BENNETT UNIVERSITY
TIMES OF INDIA GROUP

EMAT101L

Engineering Calculus

Solutions of Tutorial Sheet 8

(Applications of Integrals)

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1. Find the area of the region bounded by the given curves. (Use $\left|\int_a^b (f(x)-g(x))dx\right|$, where a and b are the x-coordinates of the intersection points)

   (a) $f(x) = 2x^2 + 5x - 3$ and $g(x) = x^2 + 4x - 1$

   Ans. $\frac{9}{2}$

   (b) $f(x) = \sin x$ and $g(x) = \cos x$ from $x = 0$ to $x = \frac{\pi}{4}$

   Ans. $\sqrt{2} - 1$

   (c) $x = 2y^2$ and $x + y = 1$

   Ans. $\frac{9}{8}$

   (d) $y = 2x, y = 5x$ and $x = 3$

   Ans. $\frac{27}{2}$

   (e) $y = x^2$ and $x = y^2$

   Ans. $\frac{1}{3}$

2. Use the Disk/Washer to find the volume of the solid of revolution formed by rotating the region about each of the given axes. Region bounded by: $y = \sqrt{x}, y = 0$ and $x = 1$. Rotated about

   (a) the x-axis Ans. $\pi \int_0^1 [\sqrt{x}]^2 dx = \frac{\pi}{2}$ unit^3

   (b) $y = 1$ Ans. $\pi \int_0^1 [1 - \sqrt{x}]^2 dx = \frac{1}{6}\pi$ unit^3

   (c) the y-axis Ans. $\pi \int_0^1 [y^2]^2 dy = \frac{\pi}{5}$ unit^3

(d) $x = 1$ Ans. $\pi \int_0^1 [1 - y^2]^2 dy = \frac{8}{15}\pi$ unit^3

3. Find the volume of the following solids of revolution using the Shell method.
   Region bounded $y = \sqrt{x}, y = 0$ and $x = 1$ and rotated about $x = 3$.
   Hint: $r(x) = 3 - x$ and $h(x) = \sqrt{x}$, Ans. $\frac{16\pi}{5}$ unit^3.

4. Find the volume of the solid of revolution where $y = \sin x$ on $[0, \pi/2]$ is revolved around the x-axis.
   Hint: Use disk method, Ans. $\pi \int_0^{\frac{\pi}{2}} [\sin(x)]^2 dx = \frac{\pi^2}{4}$ unit^3

5. Find the volume of the solid created when the area between $f(x) = x^2 + 1$ and $g(x) = x$ on $[0, 1]$ is rotated about x-axis.
   Hint: Use the Washer method, Ans. $\pi \int_0^1 \left([x^2 + 1]^2 - [x]^2\right) dx = \frac{23}{15}\pi$ unit^3

6. Find the volume of the solid created when the area contained by $f(x) = x^2$ and $g(x) = x^3$ is revolved around the x-axis.
   Hint: Use the Washer method, Ans. $\pi \int_0^1 \left([x^2]^2 - [x^3]^2\right) dx = \frac{2}{35}\pi$ unit^3

7. Using the Shell method, find the volume of the region enclosed by $y = \sqrt{x}, x = 1$ and $x = 4$ when revolved about y-axis.
   Ans. $2\pi \int_1^4 x\sqrt{x}\, dx = \frac{124}{5}\pi$ unit^3
