Started on	Tuesday, 20 July 2021, 9:02 AM
State	Finished
Completed on	Tuesday, 20 July 2021, 12:01 PM
Time taken	2 hours 58 mins
Grade	**15.50** out of 38.00 (**41**%)

Question 1

Incorrect

Mark 0.00 out of 2.00

Let $R : |x - 0| \leq 1, |y - 0| \leq 1$ be a rectangle. Consider the IVP $\frac{dy}{dx} = f(x, y), y(0) = 0$, where $f(x, y) = y^{2/9}$. Which of the following statements is/are correct?

Select one or more:

- [x] This IVP has no solution. ✗
- [ ] $f(x, y)$ does not satisfy the Lipschitz condition on R with respect to y.
- [x] This IVP has a solution in some neighborhood of 0, which may not be unique. ✓
- [ ] None of the above statements is correct.

Your answer is incorrect.

The correct answers are: $f(x, y)$ does not satisfy the Lipschitz condition on R with respect to y.

, This IVP has a solution in some neighborhood of 0, which may not be unique.

Question 2

Correct

Mark 2.00 out of 2.00

Consider the following differential equation:

$(\frac{d^2y}{dx^2})^{1/2} = x^2 \frac{d^3y}{dx^3}$.

Let O and D denote the order and degree respectively of this differential equation. Identify the correct statement(s):

Select one or more:

- [ ] $O - D = 1$ and the equation is linear.
- [ ] $O + D = 5$ and the equation is linear.
- [x] $O + D = 5$ and the equation is non-linear.
- [x] $O - D = 1$ and the equation is non-linear.

Your answer is correct.

The correct answers are: $O - D = 1$ and the equation is non-linear.

, $O + D = 5$ and the equation is non-linear.

Question 3

Incorrect

Mark -0.67 out of 2.00

Let $y_1(x)$ and $y_2(x)$ be two solutions of

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + (\sec x)y = 0 \text{ on } (-1,1)$$

with Wronskian $W(x)$. If $y_1(0) = 1,\ y_1'(0) = 0$ and $W(\frac{1}{2}) = \frac{1}{3}$, then $y_2'(0)$ equals:

Select one:

- $\frac{4}{3}$.
- $\frac{3}{4}$.
- $\frac{1}{4}$.
- 1.

Your answer is incorrect.

The correct answer is: $\frac{1}{4}$.

Question 4

Correct

Mark 2.00 out of 2.00

Consider the IVP $\frac{dy}{dx} = e^y, y(0) = 1.$

The iterate $y_2(x)$ obtained by using Picard's method of successive approximations on this IVP is given by $a + \int_0^x e^{(b+ct)}dt$, where a, b, c are some specific constants such that $a + b + c$ equals:

Select one or more:

- [ ] $1 + e.$
- [ ] $6.$
- [ ] $3.$
- [x] $2 + e.$

✓

Your answer is correct.

The correct answer is: $2 + e.$

Question 5

Incorrect

Mark 0.00 out of 3.00

Let V be a vector space of dimension n and $W_1, \cdots, W_m$ be subspaces of V. If $\epsilon = dim(W_1) + \cdots + dim(W_m) - (m-1)n \geq 1$, then

Select one or more:

- [x] there exists a non-zero $x \in V$ such that $x \in W_i$ for each $i = 1, \cdots, m.$

✓

- [ ] $dim(W_1 \cap \cdots \cap W_m) \geq \epsilon.$
- [ ] $dim(W_1 + \cdots + W_m) = \sum_{i=1}^m dim(W_i).$
- [x] $dim(W_1 \cap \cdots \cap W_m) < \epsilon.$

✗

Your answer is incorrect.

The correct answers are: $dim(W_1 \cap \cdots \cap W_m) \geq \epsilon.$

, there exists a non-zero $x \in V$ such that $x \in W_i$ for each $i = 1, \cdots, m.$

Question 6

Incorrect

Mark 0.00 out of 2.00

Consider the following differential equation: $(2y^2 + 3x)dx + 2xydy = 0$.

Identify the correct statement(s):

Select one or more:

- [ ] The differential equation is not exact and the general solution is $ax^2y + bx^3y^2 = C$, where C is an arbitrary constant and a, b are some specific constants that satisfy the relation $a + b = 8$.
- [ ] The differential equation is not exact and $5x$ is an integrating factor.
- [x] The differential equation is not exact and the general solution is $x^a y - xy + y^b = C$, where C is an arbitrary constant and a, b are some specific constants that satisfy the relation $a + b = 4$.

✗

- [x] The differential equation is not exact and x is an integrating factor.

✓

Your answer is incorrect.

