POSSESSION OF MOBILES IN EXAM IS UFM PRACTICE

Name of Student:————————————**Enrollment Number:** ————————

BENNETT UNIVERSITY, GREATER NOIDA
Supplementary Examination, December 2019

COURSE CODE :	EMAT102L	MAX. TIME: 2 Hours.
COURSE NAME :	Linear Algebra and Ordinary Differential Equations	
COURSE CREDIT:	3-1-0-4	MAX. MARKS: 100

Instructions

- There are **ten** questions in this question paper and all questions are mandatory.
- Rough work must be carried out at the back of the answer script.

1. For what values of $\lambda \in \mathbb{R}$, the following system of equations has [10]
(i) no solution, (ii) a unique solution, and (iii) infinitely many solutions?
$x + y + 2z = 3,\ 2y + \lambda z = 6,\ 4z = 8.$

2. Apply Gram-Schmidt process to the set $\{[1,\ 1,\ 0]^t, [0,\ 0,\ 1]^t, [\ 1,\ 1,\ 1]^t\}$ to obtain an orthonormal set in $\mathbb{R}^3$. [10]

3. Let A be a diagonalizable matrix such that each eigenvalue of A is equal to 2. Prove that $A = 2I$. [5]

4. The following statements are true/false. Justify your answer. [3× 5=15]

   (*a*) $W = \{A \in M_{n\times n}(\mathbb{R}) : A \text{ is singular}\}$ is a subspace of $M_{n\times n}(\mathbb{R})$.

   (*b*) If the eigenvalues of a 3×3 matrix A are $2, i$, then $traceA = 2$, $detA = -2$.

   (*c*) Let $T : M_{3\times 4}(\mathbb{R}) \to M_{2\times 3}(\mathbb{R})$ be a linear transformation which is onto, then dimension of nullspace of T is 3.

   (*d*) The vectors $(2, 0, 1, 1)$ and $(-1, 2, i, 2)$ in $\mathbb{C}^4(\mathbb{R})$ are orthogonal.

   (*e*) If f, g both are continuous functions on $[0, 1]$, then
$$\int_0^{10} f(x)g(x)dx \le \left(\int_0^{10} |f(x)|^2 dx\right)^{\frac{1}{2}} \left(\int_0^{10} |g(x)|^2 dx\right)^{\frac{1}{2}}.$$

5. Find an orthogonal basis for the subspace $W = \{p(x) \in \mathcal{P}_3(\mathbb{R}) : p(0) = p(1) = 0\}$, where the inner product is given by $\langle p, q\rangle = \displaystyle\int_{-1}^{1} p(x)q(x)dx$. [10]

6. Under what conditions on a and b, the following differential equation [8]

$$(x^3 + xy^2)dx + (ax^2y + bxy^2)dy = 0.$$

is exact?

7. Do any TWO parts. $[2 \times 8 = 16]$

(a) If y_1 and y_2 are linearly independent solutions of $xy'' + 2y' + xe^x y = 0, x \in (0, \infty)$ and if $W(y_1, y_2)(1) = 2$, find the value of $W(y_1, y_2)(5)$.

(b) Test whether the differential equation $(x + y)^2 dx - (y^2 - 2xy - x^2)dy = 0$ is exact or not and hence solve it.

(c) Discuss the existence and uniqueness of the solution for the IVP

$$\frac{dy}{dx} = 16 + y^2, \ y(0) = 0, \ |x| \leq 1, \ |y| \leq 1.$$

8. (a) Show that $y = a\cos(mx + b)$ is a solution of $\dfrac{d^2y}{dx^2} + m^2y = 0$. [4]

(b) Check whether $y_1(x) = \sin x$ and $y_2(x) = \cos x$ are linearly independent solutions of the differential equation $y'' + y = 0$, $x \in \mathbb{R}$ or not? [6]

9. By using the method of variation of parameters, find the general solution of the following differential equation. [6]

$$y'' + y = \sec x.$$

10. (a) Find the inverse Laplace transform of $\dfrac{1}{s(s+7)}$. [4]

(b) Solve the following system of differential equation using Laplace transforms [6]
$y_1' + y_2 = 2\cos x, \quad y_1 + y_2' = 0, \ \ y_1(0) = 0, \quad y_2(0) = 1.$