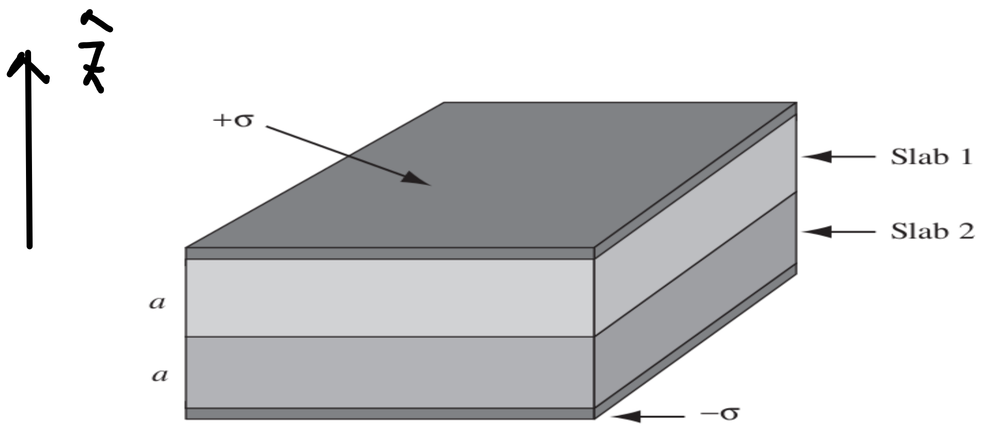
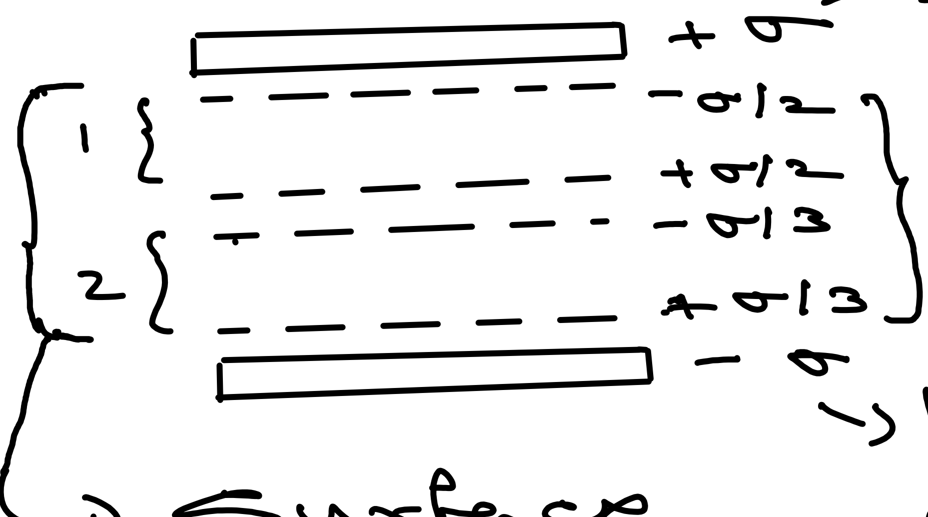
Ex (contd)



Space betn. two capacitor plates filled with dielectric slab 1 and slab 2 with dielectric const = 2 and 1.5 respectively.



$+\sigma \rightarrow$ free charge

$-\sigma \rightarrow$ free charge

$\rightarrow$ Surface bound charge density (σ_b)

We found:

$$\vec{D} = \sigma(-\hat{z})$$

$$\vec{E}_1 = \frac{\sigma}{2\epsilon_0}(-\hat{z}) \; ; \; \vec{P}_1 = \frac{\sigma}{2}(-\hat{z})$$

$$\vec{E}_2 = \frac{2\sigma}{3\epsilon_0}(-\hat{z}) \; ; \; \vec{P}_2 = \frac{\sigma}{3}(-\hat{z})$$

* Volume bound charge density

$$\rho_b = 0$$

* Potential difference, $\Delta V = \dfrac{7\sigma a}{6\epsilon_0}$

$$= \frac{7 q a}{6 A \epsilon_0}$$

$\sigma = \dfrac{q}{A}$ ($q \rightarrow$ total charge on plate; $A \rightarrow$ Area of plate)

* Capacitance, $C = \dfrac{Q}{\Delta V} = \dfrac{6 A \epsilon_0}{7 a}$

* Check the electric fields from knowledge of bound and free charges:

