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State	Finished
Completed on	Tuesday, 22 February 2022, 11:30 AM
Time taken	1 hour 29 mins
Grade	**25.33** out of 40.00 (**63**%)

Question 1

Incorrect

Mark 0.00 out of 2.00

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be differentiable at $(0,0)$. Suppose that for $U = \left(\frac{3}{5}, \frac{4}{5}\right)$ and $V = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

We have $D_U f(0,0) = 12$ and $D_V f(0,0) = -4\sqrt{2}$.

Then choose the correct option.

Select one:

- a. $f_x(0,0) = 92$ and $f_y(0,0) = -84$ ✗
- b. $f_x(0,0)$ and $f_y(0,0)$ do not exist.
- c. $f_x(0,0) = -92$ and $f_y(0,0) = 84$
- d. $f_x(0,0) = 84$ and $f_y(0,0) = -92$

Your answer is incorrect.

The correct answer is: $f_x(0,0) = -92$ and $f_y(0,0) = 84$

Question 2

Incorrect

Mark 0.00 out of 2.00

Let

$$f(x) = \lim_{n\to\infty} \frac{(1+\sin \pi x)^n - 1}{(1+\sin \pi x)^n + 1}, x \in \mathbb{R}.$$

Then choose the correct option.

Select one:

- a. $f(x)$ has removable discontinuity at $x \in \mathbb{R}$.
- b. $f(x)$ has removable discontinuity at $x \in \mathbb{Z}$.
- c. $f(x)$ has jump discontinuity at $x \in \mathbb{R}$. ✗
- d. $f(x)$ has jump discontinuity at $x \in \mathbb{Z}$.

Your answer is incorrect.

The correct answer is: $f(x)$ has jump discontinuity at $x \in \mathbb{Z}$.

Question 3

Correct

Mark 2.00 out of 2.00

Let $f_1, f_2, f_3 : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f_1(x,y) = \begin{cases} xy\cos\dfrac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases},$$

$$f_2(x,y) = \begin{cases} \dfrac{\sin(x+y)}{|x|+|y|}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) \neq (0,0) \end{cases},$$

and

$$f_3(x,y) = \begin{cases} 1, & \text{if } x > 0 \text{ and } 0 < y < x^2 \\ 0, & \text{otherwise.} \end{cases}$$

Then choose the correct option.

Select one:

a. f_1 and f_3 are continuous at $(0,0)$.

b. f_1 is discontinuous at $(0,0)$.

c. f_1 and f_2 are discontinuous at $(0,0)$.

d. f_2 and f_3 are discontinuous at $(0,0)$.

Your answer is correct.

The correct answer is: f_2 and f_3 are discontinuous at $(0,0)$.

Question **4**

Correct

Mark 2.00 out of 2.00

Find the value of $\lim_{x\to 0} \frac{d}{dx}\left(\int_{x+1}^{x^2+2} xt\ dt\right)$.

Select one:

- a. 2
- b. $\frac{2}{3}$
- c. $\frac{3}{2}$ ✓
- d. ∞
- e. 0

Your answer is correct.

The correct answer is: $\frac{3}{2}$

Question 5

Correct

Mark 2.00 out of 2.00

Let $[x]$, $\{x\}(= x - [x])$ and $sgn(x)$ denote the greatest integer function, fractional part function and signum function, respectively.

Now, choose the correct options.

Select one or more:

- [x] a. $\lim_{x\to 0^+}\left[\frac{\sin x}{x}\right] = 0,\ \lim_{x\to 0^-}\left[\frac{\sin x}{x}\right] = 0.$ ✓
- [x] b. $\lim_{x\to 0^+}\left\{\frac{\sin x}{x}\right\} = 1,\ \lim_{x\to 0^-}\left\{\frac{\sin x}{x}\right\} = 1.$ ✓
- [ ] c. $\lim_{x\to\infty} x^2 \operatorname{sgn}\cos x$ is exist.
- [ ] d. $\lim_{x\to 0^+}\sqrt{\{x\}} = 1,\ \lim_{x\to 0^-}\sqrt{\{x\}} = 0.$

Your answer is correct.

