Department of Mathematics, Bennett University
Engineering Calculus (EMAT101L)
Tutorial Sheet 5 (Continuity)

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1. Investigate the continuity at the indicated point:

   (a) at $x = c$, where $f(x) = \begin{cases} \frac{\sin(x-c)}{(x-c)} & x \neq c \\ 0 & x = c \end{cases}$

   (b) at $x = 2$, where $f(x) = \begin{cases} \frac{x^3-8}{x^2-4} & x \neq 2 \\ 3 & x = 2 \end{cases}$

   (c) at $x = 0$, where $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & x \neq 0 \\ 4 & x = 0 \end{cases}$

   (d) at $x = 0$ where $f(x) = \begin{cases} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} & x \neq 0 \\ 0 & x = 0 \end{cases}$

2. A function f defined on $\mathbb{R}$ by

$$f(x) = \begin{cases} -x^2 & x \leq 0 \\ 5x - 4 & 0 < x \leq 1 \\ 4x^2 - 3x & 1 < x < 2 \\ 3x + 4 & x \geq 2 \end{cases}$$

   Examine f for continuity at $x = 0, 1, 2$. Also discuss the kind of discontinuity, if any.

3. Is the function f, where

$$f(x) = \begin{cases} \frac{x-|x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

   continuous at $x = 0$?

4. Using intermediate value theorem, show that there exists $c \in (-2, 0)$ such that

$$c^{179} + \frac{163}{1 + c^2 + \sin^2 c} = 119.$$

5. Determine if the following equations admits solutions in the interval mentioned.

   (a) $x^5 - 3x^2 = -1,\ [0, 1]$ $\qquad$ (b) $\sin^2 x - 2\cos x = -1,\ \left[0, \frac{\pi}{2}\right]$

6. a) Give an example of a function which is continuous only at one point.

   (b) Give an example of a function which is continuous everywhere.