<u>Slab 1</u>: Total surface charge above $= \sigma - \frac{\sigma}{2} = \frac{\sigma}{2}$

" " " below $= -\dfrac{\sigma}{2}$

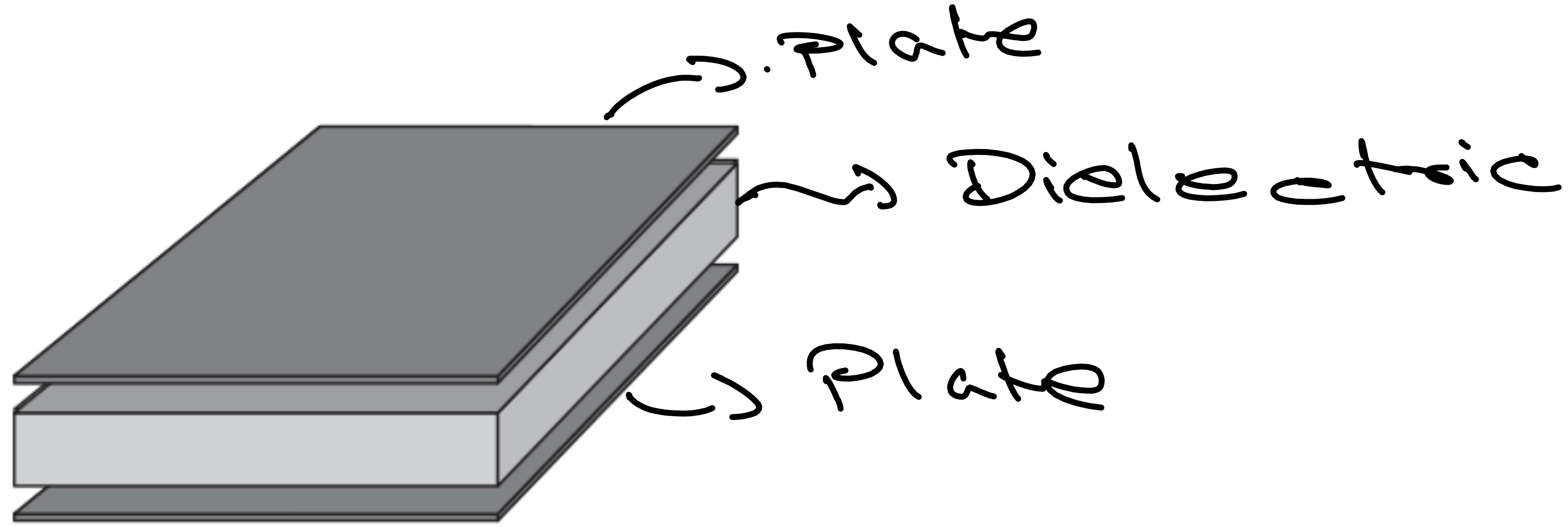
<u>Slab 2</u>: " " " above $= \dfrac{2\sigma}{3}$

" " " below $= \frac{\sigma}{3} - \sigma = -\dfrac{2\sigma}{3}$

Then, the electric fields:

$$
\begin{cases}
\vec{E}_1 = \dfrac{\sigma}{2\epsilon_0}(-\hat{z}) \\[2ex]
\vec{E}_2 = \dfrac{2\sigma}{3\epsilon_0}(-\hat{z})
\end{cases}
$$