The correct answers are: $\lim_{x\to 0^+}\left[\frac{\sin x}{x}\right] = 0,\ \lim_{x\to 0^-}\left[\frac{\sin x}{x}\right] = 0.$

, $\lim_{x\to 0^+}\left\{\frac{\sin x}{x}\right\} = 1,\ \lim_{x\to 0^-}\left\{\frac{\sin x}{x}\right\} = 1.$

Question 6

Incorrect

Mark 0.00 out of 2.00

Let $Q(x, y)$ be the Taylor's quadratic polynomial approximation of the function $f(x, y) = 3\sin 2x + 2\cos 3y$ near the origin. Then find the value of $\lim_{(x,y)\to(1,2)} Q(x, y)$.

Select one:

- a. 0
- b. -24
- c. 5 ✗
- d. -20

Your answer is incorrect.

The correct answer is: -24

Question 7

Correct

Mark 2.00 out of 2.00

Choose the correct options.

Select one or more:

- [ ] a. $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = -\pi$
- [ ] b. The integral $\displaystyle\int_{2}^{\infty} (x^2+x+1)e^{-x}\,dx$ diverges.
- [x] c. The integral $\displaystyle\int_{-2}^{2} \frac{dx}{x+1}$ diverges. ✓
- [x] d. $\displaystyle\int_{4}^{\infty} \frac{dx}{(x-2)(x-3)} = \ln 2$ ✓

Your answer is correct.

The correct answers are: The integral $\displaystyle\int_{-2}^{2} \frac{dx}{x+1}$ diverges.

, $\displaystyle\int_{4}^{\infty} \frac{dx}{(x-2)(x-3)} = \ln 2$

Question 8

Incorrect

Mark 0.00 out of 2.00

A sequence $\{u_n\}$ is defined by $u_{n+2} = \frac{1}{2}(u_{n+1} + u_n)$ for $n \geq 1$ and $0 < u_1 < u_2$.

Then choose the correct option.

Select one:

- a. The sequence $\{u_n\}$ converges to $\dfrac{3u_2 - u_1}{2}$.
- b. The sequence $\{u_n\}$ converges to $\dfrac{u_1 + 3u_2}{2}$.
- c. The sequence $\{u_n\}$ converges to $\dfrac{2u_2 - u_1}{3}$.

  ✗
- d. The sequence $\{u_n\}$ converges to $\dfrac{u_1 + 2u_2}{3}$.

Your answer is incorrect.

The correct answer is: The sequence $\{u_n\}$ converges to $\dfrac{u_1 + 2u_2}{3}$.

Question 9

Correct

Mark 2.00 out of 2.00

Choose the correct options.

Select one or more:

- [x] a. Let $f(x)$ be the sum of the power series $\sum_{n=0}^{\infty} a_n x^n$ on $(-R, R)$ for $R > 0$. If $f'(x) = f(x)$ for all $x \in (-R, R)$ and $f(0) = 1$, then $a_n = \dfrac{1}{n!}$ for all $n \in \mathbb{N}$.

✓

- [ ] b. Let $f(x)$ be the sum of the power series $\sum_{n=0}^{\infty} a_n x^n$ on $(-R, R)$ for $R > 0$. If $f'(x) = f(x)$ for all $x \in (-R, R)$ and $f(0) = 1$, then $a_n = \dfrac{2}{n!}$.

- [x] c. Let $f(x)$ be the sum of the power series $\sum_{n=0}^{\infty} a_n x^n$ on $(-R, R)$ for $R > 0$. If $f(x) + f(-x) = 0$ for all $x \in (-R, R)$, then $a_n = 0$ for all even n.

✓

Your answer is correct.

The correct answers are: Let $f(x)$ be the sum of the power series $\sum_{n=0}^{\infty} a_n x^n$ on $(-R, R)$ for $R > 0$. If $f'(x) = f(x)$ for all $x \in (-R, R)$ and $f(0) = 1$, then $a_n = \dfrac{1}{n!}$ for all $n \in \mathbb{N}$.

