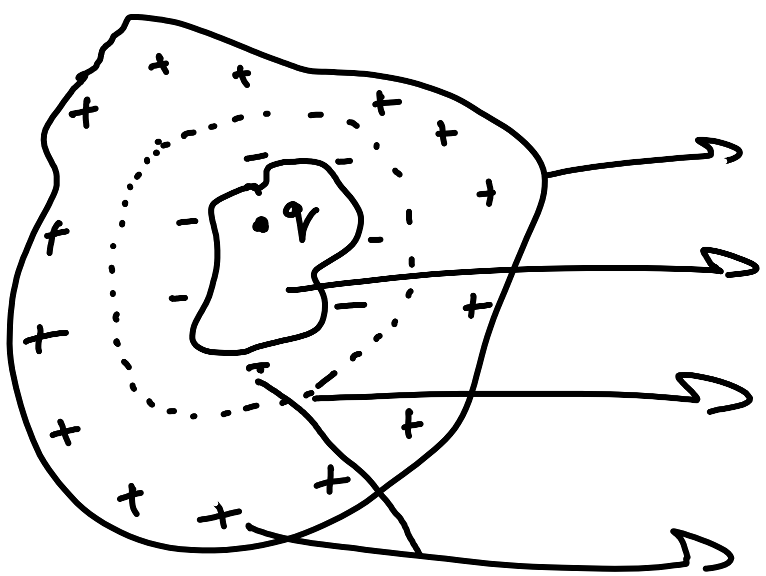
Cavity inside a conductor



→ The total charge induced on the cavity wall is equal and opposite to the charge inside.

For the Gaussian surface,

$$\oint \vec{E} \cdot d\vec{a} = 0 \Rightarrow \text{net enclosed charge} = 0$$

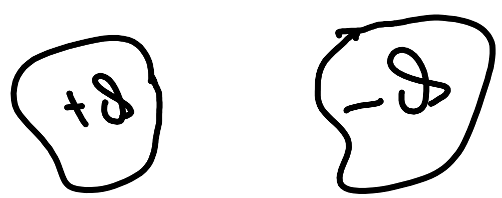
$$\Rightarrow q_{induced} = -q$$

• Field outside the conductor

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

Capacitor

→ Two conductors with charges $+q$ and $-q$.



The potential difference,

$$V = V_+ - V_- = -\int \vec{E} \cdot d\vec{l}$$

$$\vec{E} \propto q$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\rho}{r^2} \hat{r} \, d\tau$$

→ Increase ρ ⇒ We increase q

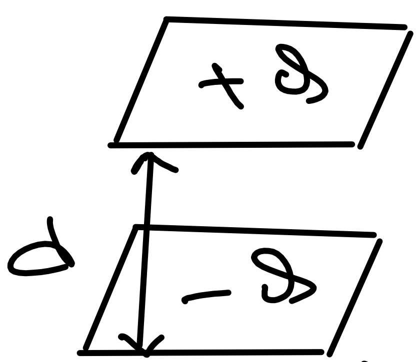
(x) V is also proportional to q.

→ The proportionality constant is called the capacitance.

$$C = \frac{q}{V}$$

↳ Determined by the sizes, shapes and separation betn. two conductors

Ex: Parallel plate capacitor



A → Area of the plates

σ → charge density (surface)

→ upper plate has charge $+q$ and lower one has $-q$

$$\vec{E} = \frac{q}{\epsilon_0 A} \Rightarrow V = \frac{q}{A\epsilon_0} d$$

d ↳ separation betn. the plates

$\Rightarrow C = \dfrac{A\epsilon_0}{d}$

⊛ Work done to charge up a capacitor to Q:

