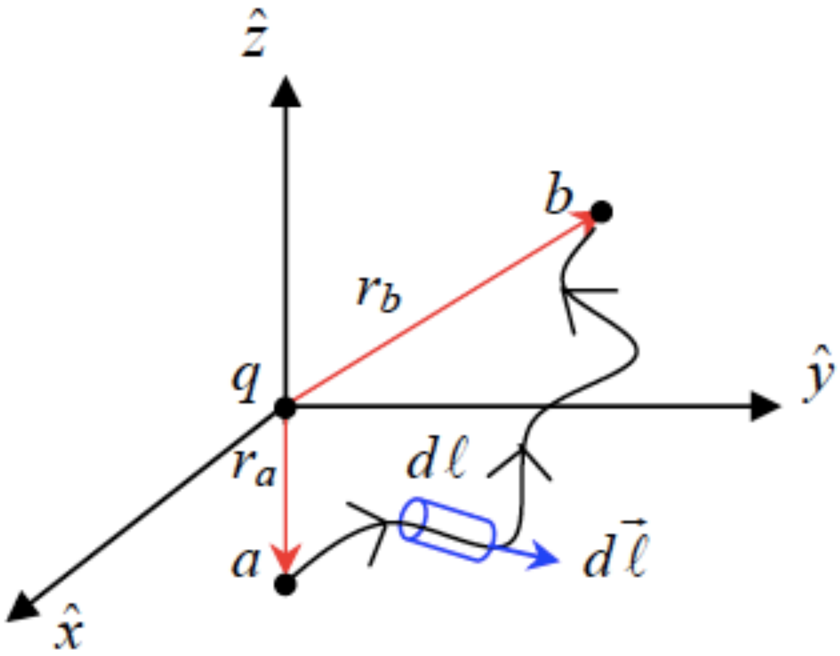
Lecture - 14



In spherical coordinates: $d\vec{\ell} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\varphi\hat{\varphi}$

$$\vec{E}(\vec{r})\bullet d\vec{\ell} = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r^2}\right)\hat{r}\bullet\left\{dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta\varphi\hat{\varphi}\right\}$$

$$\vec{E}(\vec{r})\bullet d\vec{\ell} = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r^2}\right)dr$$

$$\int_a^b \vec{E}(\vec{r})\bullet d\vec{\ell} = \frac{1}{4\pi\varepsilon_o}\int_a^b \frac{q}{r^2}dr = \frac{-1}{4\pi\varepsilon_o}\left(\frac{q}{r}\right)\Bigg|_{r_a}^{r_b} = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r_a} - \frac{q}{r_b}\right) = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

r_a = distance from origin O to point _a_. r_b = distance from origin O to point _b_.

The line integral $\int \vec{E}(\vec{r})\bullet d\vec{\ell}$ around a _closed_ contour C is zero!

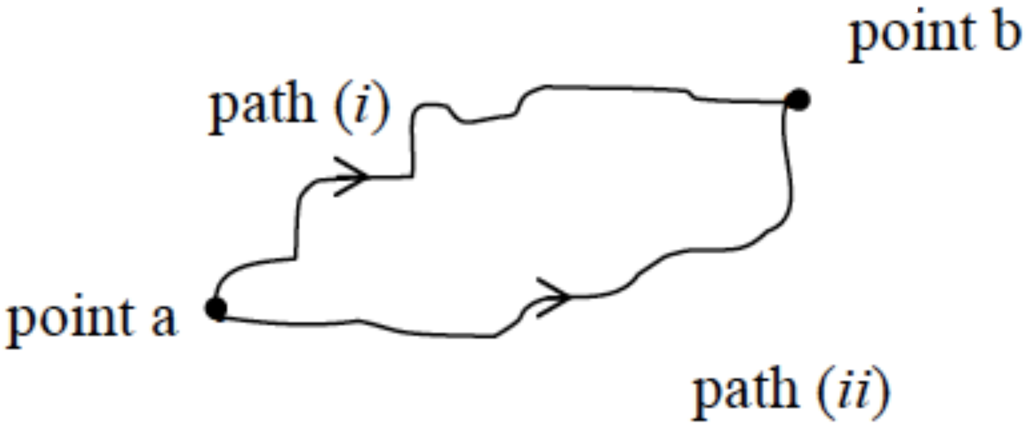
$$\boxed{\int_S \left(\vec{\nabla}\times\vec{E}(\vec{r})\right)\bullet d\vec{A} = \oint_C \vec{E}(\vec{r})\bullet d\vec{\ell} = 0}$$

arbitrary closed surface S

arbitrary closed contour C (on S)

$$\boxed{\oint_C \vec{E}(\vec{r})\bullet d\vec{\ell} = 0}$$

$$\boxed{\vec{\nabla}\times\vec{E}(\vec{r}) = 0}$$



$$\boxed{\underbrace{\int_a^b \vec{E}(\vec{r})\bullet d\vec{\ell}}_{\text{path }(i)} = \underbrace{\int_a^b \vec{E}(\vec{r})\bullet d\vec{\ell}}_{\text{path }(ii)} = \underbrace{\int_a^b \vec{E}(\vec{r})\bullet d\vec{\ell}}_{\text{any path}}}$$

because $\int_a^b \vec{E}(\vec{r})\bullet d\vec{\ell}$ is *independent* of the path taken from point $a \to b$.

