Work done to move a charge

We have a stationary configuration of charges. We want to move a test charge 'q' from point 'a' to 'b'.

Electric force on test charge q:

$$\vec{F} = q\vec{E}$$

→ The force we have we have to exert in opposition to this force $= -q\vec{E}$

Work done, $W = \int_a^b \vec{F} \cdot d\vec{l}$

$$= -q \int_a^b \vec{E} \cdot d\vec{l}$$

$$= q \left(V(b) - V(a) \right)$$

→ independent of the path taken.

$\Rightarrow$ Electrostatic force is "conservative".

$$W = q[V(r) - V(\infty)] = qV(r)$$

reference point $\equiv \infty$

$\rightarrow$ Potential difference betn. two points

$$V(b) - V(a) = \frac{W}{q}$$

Energy of a point charge distribution

$\rightarrow$ To calculate how much work it takes to assemble a collection of point charges

We are bringing the charges one by one from far away.

$\rightarrow$ For q_1, we have to do no work since there is no electric field beforehand.

$\Rightarrow W_1 = 0$

→ in order to bring in q_2,

$$W_2 = \frac{1}{4\pi\epsilon_0} q_2 \frac{q_1}{r_{12}}$$

→ similarly, to bring in q_3,

$$W_3 = \frac{1}{4\pi\epsilon_0} q_3 \left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}} \right)$$

→ for q_4,

$$W_4 = \frac{1}{4\pi\epsilon_0} q_4 \left(\frac{q_1}{r_{14}} + \frac{q_2}{r_{24}} + \frac{q_3}{r_{34}} \right)$$

The total work done to complete the charge configuration:

$$W = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_1 q_4}{r_{14}} + \frac{q_2 q_3}{r_{23}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}} \right)$$

$i\ j$

In general, for 'n' number of charges:

$$W = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \sum_{j>i}^{n} \frac{q_i q_j}{r_{ij}}$$

OR

$$W = \frac{1}{8\pi\epsilon_0} \sum_{i=1}^{n} \sum_{j\neq i}^{n} \frac{q_i q_j}{r_{ij}}$$

$$\Rightarrow W = \frac{1}{2} \sum_{i=1}^{n} q_i \underbrace{\left(\sum_{j\neq i}^{n} \frac{1}{4\pi\epsilon_0} \frac{q_j}{r_{ij}} \right)}$$

$\hookrightarrow$ Potential at point $\vec{r}_i$ due to all other charges.

Hence, $$W = \frac{1}{2} \sum_{i=1}^{n} q_i V(\vec{r}_i)$$

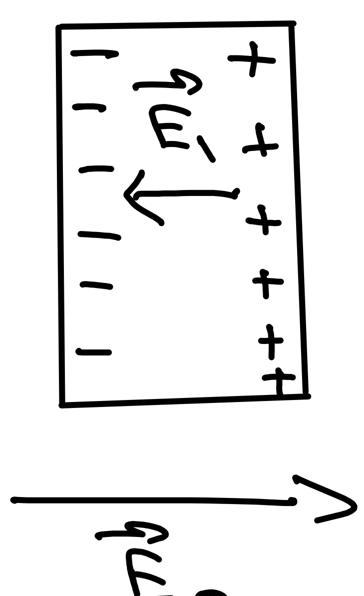
$\Rightarrow$ Represents the energy stored in the system as potential energy.

Conductors

A perfect conductor contains unlimited number of free charges.

⊗ Basic electrostatic property:

→ $\vec{E} = 0$ inside any conductor.



A conductor is put in an external electric field. The induced charges produce an electric field $\vec{E}_1$ inside the conductor such that $\vec{E}_1$ is oriented opposite to the direc^n. of $\vec{E}_0$

Then, $\vec{E}_1$ tends to cancel $\vec{E}_0$. The charges continue to flow inside until an equilibrium is reached when $\vec{E}_1$ cancels $\vec{E}_0$ exactly inside the conductor.

$$\text{Inside}, \quad \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\Rightarrow \vec{E} = 0 \Rightarrow \rho = 0$$

→ Inside the conductor, charge density is zero ⇒ equal number of positive and negative charges.

* Any net charge can only reside on the surface.

