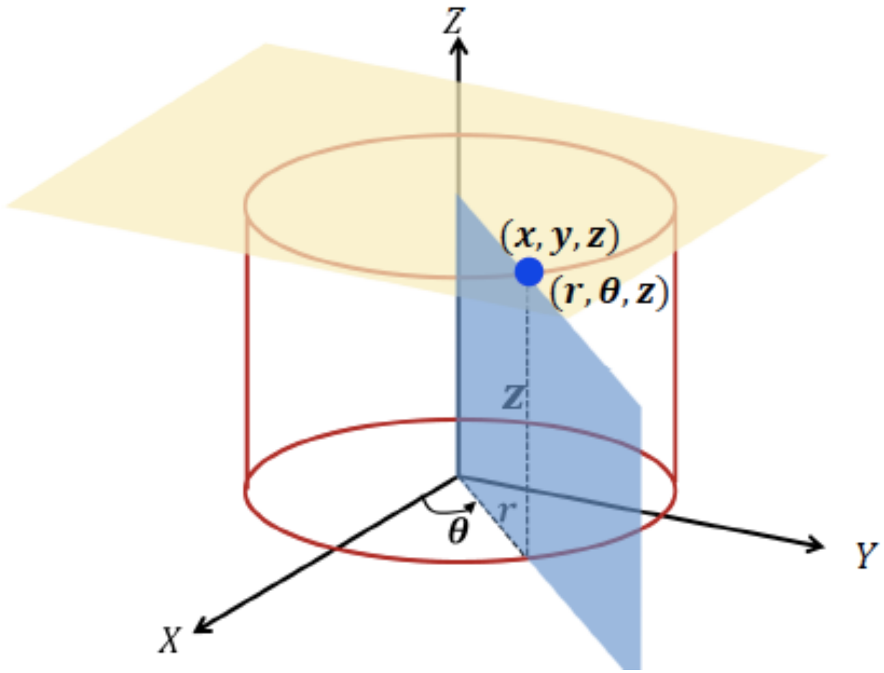
Lecture - 9

Cylindrical coordinate System



How to specify a point ***P*** in space?
(r,$\boldsymbol{\theta}$,z)

- ✓ z is the Height from the *XY*-plane
- ✓ Coordinate of the foot of the point in XY plane.

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$

Polar coordinate unit vectors $(\hat{r}, \hat{\theta})$ + additional unit vector in the z –direction.

Reverse transformation

$$r = (x^2 + y^2)^{1/2}$$
$$\theta = \tan^{-1}\frac{y}{x}$$
$$z = z$$

❑ $\hat{r}, \hat{\theta}$ **and** $\hat{z}$ are unit vectors along increasing direction of coordinates r, θ **and** z.

Line element:

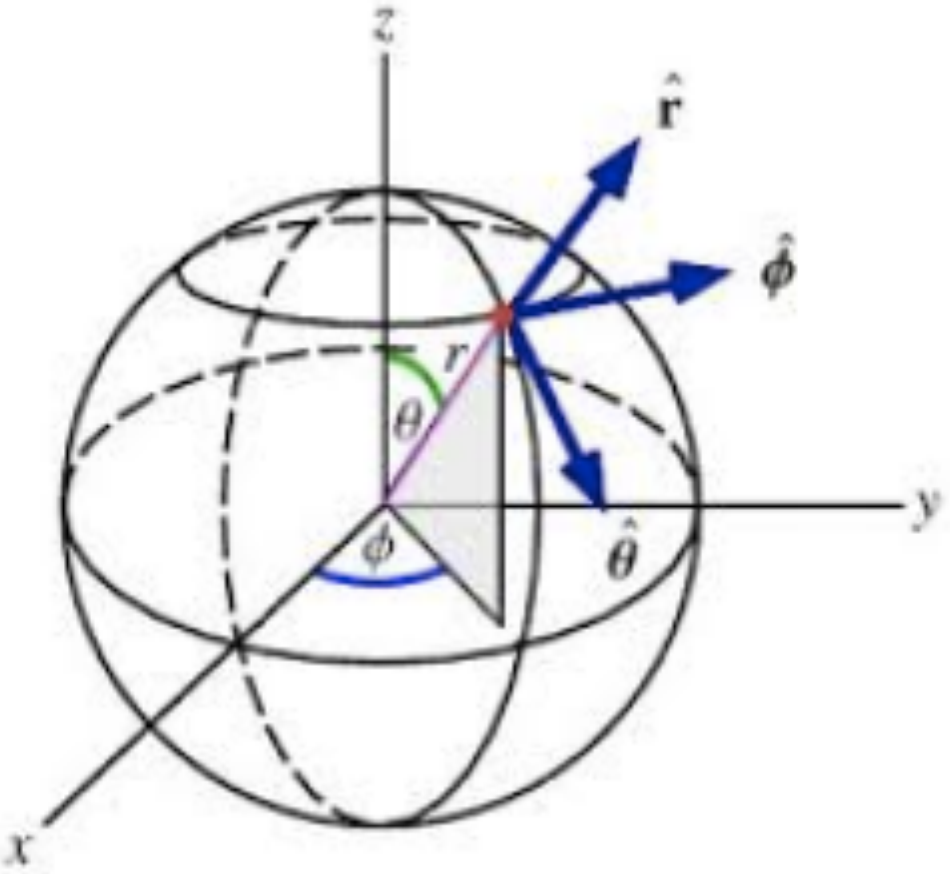
$$\vec{dl} = dr\hat{r} + rd\theta\hat{\theta} + \mathrm{d}z\hat{z}$$

