Lecture - 27-28

Electrodynamics

Pushing on the charges make a current flow. How fast the charges move depends on the nature of the materials and the forces.

$\mathbf{J}$ is proportional to the *force per unit charge*, $\mathbf{f}$:

$$\mathbf{J} = \underset{\text{conductivity}}{\underset{\uparrow}{\sigma}}\mathbf{f}.$$

In principle, the force that drives the charges to produce the current could be anything—chemical, gravitational, or trained ants with tiny harnesses. For *our* purposes, though, it's usually an electromagnetic force that does the job.

The Lorentz force drives the charges to produce current:

$$\mathbf{J} = \sigma(\mathbf{E} + \underbrace{\cancel{\mathbf{v}} \times \mathbf{B}}_{\approx 0})$$

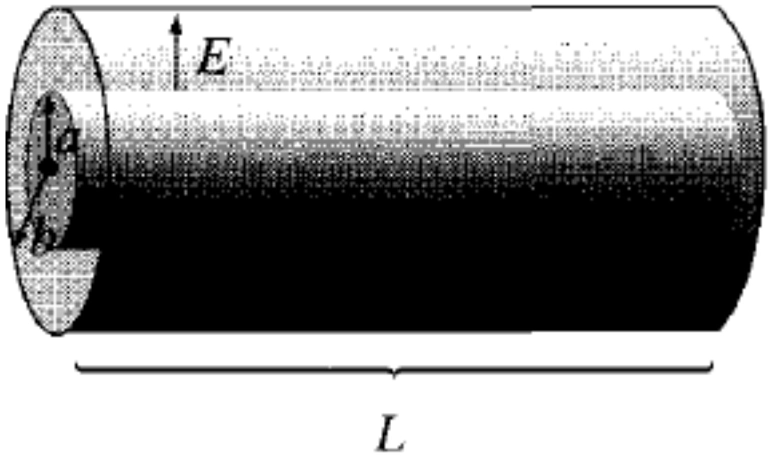
Magnetic contribution is usually small

Ohm's law (an empirical equation):

$$\boxed{\mathbf{J} = \sigma\mathbf{E}.}$$

Example 7.2

Two long cylinders (radii a and b) are separated by material of conductivity σ. If they are maintained at a potential different V, what current flows from one to the other, in a length L?



Solution: The field between the cylinders is

$$\mathbf{E} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{\mathbf{s}},$$

λ is the charge per unit length on the inner cylinder.

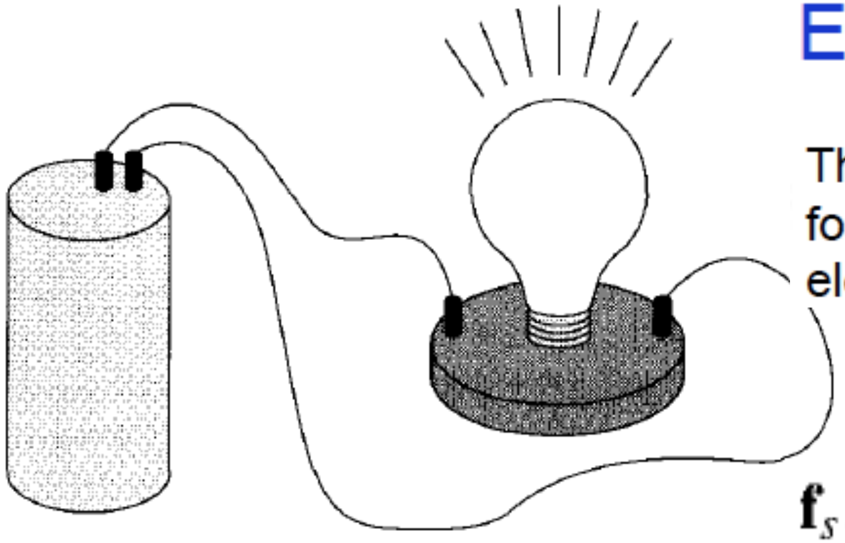
$$I = \int \mathbf{J} \cdot d\mathbf{a} = \sigma \int \mathbf{E} \cdot d\mathbf{a} = \frac{\sigma}{\epsilon_0}\lambda L.$$

$$V = -\int_b^a \mathbf{E} \cdot d\mathbf{l} = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right)$$

$$I = \frac{2\pi\sigma L}{\ln(b/a)} V.$$

$$R = \ln(b/a)/2\pi\sigma L.)$$

Electromotive Force Drives the Electrons



The battery generates the force which drives the electrons move along the loop.

