EPHY346L-2 30.9.21

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

$$\vec{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$$

$$(\hat{x}, \hat{y}, \hat{z})$$
$$\text{'' ''}$$
$$(\hat{i}, \hat{j}, \hat{k})$$

$$\vec{A} \cdot \vec{B} = (\underline{A_x} \hat{x} + A_y \hat{y} + A_z \hat{z}) \cdot (\underline{B_x} \hat{x} + \underline{B_y} \hat{y} + \underline{B_z} \hat{z})$$

$$= A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \times \vec{B} = (\underline{A_x} \hat{x} + A_y \hat{y} + A_z \hat{z}) \times (B_x \hat{x} + \underline{B_y} \hat{y} + \underline{B_z} \hat{z})$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x}(A_y B_z - A_z B_y) + \hat{y}(A_z B_x - A_x B_z) + \hat{z}(A_x B_y - A_y B_x)$$

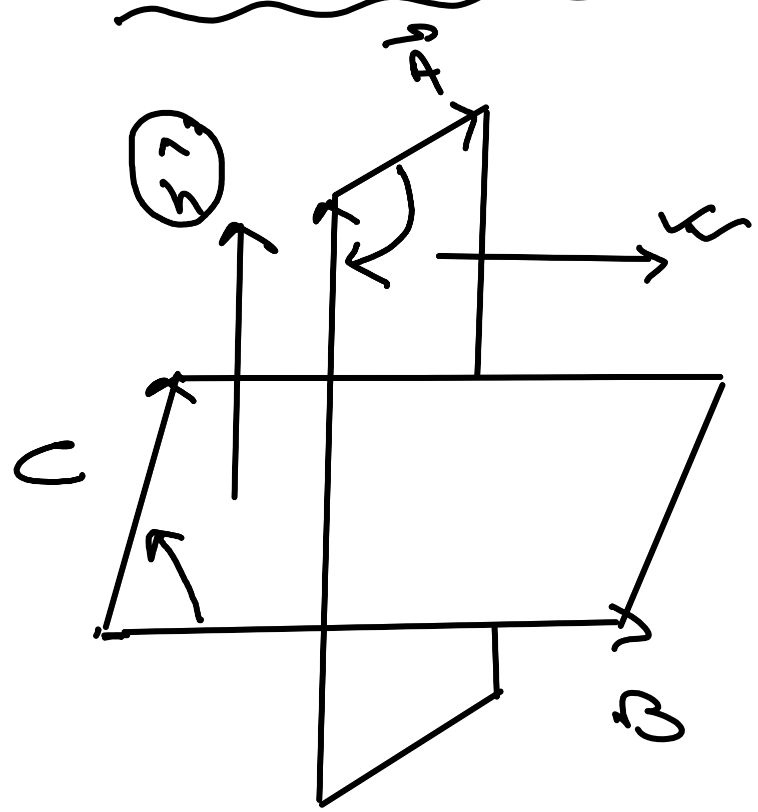
Triple product

Scalar product

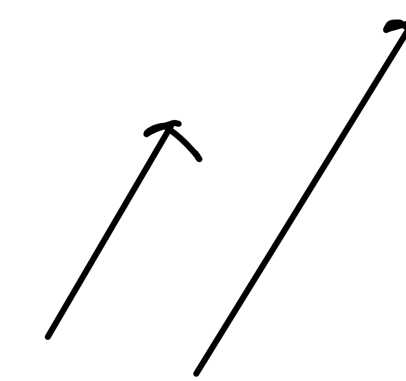
$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$$

