POSSESSION OF MOBILES IN EXAM IS UFM PRACTICE.

Name of student: .... **Enrollment No.** ....

BENNETT UNIVERSITY, GREATER NOIDA
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COURSE CODE : EMAT101L	MAX. TIME: 2 Hours
COURSE NAME: ENGINEERING CALCULUS	
COURSE CREDIT: 3-1-0	MAX. MARKS: 100

Instructions:

- This paper contains 7 questions.
- All questions are mandatory.

1. (a) Show that $\lim_{x\to 0} \cos\left(\frac{1}{x}\right)$ does not exists. [5]

   (b) Determine the point of continuity for a function $f : \mathbb{R} \to \mathbb{R}$ defined as [5]

   $$f(x) = \begin{cases} 2x^2 & \text{if } x \in \mathbb{Q} \\ 8 & \text{if } x \notin \mathbb{Q}. \end{cases}$$

   (c) Prove or disprove the uniform continuity of the function $g : (0,1) \to \mathbb{R}$ as [5]

   $$g(x) = \frac{\sin x}{x}.$$

   (d) Is the following function $f : \mathbb{R}^2 \to \mathbb{R}$ is continuous at $(1,1)$? Justify! [5]

   $$f(x,y) = \begin{cases} x^2 + y^2 & (x,y) \neq (1,1) \\ 3 & x = y = 1. \end{cases}$$

2. True/False. Justify your answer. [3 × 10=30]

   (a) The series $\sum_{n=1}^{\infty} n^{\frac{1}{n}}$ diverges.

   (b) If $f : \mathbb{R} \to \mathbb{R}$ such that f^2 is continuous then f is continuous.

   (c) Every bounded sequence is convergent.

   (d) $|\sin x| - |\sin y| > |x - y|$ for every value $x, y \in \mathbb{R}$.

   (e) $f(x) = x^2 - x\sin x - \cos x$ has exactly one root in $(0, 2\pi)$.

   (f) Mean value theorem is used only for continuous functions.

(g) $\int_0^1 \frac{dx}{x+\sqrt{x}}$ converges.

(h) If $|f|$ is Riemann integrable then f is Riemann integrable.

(i) $f(x) = \begin{cases} x[x] & 0 \leq x \leq 4 \\ 0 & x = 0, \end{cases}$ is Riemann integrable.

(j) Every differentiable function is continuous function.

3. (a) Check the convergence of the infinite series $\displaystyle\sum_{n=0}^{\infty} \frac{2n!}{(3n+1)!}$. [5]

(b) Find all the values of x for which the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n4^n}$ converges. [5]

4. Let $f(x,y) = x^2 - 2xy + 3y^2 + 10$. Then

(a) Find the linear approximation of f about the point $(1,2)$. [5]

(b) Estimate the error, while approximating $f(x,y)$ with linear approximation in the rectangle $|x-1| < 0.1$, $|y-2| < 0.2$. [5]

5. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x,y) = \begin{cases} \dfrac{y\sqrt{x^2+y^2}}{|y|} & \text{if } y \neq 0 \\ 0 & \text{if } y = 0. \end{cases}$$

(a) Prove that f is continuous at $(0,0)$. [3]

(b) Prove that all the directional derivatives of f exists at $(0,0)$. [4]

(c) Check the differentiability of f at (0,0). [3]

6. Evaluate the following integral:

(a) $\displaystyle\iint_R x^2 dA$ where R is the region bounded by $y = x^2, y = x+2$. [5]

(b) $\int_0^{\ln 2} \int_1^{\ln 5} e^{2x+y} dy\, dx$. [5]

7. In the following, sketch the region of integration, reverse the order of integration, and evaluate the integral: $\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y} dy\, dx$. [10]