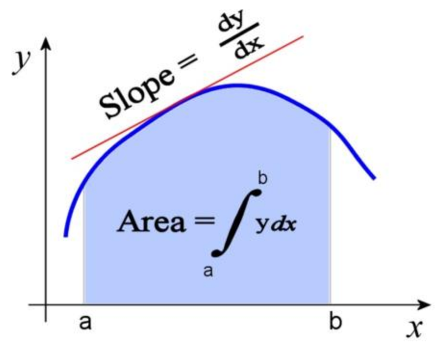
Methods of Differentiation

One Shot

Hello NVians !!



Nishant Vora

B.Tech - IIT Patna

- 7+ years Teaching experience
- Mentored 5 lac+ students
- Teaching Excellence Award

Join with us in Telegram

TELEGRAM CHANNEL

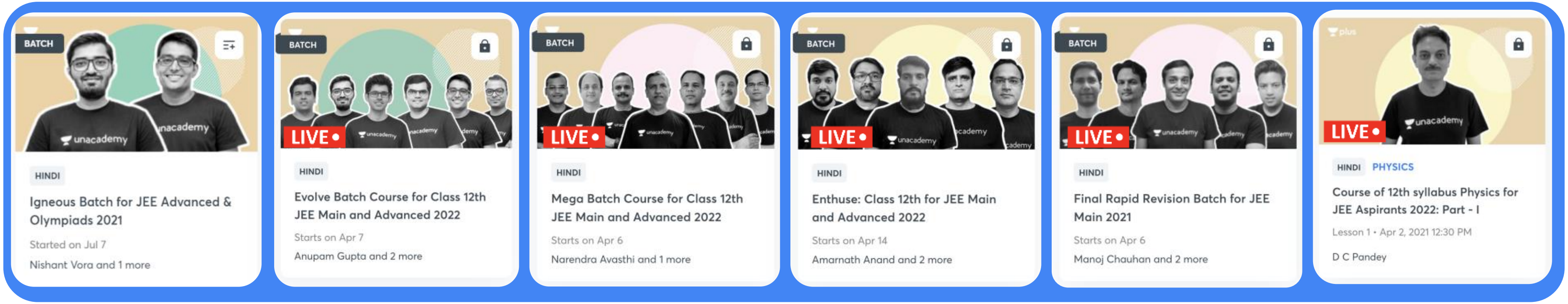
- "t.me/unacademyatoms"

COMPLETE NOTES AND LECTURES

- livedaily.me/atoms

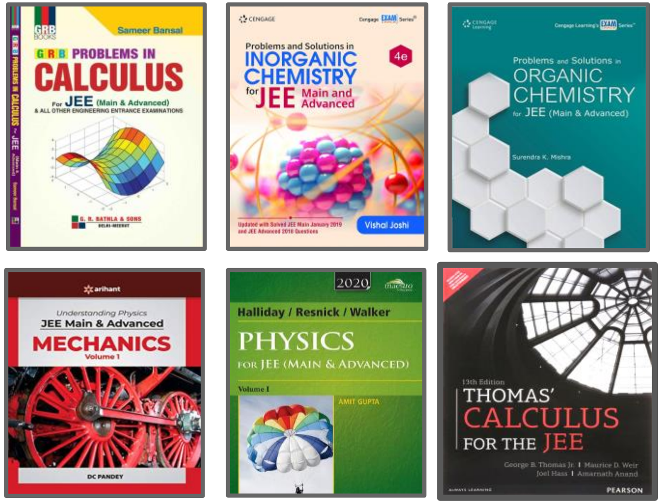
India's BEST Educators

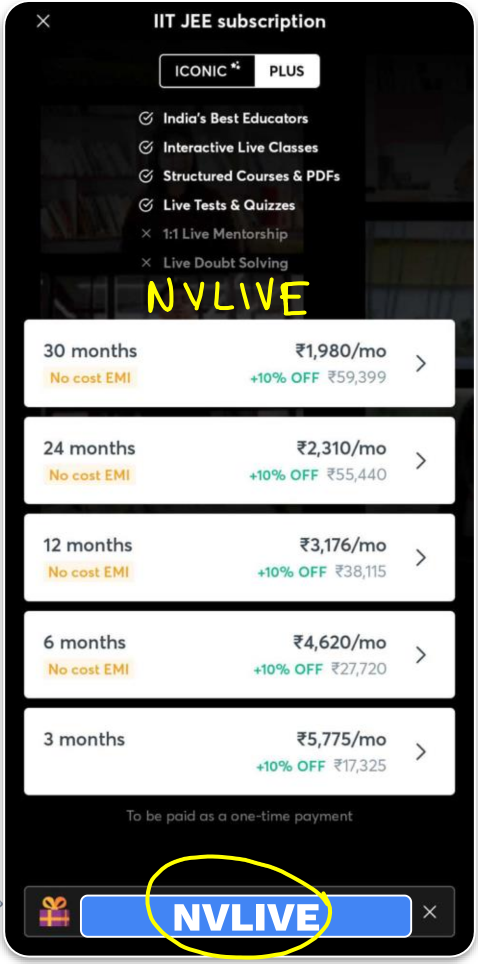
Unacademy Subscription

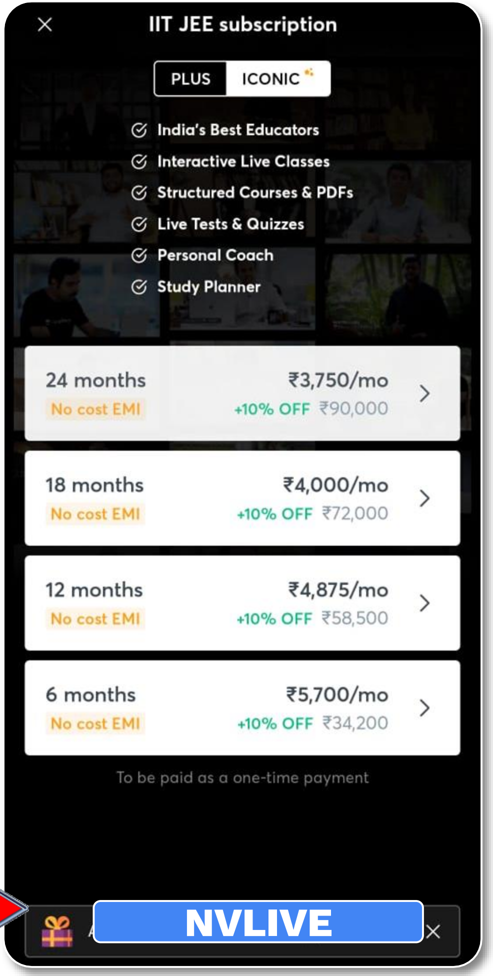


If you want to be the **BEST**
"Learn" from the **BEST**

Last min
JEE Adv
RBC 10
20
RRB




IIT JEE subscription
ICONIC
PLUS
India's Best Educators
Interactive Live Classes
Structured Courses & PDFs
Live Tests & Quizzes
1:1 Live Mentorship
Live Doubt Solving
NVLIVE
30 months ₹1,980/mo
No cost EMI +10% OFF ₹59,399
24 months ₹2,310/mo
No cost EMI +10% OFF ₹55,440
12 months ₹3,176/mo
No cost EMI +10% OFF ₹38,115
6 months ₹4,620/mo
No cost EMI +10% OFF ₹27,720
3 months ₹5,775/mo
+10% OFF ₹17,325
To be paid as a one-time payment
NVLIVE


IIT JEE subscription
PLUS
ICONIC
India's Best Educators
Interactive Live Classes
Structured Courses & PDFs
Live Tests & Quizzes
Personal Coach
Study Planner
24 months ₹3,750/mo
No cost EMI +10% OFF ₹90,000
18 months ₹4,000/mo
No cost EMI +10% OFF ₹72,000
12 months ₹4,875/mo
No cost EMI +10% OFF ₹58,500
6 months ₹5,700/mo
No cost EMI +10% OFF ₹34,200
To be paid as a one-time payment
NVLIVE

Standard Derivatives

Standard Derivatives

$$\sqrt{x} \rightarrow \frac{1}{2\sqrt{x}}$$

$$\boxed{\frac{d}{dx}(a^x) = a^x \ln a}$$

$$\boxed{\frac{d}{dx}(\ln x) = \frac{1}{x}}$$

y or f(x)	$\frac{dy}{dx}$ or f'(x)
K, K is a constant	$\boxed{\frac{d}{dx}(K) = 0}$
x^n	$\frac{d}{dx}(x^n) = \boxed{nx^{n-1}}$
$\sqrt{x}$	$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
e^x	$\frac{d}{dx}(e^x) = e^x$
a^x, $a > 0$	$\frac{d}{dx}(a^x) = \underline{a^x \log a}$

