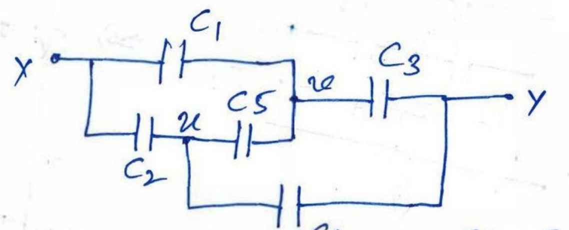
CSET102L

Tutorial Sheet - 7 (Solutions)

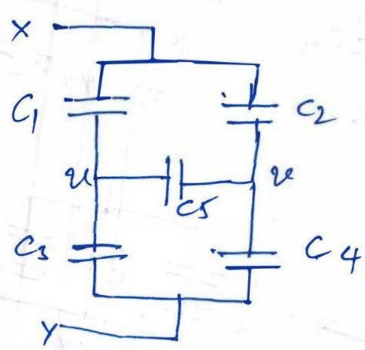
1)

Fig. 4



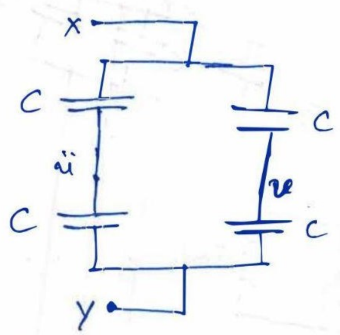
$C_i = C$

re draw the circuit



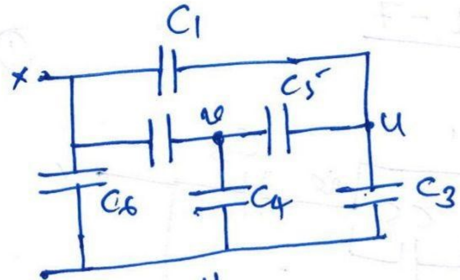
$\because C_i = C,$

Capacitor between u and v acts like a open. Then, re-drawing the circuit



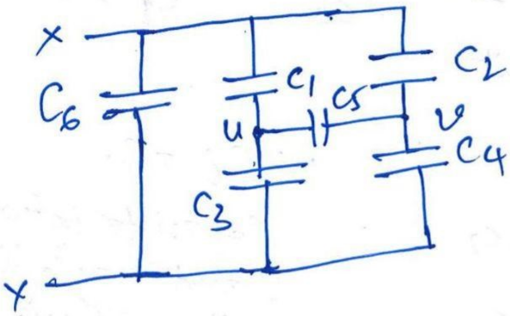
$\Rightarrow C_{xy} = C$

Note: Fig. 6 is similar except one more capacitor in parallel

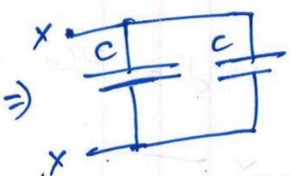


⇒ $C_i = C$

⇓

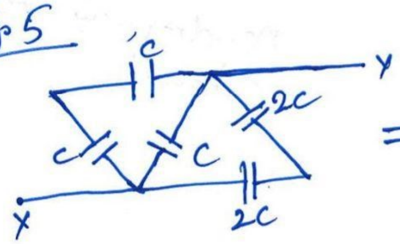


⇒

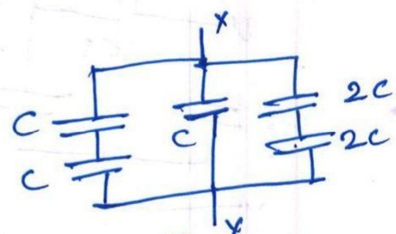


⇒ $C_{xy} = \underline{2C}$

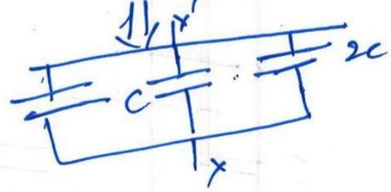
Fig 5



⟹



⇓



$C_{eq} = 2.5C$ ⇐

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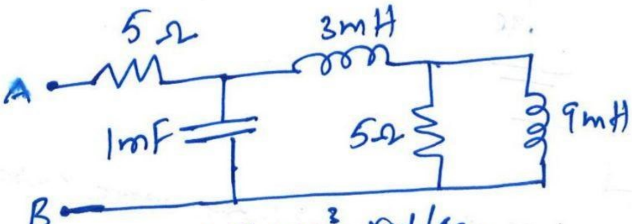


$\omega = 2.5 \times 10^3$ rad/sec.

