DO NOT WRITE ANYTHING ON QUESTION PAPER EXCEPT YOUR NAME, DEPARTMENT AND ENROLMENT No.

POSSESSION OF MOBILE IN EXAMINATION IS A UFM PRACTICE

Name of Student --- Enrolment No. -----------------

Department / School ----------------------------

BENNETT UNIVERSITY, GREATER NOIDA

End Term Examination, SPRING SEMESTER 2018-19

COURSE CODE: **EPHY108L** MAX. DURATION: **3 HOURS**

COURSE NAME: **Mechanics**

COURSE CREDIT: **3** MAX. MARKS: **50**

Note

- All questions are mandatory
- Rough work must be carried out at the back of the answer script

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Answer the following questions (1 to 8) in brief. The answers should be no longer than 2-3 lines.

1. The moment of Inertia of a rigid body about its 3 principal axes are 3, 6, 18.8 Kg.m^2. Torque-less rotation about which axis will be least stable and why?
(1 mark)

2. An observer sees three frames of references A, B and C, moving along $+x$ direction. The speeds of frames A, B and C at $t = 0$ s are 3, 2 and 2 m/s, respectively. The speeds at $t = 2$ s are 3, 3 and 2 m/s, respectively. According to the observer which frame/frames of reference is/are non-inertial? Give reason.
(1 mark)

3. Where is the centrifugal force due to earth's rotation maximum? At the equator or poles or at latitude of 45^0. Give reason.
(1 mark)

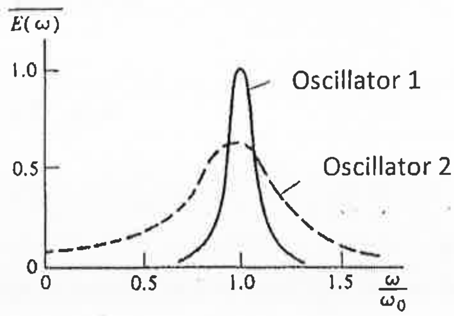
4. A planet while orbiting around the sun has a speed v at position $\mathbf{r}_1$ and it sweeps out an area α in unit time interval. At a different location in its orbit ($\mathbf{r}_2$) the planet has a speed $2v$. What is the area that it will sweep out in unit time at $\mathbf{r}_2$. Give reason.
(1 mark)

5. The path of a particle moving under the influence of gravitational force is a conic section. What should be the eccentricity and total energy so that the path is bounded, i.e. either an ellipse or a circle

(1 mark)

6. The figure below shows the response of two oscillators to variation in frequency of the driving force. Which oscillator has higher damping? Give reason.

(1 mark)



7. If a planet moves in an elliptic orbit $r = \frac{r_0}{1-\varepsilon\cos\theta}$ with sun at one of the focus. What is the closest and farthest distance of the planet from sun, in terms of eccentricity ε and constant r_0.

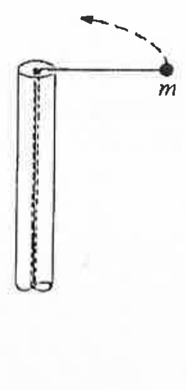
(1 mark)

8. When can a massless particle (mass measured when it is at rest = 0) have non-zero momentum? What is its momentum if its total energy = E?

(1 mark)

9. Mass m is attached to a post of radius R by a string (see figure below). Initially it is at distance r from the center of the post and is moving tangentially with speed v_0. The string passes through a hole in the center of the post at the top. The string is being pulled down and shortened by drawing it through the hole. What physical quantities are conserved in this case? Find the final speed of the mass when it hits the post.

(3 marks)



10. A geostationary orbit is one in which a satellite moves in a circular orbit at a certain height in the equatorial plane, so that its angular velocity of rotation around earth is same as earth's angular velocity. Find its angular and linear velocity. Use g = 9.8 m.s^{-2}, radius of earth R_e = 6.4 x 10^6 m and $g = \frac{GM_e}{R_e^2}$, where G = universal gravitational constant and M_e is the mass of earth. (Hint: Earth takes 24 hours to complete one rotation)
(1+2 = 3 Marks)

11. If the damping constant of a free oscillator is given by $\gamma = 2\omega_0$, the system is said to be critically damped. The motion is given by $x = (A + Bt)e^{-\gamma t/2}$, where A and B are arbitrary constants. A critically damped oscillator is at rest at equilibrium. At $t = 0$ it is given a blow of total impulse I. Find A, B and the maximum displacement from equilibrium using the given initial condition.
(4 Marks)

12. The sound intensity from a musical instrument operating at an angular frequency (ω) of 400 Hz, decreases by a factor of 5 in 4 seconds. What is the quality factor (Q) of the instrument. Note that the sound intensity is proportional to the energy of the oscillator.
(3 Marks)

13. A particle of rest mass m_0 and kinetic energy xm_0c^2, where x is some number, strikes and sticks to an identical particle at rest. What is the rest mass of the resultant particle? Assume the particles are moving with relativistic speeds.
(4 marks)

14. An object of mass 20 Kg moves in simple harmonic motion along the x axis. Initially (at t = 0) it is located at a distance of 4 meters away from the origin and has a velocity of 15 m/s and acceleration 100 m/s^2, both directed towards origin. Assume the motion is of the form $x = A\cos(\omega t + \varphi)$ and find the amplitude (A), angular frequency (ω) and phase factor (φ).
(5 marks)

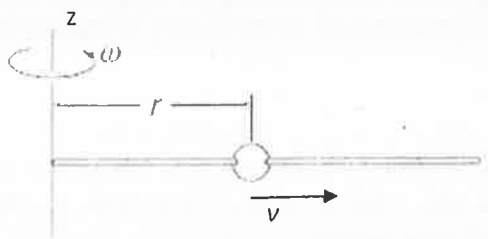
15. An event occurs in a frame of reference S at x = 6 x 10^8 m, and in another a frame of reference S' at x' = 6 x 10^8 m and t' = 4 s. Find the relative velocity of the two frames. Assume relativistic domain.
(3 marks)

16. A satellite of mass m is in circular orbit about the earth. The radius of the orbit is r_0 and the mass of the earth is M_e.
(a) Find the total energy of the satellite.

