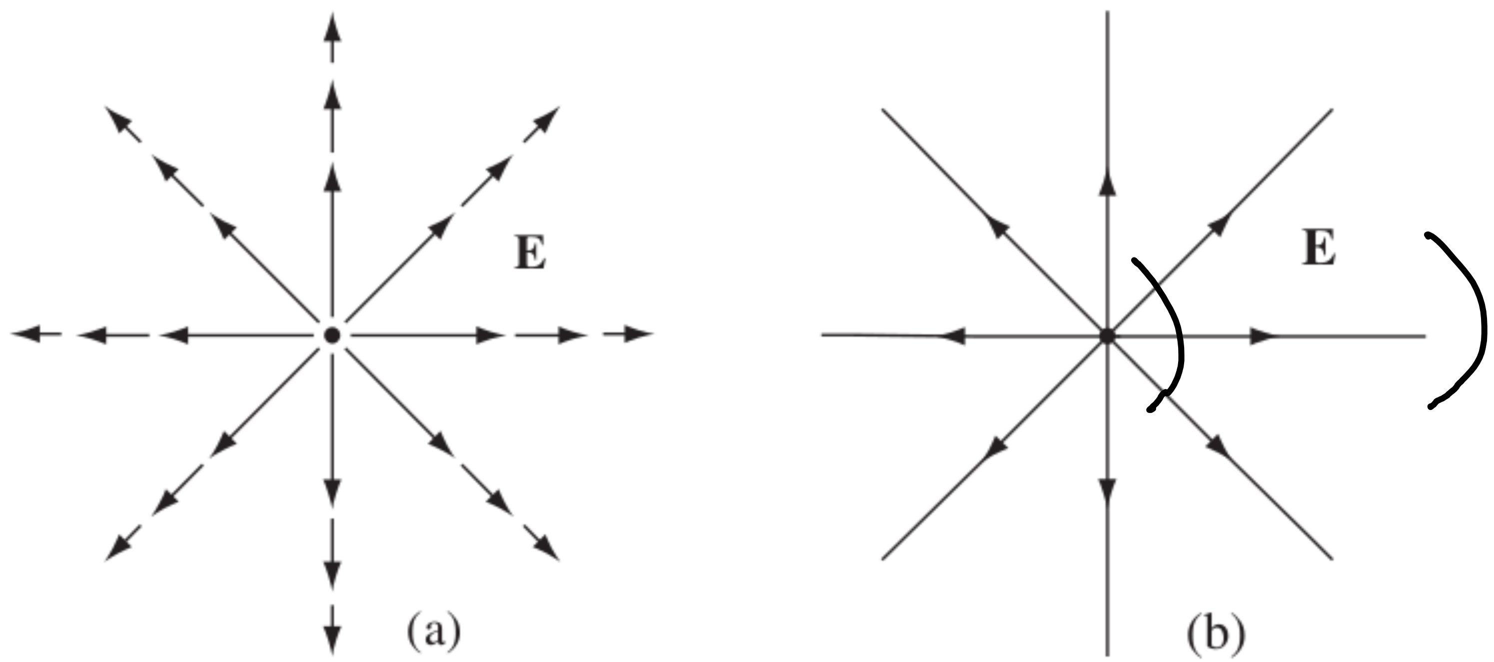
EPHY105L-10

11.11.21

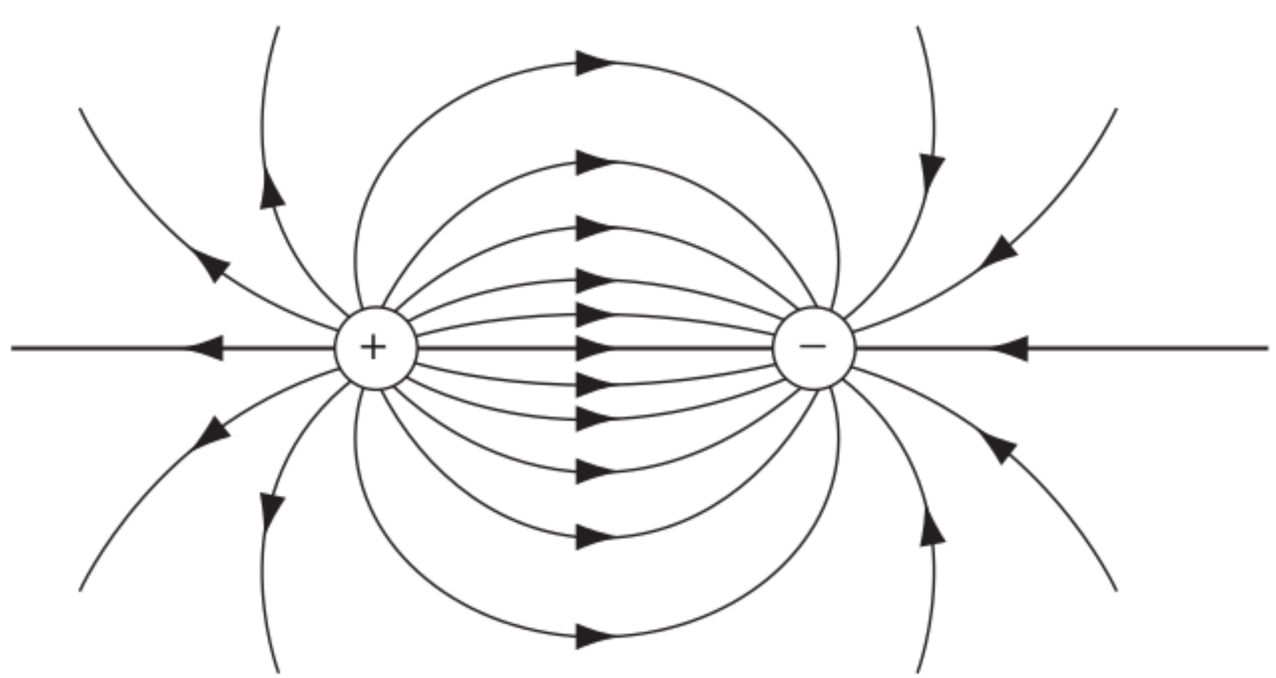
Divergence of Electric Field



$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$\sim \frac{1}{r^2}$$

→ The field strength falls as r increases



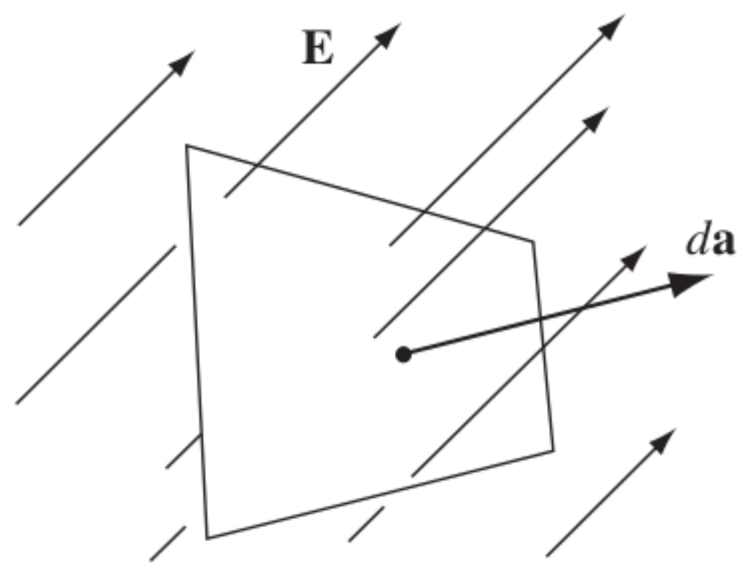