Two forces in driving the current

$$\mathbf{f} = \mathbf{f}_s + \mathbf{E}.$$

$\mathbf{f}_s$, which is ordinarily confined to one portion of the loop (a battery

and the *electrostatic* force

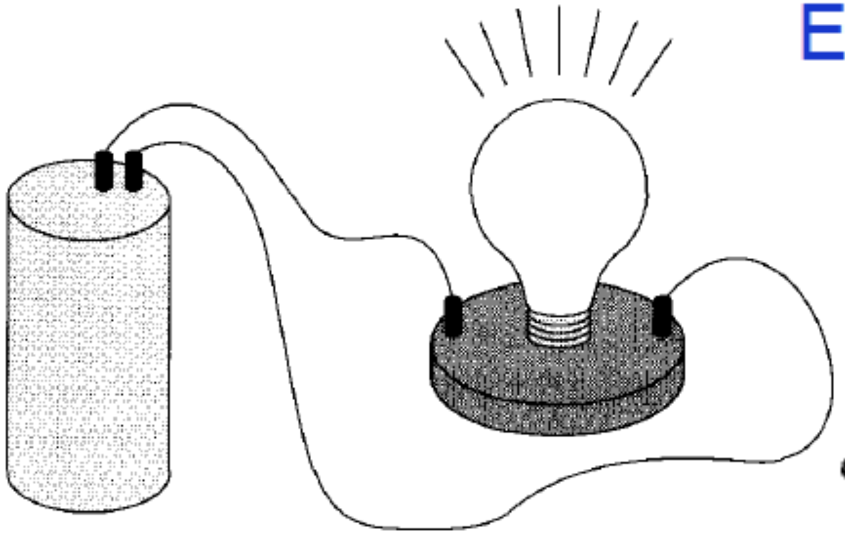
$$\boxed{\mathcal{E} \equiv \oint \mathbf{f} \cdot d\mathbf{l} = \oint \mathbf{f}_s \cdot d\mathbf{l}.}$$

$\mathcal{E}$ is called the **electromotive force**, or **emf**, of the circuit.

not a *force* at all—it's the *integral* of a *force per unit charge*.

An emf is the work per unit charge done by the source of emf in moving the charge around *a closed loop*.

Electromotive Force Drives the Electrons



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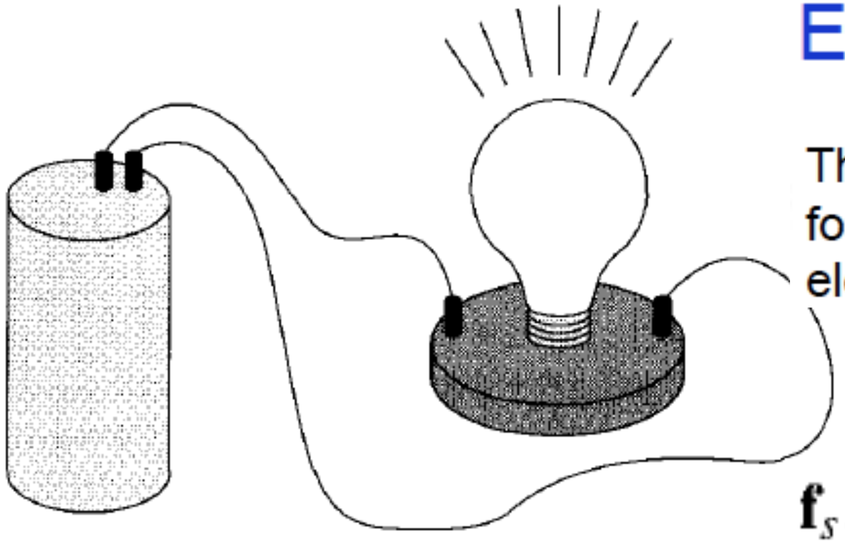
An emf is the work per unit charge done by the source of emf in moving the charge around *a closed loop*.

