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DO NOT WRITE ANYTHING ON QUESTION PAPER EXCEPT YOUR NAME, DEPARTMENT AND ENROLMENT No.

POSSESSION OF MOBILE, SMART WATCH ETC. IN EXAMINATION IS A UFM PRACTICE

Name of Student --- Enrolment No. -----------------

Department / School ----------------------------

BENNETT UNIVERSITY, GREATER NOIDA

Supplementary Examination, July 2019

COURSE CODE: **EPHY108L** MAX. DURATION: **3 HOURS**

COURSE NAME: **Mechanics**

COURSE CREDIT: **3** MAX. MARKS: **100**

Note

- All questions are mandatory
- Rough work must be carried out at the back of the answer script

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Answer the following questions (1 to 8) in brief. The answers should be no longer than 2-3 lines.

1. If a particle moving in a gravitational field has an elliptic orbit, what should be the values of its eccentricity and total energy.
(2 mark)

2. Write down the condition when a particle with zero rest mass can have non-zero momentum.
(2 mark)

3. Define non-inertial frame of reference.
(2 mark)

4. A planet while orbiting around the sun has a speed v at position $\mathbf{r}_1$ and it sweeps out an area α in unit time interval. At a different location in its orbit ($\mathbf{r}_2$) the planet has a speed $2v$. What is the area that it will sweep out in unit time at $\mathbf{r}_2$.
(2 mark)

5. Which oscillator will have a sharper resonance when driven by external force, one with higher damping or one with lower damping?
(2 mark)

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6. Equation of an elliptic orbit is: $r = \frac{r_0}{1-\varepsilon\cos\theta}$ with origin at one of the focus. What are the closest and farthest distances from the focus, in terms of eccentricity ε and constant r_0.
(2 mark)

7. The moment of Inertia of a rigid body about its 3 principal axes are 3, 6, 18.8 $Kg.m^2$. Torque-less rotation about which axis will be least stable?
(2 mark)

8. Where is the centrifugal force due to earth's rotation maximum? At the equator or poles or at latitude of 45^0.
(2 mark)

9. The sound intensity from a musical instrument operating at an angular frequency (ω) of 400 Hz, decreases by a factor of 5 in 4 seconds. What is the quality factor (Q) of the instrument? Note that the sound intensity is proportional to the energy of the oscillator.
(8 marks)

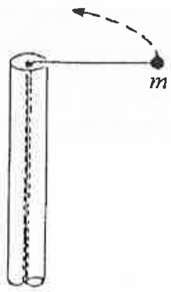
10. A coordinate system *x,y,z* is rotating w.r.t. another coordinate system *X,Y,Z* with an angular velocity $\boldsymbol{\omega} = 2t\hat{\imath} - t^2\hat{\jmath} + (2t+4)\hat{k}$, where t is time. Both systems have the same origin and *X,Y,Z* system is assumed to be inertial. The position vector of a particle in *x,y,z* system is $\mathbf{r} = (t^2+1)\hat{\imath} - 6t\hat{\jmath} + 4t^3\hat{k}$. Find its velocity in the inertial system *X,Y,Z* at $t = 1$ s.
(10 marks)

11. An object of mass 20 Kg moves in simple harmonic motion along the x axis. Initially (at $t = 0$) it is located at a distance of 4 meters away from the origin and has a velocity of 15 m/s and acceleration 100 m/s^2, both directed towards origin. Assume the motion is of the form $x = A\cos(\omega t + \varphi)$ and find the amplitude (A), angular frequency (ω) and phase factor (φ).
(12 marks)

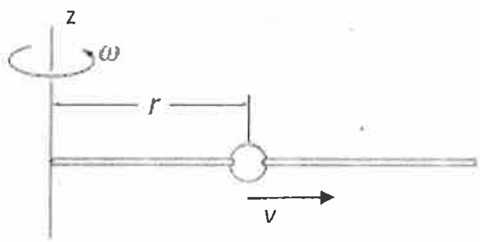
12. A satellite of mass m is in circular orbit about the earth. The radius of the orbit is r_0 and the mass of the earth is M_e. Find the total energy of the satellite. Suppose a friction force is acting on the satellite. Will the radius increase or decrease?
(6 marks)

13. Mass m is attached to a post of radius R by a string (see figure below). Initially it is at distance r from the center of the post and is moving tangentially with speed v_0. The string passes through a hole in the center of the post at the top. The string is being pulled down and

shortened by drawing it through the hole. Is this a central force problem? Find the final speed of the mass when it hits the post.
(8 marks)



14. A bead of mass m slides with a constant speed v in the outward direction on a horizontal rigid wire, which itself is rotating at constant angular speed ω about the z axis (see figure below). What is the force exerted on the bead by the wire? Note that there is variable friction acting on the bead. Neglect the weight of the bead.
(8 marks)



15. A particle of rest mass m_0 and velocity v strikes and sticks to an identical particle at rest. The rest mass of the resultant composite particle is M_0 and velocity V. All velocities are along positive x-direction. Write down the total momentum and energy before collision and the final total momentum and energy after collision. Assume the particles are moving with relativistic speeds. Will the total momentum and energy be conserved in this collision?
(8 marks)