$X_{1mF} = -\frac{j}{\omega C} = -j0.4\ \Omega$

$X_{3mH} = j\omega L = j7.5\ \Omega$

$X_{9mH} = j\omega L = j22.5\ \Omega$

$$5 \parallel X_{9mH} = \frac{5 \times j22.5}{5 + j22.5} = \frac{112.5j\,(5 - j22.5)}{(5 + j22.5)(5 - j22.5)}$$

$$= \frac{562.5j + 2531.25}{(5)^2 + (22.5)^2} = 4.76 + 1.05j$$

$$(5 \parallel X_{9mH}) + X_{3mH} = 4.76 + 1.05j + j7.5$$

$$= 4.76 + j8.55$$

$$X_{1mF} \parallel \left[(5 \parallel X_{9mH}) + X_{3mH} \right] =$$

$$= \frac{(4.76 + j8.55)(-j0.4)}{(4.76 + j8.55 - j0.4)}$$

$$= \frac{3.42 - j1.904}{(4.76 + j8.15)}$$

writing in $|z| \angle \theta$ or $r \angle \theta$ form,

$3.42 - j1.904 = 3.91 \angle -29.1^\circ$ $\quad \theta = \tan^{-1}(b/a)$

$(a) \quad (-b)$

$4.76 + j8.15 = 9.438 \angle 59.71^\circ$ $\quad r = \sqrt{a^2 + b^2}$

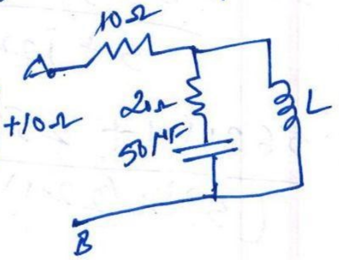
$$X = \frac{3.91 \angle -29.1}{9.438 \angle 59.71^\circ} = 0.414 \angle 88.1^\circ$$

$$= -0.414 + 0.014j$$

$$Z = 5\Omega + X = 4.586 + j0.014$$

4) given $Z_{AB} = 25 + j10\Omega$ $\omega = 4\times10^3$ rad/sec.

$$Z_{AB} = \left[j\omega L \,//\, \left(20 - \frac{j}{\omega C}\right)\right] + 10\Omega$$



$25+j10 = 10 + j\omega L$ $\omega C =$

$$20 - \frac{j}{\omega C} = 20 - j5$$

$$j\omega L \,//\, (20 - j5) = \frac{j\omega L(20-j5)}{(20 - j5 + j\omega L)}$$

$$25 + j10 - 10 = \frac{j20\omega L + 5\omega L}{(20 + j(\omega L - 5))}$$

$$(15+j10)\left[20 + j(\omega L - 5)\right] = 5\omega L + j20\omega L$$

Equating real parts

$$300 - 10\omega L + 50 = 5\omega L \Rightarrow L = \underline{5.83\,mH}$$

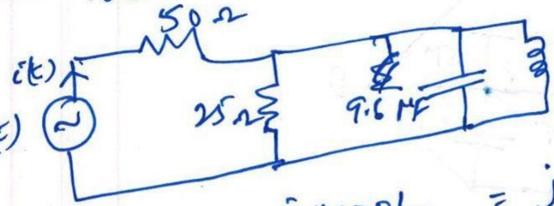
Equating imaginary parts,

$$200j - 75j + j15\omega L = 20\omega L$$

$$125 = 5\omega L \Rightarrow L = \underline{6.25\,mH}$$

So two inductance values are possible.

5) Given that $v(t)$ and $i(t)$ are in phase



$\omega = 4000$ rad/sec.

$L' = 1000L$

$$X_L = j\omega L = j4000L = j4L'$$

$$X_C = \frac{-j}{\omega C} = -26.04j$$

$$Z_{eq} = 50 + (X_L \parallel X_C \parallel 25)$$

for phase to be zero, imaginary part needs to be zero.

$$X_L \parallel X_C \parallel 25 = \frac{X_C \times X_L * 25}{X_C X_L + X_C 25 + X_L 25}$$

$$\frac{2604L'}{104.16L' + j100L' - 651j} + 50$$

$$= \frac{2604L'(104.16L' - j(100L' - 651))}{(104.16L')^2 + (100L' - 651)^2} + 50$$