→ at some point the charge on the plate $= +q$

$\Rightarrow$ Potential diff. $= \dfrac{q}{C}$

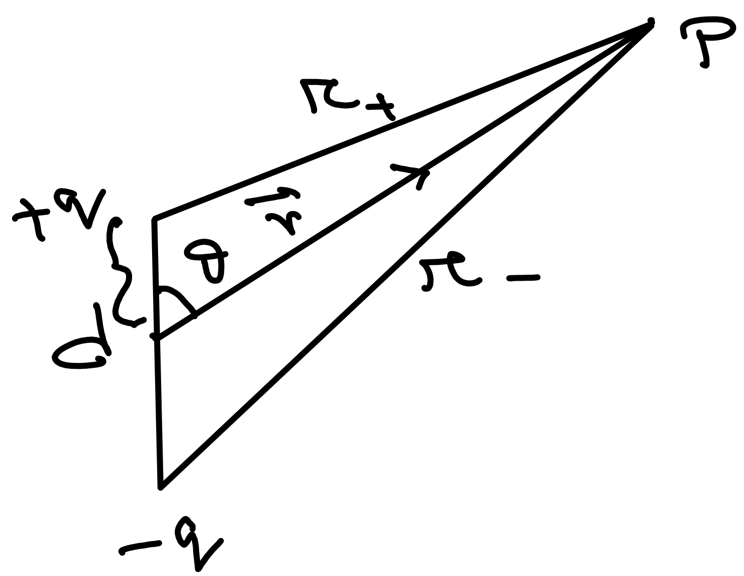
In order to bring dq (infinitesimal amount of charge) to the plate:

$$dW = \frac{q}{C}\,dq$$

The ~~total~~ work done

$$W = \int_0^Q \frac{q}{C}\,dq = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

Electric Dipole



An electric dipole with two equal and opposite charges ($\pm q$) separated by a distance 'd'.

Potential due to the dipole?

$r_- \equiv$ distance of P from $-q$

$r_+ \equiv$ " " " " $+q$

$\vec{r} \equiv$ Position vector of P.

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_+} - \frac{q}{r_-}\right)$$

$$r_\pm^2 = r^2 + \left(\frac{d}{2}\right)^2 \mp r d \cos\theta$$

$$= r^2\left(1 \mp \frac{d}{r}\cos\theta + \frac{d^2}{4r^2}\right)$$

$\frac{d^2}{4r^2} << 1$ (negligible) in the limit $\underline{r >> d}$

$$\frac{1}{r_\pm} \simeq \frac{1}{r}\left(1 \mp \frac{d}{r}\cos\theta\right)^{-1/2}$$

$$\simeq \frac{1}{r}\left(1 \pm \frac{d}{2r}\cos\theta\right)$$ (neglecting higher order terms)

Then, $$\frac{1}{r_+} - \frac{1}{r_-} \simeq \frac{d}{r^2}\cos\theta$$

$$\Rightarrow V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q\,d\cos\theta}{r^2}$$

$\rightsquigarrow$ The potential, $V(\vec{r}) \sim \frac{1}{r^2}$

$\rightarrow$ The potential falls off more rapidly than the potential of a point charge.

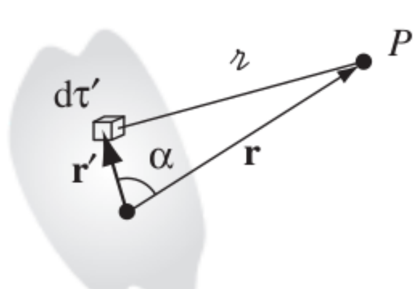
The monopole and dipole terms

The most dominant contribution in multipole expansion comes from the monopole term.

$$V_{mon.}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \qquad \Big| \; Q = \int \rho \, d\tau$$

→ If the total charge $Q = 0$, then the most dominant contribution comes from the dipole term.

$$V_{dip.}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\int r' \cos\alpha \, \rho(\vec{r}\,') \, d\tau'$$



α = Angle betn. $\vec{r}\,'$ and $\vec{r}$

$$r' \cos\alpha = \hat{r} \cdot \vec{r}\,'$$

$$\Rightarrow V_{dip.}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\hat{r} \cdot \int \vec{r}\,' \rho(\vec{r}\,') \, d\tau'$$

This integral (independent of $\vec{r}$) is called the <u>dipole moment</u> of the distribution.

$$\vec{p} = \int \vec{r}\,' \rho(\vec{r}\,')\, d\tau'$$

$$V_{dip.}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p}\cdot\hat{r}}{r^2}$$

* For a collection of point charges,

$$\vec{p} = \sum_{i=1}^{n} q_i \vec{r}_i\,'$$

* For a physical dipole with $\pm q$ charges

$$\vec{p} = q\left(\vec{r}_+\,' - \vec{r}_-\,'\right)$$

$$= q\vec{d}$$