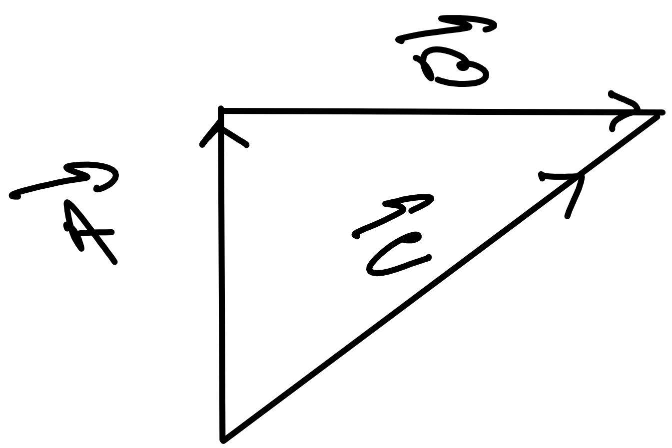
EPHY105L-1 27.9.21

Vector Operations

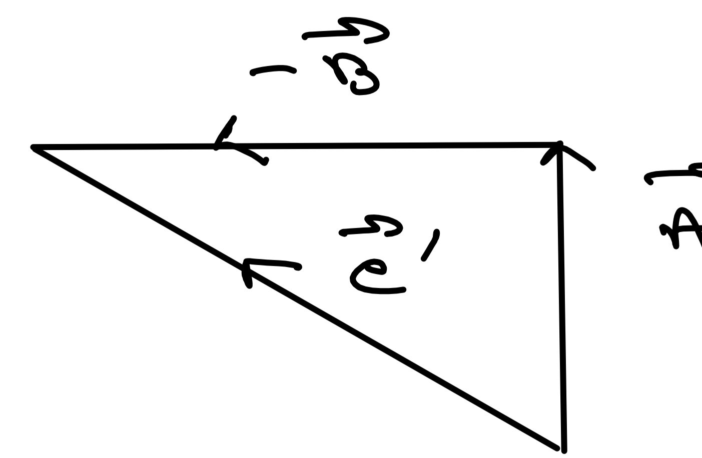
(i) Addition

Two vectors : $\vec{A}$ & $\vec{B}$

$\vec{C} = \vec{A} + \vec{B}$



Subtraction



$\vec{C}' = \vec{A} - \vec{B}'$

(*) Addition is associative

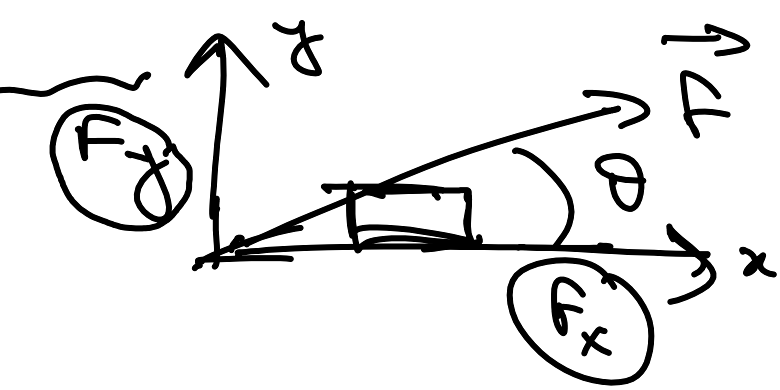
$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

② Multiplication by a scalar

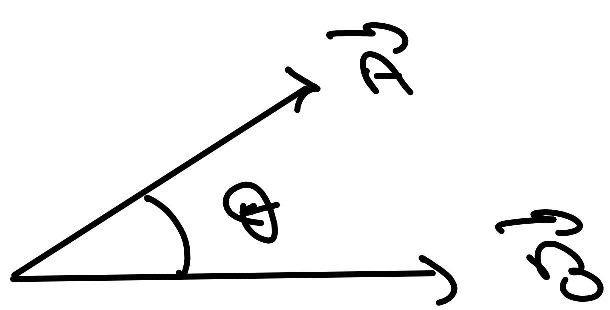
④ Multiplication is distributive

$$a(\vec{A} + \vec{B}) = a\vec{A} + a\vec{B}$$

③ Dot product of two vectors



$$\vec{A} \cdot \vec{B} = \underbrace{AB\cos\theta}_{\text{Scalar quantity}}$$



$A = |\vec{A}|$

$B = |\vec{B}|$

$F_x = \vec{F} \cdot \hat{x}$

$\vec{F} \cdot \hat{x} = F\cos\theta$

* Dot product is commutative

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

* Dot product is distributive

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

Geometrically, $\vec{A} \cdot \vec{B}$ is simply product A times the projection of $\vec{B}$ along $\vec{A}$ or vice versa.