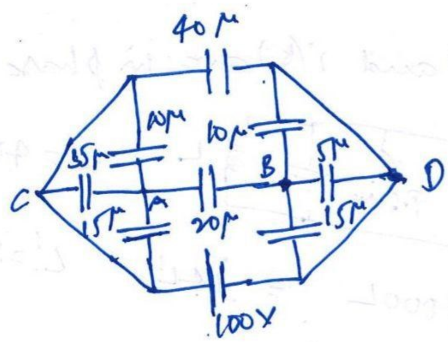
Equating imaginary part to zero,

$$\cancel{104.16L'} \quad 100L' - 651 = 0$$

$$L' = \frac{651}{100} = 6.51$$

$$L' = 100L \quad \therefore L = \underline{6.51 \text{ mH}}$$

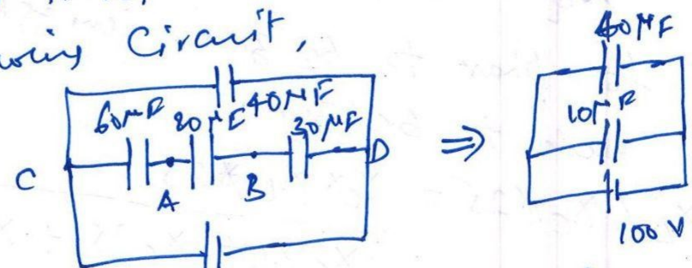
6)



10μ ∥ 35μ ∥ 15μ = 60μF

5μ ∥ 15μ ∥ 10μ = 30μF

re-drawing Circuit,



As 60μF, 20μF, 30μF are in series,

$C_{eq} = 1/\left[\frac{1}{60} + \frac{1}{20} + \frac{1}{30}\right] = 10\mu F$

$10\mu F \parallel 40\mu F = 50\mu F$

$Q = C_{eq} V = 50 \times 10^{-6} \times 100 =$
$5000\mu C$

Charge in 10μF capacitance is
100 × 10μF = 1000μC

So charge on 20μF capacitance is
~~200~~ 1000μC

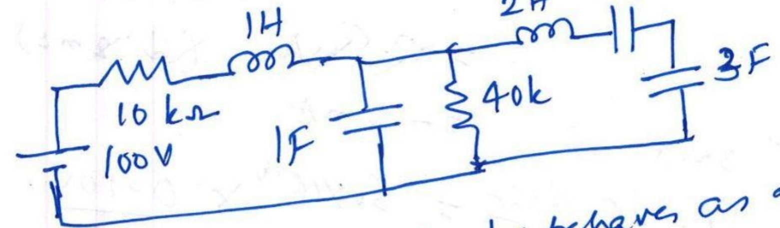
So voltage across 20μF capacitance is

$V = \frac{Q}{C} = 50V$

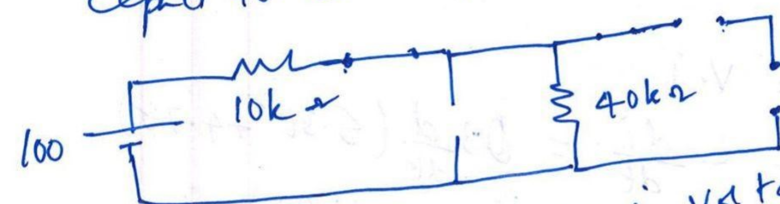
So energy stored in $\frac{1}{2} CV^2 = \underline{0.025 W}$

2F

7)



In steedy state, inductor behaves as a short, capacitor as a open. Redrawing the circuit



So Voltage across 40kΩ is Voltage across 2F, 3F capacitors combined.

$$V_{40k} = \frac{100 \times 40}{10 + 40} = 80\,V$$

$$V_{2F} = \frac{80}{C_2} \Big/ \left(\frac{1}{C_2} + \frac{1}{C_3}\right) = \frac{80 \times 3}{5} = \underline{48V}$$