(b) Suppose that a constant weak friction force f acts on the satellite. The satellite will slowly spiral toward the earth. Find the approximate change in radius per revolution of the satellite, Δr. Note that you can assume that the change is very small.
(2+3 marks)

17. A coordinate system x,y,z is rotating w.r.t. another coordinate system X,Y,Z with an angular velocity $\boldsymbol{\omega} = 2t\hat{\imath} - t^2\hat{\jmath} + (2t+4)\hat{k}$, where t is time. Both systems have the same origin and X,Y,Z system is assumed to be inertial. The position vector of a particle in x,y,z system is $\mathbf{r} = (t^2+1)\hat{\imath} - 6t\hat{\jmath} + 4t^3\hat{k}$. Find:
(a) its velocity in the inertial system X,Y,Z at $t = 1$ s.
(b) the Coriolis force acting on the particle in the rotating system x,y,z at $t = 1$ s.
(5 marks)

18. A bead of mass m slides with a constant speed v in the outward direction on a horizontal rigid wire, which itself is rotating at constant angular speed ω about the z axis (see figure below). What is the force exerted on the bead by the wire. Neglect the weight of the bead and friction.
(3 marks)



19. For what values of n are circular orbits stable for central forces of the form $\mathbf{F}(r) = -\frac{C}{r^n}\hat{\mathbf{r}}$. Here C is a positive constant. Take $V(r) = -\frac{C}{r^{n-1}}$, as the potential energy.
(4 marks)

End of Question Paper

Useful relations (symbols have their usual meanings):

Vector multiplication:
$\mathbf{A}.\mathbf{B} = AB\cos\theta$, $|\mathbf{A}\times\mathbf{B}| = AB\sin\theta$
$\mathbf{A}.\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Plane polar coordinates:
$x = r\cos\theta$, $y = r\sin\theta$, $\hat{\mathbf{r}} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$, $\hat{\boldsymbol{\theta}} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$

Kinematics in polar coordinates:
$\mathbf{r} = r\hat{\mathbf{r}}$, $\mathbf{v} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}$, $\boldsymbol{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\boldsymbol{\theta}}$

Rotational motion
$\mathbf{L} = \mathbf{r} \times \mathbf{P}$, $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \frac{d\mathrm{L}}{dt}$

Rotating vector: $\frac{dA}{dt} = \boldsymbol{\omega} \times \boldsymbol{A}$

Derivative in a rotating frame of reference
$\left(\frac{d}{dt}\right)_{in} = \left(\frac{d}{dt}\right)_{rot} + \Omega\,\mathrm{x}$

Pseudo forces in a rotating frame
$\mathbf{F}_{coriolis} = -2m\Omega \times \mathbf{v}_{rot}$
$\mathbf{F}_{centrifugal} = -m\Omega \times (\Omega \times \boldsymbol{r})$

Central Force

$\mu\ddot{r} = f(r)\hat{r}$ $\quad\quad \mu = \frac{m_1 m_2}{m_1+m_2}$

$V_{eff}(r) = \frac{L^2}{2\mu r^2} + V(r)$ $\quad\quad E = \frac{1}{2}\mu\dot{r}^2 + V_{eff}(r)$ $\quad\quad L = \mu r^2\dot{\theta}$

$$\theta - \theta_0 = L\int_{r_0}^{r} \frac{dr}{r^2\sqrt{2\mu(E - V_{eff}(r))}}$$

Gravitational force $\frac{Gm_1m_2}{r^2}$
Planetary orbits

$r = \frac{r_0}{1-\varepsilon\cos\theta}$ $\quad\quad \varepsilon = \sqrt{1 + \frac{2EL^2}{\mu C^2}}$ $\quad\quad E = -\frac{C}{A}$

Harmonic Oscillator

$m\ddot{x} + kx = 0$ $\quad\quad x = B\sin\omega_0 t + C\cos\omega_0 t$ $\quad\quad x = A\cos(\omega_0 t + \varphi)$ $\quad\quad \omega_0 = \sqrt{\frac{k}{m}}$

Damped oscillator
$x = Ae^{-\gamma t/2}\cos(\omega_1 t + \emptyset)$ $\quad\quad E\,(t) = E_0 e^{-\gamma t}$
$Q = \frac{\omega_1}{\gamma} = \frac{\omega_0}{\gamma}$
Forced oscillation

$$x = \frac{F_0}{m} \frac{1}{\omega_0^2 - \omega^2} \cos \omega t$$

Resonance curves $\Delta\omega = \gamma$

Lorentz transformation for relativistic motion

$x' = \gamma(x - vt)$

$y' = y\,,\ z' = z$

$t' = \gamma\left(t - \frac{xv}{c^2}\right)$

$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

Relative velocities in the relativistic domain

$$u'_x = \frac{u_x - v}{1 - vu_x/c^2} \qquad u'_y = \frac{u_y}{\gamma[1 - vu_x/c^2]} \qquad u'_z = \frac{u_z}{\gamma[1 - vu_x/c^2]}$$

Mass, energy, momentum in relativistic motion

$m = m_0\gamma$ $\qquad$ $p = m_0 u\gamma$ $\qquad$ $E^2 = (pc)^2 + (m_0c^2)^2$

$E = mc^2$ $\qquad$ $K = mc^2 - m_0c^2$