The correct answers are: The differential equation is not exact and x is an integrating factor.

, The differential equation is not exact and $5x$ is an integrating factor.

Question 7

Correct

Mark 2.00 out of 2.00

Which of the following is/are correct statement(s) about the solution of the IVP:
$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$, where $y(0) = 0, y'(0) = 1, y''(0) = 1.$

Select one or more:

- [ ] $y''(x) \to 0$ as $x \to \infty$.
- [x] $e^{-3x}y'(x) \to -3$ as $x \to \infty$.
- [ ] $y''(x) + 2e^x < 0$ for all real x.
- [x] It is also a solution of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = -3e^{2x}.$

Your answer is correct.

The correct answers are: It is also a solution of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = -3e^{2x}.$

, $e^{-3x}y'(x) \to -3$ as $x \to \infty$.

Question 8

Incorrect

Mark 0.00 out of 2.00

Let $T : C^3 \to C^3$ be linear such that $T((1,0,0)) = (2,0,i), T((0,1,0)) = (0,3,3), T((0,0,1)) = (i,1,0)$, where C is the set of complex numbers. Then

Select one or more:

- [ ] $dim(range(T) + null(T)) = 2.$
- [x] $range(T) \cap null(T) = \{0\}.$
- [ ] $null(T) \subset range(T).$
- [x] $range(T) + null(T) \neq C^3.$

Your answer is incorrect.

The correct answers are: $range(T) \cap null(T) = \{0\}.$
, $null(T) \subset range(T).$

Question 9

Correct

Mark 2.00 out of 2.00

Consider the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \sin x$. Suppose that a particular integral of this differential equation by the method of variation of parameters is given by $a(\sin bx)e^{cx}$. Then identify the correct statement(s):

Select one or more:

- [ ] $(a+b)c = 1.$
- [x] $a + b = 0.$
- [ ] $a^2 + b^2 = 1.$
- [x] $a + b + c = 1.$

Your answer is correct.

The correct answers are: $a + b + c = 1.$
, $a + b = 0.$

Question 10

Incorrect

Mark -0.67 out of 2.00

The solution of $y'' + a_1y' + a_2y = 0$, where a_1 and a_2 are constants, approaches to zero as $x \to \infty$, then

Select one:

- ◉ $a_1 > 0, a_2 < 0.$
- ○ $a_1 < 0, a_2 < 0.$
- ○ $a_1 < 0, a_2 > 0.$
- ○ $a_1 > 0, a_2 > 0.$

Your answer is incorrect.

The correct answer is: $a_1 > 0, a_2 > 0.$

Question 11

Correct

Mark 2.00 out of 2.00

Let $R : |x - 0| \leq \frac{\pi}{2}, |y - 0| \leq 5$ be a rectangle. Consider the IVP: $\frac{dy}{dx} = y \cos 2x, y(0) = 0.$

Which of the following statements is/are correct about this IVP?

Select one or more:

- [x] This IVP has a unique solution on the interval $|x| \leq \frac{1}{2}$.
- [ ] This IVP has no solution.
- [x] This IVP has a unique solution on the interval $|x| \leq 1$.
- [ ] This IVP has infinitely many solutions on the interval $|x| \leq 1$.

Your answer is correct.

The correct answers are: This IVP has a unique solution on the interval $|x| \leq \frac{1}{2}$.

, This IVP has a unique solution on the interval $|x| \leq 1$.

Question 12

Correct

Mark 2.00 out of 2.00

Which of the following is/are correct statement(s) about the solution of the IVP $4\frac{d^3y}{dx^3} + \frac{dy}{dx} + 5y = 0$, where $y(0) = 2, y'(0) = 1, y''(0) = -1$.

Select one or more:

- [ ] $y(t) \to 0$ as $t \to \infty$.
- [x] $y(\pi/2) > 0$.
- [x] $y(t) \to \infty$ as $t \to \infty$.
- [x] $y'(\pi/2) < 0$.

Your answer is correct.

The correct answers are: $y(t) \to \infty$ as $t \to \infty$.

, $y(\pi/2) > 0$.
, $y'(\pi/2) < 0$.

Question 13

Incorrect

Mark 0.00 out of 2.00

Which of the following is/are correct statement(s) about the solution of the IVP $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0$, where $y(0) = 1, y'(0) = 0, y''(0) = 1$.

Select one or more:

- [x] It is also a solution of $\frac{d^2y}{dx^2} - 4y = -3\cos 2x$.
- [ ] It is also a solution of $\frac{d^2y}{dx^2} - 4y = 2\sin 2x$.
- [ ] $|y''(x) - 2y'(x)| \leq 10$ for all real x.
- [ ] $y''(x) \to 0$ as $x \to \infty$.