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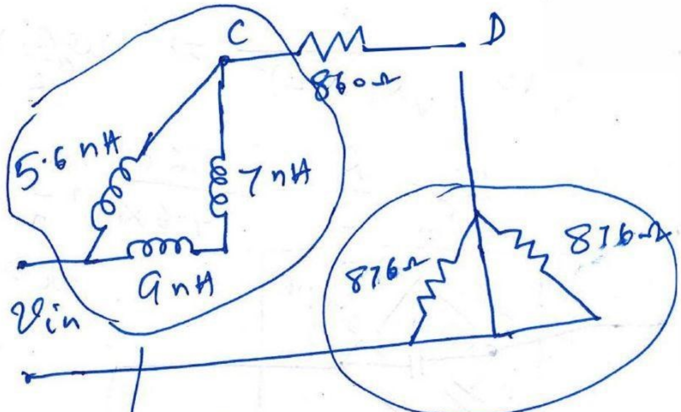
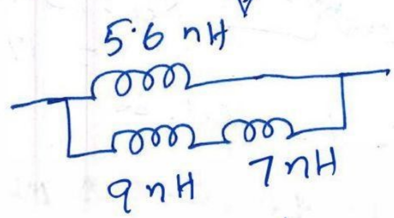
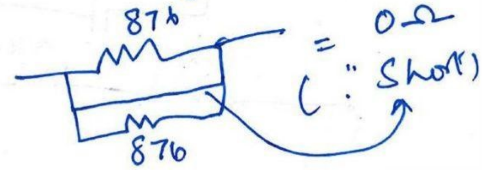
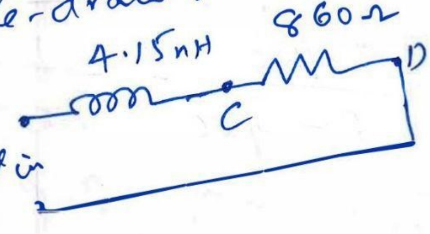






$$\frac{1}{L_{eq}} = \frac{1}{(9n+7n)} + \frac{1}{(5.6n)} \Rightarrow \underline{L_{eq} = 4.15\ nH}$$

Re-drawing the circuit



$$H(\omega) = \frac{V_R}{V_R + V_L}$$

$$= \frac{R}{R + j\omega L}$$

$$|H(\omega)| = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{1}{(1 + \omega^2 2.38 \times 10^{-23})^{1/2}}$$

$$\left.\begin{array}{l} \text{at } \omega = 0,\ |H(\omega)| = 1 \\ \text{at } \omega = \infty\ |H(\omega)| = 0 \end{array}\right\} \text{Thus a lowpass filter}$$

when $|H(\omega)| = \frac{1}{\sqrt{2}}$

$$\frac{1}{\sqrt{2}} = \frac{R}{\sqrt{R^2 + (\omega L)^2}} \Rightarrow \omega_c\left(\frac{L}{R}\right) = 1$$

$$\omega_c = \frac{R}{L} = \frac{860}{4.6 \times 10^{-9}} \frac{\Omega}{H} = \underline{207.3\,GHz}$$

f_C=32.98 GHz

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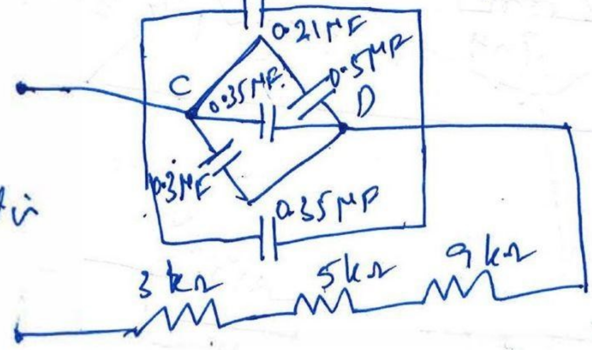
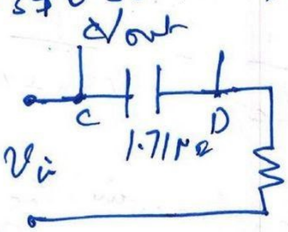


$R_{eq} = (3+5+9)\,k\Omega$
$= 17\,k\Omega$

All the capacitors are in parallel. so

$C_{eq} = (0.21 + 0.5 + 0.3 + 0.35 + 0.35)\,\mu F$
$= 1.71\,\mu F$

Redrawing the circuit



$$H(\omega) = \frac{V_C}{V_R + V_C} = \frac{-j/\omega C}{R - j/\omega C} = \frac{1}{1 + j\omega RC}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$ when $\omega = 0$

Low pass filter $\in \begin{cases} |H(\omega)| = 1 \\ |H(\omega)| = 0 \text{ for } \omega = \infty \end{cases}$

$|H(\omega)| = \frac{1}{\sqrt{2}} \Rightarrow \omega_c RC = 1$ so $f_c = \frac{1}{2\pi RC}$

$f_c = \underline{5.47\,Hz}$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (0.029)^2\omega^2}}$$

