Enrolment No. -----------------

Department / School ---------------------------

COURSE CODE: **EPHY105L** MAX. DURATION: **2 Hours**
COURSE NAME: **Electromagnetics** MAX. MARK: **45**

**A. Please write answers to all questions in the space provided in the question paper itself.**
**B. Rough work elsewhere will not be graded**
**C. There are *FIVE* questions in the question paper**
**D. ALL QUESTIONS ARE COMPULSORY**

**1. Put a tick mark on the correct answer for the following questions (Tick marks on more than one choice will be allotted zero marks): (6x1.5=9)**

a) A point charge $Q$ is placed at a point with Cartesian coordinates (1,1,0). In order to make the electrostatic field to be zero at the origin, we must
   i. Keep a charge -Q at the point with coordinates (1, -1, 0)
   ii. Keep a charge Q at a point with coordinates (-1, -1, 0)
   iii. Keep a charge -Q at a point with coordinates (-1, -1, 0)
   iv. Keep a charge Q at a point with coordinate (1, -1, 0)

b) Consider a sphere of radius $R$ carrying a total charge $Q$ distributed uniformly in the entire volume of the sphere. The value of $\nabla.\vec{E}$ at the center of the sphere will be
   i. Zero
   ii. $\frac{Q}{4\pi\epsilon_0 R^2}$
   iii. $\frac{3Q}{4\pi\epsilon_0 R^3}$
   iv. $\frac{Q}{\epsilon_0}$

c) A straight wire of radius $R$ carries a current with a current density given by $\vec{J} = J_0 \frac{r}{R} \hat{\mathbf{z}}$ where $z$ is along the axis of the wire. The value of $\nabla \times \vec{B}$ at a point at a distance $R/4$ will be
   i. $4\mu_0 J_0 \hat{\mathbf{z}}$
   ii. $\frac{\mu_0 J_0}{4} \hat{\mathbf{z}}$
   iii. Zero
   iv. $\frac{\mu_0}{4\pi} J_0 \hat{\mathbf{z}}$

d) The vector function $\vec{F} = x^2\hat{\mathbf{x}} - 2xy\hat{\mathbf{y}}$
   i. Can represent both an electrostatic field and a magnetic field
   ii. Can represent an electrostatic field but not a magnetic field
   iii. Can represent a magnetic field but not an electrostatic field
   iv. Can represent neither an electrostatic field nor a magnetic field

e) Consider an infinitely long cylinder of circular cross section of radius $a$ with a magnetization given by $\vec{M} = M_0\hat{z}$ where $z$ is along the axis of the cylinder and $M_0$ is a constant. In such a case,

i. Bound volume current density is zero while bound surface current density is non zero

ii. Bound surface current density is zero while bound volume current density is non zero.

iii. Both bound volume and surface current densities are zero.

iv. Both bound volume and surface current densities are non zero

f) Consider a closely wound solenoid with a circular cross section. The current in the solenoid is varied with time. In such a case

i. There will be no electric field both within and outside the solenoid

ii. There will be induced electric field within the solenoid but not outside the solenoid.

iii. There will be an induced electric field within and outside the solenoid which will be along the axial direction

iv. There will be induced electric field within and outside the solenoid which will be along the azimuthal direction.

2. **Give brief answers to the following questions:** **(6 x 3=18)**

a) Two equal charges $Q_1 = 1\ \mu C$ and $Q_2 = 1\ \mu C$ are located at points with Cartesian coordinates (1, 1, 0) and (-1, 0, 0) respectively. Obtain the electrostatic potential difference between two points with coordinates (0, 0, 0) and (2, 0, 0).

b) A sphere of radius $R$ carries a polarization given by $\vec{\boldsymbol{P}} = P_0 r\hat{\boldsymbol{r}}$ in spherical polar coordinates where $P_0$ is a constant. Obtain the volume and surface bound charge densities.

c) Check whether the following two vector potentials $\vec{\boldsymbol{A}}_1$ and $\vec{\boldsymbol{A}}_2 = \vec{\boldsymbol{A}}_1 + (y\hat{\boldsymbol{x}} - x\hat{\boldsymbol{y}})$ correspond to the same magnetic field

d) Consider a closely wound solenoid of circular cross section of radius $R$ and carrying a current $I$. What are the values of $\oint \vec{B}.d\vec{l}$ and $\iint \vec{B}.d\vec{a}$ integrated over a square loop of side $R/4$ placed perpendicular to the solenoid and centered on its axis.