* If the two vectors are parallel:

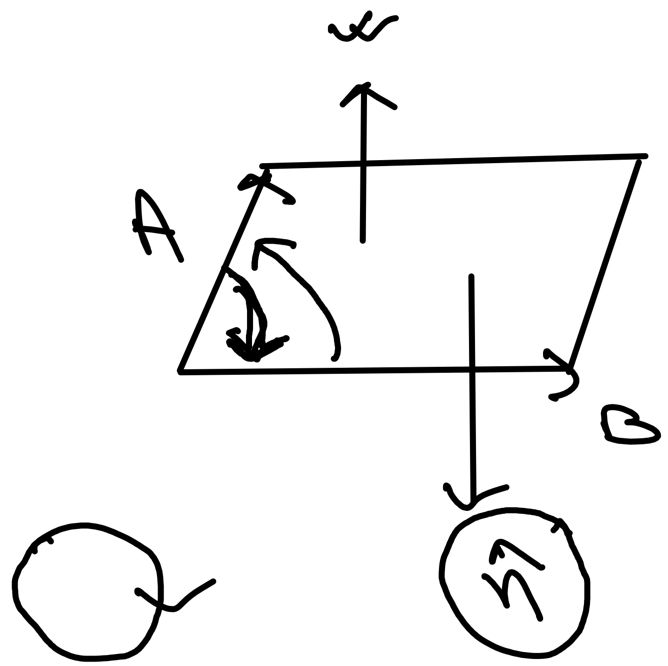
$$\vec{A} \cdot \vec{B} = AB \leftarrow$$

If they are perpendicular;

$$\vec{A} \cdot \vec{B} = 0$$

* Cross product betn. two vectors

$$\vec{A} \times \vec{B} = \underbrace{AB \sin\theta \, \hat{n}}_{\text{vector quantity}}$$



$\hat{n} \equiv$ unit vector perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$

$\vec{A} \times \vec{B} \equiv$ Points into the page

$\vec{B} \times \vec{A} \equiv$ Points out of the page.

* Cross-product is distributive

$$\vec{A} \times (\vec{B} + \vec{C}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C})$$

* Cross product is not commutative

$$\vec{B} \times \vec{A} = -(\vec{A} \times \vec{B})$$

Geometrically, $|\vec{A} \times \vec{B}|$ is the area of the parallelogram generated by $\vec{A}$ and $\vec{B}$

(x) If the vectors are parallel,

$$\vec{A} \times \vec{B} = 0$$

If they are perpendicular,

$$\vec{A} \times \vec{B} = AB\,\hat{n}$$

<u>In component form</u>

In Cartesian co-ordinate system

Basis vectors $\equiv \hat{x}, \hat{y}, \hat{z}$

$$\vec{A} = \underbrace{A_x}_{}\hat{x} + \underbrace{A_y}_{}\hat{y} + \underbrace{A_z}_{}\hat{z}$$

components

A_x, A_y, A_z are simply just projections of $\vec{A}$ along three axes.

(i) Addition

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{x} + (A_y + B_y)\hat{y} + (A_z + B_z)\hat{z}$$

(ii) Multiplication by scalar

$$a\vec{A} = (aA_x)\hat{x} + (aA_y)\hat{y} + (aA_z)\hat{z}$$

(iii) Dot product

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\hat{x} \cdot \hat{x} = \hat{y} \cdot \hat{y} = \hat{z} \cdot \hat{z} = 1$$

$$\hat{x} \cdot \hat{y} = \hat{x} \cdot \hat{z} = \hat{y} \cdot \hat{z} = 0$$

$$\vec{A} \cdot \vec{A} = |\vec{A}|^2 = A_x^2 + A_y^2 + A_z^2$$

⊗ Cross Product

$$\vec{A} \times \vec{B} = (A_x \hat{x} + A_y \hat{y} + A_z \hat{z}) \times (B_x \hat{x} + B_y \hat{y} + B_z \hat{z})$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$\hat{x} \times \hat{x} = \hat{y} \times \hat{y} = \hat{z} \times \hat{z} = 0$

$\hat{x} \times \hat{y} = -\hat{y} \times \hat{x} = \hat{z}$

$\hat{y} \times \hat{z} = -\hat{z} \times \hat{y} = \hat{x}$

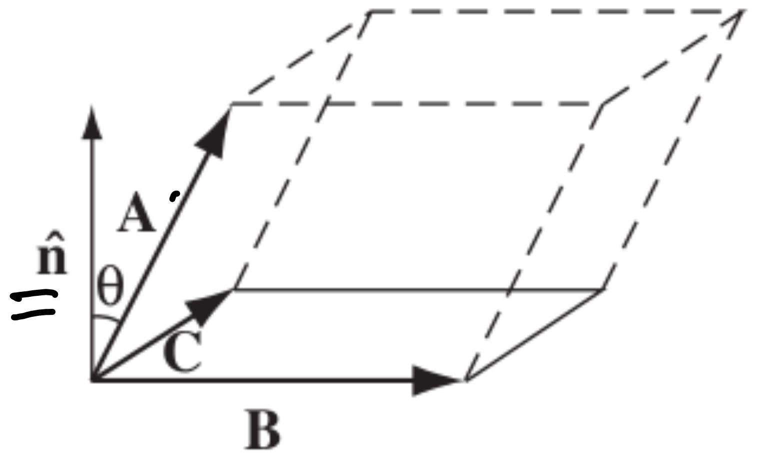
$\hat{z} \times \hat{x} = -\hat{x} \times \hat{z} = \hat{y}$

$$= \hat{x}(A_y B_z - A_z B_y) + \hat{y}(A_z B_x - A_x B_z) + \hat{z}(A_x B_y - A_y B_x)$$

Triple Product

\# Scalar triple product $\equiv \vec{A} \cdot (\vec{B} \times \vec{C})$

→ Geometrically, $|\vec{A}\cdot(\vec{B}\times\vec{C})|$ is the volume of a parallelopiped.



$|\vec{B}\times\vec{C}| \equiv$ Area of the base

$|\vec{A}\cos\theta| =$ Altitude

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$$

Link to the Recording:

https://bennettu.sharepoint.com/sites/EPHY105L-Odd2021/Shared%20Documents/General/Recordings/EPHY105L%20Theory%20Class-20210927_154620-Meeting%20Recording.mp4