BENNETT UNIVERSITY, GREATER NOIDA
Supplementary Examination, Fall SEMESTER 2019-20

Question	Marks
1	
2	
3	
4	
5	
Total	

Name of Student -- Enrolment No. ---------------------------

Department / School ------------------------------

COURSE CODE: **EPHY105L** MAX. DURATION: **2 Hours**
COURSE NAME: **Electromagnetics** MAX. MARK: **100**

A. **Please write answers to all questions in the space provided in the question paper itself.**
B. **Rough work elsewhere will not be graded**
C. **There are *FIVE* questions in the question paper**
D. **ALL QUESTIONS ARE COMPULSORY**

1. **Put a tick mark on the correct answer for the following questions (Tick marks on more than one choice will be allotted zero marks):** **(7x3=21)**

a) Two equal point charges $+Q$ are placed at points with Cartesian coordinates $(1, 0, 0)$ and $(-1, 0, 0)$. The electrostatic potential at the origin will be

i. Zero
ii. $Q/(4\pi\epsilon_o)$
iii. $Q/(2\pi\epsilon_o)$
iv. $2Q/\epsilon_o$

b) Consider a sphere of radius *R* carrying a total charge *Q* distributed uniformly in the entire volume of the sphere. The value of $\iiint \nabla.\vec{E}\, d\tau$ where the volume integration is over a sphere of radius *R*/2 concentric with the charge distribution will be

i. Zero
ii. $\frac{Q}{\epsilon_0}$
iii. $\frac{Q}{2\epsilon_0}$
iv. $\frac{Q}{8\epsilon_0}$

c) Consider a dielectric sphere of radius *R* which has a polarization given by $\vec{P} = P_0\hat{z}$ where P_0 is a constant. If ρ_b and σ_b represent bound volume and surface charge densities, then

i. $\rho_b = 0$ and $\sigma_b = 0$
ii. $\rho_b \neq 0$ and $\sigma_b = 0$.
iii. $\rho_b = 0$ and $\sigma_b \neq 0$
iv. $\rho_b \neq 0$ and $\sigma_b \neq 0$.

d) A straight cylindrical wire of radius *R* carries a current *I* distributed uniformly across its cross section. The axis of the wire is along $\hat{\mathbf{z}}$. The value of $\nabla \times \vec{B}$ at a point at a distance $R/4$ will be

i. $\frac{\mu_0 I}{\pi R^2}\hat{\mathbf{z}}$
ii. $\frac{16\mu_0 I}{\pi R^2}\hat{\mathbf{z}}$
iii. Zero
iv. $\mu_0 I\hat{\mathbf{z}}$

e) The vector function $\vec{F}_1 = xy\hat{\imath} + yz\hat{\jmath} + 2xz\hat{k}$

i. Can represent both an electrostatic field and a magnetic field
ii. Can represent an electrostatic field but not a magnetic field
iii. Can represent a magnetic field but not an electrostatic field
iv. Can represent neither an electrostatic field nor a magnetic field

f) Consider an infinitely long solenoid with circular cross section of radius R having n turns per unit length and carrying a current I. If a cylindrical rod of radius $a = R/2$ and made of a material of magnetic susceptibility χ_m is placed coaxially within the solenoid, then

i. $\vec{H}$ and $\vec{B}$ have the same value in the regions $0 < r < \frac{R}{2}$ and $\frac{R}{2} < r < R$.
ii. $\vec{H}$ has the same value in the regions $0 < r < \frac{R}{2}$ and $\frac{R}{2} < r < R$ but $\vec{B}$ values are different.
iii. $\vec{B}$ has the same value in the regions $0 < r < \frac{R}{2}$ and $\frac{R}{2} < r < R$ but $\vec{H}$ values are different
iv. $\vec{H}$ and $\vec{B}$ both have different values in the regions $0 < r < \frac{R}{2}$ and $\frac{R}{2} < r < R$.

g) An electromagnetic wave propagating in free space is described by the following expression for the electric field: $\vec{E} = \vec{E}_0 \cos[(5\pi \times 10^6 x + \omega t)]$.

i. The wavelength of the wave is $0.4\pi \times 10^{-6}$m.
ii. The direction of $\vec{E}_0$ is along $\hat{x}$.
iii. The magnetic field associated with the wave is parallel to $\vec{E}_0$.
iv. The wave is propagating along $-x$ direction.