Standard Derivatives

y or f(x)	$\frac{dy}{dx}$ or f'(x)
sin x	$\frac{d}{dx}(\sin x) = \cos x$
cos x	$\frac{d}{dx}(\cos x) = -\sin x$
tan x	$\frac{d}{dx}(\tan x) = \sec^2 x$
cot x	$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
cosec x	$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$
sec x	$\frac{d}{dx}(\sec x) = \sec x \tan x$

Standard Derivatives

$$\boxed{\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}}}$$

$$\boxed{\frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}}}$$

$$\boxed{\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}}$$

$$\boxed{\frac{d}{dx}\left(\cot^{-1}x\right)=\frac{-1}{1+x^2}}$$

$$\boxed{\frac{d}{dx}\left(\sec^{-1}x\right)=\frac{1}{x\sqrt{x^2-1}}}$$

$$\boxed{\frac{d}{dx}\left(\mathrm{cosec}^{-1}x\right)=\frac{-1}{x\sqrt{x^2-1}}}$$

Algebra of Differentiation

Algebra of Differentiation

1 $$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$$

Eg. $$\frac{d}{dx}\left(x^3 + x^2 + 5\right)$$

$$= \frac{d}{dx}(x^3) + \frac{d}{dx}(x^2) + \cancel{\frac{d}{dx}(5)}$$

$$= 3x^2 + 2x^1$$

Algebra of Differentiation

eg. $$\frac{d}{dx}\left(x^3 + 2 + \ln x + \sqrt{x} + \operatorname{cosec} x\right)$$

$$3x^2 + 0 + \frac{1}{x} + \frac{1}{2\sqrt{x}} - \operatorname{cosec} x \cot x$$

Algebra of Differentiation

2. $$\frac{d}{dx}(k\,f(x)) = k\frac{d}{dx}(f\,(x))$$

Eg. $$\frac{d}{dx}\left(5x^4 + 3e^x + 2\sin x\right)$$

$$5\frac{d}{dx}(x^4) + 3\frac{d}{dx}(e^x) + 2\frac{d}{dx}(\sin x)$$

$$5(4x^3) + 3e^x + 2\cos x$$

Product Rule

Product Rule

$$\frac{d}{dx}(\overset{\mathrm{I}}{f}(x).\overset{\mathrm{II}}{g}(x)) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

$$\boxed{(\mathrm{I}\ \mathrm{II})' = \mathrm{I}\ \mathrm{II}' + \mathrm{II}\ \mathrm{I}'}$$

Eg. $\frac{d}{dx}\left(x^3 . \sin x\right)$

$$= x^3 \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(x^3)$$

$$= \underline{x^3 \cos x + \sin x\,(3x^2)}$$

Product Rule

$$\frac{d}{dx}(x^2 e^x) = x^2 e^x + e^x(2x)$$

$$\frac{d}{dx}(\underset{\mathrm{I}}{x}\,\underset{\mathrm{II}}{\ln x}) = x \times \frac{1}{x} + \ln x\,(1)$$

$$= \boxed{1 + \ln x}$$

If $(a + \sqrt{2}b \cos x)(a - \sqrt{2}b \cos y) = a^2 - b^2$, where $a > b > 0$, then dx/dy at $(\pi/4, \pi/4)$ is :

JEE Main 2020

A. $a - 2b/ a + 2b$

B. $a - b / a + b$

C. $a + b / a - b$

D. $2a + b / 2a - b$

$$\underbrace{\left(a+\sqrt{2}b\cos x\right)}_{\mathrm{I}}\underbrace{\left(a-\sqrt{2}b\cos y\right)}_{\mathrm{II}} = a^2-b^2$$

$$\left(a+\cancel{\sqrt{2}}b\cancel{\cos}x\right)\left(+\cancel{\sqrt{2}}b\cancel{\sin}y\frac{dy}{dx}\right) - \left(a-\cancel{\sqrt{2}}b\cancel{\cos}y\right)\left(\cancel{\sqrt{2}}b\cancel{\sin}x\right) = 0$$

$$x = \frac{\pi}{4} \quad y = \frac{\pi}{4}$$

$$(a+b)\left(\cancel{b}\frac{dy}{dx}\right) - (a-b)\cancel{b} = 0$$

$$\frac{dy}{dx} = \frac{a-b}{a+b} \qquad \boxed{\frac{dx}{dy} = \frac{a+b}{a-b}}$$

Extended Product Rule

Extended Product Rule

$$(\mathrm{I}\ \mathrm{II} \cdot \mathrm{III})' = \mathrm{I}'\ \mathrm{II}\ \mathrm{III} + \mathrm{I}\ \mathrm{II}'\ \mathrm{III} + \mathrm{I}\ \mathrm{II}\ \mathrm{III}'$$

If $f(x) = \underbrace{(1 + x)}_{\mathrm{I}}(3 + x^2)^{1/2} (9 + x^3)^{1/3}$ then $f'(-1)$ is equal to

$\mathrm{I} \quad \mathrm{II} \quad \mathrm{III}$

A. 0

B. $2\sqrt{2}$

C. 4

D. 6

$$f'(x) = (1)(3+x^2)^{1/2}(9+x^3)^{1/3} + \cancel{(1+x)}^{\,0}\,\mathrm{II}'\,\mathrm{III} + \cancel{(1+x)}^{\,0}\,\mathrm{II}\,\mathrm{III}'$$

$$f'(-1) = (1)(4)^{1/2}(8)^{1/3}$$

$$= 2 \times 2$$

$$= 4$$

Quotient Rule

$N \quad D$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - g'(x)f(x)}{g^2x}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$$

Chain Rule

Chain Rule # NVStyle

$$\frac{d}{dx} f(g(x)) = f'(g(x)).g'(x)$$

$\square^3 \rightarrow 3\square^2$

P **B** **A**

Power **Base** **Angle**

Eg. $$\frac{d}{dx}\left(\sin^3(2x+5)\right)$$

$$= \boxed{3\sin^2(2x+5) \times \cos(2x+5) \times 2}$$

Chain Rule

1. $\dfrac{d}{dx}\left(\tan^5(7x+2)\right)$ $= 5\tan^4(7x+2)\ \sec^2(7x+2) \times 7$

2. $\dfrac{d}{dx}\left(\cos^2(x^3)\right)$ $= 2\cos(x^3)\left(-\sin(x^3)\right)\left(3x^2\right)$

3. $\dfrac{d}{dx}\left(\ln^5(2x+3)\right)$ $= 5\ln^4(2x+3) \times \dfrac{1}{2x+3} \times 2$

If $f(1) = 1$, $f'(1) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is :

A. 33

B. 12

C. 15

D. 9

JEE Main 2019

$= f'(f(f(1)))\, f'(f(1))\, f'(1) + 2 f(1) f'(1)$

$= f'(f(1))\, f'(1)\, f'(1) + 2(1)(3)$

$= f'(1)\, f'(1)\, f'(1) + 6$

$= 3^3 + 6$

$= 33$

"NV Tip"

First Simplify then Differentiate

i. If $y = \dfrac{x^4 + x^2 + 1}{x^2 + x + 1}$ then $\dfrac{dy}{dx}$ = ax + b find a and b

Class 8th ☆☆☆

$$\boxed{x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 - x + 1)}$$

$$y = \frac{\cancel{(x^2 + x + 1)}(x^2 - x + 1)}{\cancel{(x^2 + x + 1)}}$$

$$y = x^2 - x + 1$$

$$\boxed{\frac{dy}{dx} = 2x - 1}$$

$ax + b$

$a = 2$
$b = -1$

$$x^4 + x^2 + 1$$

$$= \underbrace{x^4 + 2x^2 + 1} - x^2$$

$$= (x^2 + 1)^2 - (x)^2$$

$$= (x^2 + 1 + x)(x^2 + 1 - x)$$

If $y(x) = \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), x \in \left(\frac{\pi}{2}, \pi\right)$,

then dy/dx at x = 5π/6 is :

A. -1/2

B. -1

C. 1/2

D. 0

$x \in \left(\frac{\pi}{2}, \pi\right)$

$\boxed{\frac{x}{2} \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)}$

$$y(x) = \cot^{-1}\left(\frac{\cancel{\cos\frac{x}{2}} + \sin\frac{x}{2} + \sin\frac{x}{2} - \cancel{\cos\frac{x}{2}}}{\cos\frac{x}{2} + \cancel{\sin\frac{x}{2}} - \cancel{\sin\frac{x}{2}} + \cos\frac{x}{2}}\right)$$

$$= \cot^{-1}\left(\frac{\cancel{2}\sin\frac{x}{2}}{\cancel{2}\cos\frac{x}{2}}\right)$$

