DEPARTMENT OF MATHEMATICS

Bennett University

Linear Algebra and Ordinary Differential Equations (EMAT102L)

Mid Term Examination
June 6, 2021

Time: 1 hour 30 minute | *MID TERM EXAMINATION* | Maximum Marks: 30

1. If A is skew-symmetric matrix, then A^2 is a

   Answer: symmetric matrix [1]

2. Let $W = \{(x_1, x_2, x_3) \in R^3 : x_1 + x_2 + x_3 = 1\}$, then W is a subspace of R^3

   Answer: False [1]

3. The linear span of the vectors $(1, 2), (3, 4)$ is R^2.

   Answer: True [1]

4. The set $\{(0, 0), (1, 0), (0, 1)\}$ is linearly independent.

   Answer: False [1]

5. Write down the dimension of the nullspace of the following matrix $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$

   Answer: 2 [1]

6. The mapping $T : R^2 \to R^2$ is defined by $T(x_1, x_2) = (x_1 + x_2, x_2^2)$ is a linear mapping.

   Answer: False [1]

7. Let the linear mapping $T : R^2 \to R^3$ be defined by $T(x_1, x_2) = (x_1, x_1 + x_2, x_2)$. Then the nullity of T is

   Answer: 0 [1]

8. The distinct eigen values of the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

   Answer: 0 and 2 [1]

9. The number of linearly independent eigenvectors of the matrix $\begin{bmatrix} 2 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is

   Answer: 4 [1]

10. The dimension of the subspace $W = \{(x_1, x_2, x_3, x_4, x_5) : 3x_1 - x_2 + x_3 = 0\}$ of R^5 is
    Answer: 4

[1]

11. Determine the rank of the following matrix $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 7 & 8 \\ 3 & 6 & 9 & 12 & 15 \\ 4 & 8 & 12 & 14 & 16 \end{bmatrix}$

    Answer: 2 [2]

12. Investigate for what values of λ and μ the following equations have an infinite number of solutions

$$x + y + z = 6$$

$$x + 2y + 3y = 10$$

$$x + 2y + \lambda z = \mu$$

    Answer: $\lambda = 3$ and $\mu = 10$ [2]

13. Determinant value of the matrix $\begin{pmatrix} a+d & a+d+k & a+d+c \\ c & c+b & c \\ d & d+k & d+c \end{pmatrix}$ is

    Answer: abc [2]

14. A linear mapping $T : R^3 \to R^3$ is defined by
    $T(x_1, x_2, x_3) = (x_1 + x_2 - x_3, 2x_1 - x_2 + x_3, x_1 - 2x_2 + 2x_3)$. Find $Ker(T)$

    Answer: $Ker(T) = 1$ [2]

15. Let $\{(1,1,0), (1,0,0), (1,1,1)\}$ is a basis of R^3, Then find the orthonormal basis for R^3 using Gram-Schmidt process with the following inner product $< x, y >= (x_1y_1 + x_2y_2 + x_3y_3)$ where $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3) \in R^3$

    Answer: $\{\frac{1}{\sqrt{(2)}}(1,1,0), \frac{1}{\sqrt{(2)}}(1,-1,0), (0,0,1)\}$ [2]

16. If the nullity of the matrix $\begin{bmatrix} k & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 4 \end{bmatrix}$ is 1 , then the value of k is

    Answer: -1 [2]

17. Let $T : R^4 \to R^4$ be the linear map, satisfying
    $T(1,0,0,0) = (0,1,0,0)$
    $T(0,1,0,0) = (0,0,1,0)$
    $T(0,0,1,0) = (0,0,0,0)$
    $T(0,0,0,1) = (0,0,1,0)$,
    where $(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)$ is the ordered basis of R^4. Then

    Answer: $Rank(T) = 2$

[2]

18. A basis of
$V = \{(x_1, x_2, x_3, x_4) \in R^4 : x_1 + x_2 - x_3 = 0, x_2 + x_3 + x_4 = 0, 2x_1 + x_2 - 3x_3 - x_4 = 0\}$ is

    Answer: $\{(2, -1, 1, 0), (1, -1, 0, 1)\}$ [2]

19. A linear mapping $T : R^3 \to R^3$ is defined by $T(x_1, x_2, x_3) = (x_1+x_2-x_3, x_1+x_2+x_3, x_2-x_3)$. Find the matrix of T with respect to the ordered basis $\{(0, 1, 0), (0, 0, 1), (1, 0, 0)\}$ of R^3

    Answer: $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$ [2]

20. Let A be a 3×3 matrix. Suppose that the eigen values of A are $-1, 0, 1$ with respective eigen vectors $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Then $6A$ equals

    Answer: $\begin{bmatrix} -1 & 5 & 2 \\ 5 & -1 & 2 \\ 2 & 2 & 2 \end{bmatrix}$ [2]