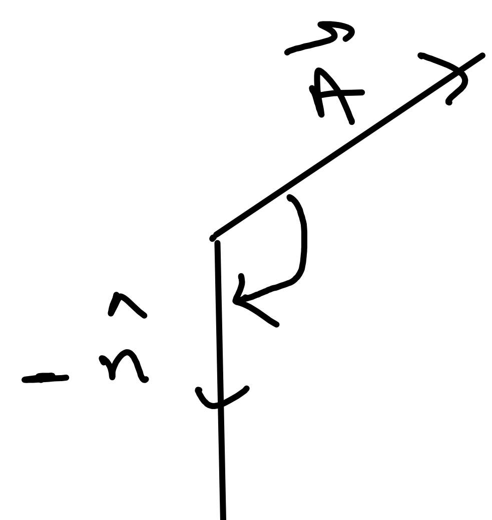
Vector product

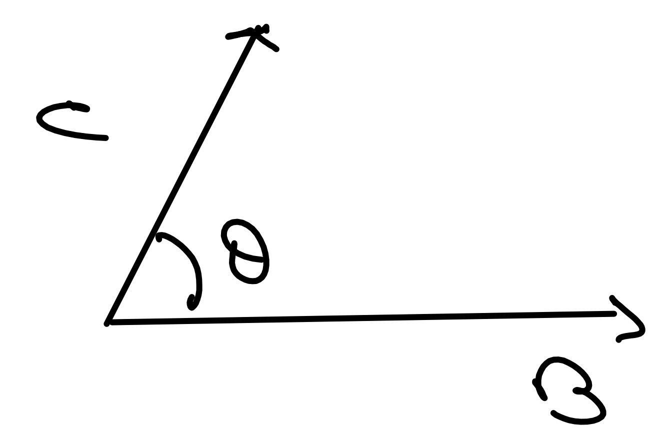
$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$



$$x\vec{B} - y\vec{C}$$

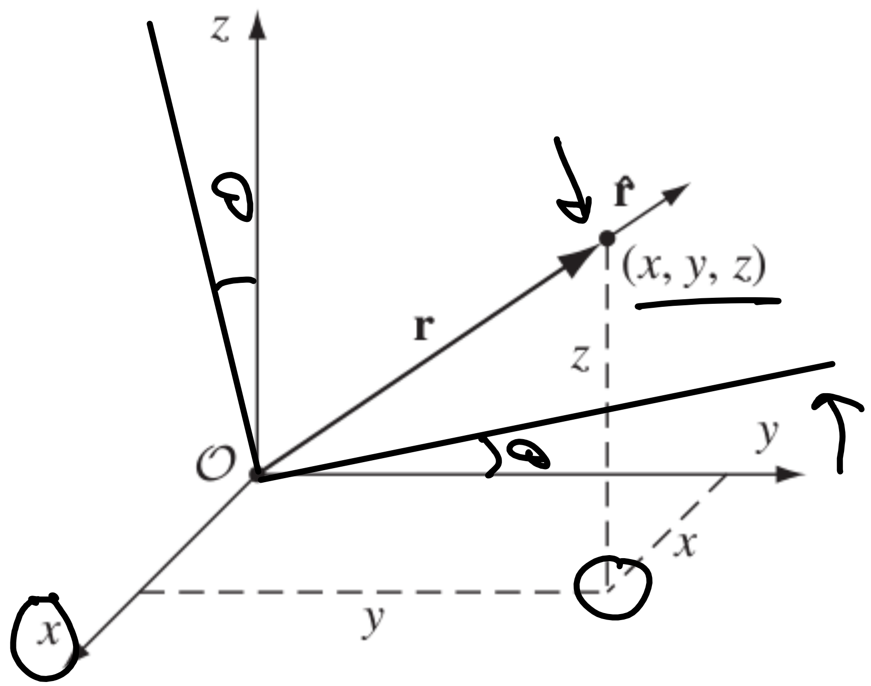






$\vec{A}\times(-\hat{n}) =$ on BC plane
(pointed towards right)

Position Vector

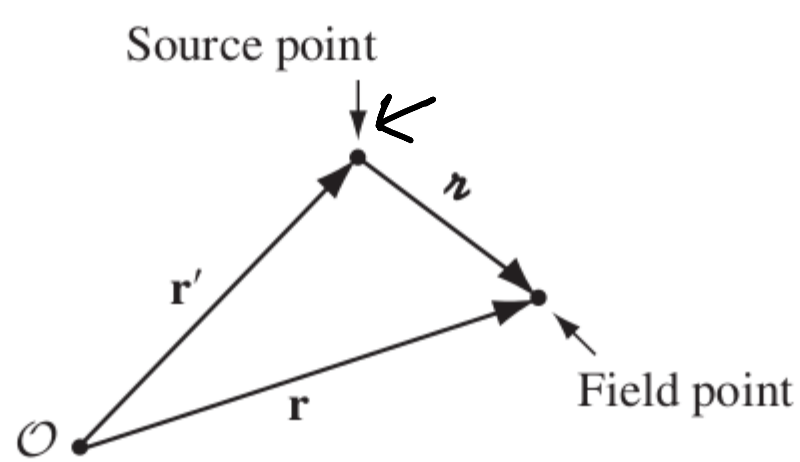


$$\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$$

$$r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{x} + y\hat{y} + z\hat{z}}{\sqrt{x^2 + y^2 + z^2}}$$

