Department of Mathematics, Bennett University
Engineering Calculus (EMAT101L)
Tutorial Sheet 2

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1. Find the limit of the following sequences.
   (a) $x_n = \frac{3n^2+2n+1}{n^2+1}$ (b) $x_n = \frac{(3n+1)(n-2)}{n(n+3)}$ (c) $x_n = (-1)^n \left(\frac{2}{n+2}\right)$ (d) $x_n = \frac{n+1}{2n+3}$
   (e)$x_n = \sqrt{4n^2+n} - 2n$. (f) $x_n = \sqrt{n^2+n} - \sqrt{n^2+1}$.

2. Use Sandwich theorem to prove that
   (a) $\lim \frac{1}{n} \sin^2 n = 0$.
   (b) $\lim \left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(n+n)^2}\right] = 0.$
   (c) $\lim \left[\frac{n}{n^3+1} + \frac{2n}{n^3+2} + \cdots + \frac{n^2}{n^3+n}\right] = \frac{1}{2}$
   (d) $\lim \sqrt[n]{(a^n+b^n)} = b$, where $0 < a < b$

3. Use Monotone convergence theorem to prove that $\{x_n\} = \{\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}\}$ is convergent.

4. Show that the sequence $\{\frac{1}{n}\}$ is a Cauchy sequence.

5. Show that the sequence $\{x_n\} = \{ny^{n-1}\}$, where $y \in (0,1)$ is a convergent sequence.

6. Show that the sequence $\{x_n\} = \{\frac{4^{3n}}{3^{4n}}\}$ converges to zero.

7. Show that $\lim_{n\to\infty} \sqrt[n]{n+1} = 1$.

8. State whether the following statements are true/false. Give proper justifications.
   (a) A sequence can have exactly two limits.
   (b) A sequence must have at least one limit.
   (c) A bounded sequence must have a limit.
   (d) An unbounded sequence will never have a limit.
   (e) A monotone sequence must have a limit.
   (f) A monotone sequence which is bounded above, must have a limit.
   (g) A bounded monotone sequence must have a limit.