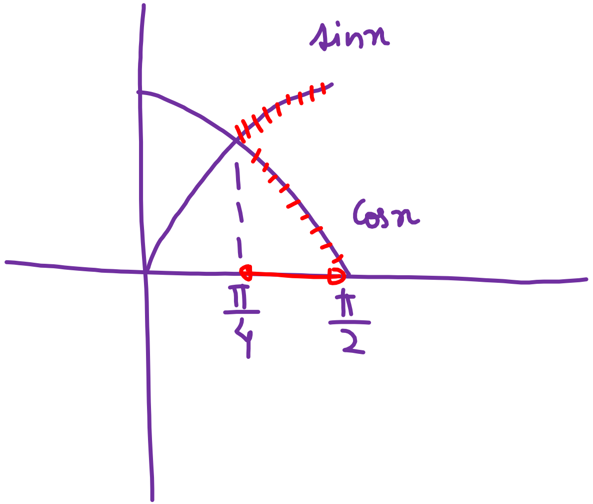
$$= \cot^{-1}\left(\tan\frac{x}{2}\right)$$

$$= \frac{\pi}{2} - \tan^{-1}\left(\tan\frac{x}{2}\right) = \frac{\pi}{2} - \frac{x}{2}$$

JEE Main 2021

$y(x) = \frac{\pi}{2} - \frac{x}{2}$

$\boxed{y'(x) = \frac{-1}{2}}$



sinx
cosx
$\frac{\pi}{4}$
$\frac{\pi}{2}$

If $f(x) = \sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ and its first derivative with respect to x is $-b/a \log_e 2$ when $x = 1$, where a and b are integers, then the minimum value of $|a^2 - b^2|$ is _

$$\boxed{f'(1) = -\frac{b}{a}\ln 2}$$

JEE Main 2021

$$f(x) = \sin\cos^{-1}\left(\frac{1-(2^x)^2}{1+(2^x)^2}\right)$$

$$\boxed{2^x = \tan\theta}$$

$$= \sin\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$= \sin\cos^{-1}\cos 2\theta$$

$$= \sin 2\theta$$

$$= \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\cdot 2^x}{1+(2^x)^2}$$

$$f(x) = \frac{2^{x+1}}{1+4^x}$$

$$f'(x) = \frac{2^{x+1}\ln 2\,(1+4^x) - (4^x \ln 4)(2^{x+1})}{(1+4^x)^2}$$

$$f'(1) = \frac{4\ln 2\,(5) - (4 \times 2\ln 2)(4)}{5^2}$$

$$f'(1) = \frac{-12\ln 2}{25} = -\frac{b}{a}\ln 2$$

$b = 12$

$a = 25$

$a^2 - b^2$

$$\begin{array}{r} 625 \\ -144 \\ \hline 481 \end{array}$$

If $y(\alpha) = \sqrt{2\left(\frac{\tan\alpha + \cot\alpha}{\boxed{1+\tan^2\alpha}}\right) + \frac{1}{\sin^2\alpha}}, \alpha \in \left(\frac{3\pi}{4}, \pi\right)$,

then $dy/d\alpha$ at $\alpha = 5\pi/6$ is :

JEE Main 2020

A. 4

B. 4/3

C. -4

D. -1/4

$\alpha > \frac{3\pi}{4}$

$\cot\alpha < \cot\frac{3\pi}{4}$

$\cot\alpha < -1$

$\cot\alpha + 1 < 0$

$$y(\alpha) = \sqrt{2\left(\frac{\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}}{\frac{1}{\cos^2\alpha}}\right) + \frac{1}{\sin^2\alpha}}$$

$$= \sqrt{2\cot\alpha + 1 + \cot^2\alpha}$$

$$= \sqrt{(\cot\alpha + 1)^2}$$

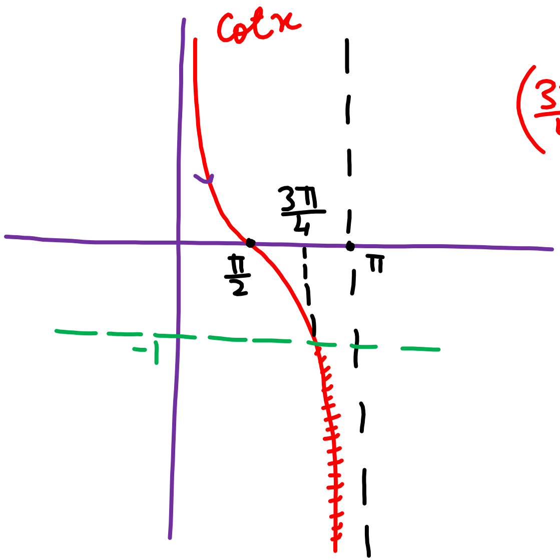
$$= |\cot\alpha + 1| = -\cot\alpha - 1$$

$$\frac{dy}{d\alpha} = \operatorname{cosec}^2\alpha$$

$$= \operatorname{cosec}^2\left(\frac{5\pi}{6}\right)$$

$$= 4$$

$\downarrow$ func



$\left(\frac{3\pi}{4}, \pi\right)$

$\longleftrightarrow$

$\cot\alpha < -1$

$\underline{1 + \cot\alpha < 0}$

Let $y = y(x)$ be a function of x satisfying $y\sqrt{1 - x^2} = k - x\sqrt{1 - y^2}$ where k is a constant and $y(1/2) = -1/4$. Then dy/dx at $x = 1/2$, is equal to :

$x \qquad y$

A. $-\sqrt{5}/4$

B. $-\sqrt{5}/2$ ✓

C. $2/\sqrt{5}$

D. $\sqrt{5}/2$ ✓

JEE Main 2020

$$y\sqrt{1-x^2} + x\sqrt{1-y^2} = K$$

$$x = \sin A \qquad y = \sin B$$

$$\sin B \cos A + \sin A \cos B = K$$

$$\sin(A+B) = K$$

$$A + B = \underbrace{\sin^{-1}(K)}$$

$$\sin^{-1}x + \sin^{-1}y = c$$

$$\sin^{-1}x + \sin^{-1}y = c$$

$$\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-\sqrt{1-y^2}}{\sqrt{1-x^2}} = \frac{-\sqrt{1-\frac{1}{16}}}{\sqrt{1-\frac{1}{4}}} = \frac{-\sqrt{15}}{4} \times \frac{2}{\sqrt{3}}$$

$$= -\frac{\sqrt{5}}{2}$$

Logarithmic Differentiation

Logarithmic Differentiation

$$y = f_1(x)f_2(x)f_3(x) \;.....$$

OR

$$y = (f(x))^{g(x)}$$

x^x

$x^{\ln x}$

$x^{\sin x}$

$f(0) = 1 \times 2 \times 3 \times \ldots n = n!$

If $\boxed{f(x) = (x + 1)(x + 2)(x + 3) \ldots\ldots (x + n)}$ then $f'(0)$ is

$\ln f(x) = \ln(x+1) + \ln(x+2) + \ldots + \ln(x+n)$

A. $n!$

$\frac{f'(x)}{f(x)} = \frac{1}{x+1} + \frac{1}{x+2} + \ldots + \frac{1}{x+n}$

B. $\frac{n(n+1)}{2}$

C. $(n!)(\ln n!)$

$\Rightarrow f'(x) = f(x)\left(\frac{1}{x+1} + \frac{1}{x+2} + \ldots + \frac{1}{x+n}\right)$

D. $n!\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{n}\right)$

$f'(0) = \underbrace{f(0)}_{n!}\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}\right)$

Find $\frac{dy}{dx}$ for $y = x^x$.

$$y = x^x$$

$$\ln y = \ln x^{x}$$

$$\ln y = \underline{x}\ \underline{\ln x}$$

$$\frac{1}{y} y' = x \times \frac{1}{x} + \ln x \times 1$$

$$y' = y(1 + \ln x)$$

$$\boxed{y' = x^x(1 + \ln x)}$$

if $y = 2^{\log_2 x^{2x}} + \left(\tan\frac{\pi x}{4}\right)^{\frac{4}{\pi x}}$ then $\left.\frac{dy}{dx}\right]_{x=1}$ is