Separation Vector



$\vec{r} \equiv$ Position vector at which we wish to calculate the field

$\vec{r}' \equiv$ Position vector of the source

$\vec{\mathscr{r}} \equiv$ Separation vector.

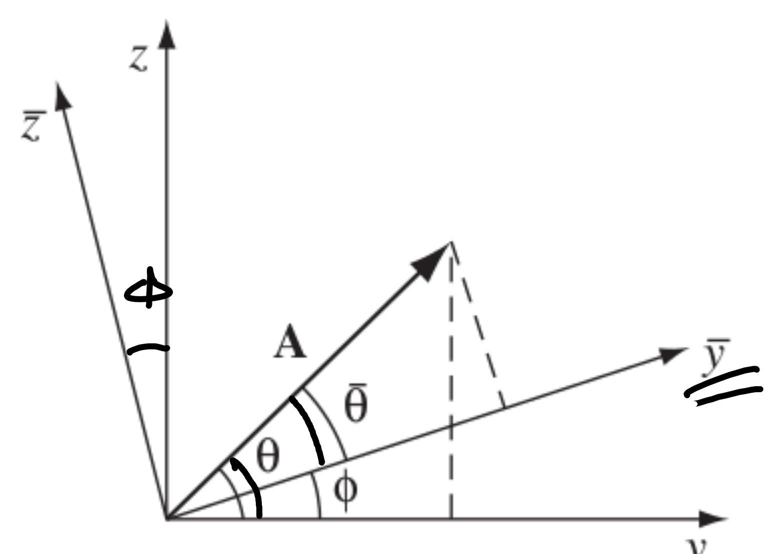
$= \vec{r} - \vec{r}'$

Transformation of Vectors

In order for $\vec{A}$ to be called a vector it has to transform "like a vector"

Say, we are going from (x, y, z) to $(\bar{x}, \bar{y}, \bar{z})$

→ Rotation by ϕ relative to (x, y, z) about $x = \bar{x}$



$\theta = \phi + \bar{\theta}$

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

$$\vec{A} = \bar{A}_x \hat{\bar{x}} + \bar{A}_y \hat{\bar{y}} + \bar{A}_z \hat{\bar{z}}$$

$$\bar{A}_y = A \cos\bar{\theta} = A \cos(\theta - \phi)$$

$$= A(\cos\theta\cos\phi + \sin\theta\sin\phi)$$

$$= A_y \cos\phi + A_z \sin\phi$$

$$\bar{A}_z = A\sin\bar{\theta} = A\sin(\theta-\phi)$$

$$= A(\sin\theta\cos\phi - \cos\theta\sin\phi)$$

$$= A_z\cos\phi - A_y\sin\phi$$

$$\bar{A}_x = A_x$$

This can be written as

$$\begin{pmatrix} \bar{A}_y \\ \bar{A}_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A_y \\ A_z \end{pmatrix}$$

More generally,

$$\begin{pmatrix} \bar{A}_x \\ \bar{A}_y \\ \bar{A}_z \end{pmatrix} = \begin{pmatrix} \overset{=1}{R_{xx}} & \overset{=0}{R_{xy}} & \overset{=0}{R_{xz}} \\ 0 = R_{yx} & R_{yy} & R_{yz} \\ 0 = R_{zx} & R_{zy} & R_{zz} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

In compact form:

$$\vec{A}_i = \sum_{j=1}^{3} R_{ij} A_j \qquad \begin{cases} j=1 \equiv x \\ j=2 \equiv y \\ j=3 \equiv z \end{cases}$$

Link to the Recording:

https://bennettu.sharepoint.com/sites/EPHY105L-Odd2021/Shared%20Documents/General/Recordings/Meeting%20in%20_General_-20210930_134250-Meeting%20Recording.mp4