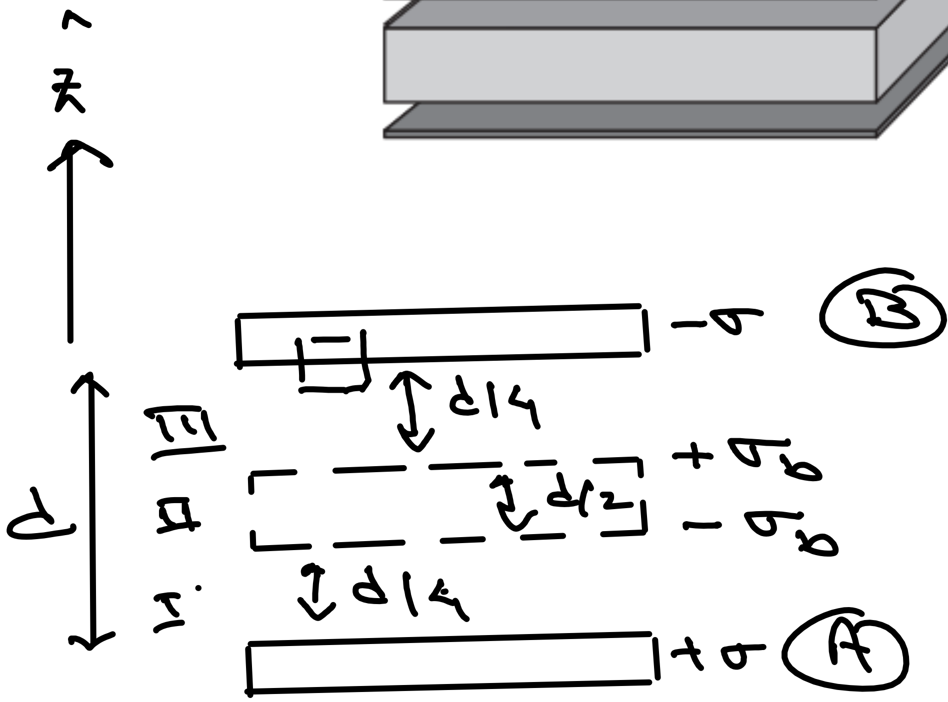
Ex:



* The space betn. the two plates is <u>half filled</u> with a dielectric slab.

→ Spacing betn the two plates = d

⇒ Thickness of the dielectric slab = $\frac{d}{2}$



* Electric Displacement, $(\vec{D})$

$$\oint \vec{D} \cdot d\vec{a} = (q_f)_{enc.}$$

$$\Rightarrow |\vec{D}|\, A = \sigma A$$

$$\Rightarrow \vec{D} = \sigma \hat{z} \quad \text{(in all regions)}$$

⊛ $\vec{E} = \dfrac{\sigma}{\epsilon_0}\,\hat{z}$ (I)

$= \dfrac{\sigma}{\epsilon}\,\hat{z}$ (II)

$= \dfrac{\sigma}{\epsilon_0}\,\hat{z}$ (III)

⊛ Polarization: $\vec{P} = 0$ (I)

$= 0$ (III)

in region (II)

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$= \frac{\epsilon_0 \chi_e \sigma}{\epsilon}\,\hat{z}$$

$$= \frac{\epsilon_0 \chi_e \sigma}{\epsilon_0 (1+\chi_e)}\,\hat{z} = \frac{\chi_e \sigma}{1+\chi_e}\,\hat{z}$$

(x) Bound charge densities:

$$\rho_b = 0$$ (polarisation is uniform)

$$\sigma_b = \vec{P} \cdot \hat{n} = \frac{\chi_e \sigma}{1+\chi_e} \hat{z} \cdot \hat{z}$$ (top plane of dielectric)

$$= \frac{\chi_e \sigma}{1+\chi_e}$$

$$\sigma_b = \frac{\chi_e \sigma}{1+\chi_e} \hat{z} \cdot (-\hat{z})$$ (bottom plane of dielectric)

$$= - \frac{\chi_e \sigma}{1+\chi_e}$$

(x) $$\Delta V = - \int_B^A \vec{E} \cdot d\vec{l} = \int_A^B \vec{E} \cdot d\vec{l}$$

$$= \underset{\mathrm{I}}{\int} \frac{\sigma}{\epsilon_0}\, dz + \underset{\mathrm{II}}{\int} \frac{\sigma}{\epsilon}\, dz + \underset{\mathrm{III}}{\int} \frac{\sigma}{\epsilon_0}\, dz$$

$$= \frac{\sigma}{\epsilon_0} \times \frac{d}{4} + \frac{\sigma}{\epsilon} \times \frac{d}{2} + \frac{\sigma}{\epsilon_0} \times \frac{d}{4}$$

$$= \frac{\sigma}{\epsilon_0} \frac{d}{2} + \frac{\sigma}{\epsilon} \frac{d}{2}$$