A. 4

B. 5/2

C. 3

D. Not defined

$$y = \underbrace{x^{2x}}_{u} + \underbrace{\left(\tan\frac{\pi x}{4}\right)^{\frac{4}{\pi x}}}_{v}$$

$$y = u + v$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \left.\boxed{\frac{du}{dx}}\right|_{x=1} + \left.\boxed{\frac{dv}{dx}}\right|_{x=1}$$

$$= 2 + 2$$

$$= 4$$

$$u = x^{2x}$$

$$\ln u = 2(x \ln x)$$

$$\frac{1}{u}\frac{du}{dx} = 2(1+\ln x)$$

$$\boxed{\frac{du}{dx} = 2\boxed{x^{2x}}(1+\ln x)}$$

$$\left.\frac{du}{dx}\right|_{x=1} = 2\times 1\times 1$$

$$= 2$$

$$v = \left(\tan\frac{\pi x}{4}\right)^{\frac{4}{\pi x}}$$

$$\ln v = \frac{4}{\pi x}\ln\left(\tan\left(\frac{\pi x}{4}\right)\right)$$

$$\frac{1}{v}\frac{dv}{dx} = \frac{4}{\pi}\left\{\frac{1}{x}\,\frac{\sec^2\left(\frac{\pi x}{4}\right)}{\tan\left(\frac{\pi x}{4}\right)}\times\frac{\pi}{4} + \underbrace{\ln\left(\tan\frac{\pi x}{4}\right)}\left(\frac{-1}{x^2}\right)\right\}$$

$$\frac{dv}{dx} = \frac{4}{\pi}\boxed{\left(\tan\frac{\pi x}{4}\right)^{\frac{4}{\pi x}}}\left\{\frac{1}{1}\,\frac{\sec^2\frac{\pi}{4}}{\tan\frac{\pi}{4}}\times\frac{\pi}{4} + 0\right\}$$

$$= \cancel{\frac{4}{\pi}}\times 1\,\frac{2}{1}\times\cancel{\frac{\pi}{4}}$$

$$= 2$$

$y = x^3 + 2\sin x + 1$

$y = f(x)$

Derivative of Implicit Functions

Implicit functions

Example

i. $\underbrace{x^2 + 2xy + y^3}$ $= 4$

$\boxed{y \neq f(x)}$

ii. $x + y + \sin(xy) = 1$

If $x^2 + 2xy + y^3 = 4$, find $\dfrac{dy}{dx}$

f

$2 x K$

$2k(1)$

$\times \quad 2x + 2(xy' + y) + 3y^2 y' = 0$

$\times \quad 2x + \underline{\underline{2xy'}} + 2y + \underline{\underline{3y^2 y'}} = 0$

$\checkmark \quad \boxed{y' = \dfrac{-(2x+2y)}{(2x+3y^2)}}$

NVstyle

$$\frac{dy}{dx} = \frac{-\dfrac{\partial f}{\partial x}}{\dfrac{\partial f}{\partial y}} = \frac{-(2x + 2y(1) + 0 + 0)}{(0 + 2x(1) + 3y^2 + 0)}$$

If $y^5 + xy^2 + x^3 = 4x + 3$, then find $\frac{dy}{dx}$ at $(2, 1)$

$$f \equiv y^5 + xy^2 + x^3 - 4x - 3$$

$$\frac{dy}{dx} = \frac{-\left.\frac{\partial f}{\partial x}\right|_{y=k}}{\left.\frac{\partial f}{\partial y}\right|_{x=k}} = \frac{-\left(0 + y^2 + 3x^2 - 4\right)}{\left(5y^4 + x(2y)\right)}$$

$$= \frac{-\left(1 + 12 - 4\right)}{(5 + 4)} = -1$$

If $y = \sqrt{\sin x + \boxed{\sqrt{\sin x + \sqrt{\sin x + \cdots \infty}}}}$ find $\frac{dy}{dx}$

$$y = \sqrt{\sin x + y}$$

$$y^2 = \sin x + y$$

$$2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{\cos x}{2y-1}}$$

If $ax^2 + 2hxy + by^2 = 0$ then prove that $\dfrac{dy}{dx} = \dfrac{ax + hy}{hx + by} = \dfrac{y}{x}$

NVStyle

$$2x^3 + 55x^2y^1 + 2022xy^2 + y^3 = 0$$

$$\boxed{\frac{dy}{dx} = \frac{y}{x}}$$

If $y = y(x)$ is an implicit function of x such that $\boxed{\log_e (x + y) = 4xy}$, then d^2y/dx^2 at $x = 0$ is equal to.

JEE Main 2021

$x=0 \quad y=1$

$$x + y = e^{4xy} \quad \text{—— (1)}$$

$1 + y' = 1 \times 4 \times 1$

$\boxed{y' = 3}$

$$1 + y' = \underset{\mathrm{I}}{e^{4xy}} \times 4\,\underset{\mathrm{II}}{\underline{(xy' + y)}} \quad \text{—— (2)}$$

$\underbrace{x=0 \quad y=1 \quad y'=3}$

$$y'' = 4\left\{ e^{4xy}\left(\cancel{x}y'' + y' + y'\right) + \left(\cancel{x}y' + y\right)^2 e^{4xy} \times 4 \right\}$$

$$y'' = 4\{1 \times (6) + 1 \times 1 \times 4\}$$

$$= \boxed{40}$$

If $x \log_e(\log_e x) - x^2 + y^2 = 4$ $(y > 0)$, then dy/dx at $x = e$ is equal to :

$x = e \quad -e^2 + y^2 = 4 \Rightarrow y^2 = 4 + e^2 \Rightarrow \boxed{y = \sqrt{4+e^2}}$

A. $\dfrac{(1+2e)}{2\sqrt{4+e^2}}$

B. $\dfrac{(2e-1)}{2\sqrt{4+e^2}}$

C. $\dfrac{(1+2e)}{\sqrt{4+e^2}}$

D. $\dfrac{e}{\sqrt{4+e^2}}$

$x\left(\dfrac{1}{\ln x} \times \dfrac{1}{x}\right) + \ln(\ln x) \times 1 - 2x + 2yy' = 0$

$\cancel{e}\left(\dfrac{1}{1} \times \dfrac{1}{\cancel{e}}\right) + 0 - 2(e) + 2\sqrt{4+e^2}\,y' = 0$

$1 - 2e + 2\sqrt{4+e^2}\,y' = 0$

$y' = \dfrac{2e-1}{2\sqrt{4+e^2}}$

JEE Main 2019

For $x > 1$, if $(2x)^{2y} = 4e^{\boxed{2x-2y}}$, then $(1 + \log_e 2x)^2 \, dy/dx$ is equal to :

JEE Main 2019

SKIP

A. $\dfrac{x\log_e 2x - \log_e 2}{x}$

B. $\log_e 2x$

C. $\dfrac{x\log_e 2x + \log_e 2}{x}$

D. $x\log_e 2x$

$$(2x)^{2y} = 2^2 e^{2x-2y}$$

$$\boxed{2y\ln(2x) = 2\ln 2 + 2x - 2y}$$

$$\cancel{2}\left(y\left(\frac{\cancel{2}}{\cancel{2}x}\right) + \ln(2x)\,y'\right) - \cancel{2} + \cancel{2}y' = 0$$

$$\frac{y}{x} + y'(\ln 2x) - 1 + y' = 0$$

$$\cancel{2}y(\ln 2x + 1) = \cancel{2}\ln 2 + \cancel{2}x$$

$$\boxed{y = \frac{\ln 2 + x}{\ln 2x + 1}}$$

$$y'(\ln 2x+1) = 1 - \frac{1}{x}\left(\frac{\ln 2 + x}{\ln 2x + 1}\right)$$

$$y'(\ln 2x+1)^2 = \frac{x(\ln 2x + \cancel{1}) - \ln 2 - \cancel{x}}{x}$$

$$= \frac{x\ln 2x - \ln 2}{x}$$

If $e^y + \cancel{x}y = e$, the ordered pair $(dy/dx, d^2y/dx^2)$ at $x = 0$ is equal to :

