Lecture - 10

Coulomb's law

Coulomb's law quantitatively describe the interaction of charges.

Coulomb determined the force law for electrostatic charges directly by experiment.

$$F = \frac{kqQ}{\mathrm{r}^2}\hat{\mathrm{r}} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{\mathrm{r}^2}\hat{\mathrm{r}}$$

Where $k = 9 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$

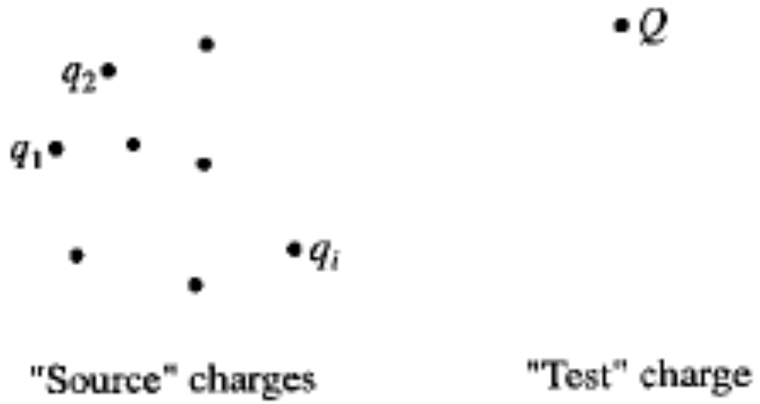
and $\varepsilon_0 = 8.86 \times 10^{-12} \frac{\mathrm{C^2}}{\mathrm{N \cdot m^2}}$

How does one particle sense the presence of the other?

The electric charge creates an electric field in the space around it. A second charged particle does not interact directly with the first; rather, it responds to whatever field it encounters. In this sense, the field acts as an *intermediary* between the particles.

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \cdots = \frac{Q}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1^{\,2}}\hat{r}_1 + \frac{q_2}{r_2^{\,2}}\hat{r}_2 + \cdots\right) = Q\mathbf{E}$$

$$\text{where } \mathbf{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1^{\,2}}\hat{r}_1 + \frac{q_2}{r_2^{\,2}}\hat{r}_2 + \cdots\right) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^{\,2}}\hat{r}_i$$



The electric field strength is defined as the force per unit charge placed at that point.

Example

On a clear day there is an electric field of approximately 100 N/C directed vertically down at the earth's surface. Compare the electrical and gravitational forces on an electron.

The magnitude of the electrical force is

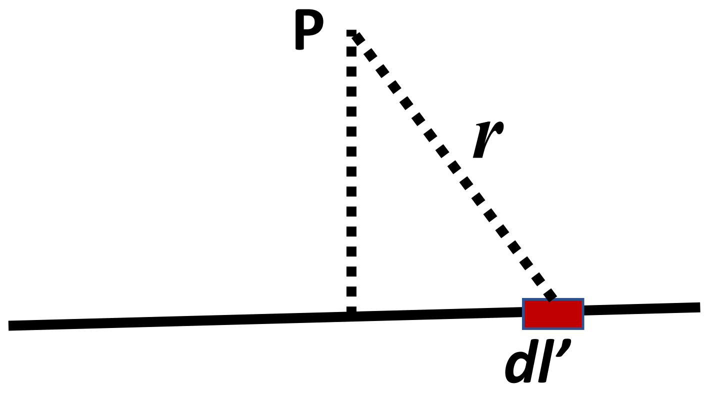
$$Fe = eE = 1.6\text{x}10^{-19}\text{x}100 = 1.6\text{x}10^{-17}\ \text{N. (upward)}$$

The magnitude of the gravitational force is

$$Fg = mg = 9.11\text{x}10^{-31}\text{x}9.8 = 8.9\text{x}10^{-30}\ \text{N. (downward)}$$

Continuous charge distribution

In order to find the electric field due to a continuous distribution of charge, one must divide the charge distribution into infinitesimal elements of charge ***dq*** which may be considered to be point charges



dq = λ dl’

$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r} \quad \Rightarrow \quad \mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$$

Thus the electric field of a line charge is

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{P}} \frac{\lambda(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, dl';$$

for a surface charge,

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{S}} \frac{\sigma(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, da';$$

and for a volume charge,

$$\boxed{\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\rho(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, d\tau'.}$$

