*All Answers are to be written on the Question paper itself*

Name of Student ---------------------------------------------------------------------- Enrolment No. ------------------

Department / School ------------------------------

**BENNETT UNIVERSITY, GREATER NOIDA**

**Mid Term Examination, Even SEMESTER 2019-20**

COURSE CODE: **EPHY108L** MAX. DURATION: **1 Hr**

COURSE NAME: **Mechanics** MAX. MARKS: **25**

- **Notes :**
- Answers are to be written on the Question paper itself
- Answers must be written only in the space provided for each answer
- Answers written elsewhere will not be given marks
- No additional sheets can be attached to the question paper
- All questions are mandatory. There are 6 questions, Q.1 has four subparts.
- Rough work must be carried out in the space provided at the back of the question paper

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**Q.1 (i)** Angular momentum of a body is $\mathbf{L} = 5\hat{\imath} + 6\hat{\jmath} + 78\hat{k}$, two external torques act on the body. One torque is $\boldsymbol{\tau}_1 = 7\hat{\imath} + 10\hat{\jmath} + 3\hat{k}$, what is the other torque? Give justification (1 Mark)

**Q. 1 (ii)** A particle moves in a circle of radius 1 m under the influence of a force. Each time it completes a turn, its kinetic energy increases by 1 J. Is the force conservative/non-conservative. Give reason. (1 Mark)

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**Q.1 (iii)** Does value of angular momentum depend on the choice of origin? Answer in yes/No. (0.5 Marks)

**Q.1 (iv)** If potential energy in a region is given by $V = 5x$, what is the y-component of force F, in that region? Justify your answer. (1 Mark)

**Q.2** An object is moving such that it has a constant acceleration $a$ and its mass is increasing at a constant rate $b$. The object starts from rest. What is the force acting on it at an instant $t$ when its mass is $m$? (2 Marks)

**Q.3** A particle is moving in a circle of radius $a$ with angular velocity $\omega = \alpha t + \beta t^2$, where $\alpha$ and $\beta$ are constants and $t$ is time. Find the tangential component of acceleration. Assume origin is at the centre of the circle. (2 Marks)

**Q.4** Suppose an object has moment of inertia tensor $I = \begin{pmatrix} I_1 & 0 \\ 0 & I_1 \end{pmatrix}$ about x and y axes. It is rotating with angular velocity $\boldsymbol{\omega} = \omega_1 \hat{j}$. What is the direction of its axis of rotation? Find the angular momentum $\mathbf{L}$ in matrix and vector forms. (1.5 Marks)

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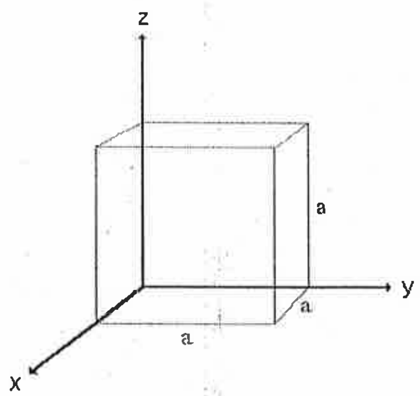
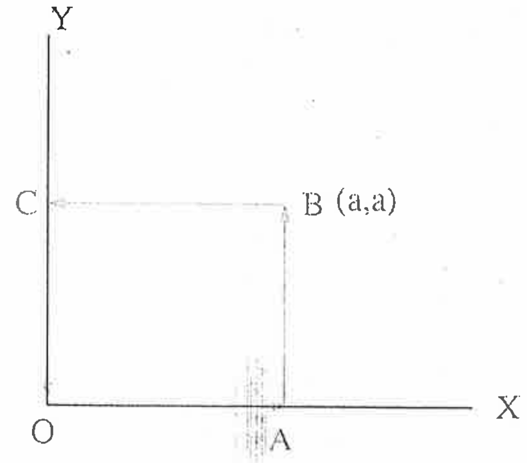
Q.5 Find the moment of inertia tensor of a solid cube of side $a$ about x, y and z axes coinciding with three intersecting edges of the cube as shown in figure below. It has a uniformly distributed mass $M$. Write the final answer in matrix form. Show all steps. (8 Marks)



**Q. 6** Consider a force $\mathbf{F} = y\hat{\imath} + 2x\hat{\jmath}$. Calculate work done by this force in going around a closed path which is a square of side $a$ in the xy plane (shown below). The motion is in anti-clockwise direction. Calculate curl of F and verify Stokes' theorem. Show all steps. (8 Marks)



## Useful relations (symbols have their usual meanings):

Vector multiplication:

$\mathbf{A}.\mathbf{B} = AB\cos\theta$, $|\mathbf{A}\times\mathbf{B}| = AB\sin\theta$

$\mathbf{A}.\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Plane polar coordinates:

$x = r\cos\theta$, $y = r\sin\theta$, $\hat{\mathbf{r}} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$, $\hat{\boldsymbol{\theta}} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$

Kinematics in polar coordinates:

$\mathbf{r} = r\hat{\mathbf{r}}$, $\mathbf{v} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}$, $\boldsymbol{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\boldsymbol{\theta}}$

Gradient of a scalar function: $\nabla V = \hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}$

Conservative Force: $\mathbf{F} = -\nabla V$

Curl of a vector: $\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

Stokes' Theorem: $\oint \mathbf{F}.d\mathbf{l} = \int_S (\nabla\times\mathbf{F})\cdot d\mathbf{S}$

Moment of Inertia tensor:

$$I = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}$$

$I_{xx} = \int (y^2+z^2)dm$, $I_{yy} = \int (x^2+z^2)dm$, $I_{zz} = \int (x^2+y^2)dm$

$I_{xy} = -\int xy\,dm$, $I_{xz} = -\int xz\,dm$, $I_{yz} = -\int yz\,dm$, $I_{yx} = I_{xy}$, $I_{zx} = I_{xz}$, $I_{zy} = I_{yz}$

Rotational motion:

$\mathbf{L} = \mathbf{r}\times\mathbf{P} = I\omega$

$\tau = \mathbf{r}\times\mathbf{F} = \frac{d\mathbf{L}}{dt}$

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Space for Rough Work