Emf is a lousy term, since it is not a force at all --- it is the integral of a force per unit charge.

Electromotive Force Drives the Electrons



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What is the physical agency responsible for $\mathbf{f}_s$?

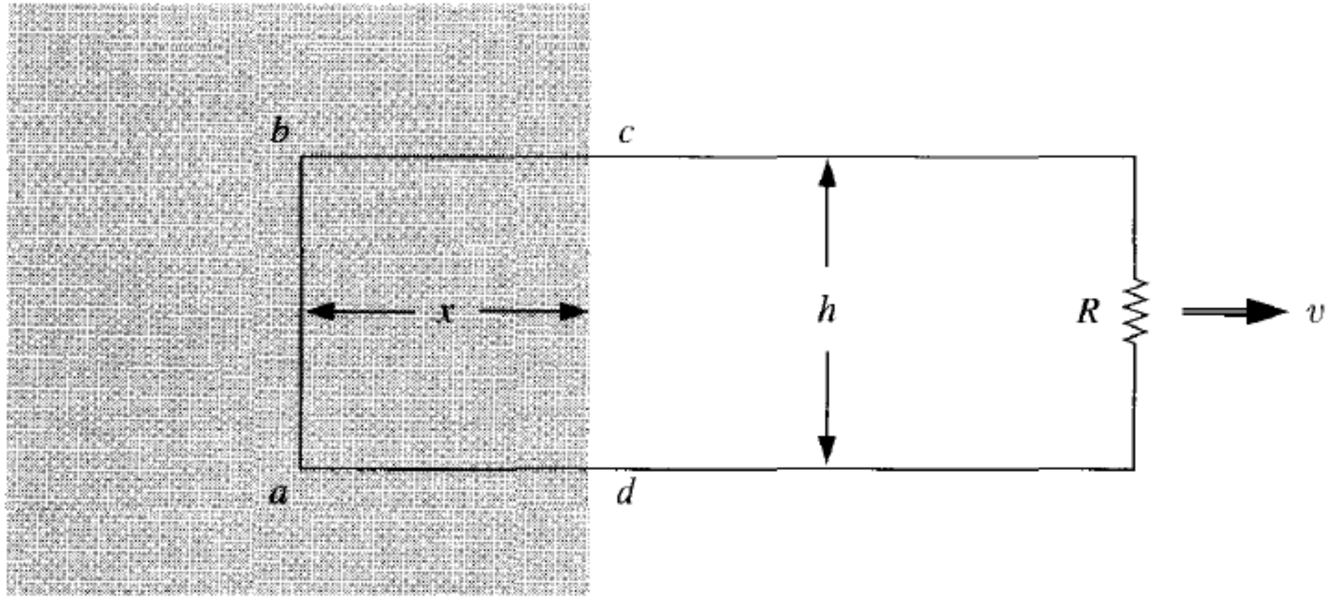
- Battery → a chemical force
- Piezoelectric crystal → mechanical pressure
- Thermal couple → temperature gradient
- Photoelectric cell → light

Motional emf

The most common source of the emfs: the generator

Generators exploit motional emf's, which arise when you move a wire through a magnetic field.