Example - 1

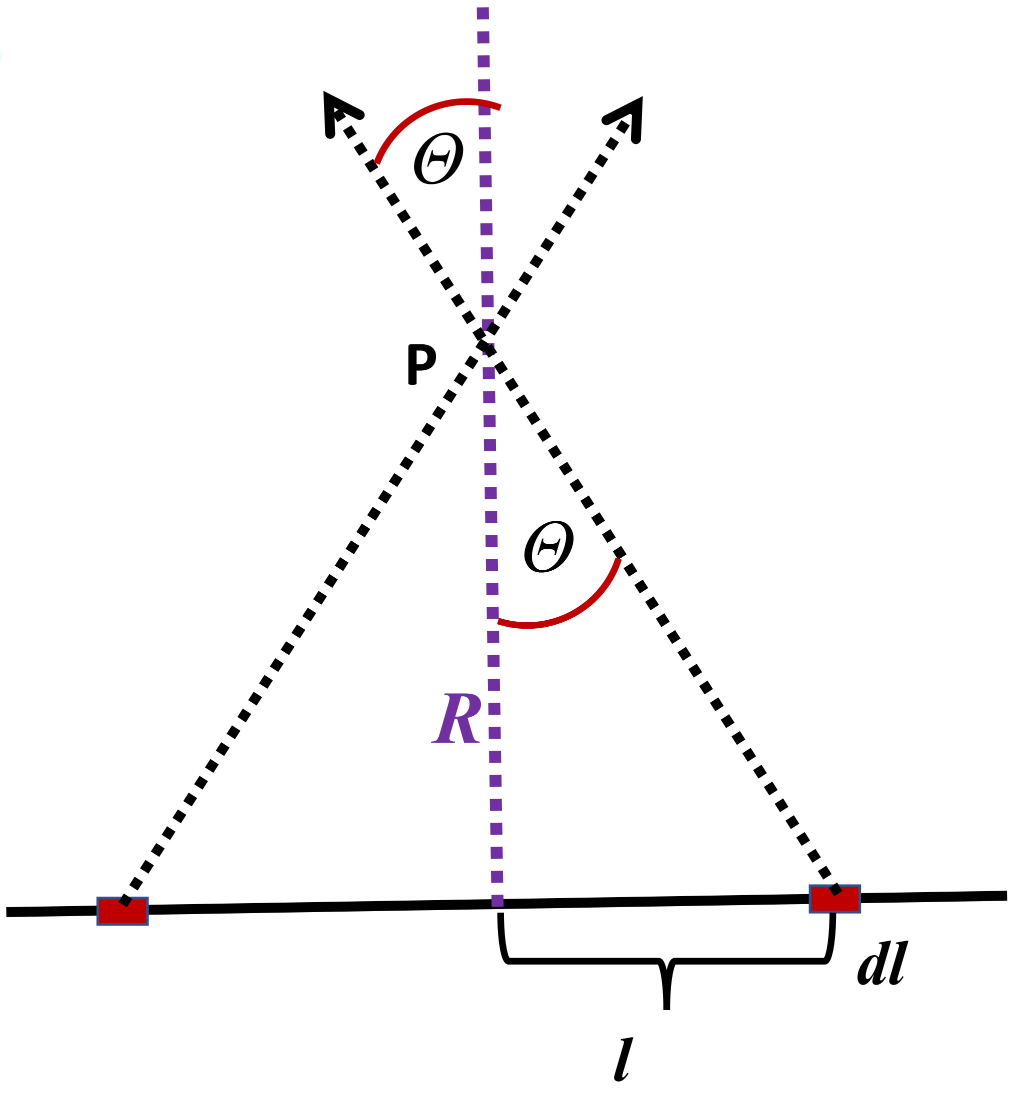
What is the field strength at a distance R from an infinite line of charge with linear charge density λ C/m.

Since the charge carrier is infinite long, the electric field in y-direction completely cancel out. Thus the resultant field is along the x-axis.

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\cos\theta d\ell}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R\sec^2\theta\cos\theta\, d\theta}{(R\sec\theta)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda\cos\theta d\theta}{R}$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{R}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta = \frac{\lambda}{2\pi\varepsilon_0 R}$$

