EPHY108L: Classical Mechanics

Tutorial 2 [QUESTIONS]

1.

A musician's tuning fork rings at A above middle C, 440 Hz. A sound level meter indicates that the sound intensity decreases by a factor of 5 in 4 seconds. What is the Q of the tuning fork?

2.

A rubber band exhibits a much lower Q than a tuning fork, primarily because of the internal friction generated by the coiling of the long-chain molecules. In one experiment, a paperweight suspended from a hefty rubber band had a period of 1.2 s and the amplitude of oscillation decreased by a factor of 2 after three periods. What is the estimated Q of this system?

3.

A 0.3-kg mass is attached to a spring and oscillates at 2 Hz with a Q of 60. Find the spring constant and damping constant.

4.

•••24 In Fig. 15-35, two springs are joined and connected to a block of mass 0.245 kg that is set oscillating over a frictionless floor. The springs each have spring constant $k = 6430$ N/m. What is the frequency of the oscillations?

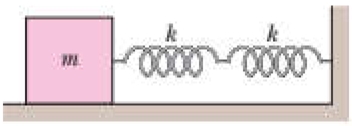


Figure 15-35 Problem 24.

5.

•••25 GO In Fig. 15-36, a block weighing 14.0 N, which can slide without friction on an incline at angle $\theta = 40.0°$, is connected to the top of the incline by a massless spring of unstretched length 0.450 m and spring constant 120 N/m. (a) How far from the top of the incline is the block's equilibrium point? (b) If the block is pulled slightly down the incline and released, what is the period of the resulting oscillations?

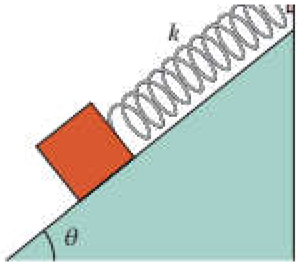


Figure 15-36 Problem 25.

6.

•27 SSM When the displacement in SHM is one-half the amplitude x_m, what fraction of the total energy is (a) kinetic energy and (b) potential energy? (c) At what displacement, in terms of the amplitude, is the energy of the system half kinetic energy and half potential energy?

7.

•28 Figure 15-38 gives the one-dimensional potential energy well for a 2.0 kg particle (the function $U(x)$ has the form bx^2 and the vertical axis scale is set by U_s = 2.0 J). (a) If the particle passes through the equilibrium position with a velocity of 85 cm/s, will it be turned back before it reaches x = 15 cm? (b) If yes, at what position, and if no, what is the speed of the particle at x = 15 cm?

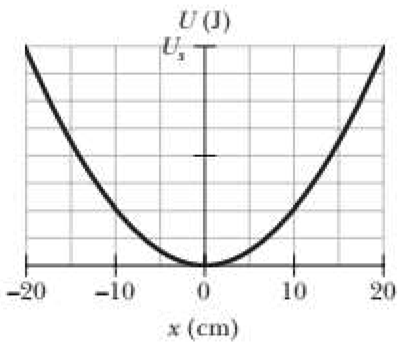


Figure 15-38 Problem 28.

8.

•29 SSM Find the mechanical energy of a block–spring system with a spring constant of 1.3 N/cm and an amplitude of 2.4 cm.

9.

•30 An oscillating block–spring system has a mechanical energy of 1.00 J, an amplitude of 10.0 cm, and a maximum speed of 1.20 m/s. Find (a) the spring constant, (b) the mass of the block, and (c) the frequency of oscillation.

10.

•31 ILW A 5.00 kg object on a horizontal frictionless surface is attached to a spring with $k = 1000$ N/m. The object is displaced from equilibrium 50.0 cm horizontally and given an initial velocity of 10.0 m/s back toward the equilibrium position. What are (a) the motion's frequency, (b) the initial potential energy of the block–spring system, (c) the initial kinetic energy, and (d) the motion's amplitude?

11.

•32 Figure 15-39 shows the kinetic energy K of a simple harmonic oscillator versus its position x. The vertical axis scale is set by $K_s = 4.0$ J. What is the spring constant?

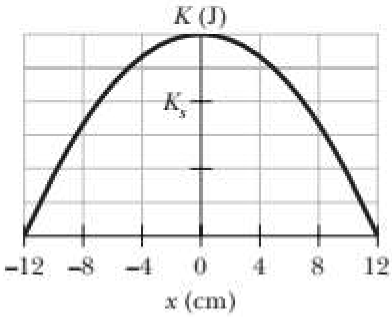


Figure 15-39 Problem 32.

