Name of Student -- Enrolment No. ------------------

Department / School ----------------------------

BENNETT UNIVERSITY, GREATER NOIDA

Supplementary Examination, July 2019

COURSE CODE: **ECSE203L / ECSE209L**   MAX. DURATION: **2 Hours**

COURSE NAME: **Discrete Mathematical Structures**

COURSE CREDIT: **4**   MAX. MARKS: **50**

Note

- All the questions are compulsory.
- Please write precisely and neatly.
- Please make clear diagram wherever required.

Q1) (a) Compute the minimal spanning tree for the following graph G using Kruskal's and Prims's algorithm: **(3+3 = 6 marks)**
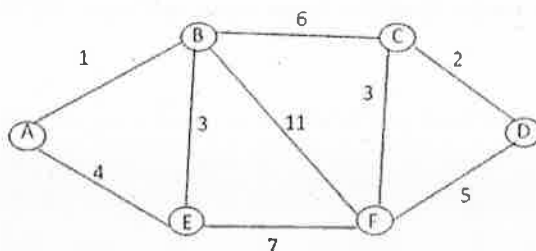

Fig 1: A weighted graph G with 6 vertices

(b) Given the preorder and inorder traversal of a binary tree, draw the unique tree:

(2 marks)

Preorder: A B D E C F G H I
Inorder: D B E A F C H G I

Q2) (a) Consider the traveling salesman problem. State whether the solution to this problem lies in finding the Hamiltonian path, Euler path, Euler circuit or Hamiltonian circuit. Justify your answer with one complete example.   **(2+3 = 5 marks)**

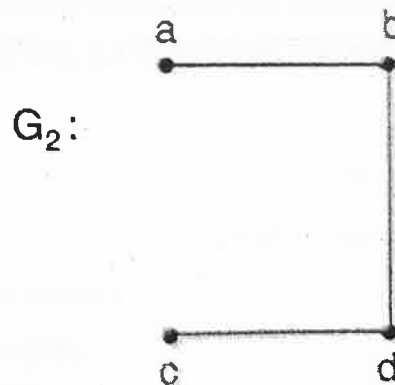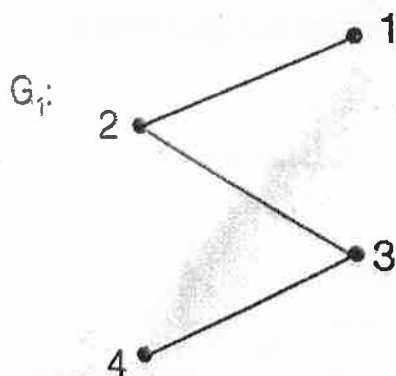(b) Show that the given pair of graphs are isomorphic: **(2 marks)**

Fig 2: A pair of graphs G_1 and G_2

Q3) Use mathematical induction to prove that **(2 marks)**

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Q4) (a) Verify the validity of the inference. If one person is more successful than another, then he has worked harder to deserve success. John has not worked harder than Peter. Therefore, john is not successful than Peter. **(2 marks)**

(b) Let P: We should be honest. Q: We should be dedicated. R: We should be overconfident. Then 'We should be honest or dedicated but not overconfident.' is best represented by: **(1 mark)**

(i) $\sim P \vee \sim Q \vee R$
(ii) $P \wedge \sim Q \wedge R$
(iii) $P \vee Q \wedge R$
(iv) $P \vee Q \wedge \sim R$

Q5) In A survey of 100 students, it was found that 30 studied Mathematics, 54 studied Statistics, 25 studied Operations Research, 1 studied all the three subjects, 20 studied Mathematics and Statistics, 3 studied Mathematics and Operation Research and 15 studied Statistics and Operation Research. Find how many students studied none of these subjects and how many students studied only Mathematics? Draw an appropriate Venn diagram
. **(2 marks)**

Q6) (a) Determine the negation of the following statement:
"For all real numbers x, if $x > 3$ then $x^2 > 9$." **(1 mark)**
(b) Show that the function $f(x) = x^3$ and $g(x) = x^{1/3}$ for all $x \in R$ are inverses of one another. **(1 mark)**

Q7) Let $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Prove that $B = C$ where A, B and C are three sets. **(3 marks)**

Q8) Let m be a positive integer with $m > 1$. Show that the relation
$$R = \{ (a, b) | a \equiv b \ (mod \ m) \}$$
is an equivalence relation on the set of integers. **(3 marks)**
(Note: Here, $a \equiv b \ (mod \ m)$ is read as a is congruent to b modulo m and its equivalent form is $a - b$ is divisible by m).

Q9) State whether the following statements are true/false: **(2 marks)**
(a) A finite commutative ring R with identity is a field if R has no zero divisors.
(b) Every field is an integral domain.

Q10) A bag has some pens. If these pens were equally distributed to:
(i) four students, then three pens left in the bag.
(ii) five students, then two pens left in the bag.
(iii) seven students, then four pens left in the bag.
Find the minimum number of pens in the bag. **(3 marks)**

Q11) Describe how the following cases define functions: **(3 marks)**

(a) The formula for converting degree measure into radian measure is given by
$r = (\pi / 180) * d$.
(b) Let $P(x)$ denote the refund/income tax payment calculated on a tax form for a given year that is owed to/by the person whose social security number is x.

Q12) Let P be the proposition "a man has discovered something he will die for" and let Q be the proposition "he is fit to live". Consider the implication $(\neg P) \rightarrow (\neg Q)$: " If a man hasn't discovered something he will die for, then he isn't fit to live". **(3+1 = 4 marks)**
(a) Write the three derived implications (both symbolically and in English).
(b) Assume that the original implication is true. Briefly discuss what we know about the truth of the derived implications.

Q13) Compute the secret message produced from the message "MEET YOU IN THE PARK" using the Caesar cipher. **(2 marks)**

Q14) If $A = \{1, 2, 3\}, B = \{4,5\}, C = \{1, 2, 3, 4, 5\}$, Find **(2 marks)**
    (a) $A \times B$
    (b) $C \times B$
    (c) $B \times B$
Hence, prove that $(C \times B) - (A \times B) = (B \times B)$

Q15) Discuss and determine the number of ways possible to wear 5 rings on 4 fingers for the following two cases: **(2+2 = 4 marks)**
(a) All rings are identical and assuming all 5 rings are worn.
(b) All rings are distinct and assuming all 5 rings are worn.