Surface element with fix r:

$$\vec{dA} = rd\theta \mathrm{d}z\hat{r}$$

Volume element: $dv = rdrd\theta \mathrm{d}z$

Spherical coordinate System



$\boldsymbol{r}$ → Radial distance from origin

$\boldsymbol{\theta}$ →Angle of radial vector with z-axis.

$\boldsymbol{\varphi}$ → Angle between X-axis and the projection of radial vector in XY plane

▸ *Y*

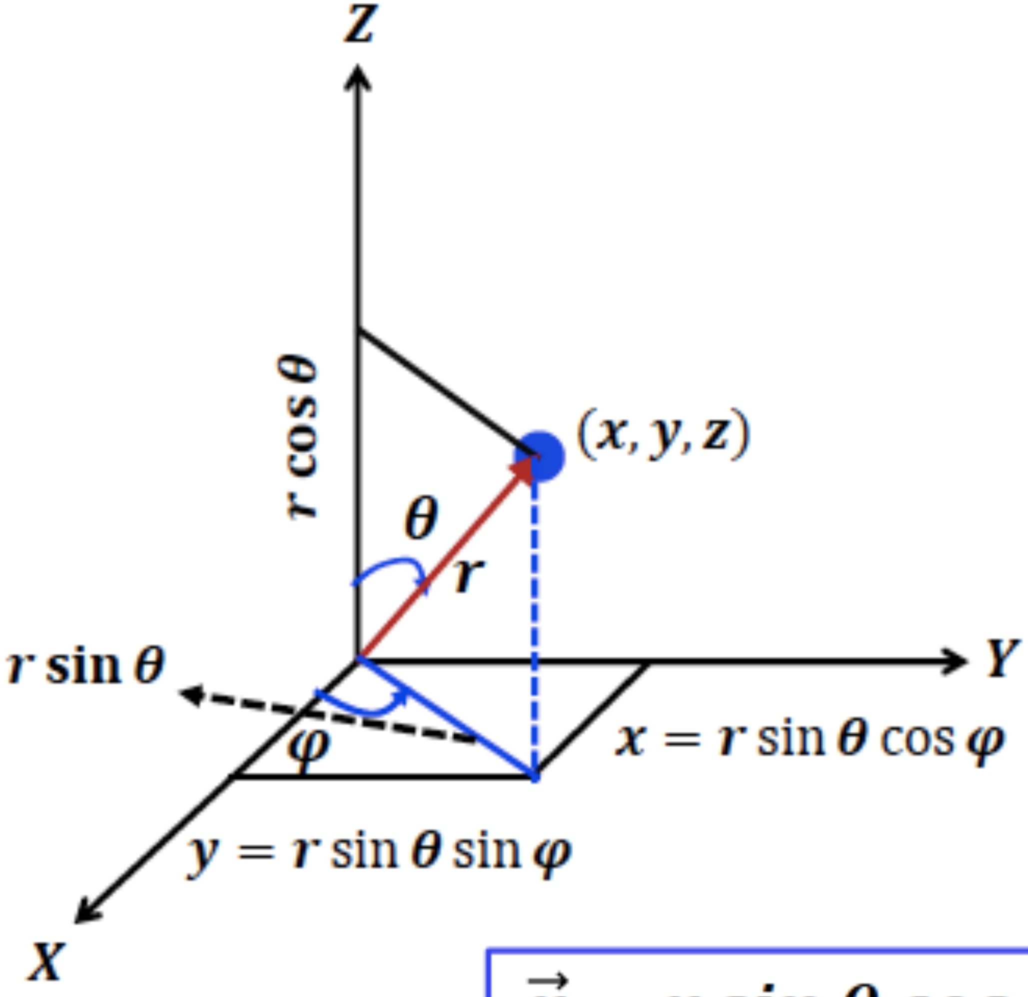
s known as spherical polar coordinate

Note that point (r, θ, φ) is at the intersection of three surfaces

- ❑ A sphere where r =Constant
- ❑ A cone about z=axis with $\boldsymbol{\theta}$=constant.
- ❑ A half plane containing z-axis and $\boldsymbol{\varphi}$= constant

Be careful, notations are different.
r and $\boldsymbol{\theta}$ are not planer coordinate.

Spherical coordinate System



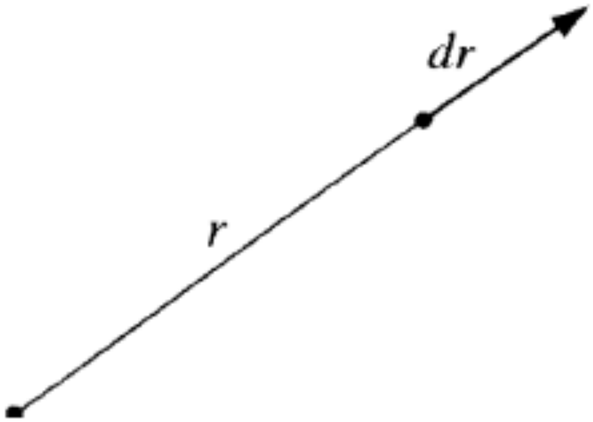
Transformation relations

