Q-1: Rules of Inferences are used to derive conclusion from the given premises. This can help to check the validity of th argument also.

List of Rules of Inference

① Modus Ponens :-

$$\frac{\begin{array}{c} P \\ P \to Q \end{array}}{\therefore Q.}$$

e.g. it is raining. If it rains then I will have tea. therefore, I will have tea.

② Modus Tollens :-

$$\frac{\begin{array}{c} \sim q \\ p \to q \end{array}}{\therefore \sim p}$$

③ Hypothetical Syllogism :-

$$\frac{\begin{array}{c} p \to q \\ q \to r \end{array}}{\therefore p \to r.}$$

④ Disjunctive Syllogism :-

$$\frac{\begin{array}{c} p \vee q \\ \sim p \end{array}}{\therefore q.}$$

⑤ Addition :-

$$\frac{p}{\therefore p \vee q.}$$

⑥ Simplification :-

$$\frac{p \wedge q}{\therefore p.}$$

⑦ Conjunction :-

$$\frac{\begin{array}{c} p \\ q \end{array}}{p \wedge q.}$$

⑧ Resolution :

$$\frac{\begin{array}{c} p \vee q \\ \sim p \vee r \end{array}}{} \therefore q \vee r.$$

the english example should be given for each rule like in rule-1

Set - A.

Q-2! Sum of first n positive integer $= \frac{n(n+1)}{2}$

Since $n \geq 1$

for $n = 1$

Sum $= \frac{1(1+1)}{2} = 1.$

for $n = k.$

assume

Sum $= \frac{k(k+1)}{2}.$

for $n = k+1$

$$\text{Sum} = \underbrace{1+2+3+4+\dots\dots k}+ (k+1)$$

$$= \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)((k+1)+1)}{2}.$$

Q.2!-

Set - B

for $n \geq 0$

$$\sum_{P=0}^{n} r^P = \frac{r^{n+1} - 1}{r-1} \quad (r \neq 1)$$

for $n = 0$

L.H.S. $\sum_{P=0}^{0} r^P = r^0 = 1$

R.H.S. $\frac{r^1 - 1}{r-1} = 1$

L.H.S = R.H.S.

for $n = k$

assume.

$$\sum_{P=0}^{K} r^P = \frac{r^{K+1} - 1}{r-1}$$

for $n = (k+1)$

L.H.S

$$\sum_{P=0}^{K+1} r^P = \sum_{P=0}^{K} r^P + r^{(K+1)}$$

$$= \frac{r^{K+1} - 1}{r-1} + r^{(K+1)}$$

$$= \frac{r^{(K+1)} - 1 + r^{(K+1)}(r-1)}{r-1}$$

$$= \frac{\cancel{r^{K+1}} - 1 + r^{K+2} - \cancel{r^{K+1}}}{r-1} = \frac{r^{(K+1)+1} - 1}{}$$

Set - A

Q-3:- $F(x,y)$:- x can fool y.

(i)

(a) $\forall y\, F(I, y)$

(b) $\forall x\, F(x,x)$

(c) $\exists x \forall y\, F(x,y)$

(d) $\exists y \exists z \{ F(Ralph, y) \wedge F(Ralph, z) \wedge (y \neq z) \}$.

(ii)

(a) Negation

$\equiv \exists y \sim F(I, y)$

$\equiv$ I can not fool someone.

(b) $\exists x \sim F(x,x)$

$\equiv$ Someone can not fool himself.

(c) $\forall x \exists y \sim F(x,y)$

$\equiv$ Everyone can not fool somebody.

(d) $\forall y \forall z \sim \{ F(Ralph, y) \wedge F(Ralph, z) \wedge (y \neq z) \}$

OR

$\equiv \forall y \forall z \{ \sim F(Ralph, y) \vee \sim F(Ralph, z) \vee \sim (y \neq z) \}$.

$\Rightarrow$ It's not true that Ralph can fool any two different people.

Set-B

Q-3: $F(x,y)$: x can fool y.

(i) (a) $\forall y \neg F(\text{George}, y)$

(b) $\forall x \neg F(x,x)$

(c) ~~$\forall x \exists y F(x,$~~ $\exists y \forall x F(x,y)$

(d) $\exists y \exists z \{F(\text{Ralph},y) \wedge F(\text{Ralph},z) \wedge (y \neq z)\}$

Negation

(a) $\exists y \; F(\text{George}, y)$

$\Rightarrow$ George can fool somebody.

(b) $\exists x \; F(x,x)$

$\Rightarrow$ Someone can fool himself.

(c) $\forall y \exists x \neg F(x,y)$

$\Rightarrow$ There is someone who can't fool everyone

(d) $\forall y \forall z \neg \{F(\text{Ralph},y) \wedge F(\text{Ralph},z) \wedge (y \neq z)\}$

$\Rightarrow$ Ralph can't fool two different set of peope.

OR

$\Rightarrow$ It's not true that Ralph can fool any two different people

Set-B

Q.4 :-

L.H.S.

$\sim(p \vee (\sim p \wedge q))$

$\Rightarrow \sim((p \vee \sim p) \wedge (p \vee q))$ Associative law

$\Rightarrow \sim(T \wedge (p \vee q))$ Negation Law.