A primitive model for a generator

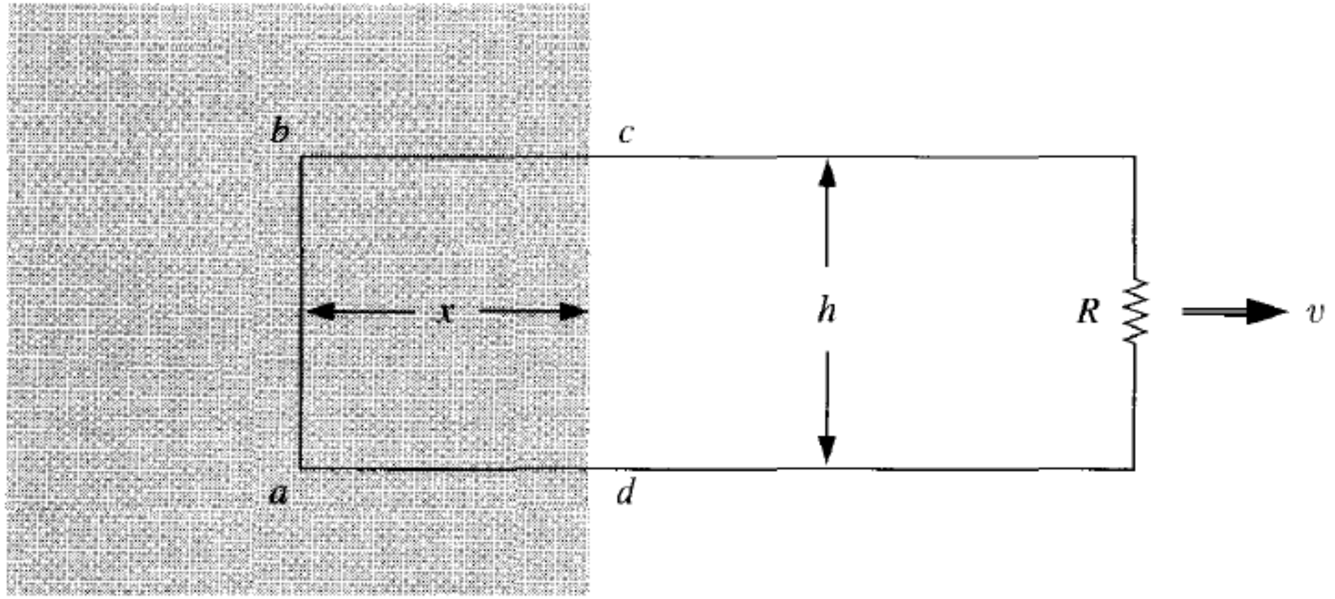


Shaded region: uniform B-field pointing into the page.

R: whatever it is, we are trying to drive current through.

$$\mathcal{E} = \oint \mathbf{f}_{\text{mag}} \cdot d\mathbf{l} = \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} = vBh$$

Please note that the ***V*** is the velocity of the loop, not the charge



A person exerts a force per unit charge on the wire by pulling it. The force counteracts the force generated by the magnetic force quB.

$$f_{\text{pull}} = uB$$

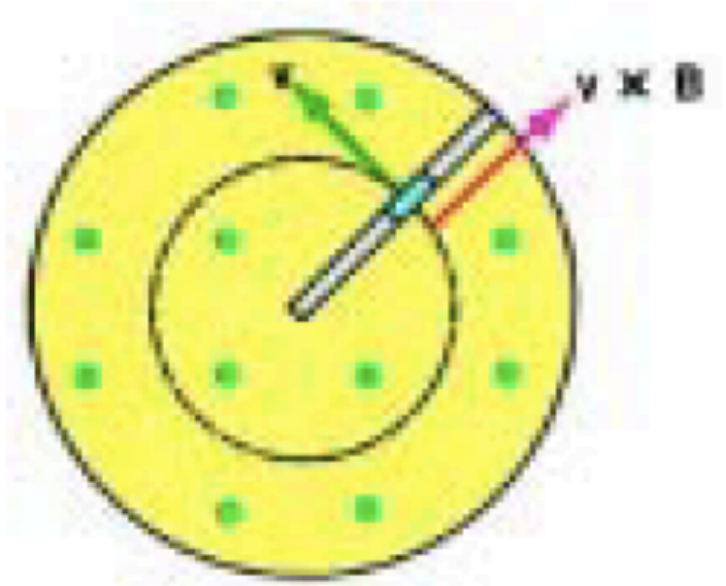
The magnetic force is responsible for establishing the emf and the emf seems to heat the resistor (i.e. do work), but magnetic fields never do work.

Who is supplying the energy that heats the resistor.

The person who's pulling on the loop!