$x=0 \quad y=1$

JEE Main 2019

A. $\left(\frac{1}{e}, -\frac{1}{e^2}\right)$

B. $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$

C. $\left(\frac{1}{e}, \frac{1}{e^2}\right)$

D. $\left(-\frac{1}{e}, -\frac{1}{e^2}\right)$

$$e^y y' + x y' + y = 0$$

$$e^1 y' + 0 + 1 = 0 \Longrightarrow \boxed{y' = \frac{-1}{e}}$$

$$e^y y'' + (y')^2 e^y + \cancel{x} y'' + y' + y' = 0$$

$$e^1 y'' + \frac{1}{e^2} \times e^1 + 2\left(\frac{-1}{e}\right) = 0$$

$$e y'' + \frac{1}{e} - \frac{2}{e} = 0$$

$$\boxed{y'' = \frac{1}{e^2}}$$

Derivatives Of Inverse Function

Derivative of inverse function

$f(x)$ = given

$$\boxed{g(x) = f^{-1}(x)}$$

$$\frac{d}{dx}(g(x)) = \frac{d}{dx}\left(f^{-1}(x)\right)$$

Property

$$g(f(x)) = f(g(x)) = x$$

$$g(f(x)) = x \qquad f(g(x)) = x$$

$$g'(f(x))\, f'(x) = 1 \qquad f'(g(x))\, g'(x) = 1$$

$$\boxed{g'(f(x)) = \frac{1}{f'(x)}} \qquad \boxed{f'(g(x)) = \frac{1}{g'(x)}}$$

If $f(x) = x^3 + x^5$ and g is the inverse of f find $g'(2)$

$f(1) = 1+1 = 2$

$$g'(f(x)) = \frac{1}{f'(x)}$$

Put $x=1$

$$g'(f(1)) = \frac{1}{f'(1)}$$

$$\boxed{g'(2) = \frac{1}{8}}$$

$f(x) = x^3 + x^5$

$f'(x) = 3x^2 + 5x^4$

$f'(1) = 3+5$

$= 8$

Let f(x) = exp($x^3 + x^2 + x$) for any real number x, and let g be the inverse for f. The value of g'(e^3) is

A. $\frac{1}{6e^3}$

B. $\frac{1}{6}$

C. $\frac{1}{34e^{39}}$

D. 6

$f(x) = e^{x^3+x^2+x}$

$g'(e^3) = ?$

$g'(f(x)) = \frac{1}{f'(x)}$

$\boxed{X = 1}$

$g'(f(1)) = \frac{1}{f'(1)}$

$$\boxed{g'(e^3) = \frac{1}{6e^3}}$$

$f(x) = e^{x^3+x^2+x}$

$f'(x) = e^{x^3+x^2+x} \times (3x^2+2x+1)$

$f'(1) = e^3 \times 6$

$= \boxed{6e^3}$

Let f and g be differentiable functions on R such that fog is the identity function. If for some a, b ∈ R, g’ (a) = 5 and g (a) = b, then f’ (b) is equal to :

A. 1/5

B. 1

C. 5

D. 2 / 5

JEE Main 2020

$f(g(x)) = x$

$f'(g(x)) = \frac{1}{g'(x)}$

$x = a$

$f'(g(a)) = \frac{1}{g'(a)}$

$\boxed{f'(b) = \frac{1}{5}}$

If the function $\boxed{f(x) = x^3 + e^{\frac{x}{2}}}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is $f(0) = 0 + e^0 = 1$

(JEE Adv. 2009)

$$g'(f(x)) = \frac{1}{f'(x)}$$

Put $x = 0$

$$g'(f(0)) = \frac{1}{f'(0)}$$

$$g'(1) = \frac{1}{(1/2)} = 2$$

$$f'(x) = 3x^2 + \frac{1}{2}e^{x/2}$$

$$f'(0) = 0 + \frac{1}{2}e^0$$

$$= 1/2$$

Let $f : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{R}$ be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in \mathbb{R}$. Then

(JEE Adv. 2016)

(a) $\boxed{g'(2) = \frac{1}{15}}$ (b) $h'(1) = 666$

(c) $h(0) = 16$ (d) $h(g(3)) = 36$

$\boxed{f'(x) = 3x^2 + 3}$

$f'(0) = 3$

$f'(1) = 6$

$f'(6) = 3(6)^2 + 3$

$= 111$

$g(f(x)) = x$

$g'(f(0)) = \frac{1}{f'(0)}$

$$\boxed{g'(2) = \frac{1}{3}}$$

$$h(g(g(x))) = x \qquad g(f(x)) = x$$

$$x \to f(x)$$

$$h(g(\underbrace{g(f(x))}))) = f(x)$$

$$\boxed{h(g(x)) = f(x)}$$

$$x \to f(x)$$

$$h(g(\underbrace{f(x)})) = f(f(x))$$

$$\boxed{h(x) = f(f(x))}$$

$$h(0) = f(f(0))$$

$$= f(2)$$

$$= 2^3 + 3(2) + 2$$

$$= 16$$

(b) $h'(x) = f'(f(x))\, f'(x)$

$$h'(1) = f'(f(1))\, f'(1)$$

$$= f'(6)\, f'(1)$$

$$= 111 \times 6$$

$$= \underline{666}$$

$$h(g(x)) = f(x)$$

$$h(g(3)) = f(3) = 3^3 + 3(3) + 2$$

$$= 27 + 11$$

$$= 38$$

Deducing new identities by using Differentiation

Deduction of new identities by differentiative a given identity

\# If $\boxed{\cos\frac{x}{2}\,.\cos\frac{x}{2^2}\,.\cos\frac{x}{2^3}\;\ldots.\cos\frac{x}{2^n}=\frac{\sin x}{2^n\sin\left(\frac{x}{2^n}\right)}}$, then prove that

$$\sum_{r=1}^{n}\frac{1}{2^r}\tan\frac{x}{2^r}=\frac{1}{2^n}\cot\frac{x}{2^n}-\cot x$$

$$\ln\cos\frac{x}{2}+\ln\cos\left(\frac{x}{2^2}\right)+\cdots+\ln\cos\left(\frac{x}{2^n}\right)=\ln\sin x-\cancel{\ln(2^n)}-\ln\sin\left(\frac{x}{2^n}\right)$$

$$-\frac{1}{2}\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}-\frac{1}{2^2}\frac{\sin\left(\frac{x}{2^2}\right)}{\cos\left(\frac{x}{2^2}\right)}+\cdots-\frac{1}{2^n}\frac{\sin\left(\frac{x}{2^n}\right)}{\cos\left(\frac{x}{2^n}\right)}=\frac{\cos x}{\sin x}-\frac{1}{2^n}\frac{\cos\left(\frac{x}{2^n}\right)}{\sin\left(\frac{x}{2^n}\right)}$$

$$-\left\{\frac{1}{2^1}\tan\frac{x}{2}+\frac{1}{2^2}\tan\frac{x}{2^2}+\cdots+\frac{1}{2^n}\tan\left(\frac{x}{2^n}\right)\right\}=\cot x-\frac{1}{2^n}\cot\left(\frac{x}{2^n}\right)$$

**

$$\boxed{\sum_{r=1}^{n}\frac{1}{2^r}\tan\left(\frac{x}{2^r}\right)=\frac{1}{2^n}\cot\left(\frac{x}{2^n}\right)-\cot x}$$

Standard Substitution

Some standard substitutions

Expressions	Substitution
$\sqrt{a^2 - x^2}$	x = a sin θ or a cos θ
$\sqrt{a^2 + x^2}$	x = a tan θ or a cot θ
$\sqrt{x^2 - a^2}$	x = a sec θ or a cosec θ
$\sqrt{\frac{a+x}{a-x}}$ or $\sqrt{\frac{a-x}{a+x}}$	x = a cos θ or $\boxed{\text{a cos 2θ}}$

RooT

Hatao!

$$\sqrt{\frac{\cancel{a}+\cancel{a}\cos 2\theta}{\cancel{a}-\cancel{a}\cos 2\theta}} = \sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}}$$

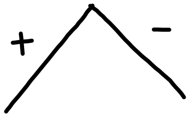
$$= \sqrt{\frac{\cancel{2}\cos^2\theta}{\cancel{2}\sin^2\theta}}$$

$$= \cot\theta$$

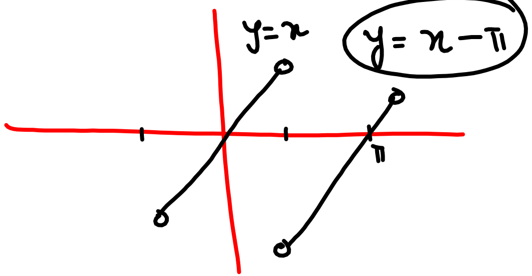
Find $\frac{dy}{dx}$ for $y = \tan^{-1}\left\{\frac{1-\cos x}{\sin x}\right\}$, where $-\pi < x < \pi$.