12.

••35 A 10 g particle undergoes SHM with an amplitude of 2.0 mm, a maximum acceleration of magnitude 8.0×10^3 m/s^2, and an unknown phase constant ϕ. What are (a) the period of the motion, (b) the maximum speed of the particle, and (c) the total mechanical energy of the oscillator? What is the magnitude of the force on the particle when the particle is at (d) its maximum displacement and (e) half its maximum displacement?

EPHY108L: Classical Mechanics

Tutorial 2 [SOLUTIONS]

1.

The sound intensity from the tuning fork is proportional to the energy of oscillation. Since the energy of a damped oscillator decreases as $e^{-\gamma t}$, we can find γ by taking the ratio of the energies at $t = 0$ and at $t = 4$ s:

$$5 = \frac{E(0)e^{(0)}}{E(0)e^{-4\gamma}} = e^{4\gamma}.$$

Hence

$$4\gamma = \ln 5 = 1.6$$
$$\gamma = 0.4\ \mathrm{s}^{-1},$$

and

$$Q = \frac{\omega_0}{\gamma} = \frac{2\pi(440\ \mathrm{s}^{-1})}{0.4\ \mathrm{s}^{-1}}$$
$$\approx 7000.$$

2.

The ratio of the amplitudes at $t = 0$ and at $t = 3(1.2\ \mathrm{s}) = 3.6$ s is

$$2 = \frac{X(0)}{X(3.6\ \mathrm{s})} = \frac{X_0 e^{(0)}}{X_0 e^{-3.6(\gamma/2)}}.$$

Solving, we have

$$1.8\gamma = \ln 2 = 0.69$$

or

$$\gamma = 0.39\ \mathrm{s}^{-1}.$$

Therefore

$$Q = \frac{\omega_0}{\gamma} = \frac{5.24\ \mathrm{s}^{-1}}{0.39\ \mathrm{s}^{-1}}$$
$$= 13.$$

3.

$$\omega = \sqrt{\frac{k}{m}}$$

$$k = m\omega^2 = (0.3\,\text{kg})\left[(2\,\text{cycles}\cdot\text{s}^{-1})(2\pi\,\text{radians}\cdot\ \text{cycle}^{-1})\right]^2$$

$$= 47.4\,\text{N}\cdot\text{m}^{-1}$$

$$Q = \frac{\omega}{\gamma} \implies \gamma = \frac{\omega}{Q}$$

$$= \frac{4\pi\,\text{rad}\cdot\text{s}^{-1}}{60} = 0.21\,\text{s}^{-1}$$

4.

24. We wish to find the effective spring constant for the combination of springs shown in the figure. We do this by finding the magnitude F of the force exerted on the mass when the total elongation of the springs is Δx. Then $k_{\text{eff}} = F/\Delta x$. Suppose the left-hand spring is elongated by Δx_ℓ and the right-hand spring is elongated by Δx_r. The left-hand spring exerts a force of magnitude $k\Delta x_\ell$ on the right-hand spring and the right-hand spring exerts a force of magnitude $k\Delta x_r$ on the left-hand spring. By Newton's third law these must be equal, so $\Delta x_\ell = \Delta x_r$. The two elongations must be the same, and the total elongation is twice the elongation of either spring: $\Delta x = 2\Delta x_\ell$. The left-hand spring exerts a force on the block and its magnitude is $F = k\Delta x_\ell$. Thus,

$$k_{\text{eff}} = k\Delta x_\ell / 2\Delta x_r = k/2\,.$$

The block behaves as if it were subject to the force of a single spring, with spring constant $k/2$. To find the frequency of its motion, replace k_{eff} in $f = (1/2\pi)\sqrt{k_{\text{eff}}/m}$ with $k/2$ to obtain

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}.$$

With $m = 0.245$ kg and $k = 6430$ N/m, the frequency is $f = 18.2$ Hz.

5.

25. (a) We interpret the problem as asking for the equilibrium position; that is, the block is gently lowered until forces balance (as opposed to being suddenly released and allowed to oscillate). If the amount the spring is stretched is x, then we examine force-components along the incline surface and find

$$kx = mg\sin\theta \;\Rightarrow\; x = \frac{mg\sin\theta}{k} = \frac{(14.0\ \text{N})\sin 40.0°}{120\ \text{N/m}} = 0.0750\ \text{m}$$

at equilibrium. The calculator is in degrees mode in the above calculation. The distance from the top of the incline is therefore (0.450 + 0.75) m = 0.525 m.