$$= \frac{\sigma d}{2} \left(\frac{1}{\epsilon_0} + \frac{1}{\epsilon} \right)$$

$$= \frac{Q d}{2 A \epsilon_0} \left(1 + \frac{\epsilon_0}{\epsilon} \right)$$

$Q \equiv$ Total charge on plate

$A \equiv$ Total area of plate

$\sigma = Q/A$

(*) Capacitance, $C = \dfrac{Q}{\Delta V}$

$$= \frac{2A\epsilon_0}{d\left(1+\frac{\epsilon_0}{\epsilon}\right)}$$

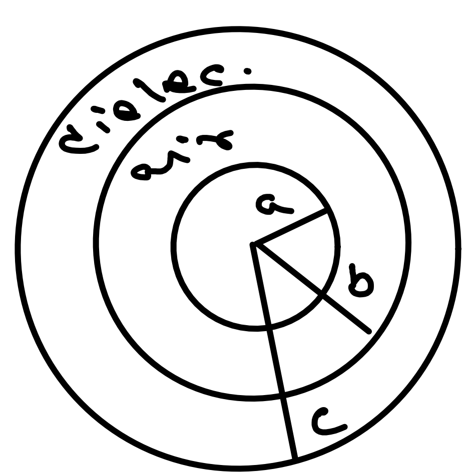
$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

$$= \frac{2\epsilon_0 A}{d} \frac{\epsilon}{\epsilon+\epsilon_0}$$

$$= \frac{2\epsilon_0 A}{d}\left(\frac{\epsilon_r}{1+\epsilon_r}\right)$$

→ Capacitance in absence of dielectric $= \frac{\epsilon_0 A}{d}$

Ex:



Coaxial cable consisting of copper wire (radius = a). surrounded by copper tube (radius = b). The space in betn. is filled with dielectric (const = ϵ_r)

↳ partially from b to c

$q \equiv$ Charge on a length 'l' on the copper wire,

$$\oint \vec{D} \cdot d\vec{a} = q$$

$$\Rightarrow \vec{D} = \frac{q}{2\pi r l} \hat{r}$$

Here, $r \equiv$ Arbitrary radial distance

$\hat{r} \equiv$ Radial unit vector.

$$\vec{E} = \frac{\vec{D}}{\epsilon_0} \quad (a < r < b)$$

$$= \frac{q}{2\pi r l \epsilon_0} \hat{r}$$

$$\vec{E} = \frac{q}{2\pi r l \epsilon} \hat{r} \quad (b < r < c)$$

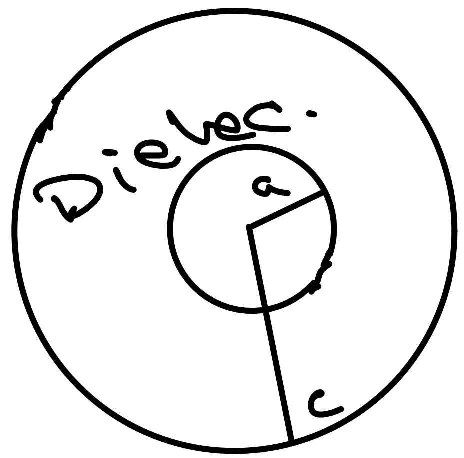
$$\Delta V = -\int_c^a \vec{E}\cdot d\vec{l} = \int_a^b \frac{\lambda}{2\pi\epsilon_0 r l}\,dr + \int_b^c \frac{\lambda}{2\pi\epsilon r l}\,dr$$

$$= \frac{\lambda}{2\pi\epsilon_0 l}\left[\ln\left(\frac{b}{a}\right) + \frac{\epsilon_0}{\epsilon}\ln\left(\frac{c}{b}\right)\right]$$

Capacitance per unit length:

$$\Rightarrow \frac{C}{l} = \frac{\lambda}{V l} = \frac{2\pi\epsilon_0}{\ln\left(\frac{b}{a}\right) + \left(\frac{1}{\epsilon_r}\right)\ln\left(\frac{c}{b}\right)}$$

Ex:



The space betn. is fully filled with dielectric.

Linear charge density on wire $= \lambda$

$Q = \lambda l$

$$\vec{E} = \frac{Q}{2\pi r l \epsilon}\hat{r} \quad (a < r < c)$$

$$= \frac{\lambda}{2\pi r \epsilon}\hat{r}$$

$$\Delta V = \int_a^b \vec{E}\cdot d\vec{r} = \int_a^b \frac{\lambda}{2\pi r \epsilon}\,dr$$

$$= \frac{\lambda}{2\pi\epsilon}\ln\left(\frac{b}{a}\right)$$

Capacitance, $$C = \frac{Q}{\Delta V} = \frac{2\pi\epsilon l}{\ln\left(\frac{b}{a}\right)}$$