$\Rightarrow \sim(p \vee q)$ Identity ~~De'Morgan's~~ Law.

$\Rightarrow \sim p \wedge \sim q$ De'Morgan's law.

Set-A.

Q:- 4. $(p\to q)\to[(p\to q)\to q]$

$\equiv (p\to q)\to[\sim(p\to q)\vee q]$ $\quad (p\to q \Rightarrow \sim p\vee q)$

$\equiv (p\to q)\to[(p\wedge\sim q)\vee q]$ $\quad$,,

$\equiv (p\to q)\to[(p\vee q)\wedge\underbrace{(\sim q\vee q)}_{T}]$ Negation Law

$\equiv (p\to q)\to(p\vee q)$ $\quad$ Ident

$\equiv \sim(p\to q)\vee(p\vee q)$ $\quad (p\to q\Rightarrow \sim p\vee q)$

$\equiv (p\wedge\sim q)\vee(p\vee q)$ $\quad (p\to q\Rightarrow\sim p\vee q$ and D'morgans)

$\equiv (p\vee(p\vee q))\wedge((p\vee q)\vee\sim q)$ Associtative law.

$\equiv ((p\vee q)\wedge p)$ $\quad$ Identity Law.

To be $((p\vee q)\wedge p)$ to be true.

$(p\vee q)$ should be true & p should be true. ba

but p can have either true or false value. Hence the given expression is not a tautology.

Set - B

5) In addition to union (∪), intersection (∩), difference (-) and power set (2A), let us add the following two operations tc
• Pairwise addition: $A \oplus B := \{a + b : a \in A, b \in B\}$ (This is also called the Minkowski addition of sets A and B.)
• Pairwise multiplication: $A \otimes B := \{a \times b : a \in A, b \in B\}$
For example,
if $A = \{1, 2\}$ and $B = \{10, 100\}$,
then
$A \oplus B = \{11, 12, 101, 102\}$
and
$A \otimes B = \{10, 20, 100, 200\}$.
Now answer the following questions:
(a) Briefly describe the following sets:
i. $N \oplus \emptyset$
ii. $N \oplus N$
iii. $N+ \oplus N+$
iv. $N+ \otimes N+$
where N is a set of all natural numbers that includes zero also, and N+ is a set of all positive natural numbers.

(a) (I) $N = \{0, 1, 2, 3, 4, 5, - - - - - +\infty\}$

$\phi = \{ \}$

$N \oplus \phi = \{ \}$ or ϕ

(II) $N \oplus N = \{0, 1, 2, 3, - - - - +\infty\} \oplus \{0, 1, 2, 3, - - +\infty\}$

$= \{0, 1, 2, 3, - - - - +\infty\} = N$

(III) $N^+ = \{1, 2, 3, 4, - - - +\infty\}$

$N^+ \oplus N^+ = \{1, 2, 3, 4, - - - +\infty\} \oplus \{1, 2, 3, 4, - - +\infty\}$

$= \{2, 3, 4, 5, - - - - +\infty\} = N^+ - \{1\}$

$= N - \{0, 1\}$

(IV) $N^+ \otimes N^+ = \{1, 2, 3, 4 - - - +\infty\} \otimes \{1, 2, 3, 4, - - - +\infty\}$

$= \{1, 2, 3, 4, - - +\infty\} = N^+$

(b) (10 points) If E is the set of all positive even numbers, what's the shortest way to write the set of all positive multiples of 4? (Use pairwise multiplication).

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set of all positive even number's = E

$$E = \{2k \,;\, k \in N^+\}$$

$$N^+ = \{1, 2, 3, 4, \cdots +\infty\}$$

$$E = \{2, 4, 6, 8, 10, \cdots +\infty\}$$

set of all positive multiples of 4 = $E \otimes E$

$$= \{4, 8, 12, 16, \cdots +\infty\}$$

Set - A

5) 5. In addition to union ($\cup$), intersection ($\cap$), difference (-) and power set (2^A), let us add the following two operations

- Pairwise addition: $A \oplus B := \{a + b : a \in A, b \in B\}$ (This is also called the Minkowski addition of sets A and B.)
- Pairwise multiplication: $A \otimes B := \{a \times b : a \in A, b \in B\}$

For example, if $A = \{1, 2\}$ and $B = \{10, 100\}$, then
$A \oplus B = \{11, 12, 101, 102\}$ and $A \otimes B = \{10, 20, 100, 200\}$.
Now answer the following questions:
(a) Briefly describe the following sets:
i). $I \oplus \emptyset$ ii). $I \oplus I$ iii). $I^+ \oplus I^+$ iv). $I^+ \otimes I^+$
where I is a set of all integer numbers and I+ is a set of all positive integer numbers.