Example 7.4

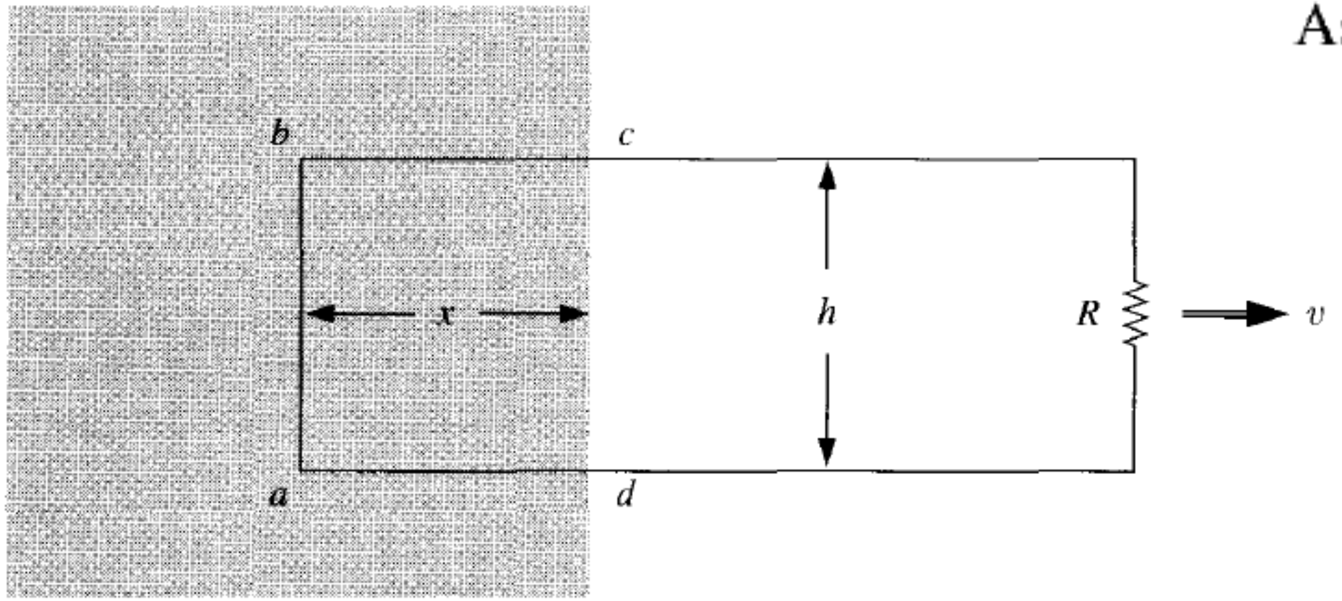
In a homopolar generator a conducting disk of radius R rotates at angular velocity ω rad/s. Its plane is perpendicular to a uniform and constant magnetic field **B**. What is the emf generated between the center and the rim?



Solution:

$$E = \oint (v \times B) \cdot d\ell = \int_0^R vBdr$$

$$= \int_0^R \omega rBdr = \frac{1}{2}\omega BR^2$$



As the loop moves, the flux decreases:

$$\frac{d\Phi}{dt} = Bh\frac{dx}{dt} = -Bhv.$$

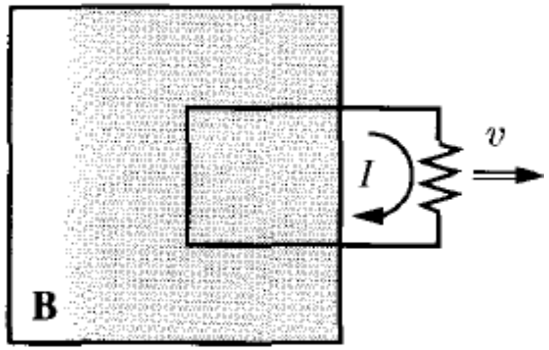
$$\mathcal{E} = \oint \mathbf{f}_{\text{mag}} \cdot d\mathbf{l} = vBh,$$

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}.}$$

In 1831 Michael Faraday reported on a series of experiments.