→ Field lines originate at positive charge and terminate at negative charge.

Flux of $\vec{E}$

flux through a surface

$$\Phi_E = \int_S \vec{E} \cdot d\vec{a}$$

↳ measures the "number of field lines" crossing through S.



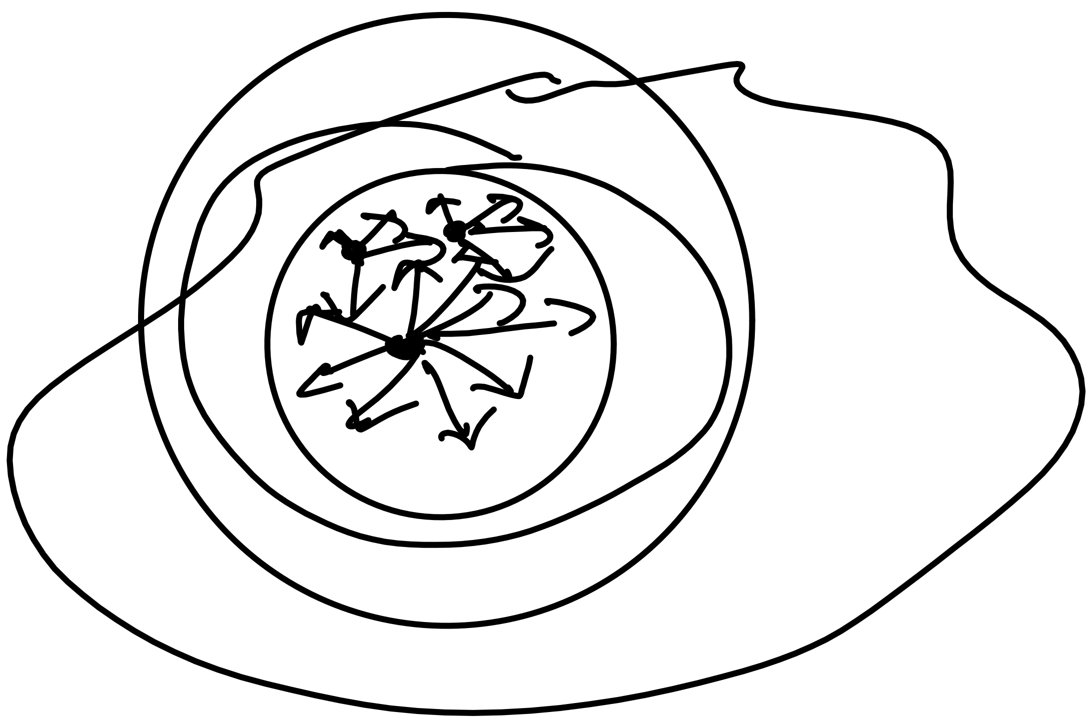
→ flux through any closed surface enclosing the charge is a measure of the electric charge.

