Mid-Semester Examination, Even Semester 2022-23

EMAT 102L

Solution File (with evaluation policy).

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1) ~~not~~ a subspace (1 mark)

reason (1 mark)

counter example:

$$(1,-1,0) \in S$$
$$(0,1,1) \in S$$

But $(1,-1,0)+(0,1,1) = (1,0,1) \notin S$.

Note: one can give other counter examples.
the counter examples are such that one point will satisfy $x+y=0$ and the other to satisfy $y-z=0$.

2) $a=1,\ b=1$ (1 mark)

$c \in \mathbb{R}$ (1 mark).

3) $\begin{cases} x+y+z=6 \\ x+2y+3z=10 \\ x+2y+\lambda z=\mu \end{cases}$ clearly $\boxed{\lambda=3,\ \mu=10}$.

(1 marks)

For procedure/ | one can use row echelon form of the augmented reason 1 mark | matrix.

4) $\alpha(1,1,1,1) + \beta(1,-1,1,-1) + \gamma(1,1,-1,-1) = (0,0,0,0)$
($\frac{1}{2}$ marks)

$$\Rightarrow \begin{cases} \alpha+\beta+\gamma = 0 \\ \alpha-\beta+\gamma = 0 \\ \alpha+\beta-\gamma = 0 \\ \alpha-\beta-\gamma = 0 \end{cases}$$ ($\frac{1}{2}$ marks)

$$\boxed{\alpha = \beta = \gamma = 0}$$ ($\frac{1}{2}$ marks)

$\therefore$ The given set of vectors are Linearly independent
($\frac{1}{2}$ marks)

Alternate procedure:

One can find rank of the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}.$$ The rank in this case should be 3. (1 mark)

LI (1 mark).

5) $T: \mathbb{R}^3 \xrightarrow{L.T.} \mathbb{R}^3$

$T(x,y,z) = (x+y,0)$.

$$\left.\begin{aligned} T(1,0,0) &= (1,0) \\ T(0,1,0) &= (1,0) \\ T(0,0,1) &= (0,0) \end{aligned}\right\}$$ (1 marks)

$R(T)$: range space of T = span$\{(1,0)\}$ (1 mark) or $\{(k,0) \mid \forall k \in \mathbb{R}\}$

$\text{rank}(T) = 1$ (1 mark).

6) a Basis $\left\{ \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}$ or $\left\{ \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right\}$ ($\frac{1}{2}$ mark)

dimension = 1 ($\frac{1}{2}$ mark)

7) $T : \mathbb{R}^2 \xrightarrow{L.T.} \mathbb{R}^2$

$T(x,y) = (x-y,\ 2x-2y)$

$T(x,y) = (0,0)$

$\Rightarrow (x-y,\ 2x-2y) = (0,0)$ ($\frac{1}{2}$ marks)

$\Rightarrow x-y=0 \Rightarrow \boxed{x=y}$

$\mathcal{N}(T) := \text{null space of } T = \text{span}\{(1,1)\}$.

or $\{(k,k)\ \forall k \in \mathbb{R}\}$

($\frac{1}{2}$ marks)

8) $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 4 & 5 & 6 \end{pmatrix} \xrightarrow{R_3 \to R_3 + 2R_1} B = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 6 & 9 & 12 \end{pmatrix}$

$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ (1 mark)

9) $(2,2,2) = \alpha(1,2,1) + \beta(1,0,1)$

$\Rightarrow \begin{cases} \alpha + \beta = 2 \\ 2\alpha = 2 \\ \alpha + \beta = 2 \end{cases}$ ($\frac{1}{2}$ marks)

Justification of your answer carries $\frac{1}{2}$ marks

$\alpha+\beta=2$ $\qquad\qquad\qquad\qquad$ $\frac{1}{2}$ mark.

$\Rightarrow \boxed{\alpha = 1, \beta = 1}$

$\therefore (2,2,2) \in \text{span}\{(1,2,1), (1,0,1)\}$ $\quad$ ($\frac{1}{2}$ mark)

10) $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$T(x,y) = (x,-y)$. $\quad$ (1 mark)