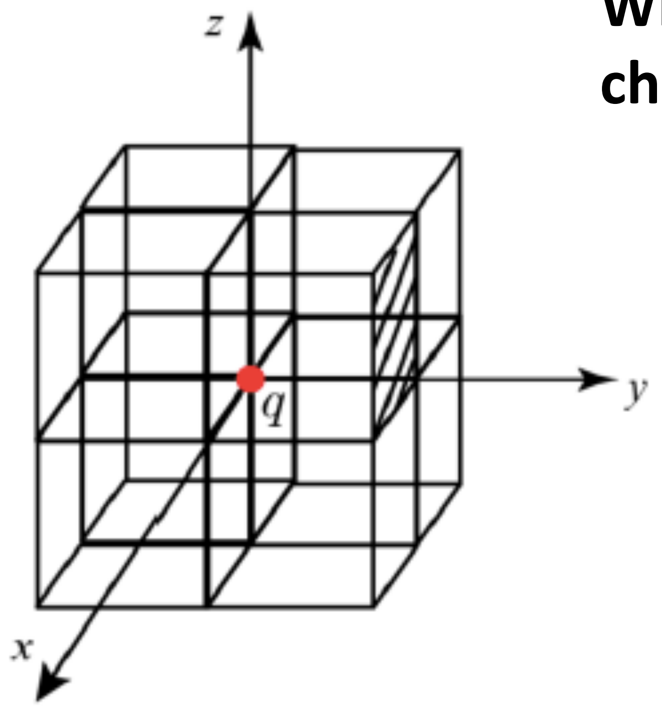
Lecture - 13

Application of Gauss's law



$$\oint_{surf} \mathbf{E} \cdot d\mathbf{a} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

What is the flux through the shaded face of the cube due to the charge q at the corner?

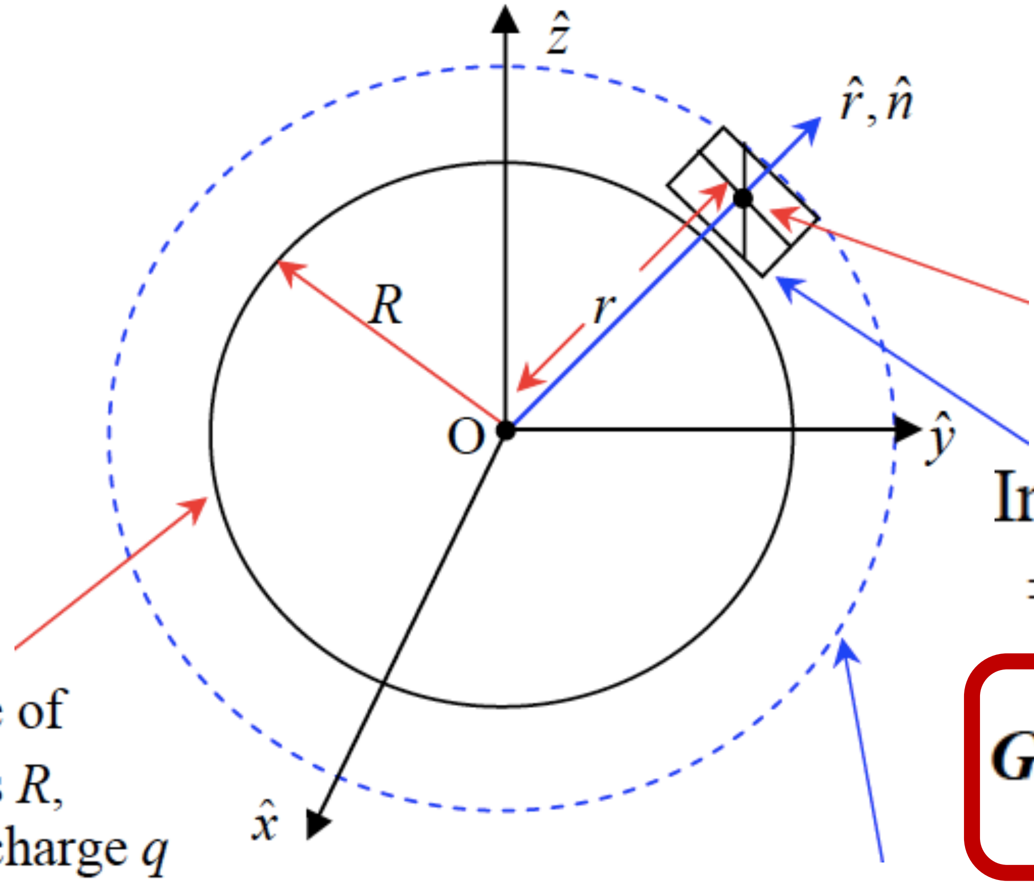
$$\int_{surf} \mathbf{E} \cdot d\mathbf{a} \;??$$

