EMAT101L

Engineering Calculus

Quiz Test 5

Group 2

Improper Integrals, Beta and Gamma Functions, and Leibniz Integral Rule

Total marks: 10 Time: 15 minutes

Each question carries 2 marks.

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1. Let
$$\int_{\frac{\pi}{2}-\alpha}^{\frac{\pi}{2}} \sin\theta \cos^{-1}(\cos\alpha \ \mathrm{cosec}\ \theta)\ d\theta = g(\alpha) + C.$$
Then

   - $g(\alpha) = -\frac{\pi}{2}\cos\alpha; \quad C = \frac{\pi}{2}$
   - $g(\alpha) = \frac{\pi}{2}\cos\alpha;\ C = \frac{\pi}{2}$
   - $g(\alpha) = -\frac{\pi}{2}\cos\alpha;\ C = -\frac{\pi}{2}$
   - $g(\alpha) = \frac{\pi}{2}\cos\alpha;\ C = -\frac{\pi}{2}$

   [Hints: Apply Leibnitz's rule, then take $\cos\alpha = t$]

2. Let
$$A = \int_0^\infty \frac{4x}{4x^2+t^2}\ dt.$$
Now choose the correct options.

   - $\frac{\mathrm{d}\alpha}{\mathrm{d}x} = \frac{4}{4x^2+t^2}$
   - $\ln A = 1$
   - A is an irrational number.
   - $\sin A = 0$

[Hints: $A = lim_{b\to\infty}\int_0^b \frac{4x}{4x^2+t^2}\ dt = lim_{b\to\infty}[\frac{4x}{2x}\tan^{-1}t]_0^b = \pi$

3. Choose the correct options.

   - $\displaystyle\int_0^1 x(1-x)^2$ converges to $\displaystyle\frac{1}{10}$
   - $\displaystyle\int_0^\infty \frac{\mathrm{d}x}{1+x^4}$ converges to $\displaystyle\frac{\pi}{2\sqrt{2}}$
   - $\displaystyle\int_0^{\frac{\pi}{2}} \sin^6\theta \mathrm{d}\theta$ converges to $\displaystyle\frac{3\pi}{32}$
   - $\displaystyle\beta(m+1,n) = \frac{n}{m+n}\beta(m,n)$

4. Which of the following is an improper integral of second kind?

   (I) $\displaystyle\int_{-1}^{0}\frac{dx}{x}$ (II) $\displaystyle\int_2^3 \frac{dx}{x^2-1}$ (III) $\displaystyle\int_0^1 \tan\left(\frac{\pi\theta}{2}\right)d\theta$

   (a) I and III only (b) III only (c) I only (d) II and III only

   In (II), $\frac{1}{x^2-1}$ is well defined and bounded in the integral limits from 2 to 3.

5. Choose the **correct** options.

   - $\displaystyle\int_0^1 x^4(1-\sqrt{x})^5\mathrm{d}x = 2\frac{\Gamma(10)\Gamma(6)}{\Gamma(16)}$ [Hints: take $\sqrt{x}=t$]
   - $\displaystyle\int_0^1 x^4(1-\sqrt{x})^5\mathrm{d}x = 2\beta(10,5)$.
   - $\displaystyle\int_0^\infty \frac{x^a}{a^x}\mathrm{d}x = \frac{1}{(\log a)^{a+1}}\Gamma(a+1)$ [Hints: take $a^x = t$]
   - $\displaystyle\int_0^\infty \frac{x^a}{a^x}\mathrm{d}x = \Gamma(a+1)$