Your answer is incorrect.

The correct answer is: $|y''(x) - 2y'(x)| \leq 10$ for all real x.

Question 14

Correct

Mark 2.00 out of 2.00

If the two roots of a cubic auxiliary equation with real coefficients are

0 and $3+i$, then what is the corresponding homogeneous differential equation?

Select one:

- $\frac{d^3y}{dx^3}+(3+i)\frac{dy}{dx}=0.$
- $\frac{d^3y}{dx^3}-6\frac{d^2y}{dx^2}+10\frac{dy}{dx}=0.$ ✓
- $\frac{d^3y}{dx^3}-(3+i)\frac{dy}{dx}=0.$
- $\frac{d^3y}{dx^3}+6\frac{d^2y}{dx^2}+10\frac{dy}{dx}=0.$

Your answer is correct.

The correct answer is: $\frac{d^3y}{dx^3}-6\frac{d^2y}{dx^2}+10\frac{dy}{dx}=0.$

Question 15

Partially correct

Mark 1.50 out of 3.00

Let V be a finite dimensional vector space and let T be a linear map on V such that $dim(null(T^2)) = dim(null(T))$. Then

Select one or more:

- [x] $range(T) \cap null(T) = \{0\}$.
- [ ] $range(T) \cap null(T) \neq \{0\}$.
- [ ] $null(T^2) = null(T)$.
- [ ] $range(T) \neq range(T^2)$.

Your answer is partially correct.

You have correctly selected 1.

The correct answers are: $range(T) \cap null(T) = \{0\}$.

, $null(T^2) = null(T)$.

Question 16

Complete

Not graded

The function $F(t)$ such that $L\{F(t)\} = \frac{12((s+1)^2-16)}{((s+1)^2+4)^3}$,

is given by

Select one:

- $F(t) = te^{-t}\sin 2t.$
- $F(t) = t^3e^{-t}\sin 2t.$
- $F(t) = t^2e^{-t}\sin 2t.$

  Bonus marks due to a typo in the question.
- $F(t) = t^2e^{t}\sin 2t.$

Your answer is correct.

Bonus marks due to a typo in the question.

The correct answer is: $F(t) = t^2e^{-t}\sin 2t.$

Question 17

Incorrect

Mark 0.00 out of 2.00

If y_1 and y_2 are two linearly independent solutions of

$e^x \frac{d^2y}{dx^2} - \frac{dy}{dx} + x^2 y = 0$ on $(0, \infty)$ and if $W(y_1, y_2)(1) = 2$, then what is the value of $W(y_1, y_2)(2)$?

Select one or more:

- [ ] $e^{e-e^{-2}}$.
- [ ] $e^{e^{-2}}$.
- [ ] $2e^{e^{-1}-e^{-2}}$.
- [x] $2e^{e^2-e}$.

Your answer is incorrect.

The correct answer is: $2e^{e^{-1}-e^{-2}}$.

Question 18

Correct

Mark 2.00 out of 2.00

By Laplace transform the particular solution of the following IVP

$X'(t) = \begin{bmatrix} 7 & -4 \\ 2 & 3 \end{bmatrix} X(t)$ where

$X \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, is given by

$*HereX(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$

Select one:

- $x(t) = e^{5t}(2\cos 2t + 4\sin 2t), y(t) = e^{5t}(-\cos 2t + 3\sin 2t),$ ✓
- $x(t) = e^{-5t}(2\cos 2t + 4\sin 2t), y(t) = e^{-5t}(-\cos 2t + 3\sin 2t),$
- $x(t) = e^{-5t}(2\cos 2t + 4\sin 2t), y(t) = e^{5t}(-\cos 2t + 3\sin 2t),$
- $x(t) = e^{5t}(2\cos 2t + 4\sin 2t), y(t) = e^{-5t}(-\cos 2t + 3\sin 2t),$

Your answer is correct.

The correct answer is: $x(t) = e^{5t}(2\cos 2t + 4\sin 2t), y(t) = e^{5t}(-\cos 2t + 3\sin 2t),$

Question 19

Incorrect

Mark -0.67 out of 2.00

Consider the vector space $V = \{f | f : \{0, 1\} \to R\}$ over R under the usual addition and scalar multiplication of functions. Then

Select one:

- V is finite dimensional and $dim(V) = 4$.
- V is finite dimensional and $dim(V) = 3$.
- V is infinite-dimensional.
- V is finite dimensional and $dim(V) = 2$.

Your answer is incorrect.

The correct answer is: V is finite dimensional and $dim(V) = 2$.