$$24 \int_{surf} \mathbf{E} \cdot d\mathbf{a} = \frac{q}{\epsilon_0}$$

$$\int_{surf} \mathbf{E} \cdot d\mathbf{a} = \frac{1}{24} \frac{q}{\epsilon_0}$$

Application of Gauss's law

<u>Griffiths Example 2.2:</u> Find / determine the electric field intensity $\vec{E}(\vec{r})$ outside a uniformly charged solid sphere of radius R and total charge q:



$$\boxed{\textbf{\textit{Gauss' Law:}} \quad \oint_S \vec{E}(\vec{r}) \bullet d\vec{A} = \frac{1}{\varepsilon_o} Q_{encl} = \frac{1}{\varepsilon_o} q = \frac{q}{\varepsilon_o}}$$

$$\oint_S \vec{E}(\vec{r})\bullet d\vec{A} = \oint_S E(\vec{r})\,dA = q/\varepsilon_o \qquad (E(\vec{r})\hat{r})\bullet(dA\hat{r}) = E(\vec{r})\,dA\underbrace{(\hat{r}\bullet\hat{r})}_{=1} = E(\vec{r})\,dA$$

$$= E(\vec{r})\oint_S dA = E(\vec{r})\left(4\pi r^2\right) = q/\varepsilon_o$$

the <u>magnitude</u> of $\vec{E}$ <u>is constant</u> $\forall$ (for all)/for any <u>fixed</u> r

$$\boxed{\vec{E}(\vec{r}) = \frac{q}{4\pi\varepsilon_o r^2}\hat{r} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}}$$

n.b. the electric field (for $r > R$) for charged sphere is equivalent / identical to that of a point charge q located at the origin!!!

Inside:

$$\mathbf{E} = \left(\frac{Q\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}\right)\frac{1}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$$

$$= \frac{Q}{4\pi\varepsilon_0 R^3} r\hat{\mathbf{r}}$$

How to apply Gauss's law?

1. Use symmetry.
2. Properly choose a Gaussian surface (E//A or E⊥A).

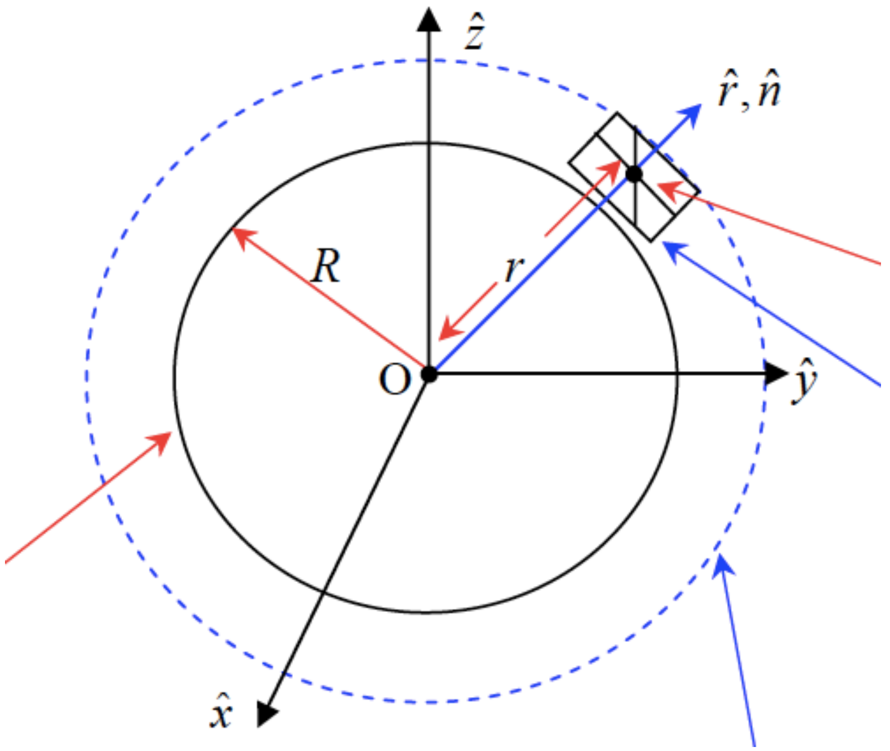
How to Choose a Good Gaussian Surface?

Gauss's Law is always true, but it is not always useful.