(b) Just as with a vertical spring, the effect of gravity (or one of its components) is simply to shift the equilibrium position; it does not change the characteristics (such as the period) of simple harmonic motion. Thus, Eq. 15-13 applies, and we obtain

$$T = 2\pi\sqrt{\frac{14.0\ \text{N}/9.80\ \text{m/s}^2}{120\ \text{N/m}}} = 0.686\ \text{s}.$$

26. To be on the verge of slipping means that the force exerted on the smaller block (at the point of maximum acceleration) is $f_{\max} = \mu_s mg$. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where $\omega = \sqrt{k/(m+M)}$ is the angular frequency (from Eq. 15-12). Therefore, using Newton's second law, we have

$$ma_m = \mu_s mg \Rightarrow \frac{k}{m+M}x_m = \mu_s g$$

which leads to

$$x_m = \frac{\mu_s g(m+M)}{k} = \frac{(0.40)(9.8\ \text{m/s}^2)(1.8\ \text{kg} + 10\ \text{kg})}{200\ \text{N/m}} = 0.23\ \text{m} = 23\ \text{cm}.$$

6.

27. **THINK** This problem explores the relationship between energies, both kinetic and potential, with amplitude in SHM.

EXPRESS In simple harmonic motion, let the displacement be

$$x(t) = x_m\cos(\omega t + \phi).$$

The corresponding velocity is

$$v(t) = dx/dt = -\omega x_m \sin(\omega t + \phi).$$

Using the expressions for $x(t)$ and $v(t)$, we find the potential and kinetic energies to be

$$U(t) = \frac{1}{2}kx^2(t) = \frac{1}{2}kx_m^2\cos^2(\omega t+\phi)$$
$$K(t) = \frac{1}{2}mv^2(t) = \frac{1}{2}m\omega^2 x_m^2\sin^2(\omega t+\phi) = \frac{1}{2}kx_m^2\sin^2(\omega t+\phi)$$

where $k = m\omega^2$ is the spring constant and x_m is the amplitude. The total energy is

$$E = U(t)+K(t) = \frac{1}{2}kx_m^2\left[\cos^2(\omega t+\phi)+\sin^2(\omega t+\phi)\right] = \frac{1}{2}kx_m^2.$$

ANALYZE (a) The condition $x(t) = x_m/2$ implies that $\cos(\omega t+\phi) = 1/2$, or $\sin(\omega t+\phi) = \sqrt{3}/2$. Thus, the fraction of energy that is kinetic is

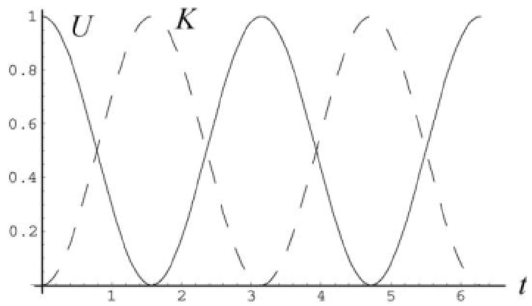
$$\frac{K}{E} = \sin^2(\omega t+\phi) = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}.$$

(b) Similarly, we have $\dfrac{U}{E} = \cos^2(\omega t+\phi) = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$.

(c) Since $E = \frac{1}{2}kx_m^2$ and $U = \frac{1}{2}kx(t)^2$, $U/E = x^2/x_m^2$. Solving $x^2/x_m^2 = 1/2$ for x, we get $x = x_m/\sqrt{2}$.

LEARN The figure to the right depicts the potential energy (solid line) and kinetic energy (dashed line) as a function of time, assuming $x(0) = x_m$. The curves intersect when $K = U = E/2$, or equivalently,

$$\cos^2\omega t = \sin^2\omega t = 1/2.$$



7.

28. The total mechanical energy is equal to the (maximum) kinetic energy as it passes through the equilibrium position ($x = 0$):

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}(2.0\text{ kg})(0.85\text{ m/s})^2 = 0.72\text{ J}.$$

Looking at the graph in the problem, we see that $U(x = 10) = 0.5$ J. Since the potential function has the form $U(x) = bx^2$, the constant is $b = 5.0\times10^{-3}\,\text{J/cm}^2$. Thus, $U(x) = 0.72$ J when $x = 12$ cm.

(a) Thus, the mass does turn back before reaching $x = 15$ cm.

(b) It turns back at $x = 12$ cm.

8.

29. **THINK** Knowing the amplitude and the spring constant, we can calculate the mechanical energy of the mass-spring system in simple harmonic motion.