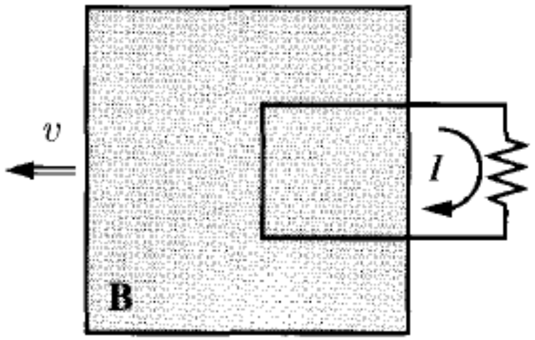
Experiment 1. He pulled a loop of wire to the right through a magnetic field

A current flowed in the loop.



Experiment 2. He moved the *magnet* to the *left*, holding the loop still

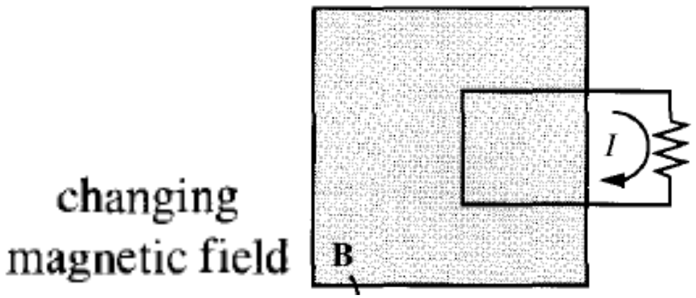
A current flowed in the loop.



Experiment 3. With both the loop and the magnet at rest

he changed the *strength* of the field (he used an electromagnet, and varied the current in the coil

A current flowed in the loop.



—stationary charges experience no magnetic forces.

What sort of field exerts a force on charges at rest? *electric* fields

A changing magnetic field induces an electric field.

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt},$$ **universal flux rule**:

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}.$$ **Faraday's law**, in integral form

$$\boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.}$$

What Faraday's discovery tells us is that there are really two distinct kinds of electric fields: those attributable directly to electric charges, and those associated with changing magnetic fields.[9] The former can be calculated (in the static case) using Coulomb's law; the latter can be found by exploiting the analogy between Faraday's law,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

Ampère's law, $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$.

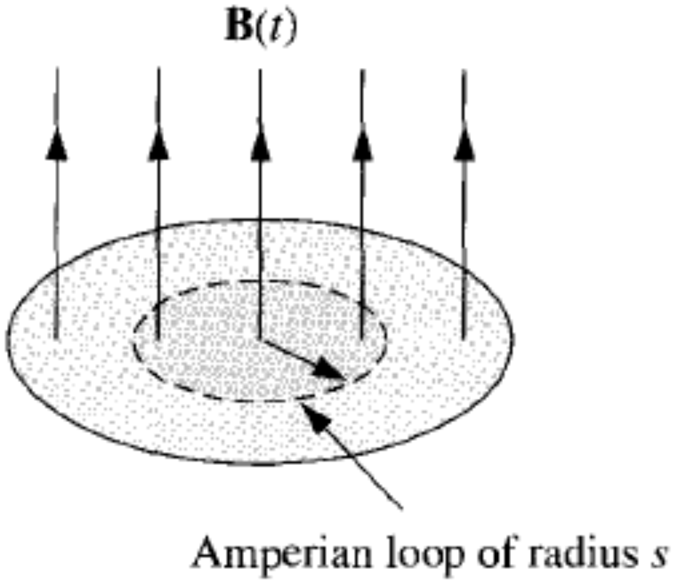
$\mathbf{E}$ is a *pure* Faraday field, due exclusively to a changing $\mathbf{B}$ $\nabla \cdot \mathbf{E} = 0,$

$$\nabla \cdot \mathbf{B} = 0$$

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}, \qquad \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}.$$

A uniform magnetic field $\mathbf{B}(t)$, pointing straight up, fills the shaded circular region

If $\mathbf{B}$ is changing with time, what is the induced electric field?



Draw an Amperian loop of radius s

Faraday's law:

$$\oint \mathbf{E} \cdot d\mathbf{l} = E(2\pi s) = -\frac{d\Phi}{dt}$$

$$= -\frac{d}{dt}\left(\pi s^2 B(t)\right)$$

$$= -\pi s^2 \frac{dB}{dt}$$

$$\mathbf{E} = -\frac{s}{2}\frac{dB}{dt}\,\hat{\boldsymbol{\phi}}.$$