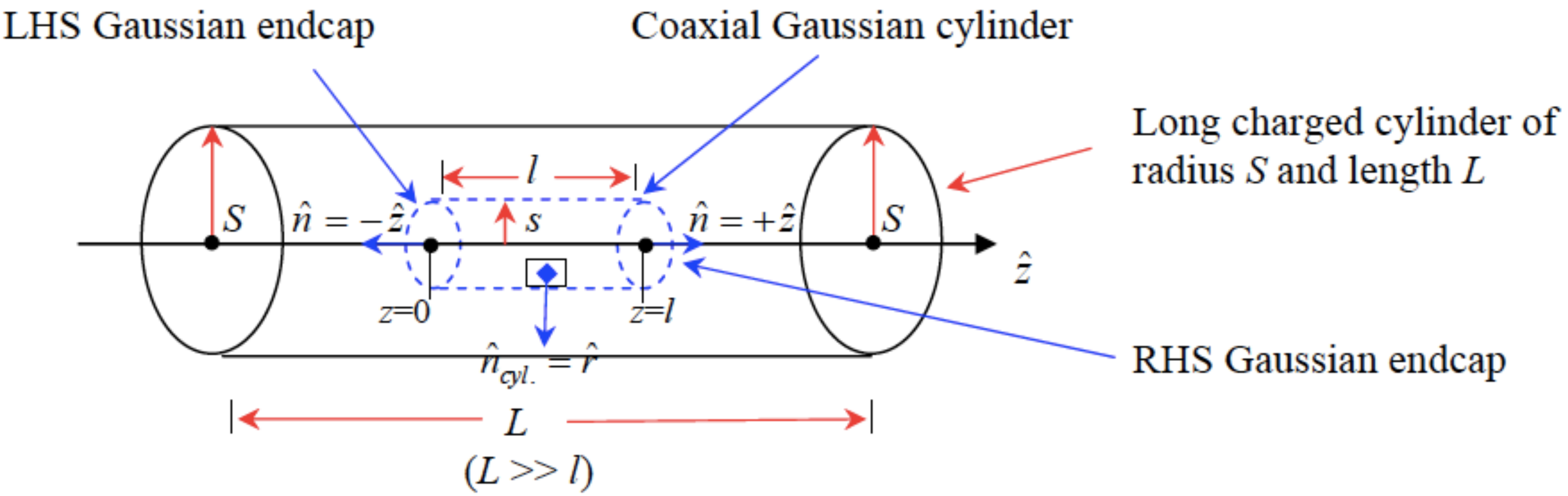
Symmetry is crucial to the application of Gauss's law.

There are only three kinds of symmetry that work:

1. Spherical symmetry: Make your Gaussian surface a concentric sphere.
2. Cylindrical symmetry: Make your Gaussian surface a coaxial cylinder.
3. Plane symmetry: Use a Gaussian "pillbox", which straddles the surface.



Griffiths Example 2.3 Consider a long cylinder (e.g. plastic rod) of length L and radius S that carries a volume charge density ρ that is proportional to the distance from the axis s of the cylinder / rod –



a) Determine the electric field $\vec{E}(\vec{r})$ inside this long cylinder / charged plastic rod

- Use a coaxial Gaussian cylinder of length l and radius s: (with $l << L$)

Gauss' Law $\oint_S \vec{E}(\vec{r})\bullet d\vec{A} = \frac{Q_{encl}}{\varepsilon_o}$

Enclosed charge: $Q_{encl} = \int_v \rho(s')d\tau' = \int_v (ks')(s'ds'd\varphi dz) \Leftarrow$ integral over Gaussian surface

$$Q_{encl} = \int_{s'=0}^{s'=s}\int_{\varphi=0}^{\varphi=2\pi}\int_{z=0}^{z=l}(ks')(s'ds'd\varphi dz) = 2\pi kl\int_{s'=0}^{s'=s} s'^2 ds'$$

$$Q_{encl} = \frac{2}{3}\pi kls^3$$

$$\underset{\substack{S\\ \text{Gaussian}\\ \text{cylinder}}}{\oint} \vec{E}(\vec{r})\bullet d\vec{A} = \underbrace{\underset{\substack{\text{Cyl.}\\ \text{Gaussian}\\ \text{surface}}}{\int} (E(\vec{r})\hat{r})\bullet(sdld\varphi\hat{r})}_{\hat{r}\bullet\hat{r}=1} + \underbrace{\cancel{\underset{\substack{\text{LHS}\\ \text{Gaussian}\\ \text{endcap}}}{\int} (E(\vec{r})\hat{r})\bullet(-sdsd\varphi\hat{z})}}_{\hat{r}\bullet\hat{z}=0} + \underbrace{\cancel{\underset{\substack{\text{RHS}\\ \text{Gaussian}\\ \text{endcap}}}{\int} (E(\vec{r})\hat{r})\bullet(+sdsd\varphi\hat{z})}}_{\hat{r}\bullet\hat{z}=0}$$

