POSSESSION OF MOBILES IN EXAM IS UFM PRACTICE

Name of Student: .... **Enrollment No.** ....
Department: ....

BENNETT UNIVERSITY, GREATER NOIDA
Mid Term Examination, SPRING SEMESTER 2018-19

COURSE CODE : EMAT102L MAX. DURATION: 1 Hour
COURSE NAME: Linear Algebra and Ordinary Differential Equations
COURSE CREDIT: 3-1-0 MAX. MARKS: 25

Instructions:

- There are **seven** questions in this question paper and all questions are mandatory.
- Rough work must be carried out at the back of the answer script.

1. Is the matrix $A = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 1 & 1 & 3 \end{bmatrix}$ in the row reduced echelon form? If not, find the row reduced echelon form and rank of A. [2]

2. Under what condition on $a \in \mathbb{R}$, the following system of linear equations has [3]

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 2 \\ x_1 + ax_2 + x_3 &= 6 \\ 4x_3 &= 8 \end{aligned}$$

   (a) a unique solution (b) infinitely many solutions (c) no solution.

3. Let V be the set of all real symmetric 4×4 matrices. Find its basis and dimension. What if V is the complex vector space of all 4×4 Hermitian matrices? [5]

OR

Let $P_4(\mathbb{R})$ be the real vector space of all the polynomials of degree less than or equal to 4. Is the set

$$W = \{p(x) \in P_4(\mathbb{R}) : p(-1) = p(1) = 0\}$$

a subspace of $P_4(\mathbb{R})$? If yes, find its basis and dimension.

4. Justify your answer, whether the following statements are true/false. [2×4=8]

   (a) The set $S = \{(x, y, z) \in \mathbb{R}^3 : x \text{ is an irrational number}\}$ is a subspace of $\mathbb{R}^3(\mathbb{R})$.

   (b) The maximum number of linearly independent vectors in $\mathbb{R}^3$ are 3.

(c) If U and W are subspaces of $\mathbb{R}^8$ such that $dim(U) = 3$, $dim(W) = 5$ and $U + W = \mathbb{R}^8$, then $U \cap W \neq \{0\}$.

(d) The map $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (|x|, y - z)$ is a linear transformation.

5. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that [3]

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\2\end{bmatrix}, \quad T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}3\\4\end{bmatrix}.$$

Then find $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right)$.

6. Determine all the linear maps $T : \mathbb{R}^4 \to \mathbb{R}^5$ which are onto. [2]

7. Let A be an $n \times n$ square matrix such that $det(A) \neq 0$, then find the basis and dimension of [2]

$$W = \{X \in \mathbb{R}^{n\times 1} : AX = 0\}.$$