16. For what values of n are circular orbits stable for central forces. Take $V(r) = -\frac{C}{r^{n-1}}$, as the potential energy. Here C is a positive constant.
(10 marks)

17. A geostationary orbit is one in which a satellite moves in a circular orbit at a certain height in the equatorial plane, so that its angular velocity of rotation around earth is same as earth's angular velocity. Find its angular and linear velocity. Use g = 9.8 m.s^{-2}, radius of earth R_e = 6.4 x 10^6 m and $g = \frac{GM_e}{R_e^2}$, where G = universal gravitational constant and M_e is the mass of earth. (Hint: Earth takes 24 hours to complete one rotation)

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(6 marks)

18. If the damping constant of a free oscillator is given by $\gamma = 2\omega_0$, the system is said to be critically damped. The motion is given by $x = (A + Bt)e^{-\gamma t/2}$, where A and B are arbitrary constants. A critically damped oscillator is at rest at equilibrium. At $t = 0$ it is given a blow of total impulse I. Find A and B using the given initial condition.
(8 marks)

End of Question Paper

Useful relations (symbols have their usual meanings):

Vector multiplication:
$\mathbf{A}.\mathbf{B} = AB\cos\theta$, $|\mathbf{A} \times \mathbf{B}| = AB\sin\theta$
$\mathbf{A}.\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$

$$\mathbf{A} \text{ X } \mathbf{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Plane polar coordinates:
$x = r\cos\theta, y = r\sin\theta, \hat{\mathbf{r}} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}, \hat{\boldsymbol{\theta}} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$

Kinematics in polar coordinates:
$\mathbf{r} = r\hat{\mathbf{r}}, \mathbf{v} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}, \boldsymbol{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\boldsymbol{\theta}}$

Rotational motion
$\mathbf{L} = \mathbf{r} \times \mathbf{P}, \boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \frac{d\mathbf{L}}{dt}$

Rotating vector: $\frac{dA}{dt} = \omega \times A$

Derivative in a rotating frame of reference
$\left(\frac{d}{dt}\right)_{in} = \left(\frac{d}{dt}\right)_{rot} + \Omega \times$

Pseudo forces in a rotating frame
$\mathbf{F}_{coriolis} = -2m\Omega \times \mathbf{v}_{rot}$
$\boldsymbol{F}_{centrifugal} = -m\Omega \times (\Omega \times \boldsymbol{r})$,
its magnitude $= -m\Omega^2 r = -m\frac{v^2}{r}$ (in simple circular motion)

Central Force

$\mu\ddot{r} = f(r)\hat{r}$ $\quad\quad \mu = \frac{m_1 m_2}{m_1+m_2}$

$V_{eff}(r) = \frac{L^2}{2\mu r^2} + V(r)$ $\quad\quad E = \frac{1}{2}\mu\dot{r}^2 + V_{eff}(r)$ $\quad\quad L = \mu r^2\dot{\theta}$

$$\theta - \theta_0 = L\int_{r_0}^{r} \frac{dr}{r^2\sqrt{2\mu(E - V_{eff}(r))}}$$

Gravitational force $\frac{Gm_1m_2}{r^2} = \frac{C}{r^2}$

Planetary orbits

$r = \frac{r_0}{1-\varepsilon\cos\theta}$ $\quad\quad \varepsilon = \sqrt{1 + \frac{2EL^2}{\mu C^2}}$ $\quad\quad E = -\frac{C}{A}$

Harmonic Oscillator

$m\ddot{x} + kx = 0$ $\quad\quad x = B\sin\omega_0 t + C\cos\omega_0 t$ $\quad\quad x = A\cos(\omega_0 t + \varphi)$ $\quad\quad \omega_0 = \sqrt{\frac{k}{m}}$

Damped oscillator

$x = Ae^{-\gamma t/2}\cos(\omega_1 t + \emptyset)$ $\quad\quad E(t) = E_0 e^{-\gamma t}$

$Q = \frac{\omega_1}{\gamma} = \frac{\omega_0}{\gamma}$

Forced oscillation

$x = \frac{F_0}{m}\frac{1}{\omega_0^2-\omega^2}\cos\omega t$

Resonance curves $\Delta\omega = \gamma$

Lorentz transformation for relativistic motion

$x' = \gamma(x - vt)$

$y' = y\,,\ z' = z$

$t' = \gamma\left(t - \frac{xv}{c^2}\right)$

$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

Relative velocities in the relativistic domain

$u_x' = \frac{u_x - v}{1-vu_x/c^2}$ $\quad\quad u_y' = \frac{u_y}{\gamma[1-vu_x/c^2]}$ $\quad\quad u_z' = \frac{u_z}{\gamma[1-vu_x/c^2]}$

Mass, energy, momentum in relativistic motion

$m = m_0\gamma$ $\quad\quad p = m_0 u\gamma$ $\quad\quad E^2 = (pc)^2 + (m_0c^2)^2$

$E = mc^2$ $\quad\quad K = mc^2 - m_0c^2$