EPHY105L-4 [Meeting Code: 9pm1e7g] 7.10.21

Del Operator

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

Operator, only has meaning when it operates on something.

Operations

$\vec{\nabla} T \equiv$ Gradient

$\vec{\nabla} \cdot \vec{A} \equiv$ Divergence

$\vec{\nabla} \times \vec{A} \equiv$ Curl

Divergence

$\vec{\nabla} \cdot \vec{A} \equiv$ Measurement of how the vector spreads out

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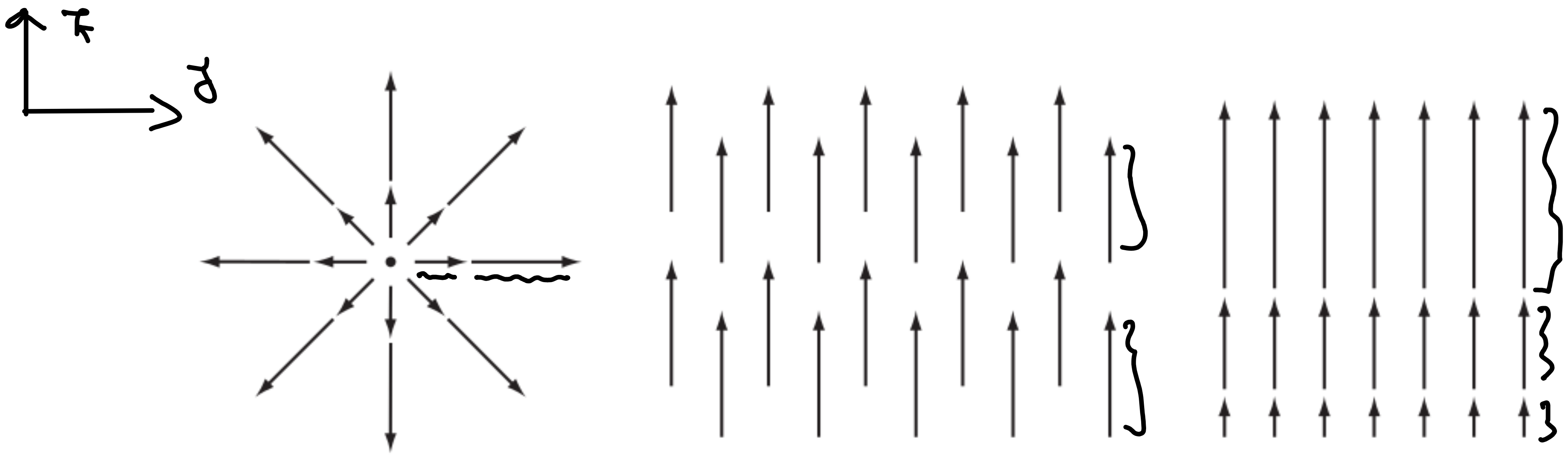

(a) (b) (c)

$\vec{A} = \underline{\underline{y}}\,\hat{y} + \underline{\underline{z}}\,\hat{z}$ $\qquad \vec{A} = c\,\hat{z}$ $\qquad \vec{A} = z\,\hat{z}$

$\vec{\nabla}\cdot\vec{A} \neq 0$ $\qquad \vec{\nabla}\cdot\vec{A} = 0$ $\qquad \vec{\nabla}\cdot\vec{A} \neq 0$

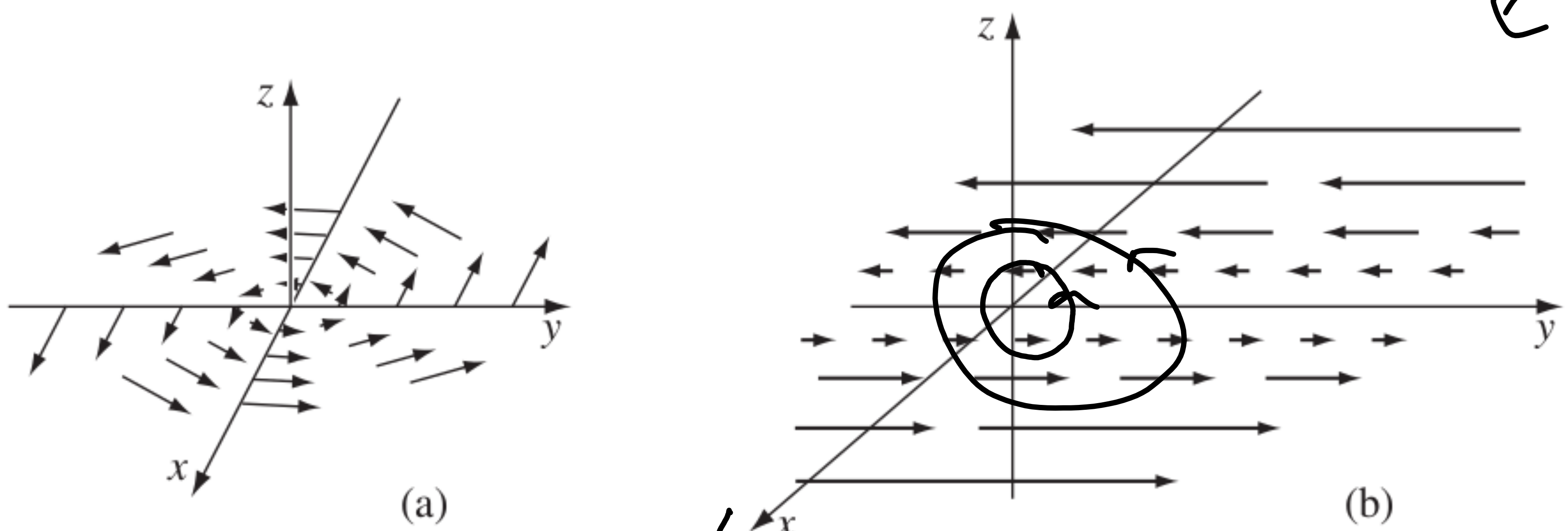
$$\vec{\nabla}\cdot\vec{A} = \frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z$$

↳ Scalar quantity.