, Let $f(x)$ be the sum of the power series $\sum_{n=0}^{\infty} a_n x^n$ on $(-R, R)$ for $R > 0$. If $f(x) + f(-x) = 0$ for all $x \in (-R, R)$, then $a_n = 0$ for all even n.

Question 10

Correct

Mark 2.00 out of 2.00

Which among the following are NOT correct values of $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\; d\theta$?

Select one or more:

- [x] a. $\Gamma\left(\frac{3}{4}, \frac{1}{4}\right)$ ✓
- [x] b. $\frac{\pi}{2}$ ✓
- [ ] c. $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{1}{4}\right)$
- [ ] d. $\int_0^{\frac{\pi}{2}} \sin^{\frac{1}{2}}\theta\; \cos^{-\frac{1}{2}}\theta\; d\theta$

Your answer is correct.

The correct answers are: $\frac{\pi}{2}$

, $\Gamma\left(\frac{3}{4}, \frac{1}{4}\right)$

Question 11

Correct

Mark 2.00 out of 2.00

Let $f(x, y) = 6\sin(2x)\cos(3y)$ and let $P = \left(\frac{\pi}{6}, -\frac{\pi}{6}\right)$. Then find the direction of maximum increase.

Select one:

a. along the direction $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

b. along the direction $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$

c. along the y-axis ✓

d. along the x-axis

Your answer is correct.

The correct answer is: along the y-axis

Question 12

Incorrect

Mark 0.00 out of 2.00

Evaluate the double integral $\iint_R (x^2 + x - 1)\, dA$, where R is the region bounded by $y = x^2$ and $x = y^2$.

Select one:

a. $-\dfrac{41}{420}$

b. $-\dfrac{11}{420}$ ✗

c. $\dfrac{1}{6}$

d. $-\dfrac{1}{6}$

Your answer is incorrect.

The correct answer is: $-\dfrac{41}{420}$

Question 13

Incorrect

Mark 0.00 out of 2.00

Let $f(x, y) = 3x^2y + x^2 - 6x - 3y - 15$. Then choose the correct options.

Select one or more:

- [x] a. The determinant of the Hessian matrix of $f(x, y)$ at $\left(1, \frac{2}{3}\right)$ is -36. ✓
- [ ] b. $\left(-1, -\frac{4}{3}\right)$ is a saddle point.
- [x] c. The function has exactly four critical points. ✗
- [ ] d. The determinant of the Hessian matrix of $f(x, y)$ at $(0, 0)$ is -36.

Your answer is incorrect.

The correct answers are: $\left(-1, -\frac{4}{3}\right)$ is a saddle point.

, The determinant of the Hessian matrix of $f(x, y)$ at $\left(1, \frac{2}{3}\right)$ is -36.

Question 14

Correct

Mark 2.00 out of 2.00

Which among the following is the correct expression for the integral $\int_0^{\frac{12}{5}} \int_y^{6-\frac{3}{2}y} x^2y\,dx\,dy,$

when the order of integration is reversed?

Select one:

a. $\int_0^{\frac{12}{5}} \int_0^{\frac{12}{5}} x^2y\,dy\,dx$

b. $\int_0^{6} \int_0^{\frac{11}{5}} x^2y\,dy\,dx$

c. $\int_0^{\frac{11}{5}} \int_0^{x} x^2y\,dy\,dx + \int_{\frac{11}{5}}^{6} \int_0^{6-\frac{3}{2}x} x^2y\,dy\,dx$

d. $\int_0^{\frac{12}{5}} \int_0^{x} x^2y\,dy\,dx + \int_{\frac{12}{5}}^{6} \int_0^{4-\frac{2}{3}x} x^2y\,dy\,dx$

Your answer is correct.

The correct answer is: $\int_0^{\frac{12}{5}} \int_0^{x} x^2y\,dy\,dx + \int_{\frac{12}{5}}^{6} \int_0^{4-\frac{2}{3}x} x^2y\,dy\,dx$

Question 15

Correct

Mark 2.00 out of 2.00

If $\{u_n\}$ be a monotone decreasing sequence of positive real numbers and $\lim u_n = 0$. Then choose the correct options for the following series.