⇒ Essence of Gauss' law

** A charge outside does not contribute since the field lines simply pass through the surface.

(x) A point charge q (at the origin) enclosed by a spherical surface.

$$\oint \vec{E} \cdot d\vec{a} = \frac{1}{4\pi\epsilon_0} \int \left(\frac{q}{r^2}\right)\hat{r} \cdot (r^2 \sin\theta \, d\theta \, d\phi) \, \hat{r}$$



$$= \frac{q}{4\pi\epsilon_0} \int_0^{\pi}\int_0^{2\pi} \sin\theta \, d\theta \, d\phi$$

$$= \frac{q}{\epsilon_0} \Rightarrow \text{independent of '}r\text{'}$$

→ The number of field lines crossing does not depend on the radius of the sphere.

→ the surface can be of any shape.

(x) flux through any surface enclosing the charge = $\frac{q}{\epsilon_0}$

→ If we have a collection of 'n' charges,

$$\vec{E} = \sum_{i=1}^{n} \vec{E}_i$$

flux through a surface enclosing all charges

$$\oint \vec{E} \cdot d\vec{a} = \sum_{i=1}^{n} \oint \vec{E}_i \cdot d\vec{a}$$

$$= \sum_{i=1}^{n} \frac{q_i}{\epsilon_0}$$

(x) for any closed surface

$$\oint \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0} Q_{enc.}$$

$\vec{E}$ → net electric field

$Q_{enc.}$ → charge enclosed

Quantitative statement of Gauss' law.

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* We can write

$$\oint_S \vec{E} \cdot d\vec{a} = \int_V (\vec{\nabla} \cdot \vec{E})\, d\tau = \frac{1}{\epsilon_0} Q_{enc.}$$

if the volume charge density of the system $= \rho$

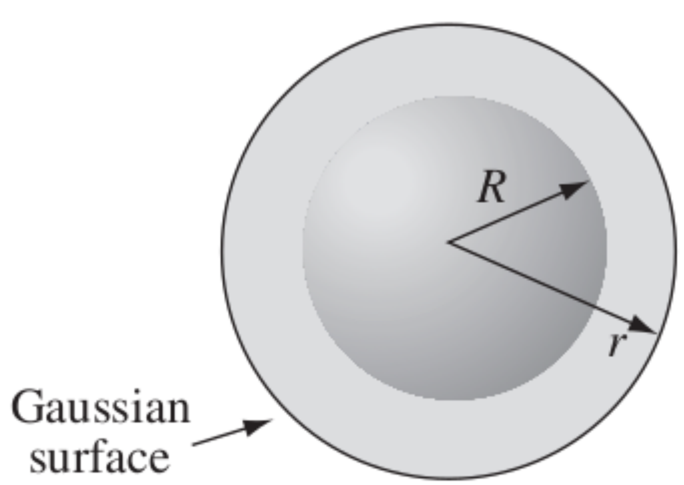
$$Q_{enc.} = \int_V \rho \, \underline{d\tau}$$

Hence, $\int_V (\vec{\nabla}\cdot\vec{E})\, d\tau = \frac{1}{\epsilon_0}\int_V \rho\, d\tau$

$$\Rightarrow \quad \underline{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}} \quad \Rightarrow \text{ Gauss' law in differential form.}$$

(*) When symmetry permits, this is a very simple way to calculate $\vec{E}$.

Ex:



Electric field outside a uniformly charged solid sphere of radius R and total charge q.

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{Q_{enc.}}{\epsilon_0} = \frac{q}{\epsilon_0}$$

→ $\vec{E}$ points radially outward, so does $d\vec{a}$

$$\oint_S \vec{E}\cdot d\vec{a} = \oint_S |\vec{E}|\, da$$

→ The magnitude of $\vec{E}$ is const. over the Gaussian surface.

Hence,
$$\oint_S |\vec{E}|\, da = |\vec{E}| \oint_S da$$
$$= |\vec{E}|\, 4\pi r^2$$

$$\Rightarrow |\vec{E}|\, 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\Rightarrow \vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\, \hat{r}$$

⊗ Gauss' law is always true, but not always useful

→ 'g' has to be uniform

→ Gaussian surface has to be symmetric.

* Different kind of symmetry:
    * Spherical
    * Cylindrical
    * Planer