Solutions – Tutorial Sheet 4

(1)(a) $\lim\limits_{x \to -1} \dfrac{(x+2)(3x-1)}{x^2+3x-2} = \dfrac{(-1+2)(-3-1)}{1-3-2}$

$= \dfrac{-4}{-4} = 1$

(b) $\lim\limits_{x \to 0} \dfrac{\sqrt{4+x}-2}{x}$

$= \lim\limits_{x \to 0} \dfrac{(\sqrt{4+x}-2)(\sqrt{4+x}+2)}{x(\sqrt{4+x}+2)}$

$= \lim\limits_{x \to 0} \dfrac{\cancel{4}+x-\cancel{4}}{x(\sqrt{4+x}+2)} = \lim\limits_{x \to 0} \dfrac{1}{\sqrt{4+x}+2}$

$= \dfrac{1}{\sqrt{4}+2} = \dfrac{1}{4}$

(c) $\lim\limits_{x \to 1} \dfrac{x^2-1}{x-1}$

$= \lim\limits_{x \to 1} \dfrac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = 2.$

(d) $\lim\limits_{x \to 2} \dfrac{4-x^2}{3-\sqrt{x^2+5}} = \lim\limits_{x \to 2} \dfrac{(4-x^2)(3+\sqrt{x^2+5})}{9-x^2-5}$

$= \lim\limits_{x \to 2} \dfrac{\cancel{(4-x^2)}(3+\sqrt{x^2+5})}{\cancel{(4-x^2)}}$

$= \lim\limits_{x \to 2} 3+\sqrt{x^2+5}$

$= 3+\sqrt{4+5} = 3+3 = 6$

(e) $$\lim_{x \to 6} \frac{\sin 2x + \sin 6x}{\sin 5x - \sin 3x}$$

$$= \lim_{x \to 0} \frac{2 \sin \frac{2x+6x}{2} \cos \frac{2x-6x}{2}}{2 \cos \frac{5x+3x}{2} \sin \frac{5x-3x}{2}}$$

$$= \lim_{x \to 6} \frac{\sin 4x \cos(-2x)}{\cos 4x \sin x}$$

$$= \lim_{x \to 0} 4 \left(\frac{\sin 4x}{4x}\right) \cdot \frac{1}{\cos 4x} \cdot \cos 2x \left(\frac{x}{\sin x}\right)$$

$$= 4 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 4 \qquad \left(\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right)$$

(f) $$\lim_{x \to a} \frac{\sin x - \sin a}{x - a}$$

$$= \lim_{x \to a} \frac{2 \cos\left(\frac{x+a}{2}\right) \sin\left(\frac{x-a}{2}\right)}{x-a}$$

$$= \lim_{x \to a} \cancel{2} \cos\left(\frac{x+a}{2}\right) \frac{\sin\left(\frac{x-a}{2}\right)}{\frac{x-a}{2} \times \cancel{2}}$$

$$= \cos\left(\frac{2a}{2}\right) \times 1 = \cos a$$

(g) $\lim_{x \to 0} \dfrac{\sec x - 1}{\tan^2 x}$

$= \lim_{x \to 0} \dfrac{\sec x - 1}{\sec^2 x - 1}$

$= \lim_{x \to 0} \dfrac{\cancel{\sec x - 1}}{(\sec x + 1)\cancel{(\sec x - 1)}}$

$= \lim_{x \to 0} \dfrac{1}{\sec x + 1} = \dfrac{1}{1+1} = \dfrac{1}{2}$

(h) $\lim_{x \to \frac{\pi}{2}} \sec x - \tan x$

$= \lim_{x \to \frac{\pi}{2}} \dfrac{\sec^2 x - \tan^2 x}{\sec x + \tan x}$

$= \lim_{x \to \frac{\pi}{2}} \dfrac{1}{\sec x + \tan x}$

$= \lim_{x \to \pi} \dfrac{1}{\sec \pi/2 + \tan \pi/2} = \dfrac{1}{\infty + \infty} = 0$

(i) $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 6x - 4 = 2 = R.H.L$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 4 - 2x = 2 = L.H.L$

$\because$ R.H.L = L.H.L = 2

$\therefore \lim_{x \to 1} f(x) = 2$

(K) $\lim_{x\to 0} \frac{\tan 3x}{\sin 8x}$

$= \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot \frac{1}{\cos 3x} \cdot \frac{8x}{\sin 8x} \cdot \frac{3}{8}$

$= 1 \cdot 1 \cdot 1 \cdot \frac{3}{8} = \frac{3}{8}$

2(a) $\lim_{x\to 1} \frac{x-1}{|x-1|}$

Now $\lim_{x\to 1^+} \frac{x-1}{x-1} = 1 = \text{R.H.L}$

$\lim_{x\to 1^-} \frac{x-1}{-(x-1)} = -1 = \text{L.H.L}$

$\therefore$ L.H.L $\neq$ R.H.L $\Rightarrow \lim_{x\to 1} \frac{x-1}{|x-1|}$ not exist.

(b) $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} -2x+5 = 1 = \text{R.H.L}$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x+3 = 5 = \text{L.H.L}$

$\therefore$ L.H.L $\neq$ R.H.L $\Rightarrow \lim_{x\to 2} f(x)$ not exist.

(c) $\lim_{x\to 0^+} e^{\frac{1}{x}} = e^{\lim_{x\to 0^+} \frac{1}{x}} = e^{\infty} = \infty = \text{R.H.L}$

$\lim_{x\to 0^-} e^{\frac{1}{x}} = e^{\lim_{x\to 0^-} \frac{1}{x}} = e^{-\infty} = \frac{1}{e^{\infty}} = 0 = \text{L.H.L}$

$\therefore$ L.H.L $\neq$ R.H.L $\Rightarrow \lim_{x\to 0} e^{1/x}$ not exist

2) (d) $\lim\limits_{x \to 0^+} x + sgn(x) = 0 + 1 = 1 = R.H.L$

$\lim\limits_{x \to 0^-} x + sgn(x) = 0 - 1 = -1 = L.H.L$

$\therefore$ $R.H.L \neq L.H.L \Rightarrow \lim\limits_{x \to 0} x + sgn(x)$ not exist

(e) $\lim\limits_{x \to 0^+} \frac{\sin x}{|x|} = \lim\limits_{x \to 0^+} \frac{\sin x}{x} = 1 = R.H.L$

$\lim\limits_{x \to 0^-} \frac{\sin x}{|x|} = \lim\limits_{x \to 0^-} \frac{\sin x}{-x} = -1 = L.H.L$

$\therefore$ $R.H.L \neq L.H.L \Rightarrow \lim\limits_{x \to 0} \frac{\sin x}{|x|}$ not exist.

(3) $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x)$

$\Rightarrow \lim\limits_{x \to 2^-} 12 = \lim\limits_{x \to 2^+} a^2 x - 2a$

$\Rightarrow 12 = 2a^2 - 2a$

$\Rightarrow a^2 - a = 6$

$\Rightarrow a^2 - 3a + 2a - 6 = 0$

$\Rightarrow a(a-3) + 2(a-3) = 0$

$\Rightarrow (a-3)(a+2) = 0$

$\Rightarrow a = 3$ & $a = -2$

4. $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x)$

$\Rightarrow \lim_{x \to 3^+} 2ax = \lim_{x \to 3^-} x^2 - 1$

$\Rightarrow 6a = 9 - 1$

$\Rightarrow a = \frac{8}{6} = \frac{4}{3}$