EXPRESS In simple harmonic motion, let the displacement be $x(t) = x_m \cos(\omega t + \phi)$. The corresponding velocity is

$$v(t) = dx/dt = -\omega x_m \sin(\omega t + \phi).$$

Using the expressions for $x(t)$ and $v(t)$, we find the potential and kinetic energies to be

$$U(t) = \frac{1}{2}kx^2(t) = \frac{1}{2}kx_m^2\cos^2(\omega t + \phi)$$
$$K(t) = \frac{1}{2}mv^2(t) = \frac{1}{2}m\omega^2 x_m^2\sin^2(\omega t + \phi) = \frac{1}{2}kx_m^2\sin^2(\omega t + \phi)$$

where $k = m\omega^2$ is the spring constant and x_m is the amplitude. The total energy is

$$E = U(t) + K(t) = \frac{1}{2}kx_m^2\left[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right] = \frac{1}{2}kx_m^2.$$

ANALYZE With $k = 1.3\text{ N/cm} = 130\text{ N/m}$ and $x_m = 2.4\text{ cm} = 0.024\text{ m}$, the mechanical energy is

$$E = \frac{1}{2}kx_m^2 = \frac{1}{2}\left(1.3\times10^2\text{ N/m}\right)\left(0.024\text{ m}\right)^2 = 3.7\times10^{-2}\text{ J}.$$

LEARN An alternative to calculate E is to note that when the block is at the end of its path and is momentarily stopped ($v = 0 \Rightarrow K = 0$), its displacement is equal to the amplitude and all the energy is potential in nature ($E = U + K = U$). With the spring potential energy taken to be zero when the block is at its equilibrium position, we recover the expression $E = kx_m^2/2$.

9.

30. (a) The energy at the turning point is all potential energy: $E = \frac{1}{2}kx_m^2$ where E = 1.00 J and x_m = 0.100 m. Thus,

$$k = \frac{2E}{x_m^2} = 200 \text{ N/m}.$$

(b) The energy as the block passes through the equilibrium position (with speed v_m = 1.20 m/s) is purely kinetic:

$$E = \frac{1}{2}mv_m^2 \Rightarrow m = \frac{2E}{v_m^2} = 1.39 \text{ kg}.$$

10.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1000 \text{ N/m}}{5.00 \text{ kg}}} = 2.25 \text{ Hz}.$$

(b) With x_0 = 0.500 m, we have $U_0 = \frac{1}{2}kx_0^2 = 125 \text{ J}$.

(c) With v_0 = 10.0 m/s, the initial kinetic energy is $K_0 = \frac{1}{2}mv_0^2 = 250 \text{ J}$.

(d) Since the total energy $E = K_0 + U_0$ = 375 J is conserved, then consideration of the energy at the turning point leads to

$$E = \frac{1}{2}kx_m^2 \Rightarrow x_m = \sqrt{\frac{2E}{k}} = 0.866 \text{ m}.$$

11.

32. We infer from the graph (since mechanical energy is conserved) that the *total* energy in the system is 6.0 J; we also note that the amplitude is apparently x_m = 12 cm = 0.12 m. Therefore we can set the maximum *potential* energy equal to 6.0 J and solve for the spring constant k:

$$\tfrac{1}{2}k\,x_m{}^2 = 6.0 \text{ J} \quad \Rightarrow \quad k = 8.3 \times 10^2 \text{ N/m}.$$

12.

acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency and x_m = 0.0020 m is the amplitude. Thus, a_m = 8000 m/s^2 leads to ω = 2000 rad/s. Using Newton's second law with m = 0.010 kg, we have

$$F = ma = m\left(-a_m\cos(\omega t + \phi)\right) = -(80 \text{ N})\cos\left(2000t - \frac{\pi}{3}\right)$$

where t is understood to be in seconds.

$T = 2\pi/\omega = 3.1 \times 10^{-3}$ s.

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2 \quad \Rightarrow \quad v_m = x_m\sqrt{\frac{k}{m}} = 4.0 \text{ m/s}.$$

(c) The total energy is $\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2 = 0.080$ J.

(d) At the maximum displacement, the force acting on the particle is

$$F = kx = (4.0\times10^4 \text{ N/m})(2.0\times10^{-3}\text{m}) = 80 \text{ N}.$$

(e) At half of the maximum displacement, $x = 1.0$ mm, and the force is

$$F = kx = (4.0\times10^4 \text{ N/m})(1.0\times10^{-3}\text{m}) = 40 \text{ N}.$$