$$y = \tan^{-1}\left\{\frac{\cancel{2}\sin^{\cancel{2}}\frac{x}{2}}{\cancel{2}\,\cancel{\sin\frac{x}{2}}\cos\frac{x}{2}}\right\}$$

$$= \tan^{-1}\left(\tan\boxed{\frac{x}{2}}\right)$$



$$-\frac{\pi}{2} < \frac{x}{2} < \frac{\pi}{2}$$



$$y = \boxed{\frac{x}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}$$

$y = \tan^{-1}\left(\frac{\sqrt{\boxed{1+x^2}}-1}{x}\right)$ Find $\frac{dy}{dx}$ at x = 1

A. 1/2

B. √3/2

C. 1

D. 1/4

M C M

$\boxed{x = \tan\boxed{\theta}}$

$y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$= \tan^{-1}\left(\frac{\cancel{2}\sin^2\frac{\theta}{2}}{\cancel{2}\cancel{\sin\frac{\theta}{2}}\cos\frac{\theta}{2}}\right)$

$\boxed{y = \frac{\theta}{2}}$

M-1 $y = \frac{1}{2}\tan^{-1}x$

$\left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{2}\times\frac{1}{1+x^2} = \frac{1}{4}$

M-2 $\frac{dy}{\cancel{d\theta}} = \frac{1}{2}$

$\frac{dx}{\cancel{d\theta}} = \sec^2\theta$

$\frac{dy}{dx} = \frac{1}{2(\sec^2\theta)}$

$= \frac{1}{2(1+x^2)}$

If $f(x) = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$, find $f'(0)$.

A. ln2

B. ln4

C. -ln2

D. -ln4

$$f(x) = \sin^{-1}\left(\frac{2 \cdot 2^x}{1+(2^x)^2}\right)$$

$$= \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= 2\theta$$

$$\boxed{f(x) = 2\tan^{-1}(2^x)}$$

$$\boxed{2^x = \tan\theta}$$

$$\tan^{-1}(2^x) = \theta$$

$$f'(x) = 2\,\frac{1}{1+(2^x)^2} \times 2^x \ln 2$$

$$f'(0) = \cancel{2} \times \frac{1}{\cancel{2}} \times 1 \ln 2$$

$$= \ln 2$$

$y \neq f(x)$

$x = f(t)$
$y = f(t)$

Parametric Differentiation

Parametric differentiation

MCM

$x = at^2 \quad y = 2at, \quad \frac{dy}{dx} = ?$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{2a}{2at} = \frac{1}{t}$$

$\frac{d^2y}{dx^2} \neq \frac{-1}{t^2}$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \boxed{\frac{d}{dt}\left(\frac{1}{t}\right)}\frac{dt}{dx}$$

(# chalaki)

$$= \frac{-1}{t^2} \times \frac{1}{2at} = \frac{-1}{2at^3}$$

Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$.

A. $\cot\left(\frac{\theta}{2}\right)$

B. $\tan\left(\frac{\theta}{2}\right)$

C. $\sin\left(\frac{\theta}{2}\right)$

D. $\cos\left(\frac{\theta}{2}\right)$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{+\cancel{a}(\sin\theta)}{\cancel{a}(1-\cos\theta)} = \frac{\cancel{2}\,\cancel{\sin\frac{\theta}{2}}\cos\frac{\theta}{2}}{\cancel{2}\sin^{\cancel{2}}\frac{\theta}{2}}$$

$$= \cot\left(\frac{\theta}{2}\right)$$

If $x = 3\tan t$ and $y = 3\sec t$, then the value of d^2y/dx^2 at $t = \pi/4$, is :

A. $1 / 3\sqrt{2}$

B. $1 / 6\sqrt{2}$

C. $3 / 2\sqrt{2}$

D. $1 / 6$

JEE Main 2019

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3\sec t\tan t}{3\sec^2 t} = \sin t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \boxed{\frac{d}{dt}(\sin t)}\,\frac{dt}{dx}$$

$$= (\cos t) \times \frac{1}{3\sec^2 t}$$

$$= \frac{\cos^3 t}{3} = \frac{\left(\frac{1}{\sqrt{2}}\right)^3}{3} = \frac{1}{6\sqrt{2}}$$

If $x = 2\sin\theta - \sin 2\theta$ and $y = 2\cos\theta - \cos 2\theta$, $\theta \in [0, 2\pi]$, then d^2y/dx^2 at $\theta = \pi$ is :

JEE Main 2020

A. 3/4

B. 3/8

C. 3/2

D. -3/4

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{-\cancel{2}\sin\theta + \cancel{2}\sin 2\theta}{\cancel{2}\cos\theta - \cancel{2}\cos 2\theta}$$

$$= \frac{\sin 2\theta - \sin\theta}{\cos\theta - \cos 2\theta}$$

$$= \frac{\cancel{2}\cos\left(\frac{3\theta}{2}\right)\cancel{\sin\left(\frac{\theta}{2}\right)}}{+\cancel{2}\sin\left(\frac{3\theta}{2}\right)\cancel{\sin\left(\frac{+\theta}{2}\right)}}$$

$$= \cot\left(\frac{3\theta}{2}\right)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\cot\frac{3\theta}{2}\right)$$

$$= \frac{d\left(\cot\frac{3\theta}{2}\right)}{d\theta} \quad \frac{d\theta}{dx}$$

$$\left.\frac{d^2y}{dx^2}\right|_{\theta=\pi} = -\frac{3}{2}\underbrace{\operatorname{cosec}^2\left(\frac{3\theta}{2}\right)} \times \frac{1}{2\cos\theta - 2\cos 2\theta}$$

$$= \frac{-3}{2} \times 1 \times \frac{1}{2(-1) - 2(1)}$$

$$= \frac{3}{8}$$

Differentiation of one function w.r.t other

Differentiation of one function w.r.t. other function

$$\frac{dy}{dx}$$

$$\frac{d\,f(x)}{d\,g(x)} = \frac{\frac{d(f(x))}{dx}}{\frac{d(g(x))}{dx}} = \frac{f'(x)}{g'(x)}$$

$\ln x$ w.r.t $\sin x \Rightarrow \dfrac{\frac{1}{x}}{\cos x}$

Differentiate log sin x w.r.t $\sqrt{\cos x}$

$$\Rightarrow \frac{\frac{1}{\sin x} \times \cos x}{\frac{1}{2\sqrt{\cos x}} \times -\sin x}$$

The derivative of $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\dfrac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=\dfrac{1}{2}$ is :

A. 2√3 / 5

B. √3 / 12

C. 2√3 / 3

D. √3 / 10

JEE Main 2020

$x = \tan\theta$

$\Rightarrow \tan^{-1}\left(\dfrac{\sec\theta - 1}{\tan\theta}\right)$

$\Rightarrow \tan^{-1}\left(\dfrac{1-\cos\theta}{\sin\theta}\right)$

$\Rightarrow \tan^{-1}\left(\tan\dfrac{\theta}{2}\right)$

$\Rightarrow \dfrac{\theta}{2}$

$\Rightarrow \boxed{\dfrac{\tan^{-1}x}{2}}$

$x = \sin\theta$

$\Rightarrow \tan^{-1}\left(\dfrac{2\sin\theta\cos\theta}{\cos 2\theta}\right)$

$\Rightarrow \tan^{-1}(\tan 2\theta)$

$\Rightarrow 2\theta$

$\Rightarrow \underline{2\sin^{-1}x}$

$$\left.\frac{\frac{1}{2}\times\frac{1}{1+x^2}}{2\times\frac{1}{\sqrt{1-x^2}}}\right|_{x=1/2}$$

$$\frac{1}{4}\times\frac{\sqrt{1-\frac{1}{4}}}{1+\frac{1}{4}}$$

$$\frac{1}{\cancel{4}}\times\frac{\sqrt{3}}{2}\times\frac{\cancel{4}}{5} = \frac{\sqrt{3}}{10}$$

The derivative of $\tan^{-1}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$, with respect to x/2, where $\left(x \in \left(0, \frac{\pi}{2}\right)\right)$ is :

JEE Main 2019

A. 1

B. 2/3

C. 1/2

D. 2

$\Rightarrow \tan^{-1}\left(\frac{\tan x - 1}{\tan x + 1}\right)$

$\Rightarrow \tan^{-1}\left(\tan\left(x - \frac{\pi}{4}\right)\right)$

$\Rightarrow x - \frac{\pi}{4}$

$\Rightarrow \frac{x}{2}$

$\frac{1}{\left(\frac{1}{2}\right)} = 2$

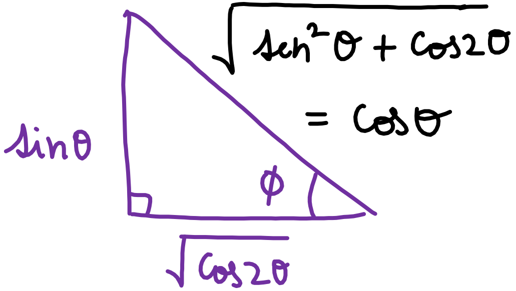
Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$.