$\oint \vec{B}.d\vec{l} =$

$\iint \vec{B}.d\vec{a} =$

e) A circular current loop having a radius $R$ and carrying a current $I$ is placed at the center of a sphere of radius $4R$. What is the net magnetic flux passing through the sphere?

f) Consider a parallel plate capacitor with circular plates of radius $R$ and separated by a distance $d$ with free space between the plates. The charge on capacitor changes with time according to $Q(t) = Q_0 \sin \omega t$ where $Q_0$ and $\omega$ are constants. What is the displacement current density between the plates of the capacitor?

3. Consider a parallel plate capacitor with plate separation of $d$. The space between the plates is partially filled with a slab of a dielectric (with its faces parallel to the capacitor plates) with thickness $d/4$ and having a dielectric constant $K$. It is given that the free surface charge density on the upper plate is $+\sigma_f$ and that on the lower plate is $-\sigma_f$.

(a) From Gauss's law obtain the electric field $\vec{E}$ in the air gap and the dielectric slab. (4)

(b) Obtain the surface bound charge density on the two surfaces of the dielectric. (2)

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4. An infinitely long cylindrical wire of radius $R$ made of a medium having a magnetic susceptibility given by $\chi_m$ carries a current $I$ with a uniform volume current density and $\hat{z}$ is the unit vector along the axis of the cylinder.

(a) From Ampere's law obtain the fields $\vec{H}$ and $\vec{B}$ in the region $r < R$. (4)

(b) What will be the bound volume current density within the wire? (2)

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5. Consider an infinitely long circular cross section solenoid having a radius $R$ with windings of $n$ turns per meter and with its axis along the $z$-direction. The current in the solenoid is increased according to the equation $I = I_0\left(1 - \frac{t}{T}\right); 0 < t < T$ . Using Faraday's law obtain an expression for the induced electric field $\vec{E}$ within the solenoid. **(6)**

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**Some useful formulas**

- In Cartesian coordinates:

$$\nabla f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}$$

$$\nabla . \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\nabla \times \vec{F} = \left[\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right]\hat{\imath} + \left[\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right]\hat{\jmath} + \left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right]\hat{k}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

- In spherical polar coordinates:

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\nabla . \vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$$

$$\nabla \times \vec{F} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, F_\phi) - \frac{\partial F_\theta}{\partial \phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial F_r}{\partial \phi} - \frac{\partial}{\partial r}(rF_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial F_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

- In cylindrical coordinates:

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{z}$$

$$\nabla . \vec{F} = \frac{1}{r}\frac{\partial}{\partial r}(rF_r) + \frac{1}{r}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$$

$$\nabla \times \vec{F} = \left[\frac{1}{r}\frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z}\right]\hat{r} + \left[\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right]\hat{\boldsymbol{\phi}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rF_\phi) - \frac{\partial F_r}{\partial \phi}\right]\hat{z}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

- $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N\text{-}m^2}$
- Theorem on Gradients: $\int_a^b \nabla f . d\vec{l} = f(b) - f(a)$
- Divergence theorem: $\iiint \nabla . \vec{F}\, d\tau = \oiint \vec{F} . d\vec{a}$
- Stokes theorem: $\iint \nabla \times \vec{F} . d\vec{a} = \oint \vec{F} . d\vec{l}$
- **Maxwell's equations:**

$$\nabla . \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla . \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$