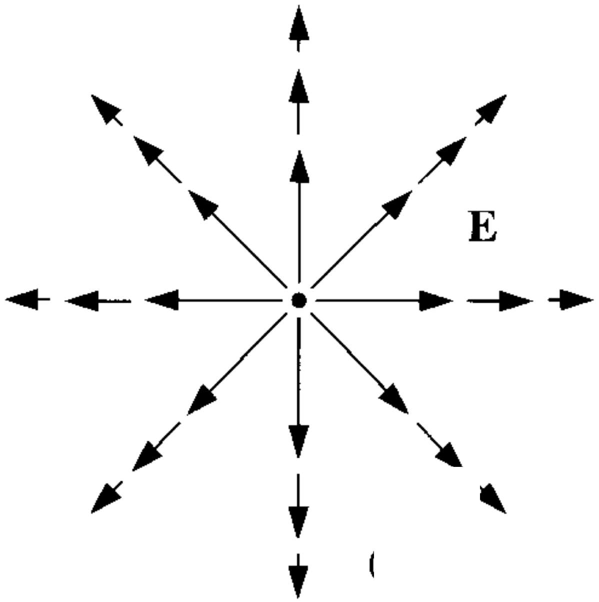


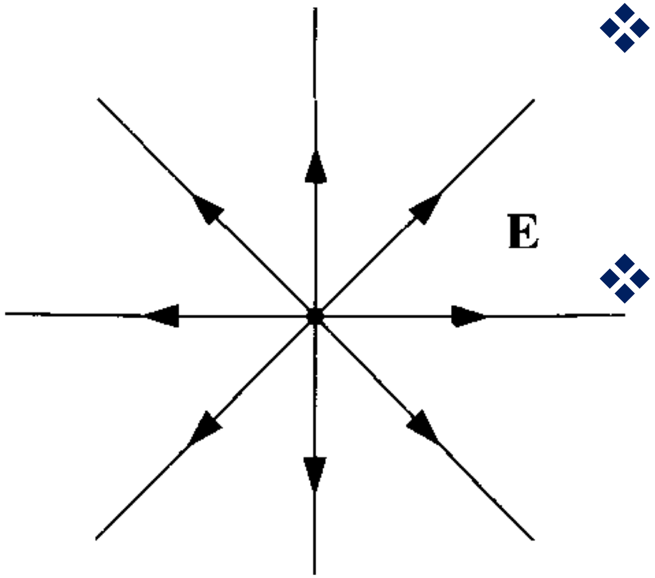
Field lines

How to express the magnitude and vector properties of the field strength?

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}.$$

The field strength at any point could be represented by an arrow drawn to scale. However, when several charges are present, the use of arrows of varying length and orientations becomes confusing. Instead we represent the electric field by continuous field lines or lines of force.





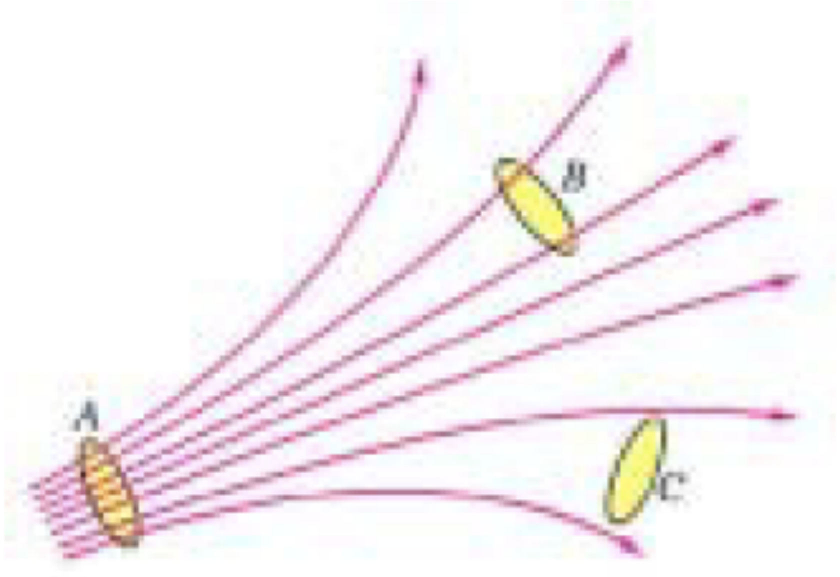
- ❖ The magnitude of the field is indicated by the density of the field lines.
- ❖ The density of lines is the total number divided by the area of the sphere.

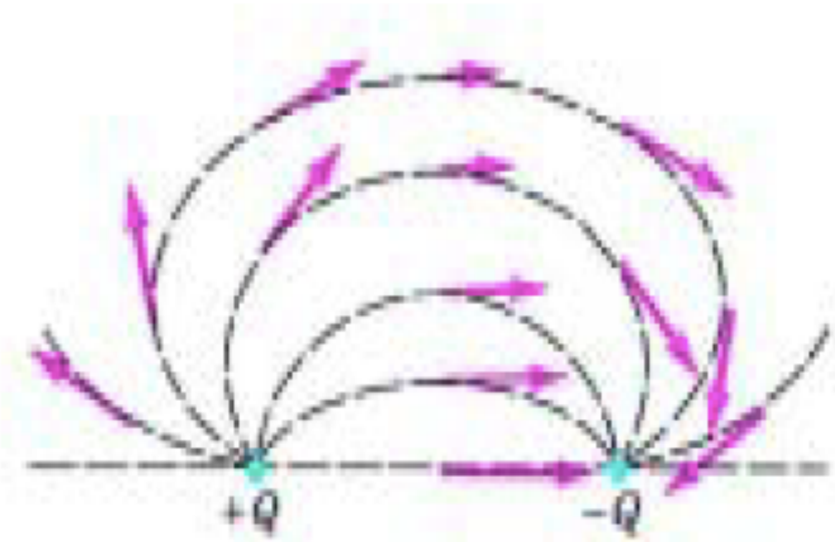
$$\frac{n}{4\pi r^2}$$

Field lines

How to determine the field strength from the field lines?

The lines are crowed together when the field is strong and spread apart where the field is weaker. The field strength is proportional to the density of the lines.





(a)Symmetry

(b)Near field

(c)Far field

(d)Null point

(e)Number of lines

Be consistent: If your charge is doubled, number of lines need to be doubled