$\sqrt{\sin^2\theta + 1 - 2\sin^2\theta}$

$= \sqrt{1-\sin^2\theta} = \cos\theta$

Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is

(JEE Adv. 2011)



$\underbrace{f(\theta)}$ wrt $\underbrace{\tan\theta}$

$f(\theta) = \sin\sin^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$

$= \tan\theta$

1

$$y' = \frac{dy}{dx} = y_1 = D_1(y) = f'(x)$$

$$y'' = \frac{d^2y}{dx^2} = y_2 = D_2(y) = f''(x)$$

Successive Differentiation

Successive Differentiation

Successive Differentiation

1. $\boxed{\dfrac{dx}{dy} = \dfrac{1}{\left(\dfrac{dy}{dx}\right)}}$

$\dfrac{dx}{dy} = \dfrac{1}{\left(\dfrac{dy}{dx}\right)}$

$\dfrac{\Delta y}{\Delta x} = \dfrac{1}{\left(\dfrac{\Delta x}{\Delta y}\right)}$

$\boxed{\dfrac{d^2x}{dy^2} = \dfrac{d}{dy}\left(\dfrac{dx}{dy}\right)}$

2. $\boxed{\dfrac{d^2x}{dy^2} \neq \dfrac{1}{\dfrac{d^2y}{dx^2}}}$

Successive Differentiation

3. $$\boxed{\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)}$$

4. $$\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right)$$

$\dfrac{d^2x}{dy^2}$ equals:

$y = f(x) \qquad y' = f'(x)$

(JEE Adv. 2007)

(JEE Main)

A. $-\left(\dfrac{d^2y}{dx^2}\right)^{-1}\left(\dfrac{dy}{dx}\right)^{-3}$

B. $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-2}$

C. $-\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-3}$

D. $\left(\dfrac{d^2y}{dx^2}\right)^{-1}$

$$\frac{d^2x}{dy^2}$$

$$= \frac{d}{dy}\left(\frac{dx}{dy}\right)$$

$$= \boxed{\frac{d}{dx}\left(\frac{dy}{dx}\right)^{-1}}\ \frac{dx}{dy}$$

$$= -1\left(\frac{dy}{dx}\right)^{-2} \times \frac{d^2y}{dx^2} \times \left(\frac{dy}{dx}\right)^{-1}$$

$$= -\left(\frac{dy}{dx}\right)^{-3}\left(\frac{d^2y}{dx^2}\right)$$

If $y = \sin(\sin x)$ then prove that $\underset{y''}{y_2} + (\tan x)\underset{y'}{y_1} + y \cos^2 x = 0$

$$y_1 = \boxed{\cos(\sin x)}\ \cos x$$

$$y_2 = \boxed{\cos(\sin x)}\,(-\sin x) + (\cos x)^2\left(-\sin(\sin x)\right)$$

$$y_2 = \left(\frac{y_1}{\cos x}\right)(-\sin x) + \cos^2 x\,(-y)$$

$$\boxed{y_2 + \tan x\, y_1 + \cos^2 x\, y = 0}$$

If $y^2 + 2\log_e(\cos x) = y$, $x \in (-\pi/2, \pi/2)$, then :

$x = 0 \quad y = 0, 1$

A. $y''(0) = 0$

B. $|y'(0)| + |y''(0)| = 1$

C. $|y''(0)| = 2$

D. $|y'(0)| + |y''(0)| = 3$

JEE Main 2020

$y^2 + 0 = y$

$y^2 - y = 0$

$\boxed{y = 0, 1}$

$$\boxed{2yy' + 2\frac{(-\sin x)}{\cos x} = y'}$$

$x = 0$

$2yy' = y'$

$(2y-1)y' = 0$

$\neq 0$

$\boxed{y' = 0}$

$$2\left(yy'' + (y')^2\right) - 2\sec^2 x = y''$$

$$2(yy'' + 0) - 2 = y''$$

$$\boxed{y'' = \frac{2}{2y-1}}$$

$y = 0 \rightarrow -2$

$y = 1 \rightarrow +2$

$$\boxed{|y''| = 2}$$

If $f : S \to S$ where $S = (0, \infty)$ be a twice differentiable function such that $f(x + 1) = xf(x)$ If $g : S \to R$ be defined as $g(x) = \log_e f(x)$, then the value of $|g''(5) - g''(1)|$ is equal to :

A. 205/144

B. 197/144

C. 187/144

D. 1

JEE Main 2021

Let $g(x) = \log f(x)$ where $f(x)$ is twice differentible positive function on $(0, \infty)$ such that $f(x+1) = x f(x)$. Then, for $N = 1, 2, 3, \ldots\ldots\ldots$

★★★★ MOD

$$g''\left(N+\frac{1}{2}\right) - g''\left(\frac{1}{2}\right) =$$

(JEE Adv. 2008)

(a) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots..+\frac{1}{(2N-1)^2}\right\}$

(b) $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots..+\frac{1}{(2N-1)^2}\right\}$

(c) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots..+\frac{1}{(2N+1)^2}\right\}$

(d) $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots..+\frac{1}{(2N+1)^2}\right\}$

$$\frac{f(x+1)}{f(x)} = x$$

$$\ln f(x+1) - \ln f(x) = \ln x$$

$$g(x+1) - g(x) = \ln x$$

$$g'(x+1) - g'(x) = \frac{1}{x}$$

$$g''(x+1) - g''(x) = \frac{-1}{x^2}$$

$$\boxed{x \to x - \frac{1}{2}}$$

$$g''\left(x-\frac{1}{2}+1\right) - g''\left(x-\frac{1}{2}\right) = \frac{-1}{\left(x-\frac{1}{2}\right)^2}$$

$$g''\left(x+\tfrac{1}{2}\right) - g''\left(x-\tfrac{1}{2}\right) = \frac{-4}{(2x-1)^2}$$

$x=1$ $$\cancel{g''\left(\tfrac{3}{2}\right)} - g''\left(\tfrac{1}{2}\right) = \frac{-4}{1^2}$$

$x=2$ $$\cancel{g''\left(\tfrac{5}{2}\right)} - \cancel{g''\left(\tfrac{3}{2}\right)} = \frac{-4}{3^2}$$

$x=3$ $$\cancel{g''\left(\tfrac{7}{2}\right)} - \cancel{g''\left(\tfrac{5}{2}\right)} = \frac{-4}{5^2}$$

$x=N$ $$g''\left(N+\tfrac{1}{2}\right) - \cancel{g''\left(N-\tfrac{1}{2}\right)} = \frac{-4}{(2N-1)^2}$$

$$g''\left(N+\tfrac{1}{2}\right) - g''\left(\tfrac{1}{2}\right) = -4\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}\cdots+\frac{1}{(2N-1)^2}\right)$$

Bonus Concept

$\boxed{(\text{Odd})' = \text{Even}}$ ✓

$\boxed{(\text{Even})' = \text{Odd}}$ ✓

Even $f(-x) = f(x)$

odd $f(-x) = -f(x)$

$f \rightarrow$ Even

$f' \rightarrow$ Odd

$f(x) = x^4$ (Even)

$f'(x) = 4x^3$ (odd)

$f''(x) = 12x^2$ (Even)

$f'''(x) = 24x$ (odd)

$f''''(x) = 24$ (Even)