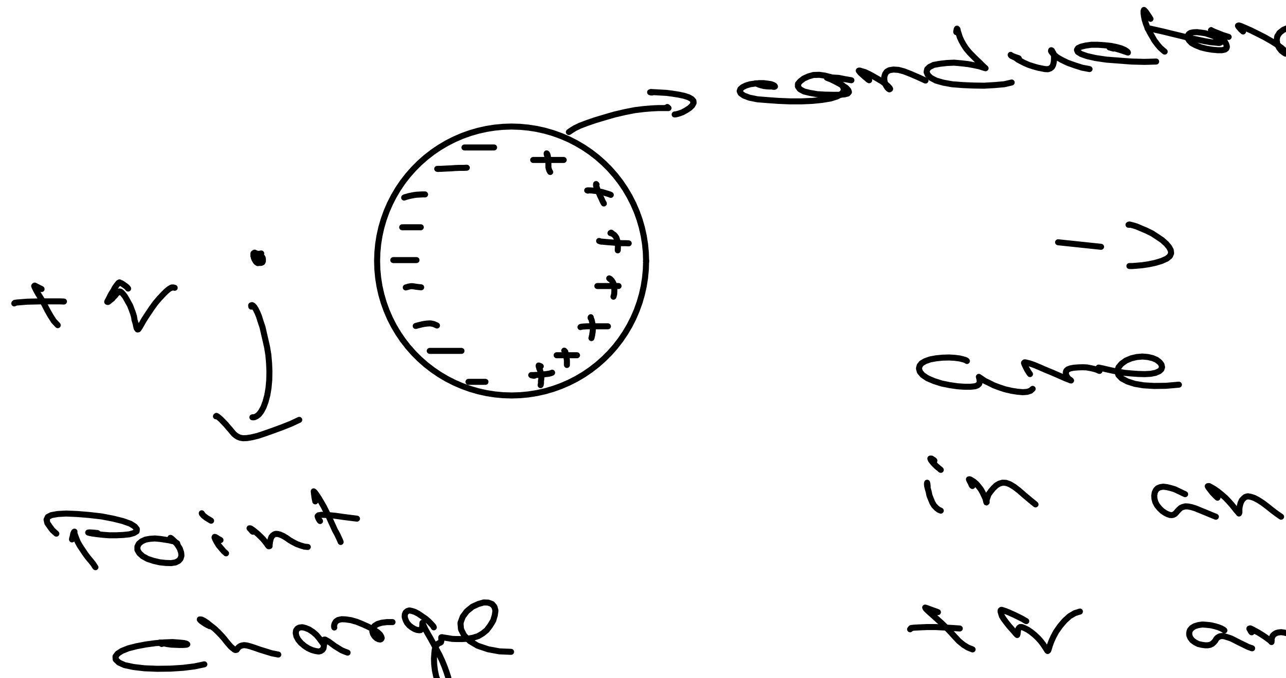
→ Conductors are equipotential

for any two points on the surface of the conductor, $V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{l} = 0$

$$\Rightarrow V(b) = V(a)$$

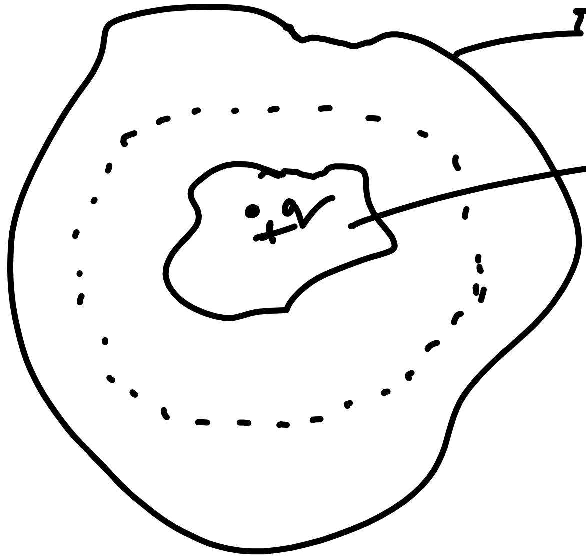
- The electric field $\vec{E}$ is perpendicular to the surface.
- The electrostatic energy is at its min^{m} when the charge is spread over the surface.

Induced Charges -



→ Negative induced charges are closer to +q resulting in an attractive force bet^{n} +q and the conductor.

- **+q charge inside the cavity of a conductor**



- Inside the cavity

  $\vec{E} \neq 0$

- Outside the cavity, but inside the conductor

  $\vec{E} = 0$

- Outside the conductor,

  $\vec{E} \neq 0$