$$x = r\sin\theta\cos\varphi$$
$$y = r\sin\theta\sin\varphi$$
$$z = r\cos\theta$$

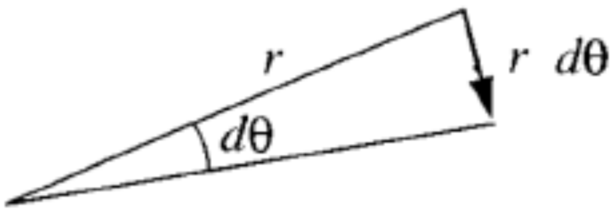
Hence

$$r = \left(x^2 + y^2 + z^2\right)^{1/2}$$
$$\theta = \tan^{-1}\frac{\left(x^2 + y^2\right)^{1/2}}{z}$$
$$\varphi = \tan^{-1}\frac{y}{x}$$

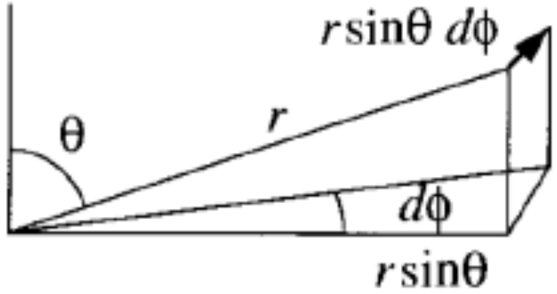
$$\boxed{\vec{r} = r\sin\theta\cos\varphi\,\hat{x} + r\sin\theta\sin\varphi\,\hat{y} + r\cos\theta\,\hat{z}}$$



$$dl_r = dr$$



$$dl_\theta = r d\theta$$



$$dl_\phi = r \sin\theta d\phi$$

Line element: $\mathbf{dl} = dr\hat{\mathbf{r}} + r d\theta \hat{\theta} + r \sin\theta d\phi \hat{\phi}$

Voume element: $dV = dl_r dl_\theta dl_\phi = r^2 \sin\theta dr d\theta d\phi$

Surface element for fix *r*: $d\boldsymbol{A} = r^2 sin\theta d\theta d\phi \hat{r}$

Well, We are done with the necessary mathematical concepts!

Ok, Now in to Physics!