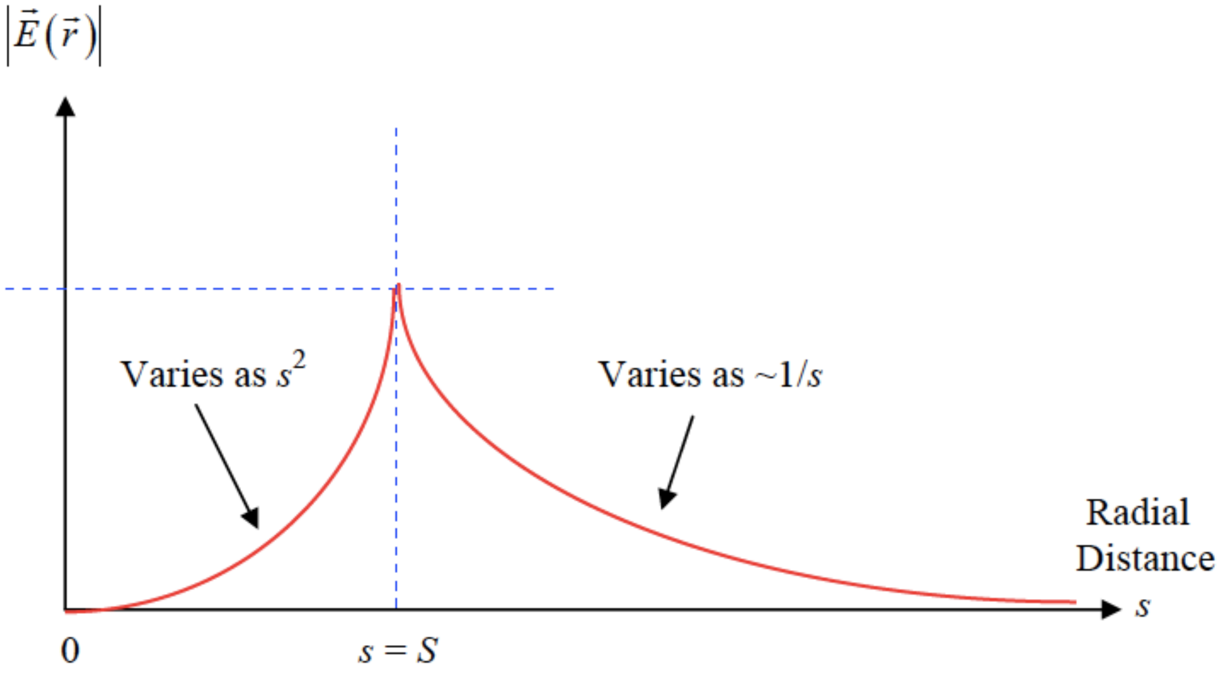
$$\underset{\substack{S\\ \text{Gaussian}\\ \text{cylinder}}}{\oint} \vec{E}(\vec{r})\bullet d\vec{A} = \underset{\substack{\text{cylindrical}\\ \text{Gaussian}\\ \text{surface}}}{\int} \overbrace{E(\vec{r})}^{\text{Constant here}} sdld\varphi = E(\vec{r})s\int_{z=0}^{z=l}\int_{\varphi=0}^{\varphi=2\pi} dld\varphi = E(\vec{r})sl(2\pi) = 2\pi slE(\vec{r})$$

$$\cancel{2\pi}\,\cancel{s}\,\cancel{l}\,E(\vec{r}) = \frac{\cancel{2\pi} k s^{\cancel{4}\,3}\,\cancel{l}}{3\varepsilon_o}$$

$$\boxed{\begin{array}{l}\text{inside} \\ \vec{E}_{in}(\vec{r}) = \dfrac{ks^2}{3\varepsilon_o}\hat{r} \\ (s = r < S)\end{array}}$$

$$\oint_S \vec{E}(\vec{r})\bullet d\vec{A} = \int_{\substack{\text{cylindrical} \\ \text{Gaussian} \\ \text{surface}}} \left(E(\vec{r})\hat{r}\right)\bullet\left(dA_{cyl}\hat{r}\right)$$

$$= E(\vec{r})\int_{z=0}^{z=l}\int_{\varphi=0}^{\varphi=2\pi} s\,dl\,d\varphi = 2\pi s l E(\vec{r})$$



Electric field outside charged rod $(s = r > S)$:

$$E_{out}(\vec{r}) = \frac{\cancel{2\pi} k \cancel{l} S^3}{3\bullet\cancel{2\pi} s \cancel{l} \varepsilon_o}\hat{r} = \frac{kS^3}{3s\varepsilon_o}$$