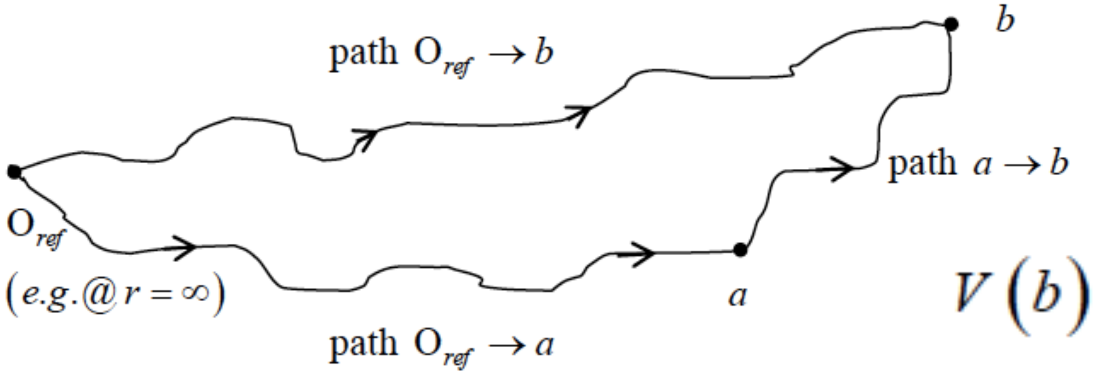
We now define a *scalar point function*, $V(\vec{r})$ known as the electric potential,

$$\boxed{V(\vec{r}) \equiv -\int_{\mathrm{O}_{ref}}^{r} \vec{E}(\vec{r})\bullet d\vec{\ell}}$$

Electric Potential (integral version)	$V(\vec{r}) \equiv -\int_{O_{ref}}^{r} \vec{E}(\vec{r}) \cdot d\vec{\ell}$

By convention, the point $r = O_{ref}$ is taken to be a *standard reference point* of electric potential, $V(\vec{r})$ where $V(\vec{r} = O_{ref}) = 0$ (usually $r = \infty$).

$V(\vec{r})$ depends only on point $\vec{r}$.



$$V(b) - V(a) = \left(-\int_{O_{ref}}^{b} \vec{E}(\vec{r}) \cdot d\vec{\ell}\right) - \left(-\int_{O_{ref}}^{a} \vec{E}(\vec{r}) \cdot d\vec{\ell}\right)$$

$$= -\int_{O_{ref}}^{b} \vec{E}(\vec{r}) \cdot d\vec{\ell} + \int_{O_{ref}}^{a} \vec{E}(\vec{r}) \cdot d\vec{\ell}$$

$$= -\int_{O_{ref}}^{b} \vec{E}(\vec{r}) \cdot d\vec{\ell} - \int_{a}^{O_{ref}} \vec{E}(\vec{r}) \cdot d\vec{\ell}$$

$$= -\int_{a}^{O_{ref}} \vec{E}(\vec{r}) \cdot d\vec{\ell} - \int_{O_{ref}}^{b} \vec{E}(\vec{r}) \cdot d\vec{\ell}$$

$$\boxed{\Delta V_{ab} \equiv V(\vec{r} = b) - V(\vec{r} = a) = -\int_{a}^{b} \vec{E}(\vec{r}) \cdot d\vec{\ell}}$$

The <u>fundamental theorem for gradients</u> states that:

Potential difference: $\Delta V_{ab} \equiv V(r=b) - V(r=a) = \int_a^b \vec{\nabla} V(\vec{r}) \bullet d\ell = -\int_a^b \vec{E}(\vec{r}) \bullet d\vec{\ell}$

This is true for <u>any</u> end-points <u>a</u> & <u>b</u> (and any contour from a → b). Thus the two *<u>integrands</u>* ***<u>must</u>*** be equ

$\vec{E}(\vec{r}) = -\vec{\nabla} V(\vec{r})$	Differential Version

It is often easier to analyze a physical situation in terms of ***potential***, which is a ***scalar***, rather than the ***electric field strength***, which is a ***vector***.

$\Rightarrow$ Knowing $V(\vec{r})$ enables you to specify/calculate $\vec{E}(\vec{r})$!!