Curl

$$\underline{\underline{\vec{\nabla}\times\vec{A}}} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \\ 0 & x & 0 \end{vmatrix} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \underline{\underline{\hat{z}}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

→ Measurement of how much the vector swirls around a point.



$\vec{A} = -y\hat{x} + x\hat{y}$

$\vec{\nabla} \times \vec{A} \neq 0$

$\vec{A} = x\hat{y}$

$\vec{\nabla} \times \vec{A} \neq 0$

Product Rules

$$\vec{\nabla}(f+g) = \vec{\nabla}f + \vec{\nabla}g$$

$$\vec{\nabla}\cdot(\vec{A}+\vec{B}) = \vec{\nabla}\cdot\vec{A} + \vec{\nabla}\cdot\vec{B}$$

$$\vec{\nabla}\times(\vec{A}+\vec{B}) = \vec{\nabla}\times\vec{A} + \vec{\nabla}\times\vec{B}$$

$\rightarrow \vec{\nabla}(kf) = k\vec{\nabla}f + f\vec{\nabla}k$

$\rightarrow \vec{\nabla}\cdot(k\vec{A}) = k(\vec{\nabla}\cdot\vec{A}) + \vec{A}\cdot(\vec{\nabla}k)$

$\vec{\nabla}\times(k\vec{A}) = k(\vec{\nabla}\times\vec{A}) - \vec{A}\times(\vec{\nabla}k)$

Accordingly, there are *six* product rules, two for gradients:

(i) $$\nabla(fg) = f\nabla g + g\nabla f,$$

(ii) $$\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A},$$

two for divergences:

(iii) $$\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f),$$

(iv) $$\nabla\cdot(\underline{\mathbf{A}\times\mathbf{B}}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B}), \leftarrow$$

and two for curls:

(v) $$\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f),$$

(vi) $$\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A}).$$

Second Derivative

1. $\vec{\nabla} \cdot (\vec{\nabla} T) \equiv$ Divergence of a gradient
2. $\vec{\nabla} \times (\vec{\nabla} T) \equiv$ Curl of a gradient
3. $\vec{\nabla} (\vec{\nabla} \cdot \vec{A}) \equiv$ Gradient of a divergence
4. $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) \equiv$ Divergence of a curl
5. $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) \equiv$ Curl of a curl

① $\vec{\nabla} \cdot (\vec{\nabla} T) = \nabla^2 T$

$$= \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

↳ Laplacian of a scalar.

② $\vec{\nabla} \times (\vec{\nabla} T) = 0$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \dfrac{\partial T}{\partial x} & \dfrac{\partial T}{\partial y} & \dfrac{\partial T}{\partial z} \end{vmatrix} = \hat{x}\left(\cancel{\frac{\partial^2 T}{\partial y \partial z}} - \cancel{\frac{\partial^2 T}{\partial z \partial y}}\right) + \hat{y}\left(\cancel{\frac{\partial^2 T}{\partial x \partial z}} - \cancel{\frac{\partial^2 T}{\partial z \partial x}}\right) + \hat{z}\left(\cancel{\frac{\partial^2 T}{\partial x \partial y}} - \cancel{\frac{\partial^2 T}{\partial y \partial x}}\right)$$

$$= \underline{\underline{0}}\,\hat{x} + \underline{\underline{0}}\,\hat{y} + \underline{\underline{0}}\,\hat{z}$$

$$= \text{Null vector.}$$

③ $\vec{\nabla}(\underset{T}{\underset{\shortparallel}{\vec{\nabla}\cdot\vec{A}}})$ (not important)

$$= \hat{x}\frac{\partial T}{\partial x} + \hat{y}\frac{\partial T}{\partial y} + \hat{z}\frac{\partial T}{\partial z}$$

(4) $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x}\right)$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = \frac{\partial}{\partial x}\left(\cancel{\frac{\partial A_z}{\partial y}} - \cancel{\frac{\partial A_y}{\partial z}}\right) + \frac{\partial}{\partial y}\left(\cancel{\frac{\partial A_x}{\partial z}} - \cancel{\frac{\partial A_z}{\partial x}}\right) + \frac{\partial}{\partial z}\left(\cancel{\frac{\partial A_x}{\partial y}} - \cancel{\frac{\partial A_y}{\partial x}}\right)$$

$$= 0$$

5 $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$

$\equiv$ 3

$\nabla^2 \vec{A} = \hat{x}(\nabla^2 A_x) + \hat{y}(\nabla^2 A_y) + \hat{z}(\nabla^2 A_z)$

Link to the recording:

https://bennettu.sharepoint.com/sites/EPHY105L-Odd2021/Shared%20Documents/Forms/AllItems.aspx?id=%2Fsites%2FEPHY105L%2DOdd2021%2FShared%20Documents%2FGeneral%2FRecordings%2FEPHY105L%20Theory%20Class%2D20211007%5F133314%2DMeeting%20Recording%2Emp4&parent=%2Fsites%2FEPHY105L%2DOdd2021%2FShared%20Documents%2FGeneral%2FRecordings