$f(-x) = f(x)$

$f'(-x) \times (-1) = f'(x)$

$\underline{f'(-x) = -f'(x)}$

Join with us in Telegram

TELEGRAM CHANNEL

S N

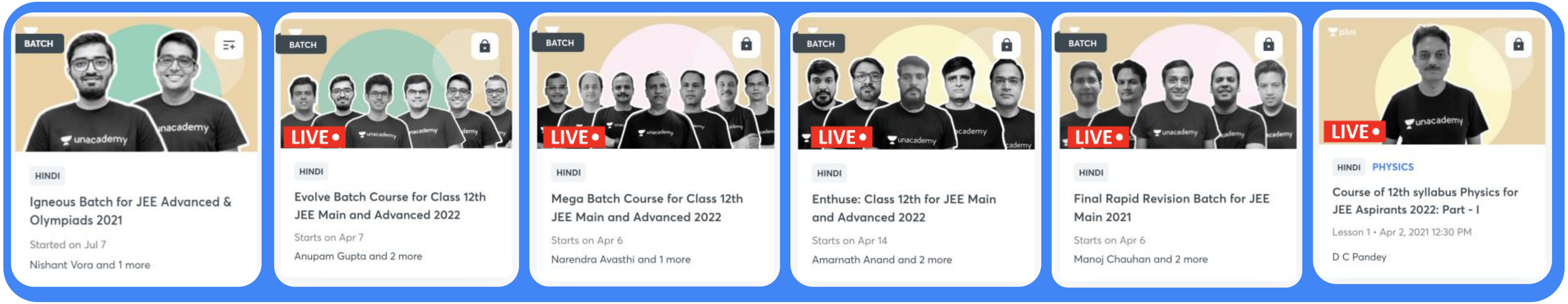
- t.me/unacademyatoms ☆ PDF

COMPLETE NOTES AND LECTURES

- livedaily.me/atoms

India's **BEST** Educators

Unacademy Subscription



If you want to be the **BEST**
"Learn" from the **BEST**

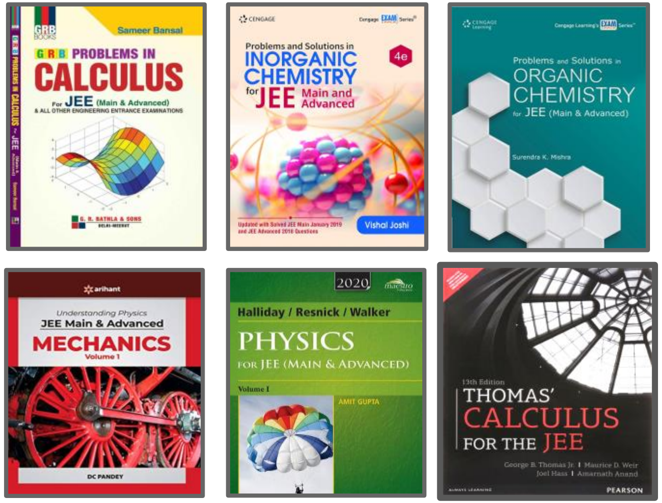
AOD

Last Min*

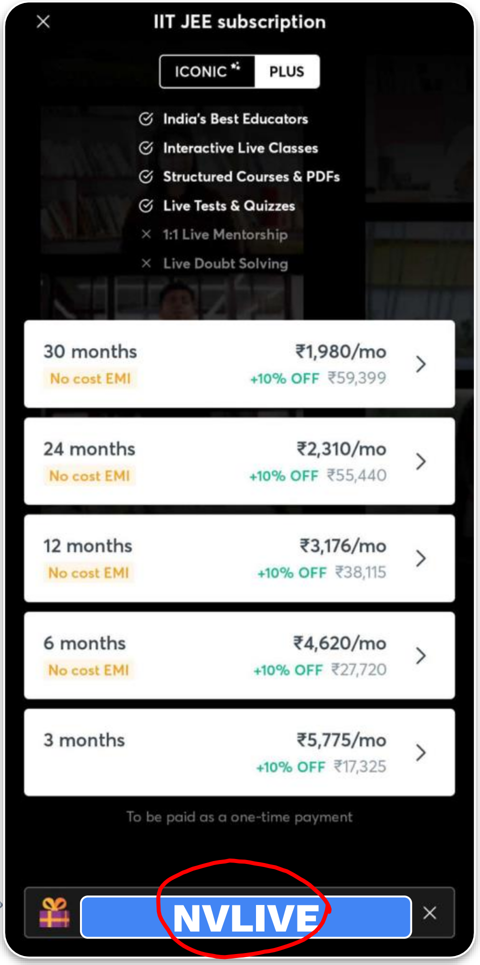
JEE Adv

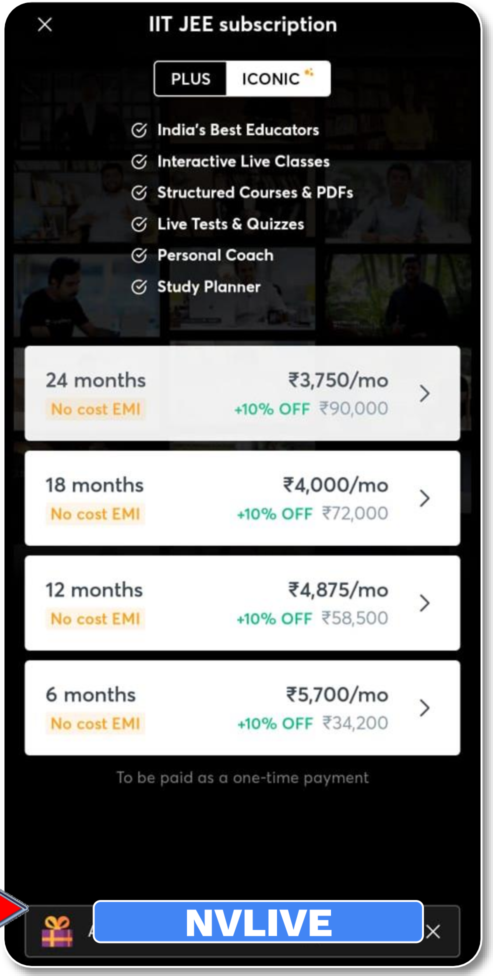
R B.C

R R.B



NVLIVE


IIT JEE subscription
ICONIC
PLUS
India's Best Educators
Interactive Live Classes
Structured Courses & PDFs
Live Tests & Quizzes
1:1 Live Mentorship
Live Doubt Solving
30 months ₹1,980/mo
No cost EMI +10% OFF ₹59,399
24 months ₹2,310/mo
No cost EMI +10% OFF ₹55,440
12 months ₹3,176/mo
No cost EMI +10% OFF ₹38,115
6 months ₹4,620/mo
No cost EMI +10% OFF ₹27,720
3 months ₹5,775/mo
+10% OFF ₹17,325
To be paid as a one-time payment
NVLIVE


IIT JEE subscription
PLUS
ICONIC
India's Best Educators
Interactive Live Classes
Structured Courses & PDFs
Live Tests & Quizzes
Personal Coach
Study Planner
24 months ₹3,750/mo
No cost EMI +10% OFF ₹90,000
18 months ₹4,000/mo
No cost EMI +10% OFF ₹72,000
12 months ₹4,875/mo
No cost EMI +10% OFF ₹58,500
6 months ₹5,700/mo
No cost EMI +10% OFF ₹34,200
To be paid as a one-time payment
NVLIVE

Personal Guidance

Get one on one guidance from top exam experts

Study Planner

Customized study plan with bi-weekly reviews

ICONIC PLUS

Rec

Live Classes

Weekly Tests

V C
D CS
M.T

(IIT)

Structured Courses

Unlimited Access

Test Analysis

Get one on one guidance from top exam experts

Study Material

Specialised Notes & Practice Sets

Experts' Guidelines

Study booster workshops by exam experts

Upcoming Batches in December

unacademy

All Stars Crash Course 3.0 Batch for Class 11th JEE Main and Advanced 2023	Starts on 1st Dec. 2021
Rapid revision for JEE Main and Advanced 2022	Starts on 8th Dec. 2021
Edge for JEE Main and Advanced 2023 - Early class 12th	Starts on 15th Dec. 2021
Rapid revision for JEE Main and Advanced 2022 - 2.0	Starts on 15th Dec. 2021
Striker Batch: JEE Main 2021	Starts on 15th Dec. 2021
Excellence for JEE Main and Advanced 2023	Starts on 22nd Dec. 2021
Striker Batch: JEE Main 2021 - 2.0	Starts on 22nd Dec. 2021
Edge for JEE Main and Advanced 2023 - Early class 12th - 2.0	Starts on 29th Dec. 2021
Excellence for JEE Main and Advanced 2023 - 2.0	Starts on 5th jan. 2022

unacademy | UNACADEMY CHAMPIONS LEAGUE

Ace UCL for JEE

Stand a chance to meet and get inspired by

Sachin Tendulkar

Aptitude Test on Dec 18, 19, 26 | 1 PM Onwards

Enroll Now

Use Invite Code

unacademy | UNACADEMY CHAMPIONS LEAGUE

Unacademy Champions League

Is Back, once again for You !!

Participate and Win!! | Mega Prizes

Apple Macbook Air

Apple watch

Airpod

13 Participants win Amazon echo dot

60 Participants to get 1000 AGV

Winners will be decided based on lucky draw of all participants of December 26th test

Enroll Now

*Plus subscribers are not eligible

Thank You!

+ SUBSCRIBE

LIKE & SHARE

PDF

Download Now !