2. Give brief answers to the following questions: **(6 x 6=36)**

a) Consider a point charge $+Q$ located at a point with Cartesian coordinates $(x_1, 0, 0)$. What will be the force $\vec{F}$ on a charge $-Q$ placed at a point with coordinates $(0, y_2, z_2)$?

b) An infinite dielectric slab with parallel surfaces and of thickness d and dielectric constant K is placed in a uniform electric field $\vec{E} = E_0\hat{z}$ pointing perpendicular to the surfaces. Calculate the bound surface charge density on the surface of the dielectric in terms of E_0.

c) In a certain region of space the electrostatic potential (in spherical polar coordinates) is given by $V(r, \theta, \phi) = -Cr^2$ where C is a constant. Obtain the electrostatic field $\vec{E}$ and the charge density ρ in this region.

d) A long cylindrical wire of radius R carries a current I distributed uniformly across its cross section with the unit vector $\hat{\mathbf{z}}$ pointing along the axis of the cylinder. Write down the values of $\oint \vec{\boldsymbol{B}}.d\vec{\boldsymbol{l}}$ and $\iint \vec{\boldsymbol{B}}.d\vec{\boldsymbol{a}}$ integrated over a circular loop of radius $R/2$ centered on the axis of the cylinder and placed parallel to the *x-y* plane.

$\oint \vec{\boldsymbol{B}}.d\vec{\boldsymbol{l}} =$

$\iint \vec{\boldsymbol{B}}.d\vec{\boldsymbol{a}} =$

e) Consider an infinitely long cylinder of circular cross section of radius R and axis along $\hat{\mathbf{z}}$ which is uniformly magnetized with magnetization $\vec{\boldsymbol{M}} = M_0\hat{\mathbf{z}}$ where M_0 is a constant. Obtain the corresponding bound volume and surface current densities.

f) A parallel plate capacitor with plate separation of 10 mm and filled with free space has a time varying applied voltage given by $V = V_0 \sin 2\pi f t$, with $V_0 = 10$ V and $f = 10^8$ Hz. Obtain the peak value of displacement current density between the plates.

3. A point charge Q is placed at the center of a dielectric sphere of radius R and dielectric constant K.

(a) Starting from Gauss's law, obtain the displacement vector $\vec{D}$ and the electrostatic field $\vec{E}$ in the regions $r < R$ and $r > R$. **(8)**

(b) Obtain the bound volume charge density within the volume and the surface charge density on the outer surface of the dielectric. **(6)**

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4. An infinitely long cylindrical tube with inner radius R_1 and outer radius R_2 made of a medium having a magnetic susceptibility given by χ_m carries a current I which is distributed uniformly across its cross section.
(a) From Ampere's law obtain the fields $\vec{H}$ and $\vec{B}$ in the region $r < R_1$ and $R_1 < r < R_2$ (9)
(b) Calculate the bound surface and volume current densities in the medium. (6)

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5. Consider a parallel plate capacitor with circular plates of radius R filled with free space. The conduction current charging the capacitor is given by $I_c(t)$.

   a) Show that the total displacement current between the capacitor plates will be equal to the conduction current flowing along the wire. **(6)**

   b) Using the appropriate Maxwell's equation, obtain an expression for the magnetic field generated between the capacitor plates. **(8)**

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Some useful formulas

- In Cartesian coordinates:

$$\nabla f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}$$

$$\nabla.\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\nabla \times \vec{F} = \left[\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right]\hat{\imath} + \left[\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right]\hat{\jmath} + \left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right]\hat{k}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

- In spherical polar coordinates:

$$\nabla f = \frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\nabla.\vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$$

$$\nabla \times \vec{F} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, F_\phi) - \frac{\partial F_\theta}{\partial \phi}\right]\hat{\boldsymbol{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial F_r}{\partial \phi} - \frac{\partial}{\partial r}(rF_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial F_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

- In cylindrical coordinates:

$$\nabla f = \frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$$

$$\nabla.\vec{F} = \frac{1}{r}\frac{\partial}{\partial r}(rF_r) + \frac{1}{r}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$$

$$\nabla \times \vec{F} = \left[\frac{1}{r}\frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z}\right]\hat{\boldsymbol{r}} + \left[\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right]\hat{\boldsymbol{\phi}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rF_\phi) - \frac{\partial F_r}{\partial \phi}\right]\hat{\boldsymbol{z}}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

- $\epsilon_0 = 8.85 \times 10^{-12}$ C^2/N-m^2
- Theorem on Gradients: $\int_a^b \nabla f.d\vec{l} = f(b) - f(a)$
- Divergence theorem: $\iiint \nabla.\vec{F}\, d\tau = \oiint \vec{F}.d\vec{a}$
- Stokes theorem: $\iint \nabla \times \vec{F}.d\vec{a} = \oint \vec{F}.d\vec{l}$
- **Maxwell's equations:**

$$\nabla.\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla.\vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial \vec{E}}{\partial t}$$