Department of Mathematics, Bennett University
Engineering Calculus (EMAT101L)
Tutorial Sheet 6 (Differentiability)

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1. Determine if the following functions are differentiable at 0. Find $f'(0)$ if exists

   (a) $f(x) = \begin{cases} x, & x \in \mathbb{Q} \\ \sin x, & x \notin \mathbb{Q}. \end{cases}$ (b) $f(x) = \begin{cases} e^{-\frac{1}{x^2}}, & x \neq 0 \\ 0, & x = 0. \end{cases}$

   (c) $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$ (d) $f(x) = e^{-|x|}$.

2. Determine if $f'(x)$ is continuous at 0 for the following functions:

   (a) $f(x) = \begin{cases} x^3 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$ (b) $f(x) = \begin{cases} x^2 \cos \frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$

3. Evaluate the following limits:

   $(a)\ \lim_{x \to 0} \frac{x - \tan x}{x^3}$, $(b)\ \lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$, $(c)\ \lim_{x \to \infty} \frac{e^x}{x^2}$

4. Determine all the number(s) c which satisfy the conclusion of Rolle's Theorem for $f(x) = x^2 - 2x - 8$ on $[-1, 3]$.

5. Verify Lagranges Mean Value Theorem for the function $f(x) = x + \frac{1}{x}$ in the interval $[1, 3]$.

6. Show that $\log(1 + x)$ lies between $x - \frac{x^2}{2}$ and $x - \frac{x^2}{2(1+x)}$ for all $x > 0$

7. Find all critical points and determine whether or not they are local minimums or maximums for

   - (i) $f(x) = x^3 - 3x^2 + 1$ (ii) $f(x) = x^3 - 12x + 1$
   - (iii) $f(x) = 3x^3 - 9x^2 - 27x + 15$