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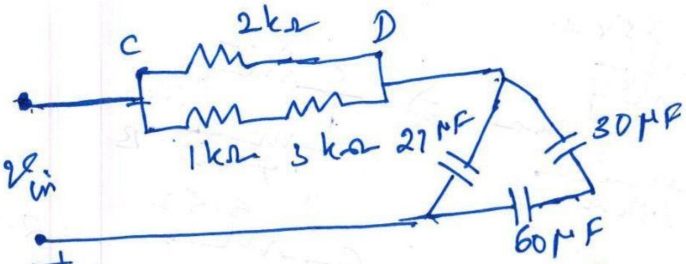
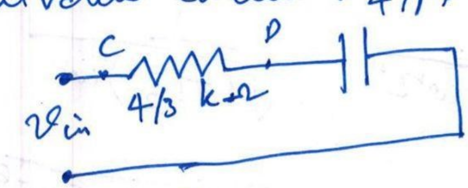


$$R_{eq} = \frac{1}{2k} + \frac{1}{(1k+3k)} \Rightarrow R_{eq} = \frac{4}{3}\ k\Omega$$

$$C_{eq} = 27\mu F + \frac{1}{\left(\frac{1}{60\mu} + \frac{1}{30\mu}\right)} = (27+20)\mu F = 47\mu F$$

Equivalent circuit:



$$H(\omega) = \frac{V_R}{V_R + V_C}$$

$$|H(\omega)| = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} = \frac{1}{\sqrt{1 + \frac{1}{(\omega RC)^2}}} = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

$$\left.\begin{array}{l} \omega \to 0 \Rightarrow H(\omega) \to 0 \\ \omega \to \infty \Rightarrow H(\omega) \to 1 \end{array}\right\} \rightarrow \text{High pass filter}$$

$$|H(\omega)| = \frac{0.063\omega}{\sqrt{1 + 4\times10^{-3}\omega^2}} \qquad f_c = \frac{1}{2\pi RC} = 2.54\ Hz$$

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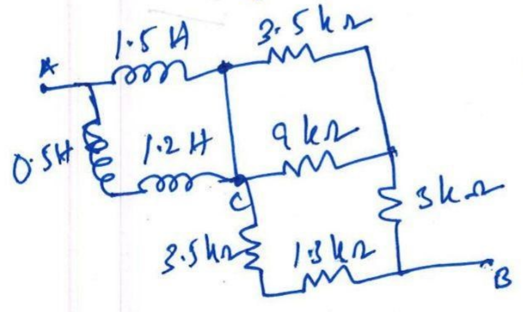
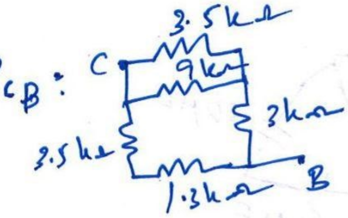


$$\frac{1}{L_{eq}} = \frac{1}{1.5} + \frac{1}{(1.2+0.5)}$$

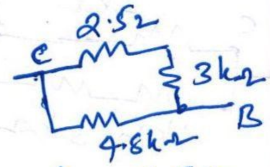
$$\Rightarrow L_{eq} \simeq 0.8H$$

(4)

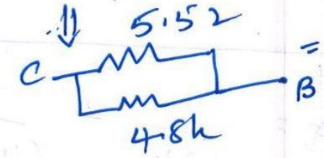
R_{CB} :



$\Rightarrow$

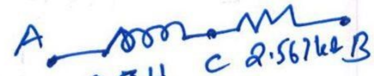


$\Downarrow$



$= 2.567 k\Omega$

Equivalent circuit:



$$H(\omega) = \frac{V_R}{V_L + V_C} = \frac{R}{j\omega L + R}$$

$$|H(\omega)| = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{1}{\sqrt{1 + \left(\frac{L}{R}\right)^2 \omega^2}}$$

$$= \frac{1}{\sqrt{1 + 0.097\omega^2 \times 10^{-6}}}$$

$$\left.\begin{array}{l} \omega \to 0 \quad |H(\omega)| \to 1 \\ \omega \to \infty \quad |H(\omega)| \to 0 \end{array}\right\} \Rightarrow \text{low pass filter}$$

$$f_c : \quad |H(\omega)| = \frac{1}{\sqrt{2}} \Rightarrow \frac{1}{2} = \frac{1}{1 + 0.097\omega^2 \times 10^{-6}}$$

$$\omega_c = \frac{1}{\sqrt{0.97}} \quad f_c = \frac{1}{2\pi \times \sqrt{0.097 \times 10^{-6}}}$$

$$= \underline{510\ Hz}$$