(a) (I) $I = \{-\infty \cdots -2, -1, 0, 1, 2, \cdots +\infty\}$

$\emptyset = \{\}$

$I \oplus \emptyset = \{\}$ or $\emptyset$

(II) $I \oplus I = \{-\infty \cdots -2, -1, 0, 1, 2, \cdots +\infty\}$

$\oplus \{-\infty, \cdots -2, -1, 0, 1, 2, \cdots +\infty\}$

$= \{-\infty, \cdots -2, -1, 0, 1, 2, \cdots +\infty\} = I$

(III) $I^+ = \{1, 2, 3, 4, \cdots +\infty\}$

$I^+ \oplus I^+ = \{1, 2, 3, \cdots +\infty\} \oplus \{1, 2, 3, \cdots +\infty\}$

$= \{2, 3, 4, \cdots +\infty\}$

$= I^+ - \{1\}$

(IV) $I^+ \otimes I^+ = \{1, 2, 3, \cdots +\infty\} \otimes \{1, 2, 3, \cdots +\infty\}$

$= \{1, 2, 3, \cdots +\infty\}$

$= I^+$

(b) (10 points) If E is the set of all positive even numbers, what's the shortest way to write the set of all positive multiples of 4? (Use pairwise multiplication).

=

E = set of all positive even numbers's.

$$E = \{2k,\ k \in I^+\}$$

$$I^+ = \{1, 2, 3, 4, \ldots +\infty\}$$

$$E = \{2, 4, 6, 8, \ldots +\infty\}$$

set of all positive multiples of 4 $= E \otimes E$

$$= \{4, 8, 12, 16, \ldots +\infty\}$$

6) Let Triangle(x), Circle(x), and Square(x) mean “x is a triangle,
” “x is a circle,” and “x is a square”;
let White(x), Gray(x), and Black(x) mean “x is white,” “x is gray,”
and “x is black”;
let RightOf(x, y), Above(x, y), and SameColorAs(x, y) mean
“x is to the right of y,” “x is above y,” and “x has the same color as y”;
and use the notation x = y to denote the predicate “x is equal to y”.
Let the common domain D of all variables be the set of all the objects in the Tarski world.
Use formal, logical notation to write each of the following statements,
and write a formal negation for each statement.

a. For all circles x, x is above f .
b. There is a square x such that x is black.
c. For all circles x, there is a square y such that x and y have the same color.
d. There is a square x such that for all triangles y, x is to right of y.

a. *Statement:* $\forall x(\text{Circle}(x) \to \text{Above}(x, f))$.
Negation: $\sim(\forall x(\text{Circle}(x) \to \text{Above}(x, f)))$

$\equiv \exists x \sim (\text{Circle}(x) \to \text{Above}(x, f))$

by the law for negating a $\forall$ statement

$\equiv \exists x(\text{Circle}(x) \wedge \sim\text{Above}(x, f))$

by the law of negating an if-then statement

b. *Statement:* $\exists x(\text{Square}(x) \wedge \text{Black}(x))$.
Negation: $\sim(\exists x(\text{Square}(x) \wedge \text{Black}(x)))$

$\equiv \forall x \sim (\text{Square}(x) \wedge \text{Black}(x))$

by the law for negating a $\exists$ statement

$\equiv \forall x(\sim\text{Square}(x) \vee \sim\text{Black}(x))$

by De Morgan’s law

c. *Statement:* $\forall x(\text{Circle}(x) \to \exists y(\text{Square}(y) \wedge \text{SameColor}(x, y)))$.
Negation: $\sim(\forall x(\text{Circle}(x) \to \exists y(\text{Square}(y) \wedge \text{SameColor}(x, y))))$

$\equiv \exists x \sim (\text{Circle}(x) \to \exists y(\text{Square}(y) \wedge \text{SameColor}(x, y)))$

by the law for negating a $\forall$ statement

$\equiv \exists x(\text{Circle}(x) \wedge \sim(\exists y(\text{Square}(y) \wedge \text{SameColor}(x, y))))$

by the law for negating an if-then statement

$\equiv \exists x(\text{Circle}(x) \wedge \forall y(\sim(\text{Square}(y) \wedge \text{SameColor}(x, y))))$

by the law for negating a $\exists$ statement

$\equiv \exists x(\text{Circle}(x) \wedge \forall y(\sim\text{Square}(y) \vee \sim\text{SameColor}(x, y)))$

by De Morgan's law

d. *Statement:* $\exists x(\text{Square}(x) \wedge \forall y(\text{Triangle}(y) \to \text{RightOf}(x, y)))$.

Negation: $\sim(\exists x(\text{Square}(x) \wedge \forall y(\text{Triangle}(y) \to \text{RightOf}(x, y))))$

$\equiv \forall x \sim (\text{Square}(x) \wedge \forall y(\text{Triangle}(x) \to \text{RightOf}(x, y)))$

by the law for negating a $\exists$ statement

$\equiv \forall x(\sim\text{Square}(x) \vee \sim(\forall y(\text{Triangle}(y) \to \text{RightOf}(x, y))))$

by De Morgan's law

$\equiv \forall x(\sim\text{Square}(x) \vee \exists y(\sim(\text{Triangle}(y) \to \text{RightOf}(x, y))))$

by the law for negating a $\forall$ statement

$\equiv \forall x(\sim\text{Square}(x) \vee \exists y(\text{Triangle}(y) \wedge \sim\text{RightOf}(x, y)))$

by the law for negating an if-then statement