Select one or more:

- [ ] a. $u_1 - \frac{1}{2}(u_1 + u_2) + \frac{1}{3}(u_1 + u_2 + u_3) - \ldots\ldots$ is a divergent series.
- [x] b. $u_1 - \frac{1}{2}(u_1 + u_3) + \frac{1}{3}(u_1 + u_3 + u_5) - \ldots\ldots$ is a convergent series. ✓
- [x] c. $u_1 - \frac{1}{2}(u_1 + u_2) + \frac{1}{3}(u_1 + u_2 + u_3) - \ldots\ldots$ is a convergent series. ✓
- [ ] d. $u_1 - \frac{1}{2}(u_1 + u_3) + \frac{1}{3}(u_1 + u_3 + u_5) - \ldots\ldots$ is a divergent series.

Your answer is correct.

The correct answers are: $u_1 - \frac{1}{2}(u_1 + u_2) + \frac{1}{3}(u_1 + u_2 + u_3) - \ldots\ldots$ is a convergent series.

, $u_1 - \frac{1}{2}(u_1 + u_3) + \frac{1}{3}(u_1 + u_3 + u_5) - \ldots\ldots$ is a convergent series.

Question 16

Correct

Mark 2.00 out of 2.00

Consider the function $f(x) = x^{15} + 100x^5 - 5e^{-x}$. Then choose the correct option.

Select one:

a. $f(x) = 0$ has two real solutions.

b. $f(x) = 0$ has exactly one real solution.

c. $f(x) = 0$ has 15 real solutions.

d. $f(x) = 0$ has no real solution.

Your answer is correct.

The correct answer is: $f(x) = 0$ has exactly one real solution.

Question 17

Correct

Mark 2.00 out of 2.00

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} 0, & \text{if } x \in \mathbb{Q},\ y \in \mathbb{Q} \\ xy, & \text{otherwise.} \end{cases}$$

Then choose the correct option.

Select one:

- a. f is continuous only on x-axis and y-axis. ✓
- b. f is continuous only at $(0, 0)$.
- c. f is continuous for all (x, y) in $\mathbb{R}^2$.
- d. f is discontinuous for all (x, y) in $\mathbb{R}^2$.

Your answer is correct.

The correct answer is: f is continuous only on x-axis and y-axis.

Question 18

Correct

Mark 2.00 out of 2.00

Find the signed volume under the plane $z = 4 - x - 2y$ over the disk with equation $x^2 + y^2 \leq 4$.

Select one:

- a. 8π
- b. 2π
- c. 4π
- d. 16π

Your answer is correct.

The correct answer is: 16π

Question 19

Partially correct

Mark 1.00 out of 2.00

Let $z = x^2y + xy^2$ and $x = 3 + t^4, y = 1 - t^3$. Then choose the correct options.

Select one or more:

- [ ] a. $\left.\dfrac{dz}{dt}\right|_{\text{at } t=1} = 9$
- [ ] b. $\left.\dfrac{dz}{dt}\right|_{\text{at } t=0} = 4$
- [ ] c. $\left.\dfrac{dz}{dt}\right|_{\text{at } t=1} = -27$
- [x] d. $\left.\dfrac{dz}{dt}\right|_{\text{at } t=0} = 0$

✓

Your answer is partially correct.

You have correctly selected 1.

The correct answers are: $\left.\dfrac{dz}{dt}\right|_{\text{at } t=0} = 0$

, $\left.\dfrac{dz}{dt}\right|_{\text{at } t=1} = -27$

Question 20

Partially correct

Mark 0.33 out of 2.00

Consider the function $f(x) = \dfrac{2x^3 - x^2 + 9}{2(9 - x^2)}$. Then choose the correct options.

Select one or more:

- [ ] a. $y = \pm 3$ are horizontal asymptotes.
- [x] b. f has local minimum at $x = -3\sqrt{3}$ ✓
- [ ] c. f has local maximum at $x = 3\sqrt{3}$
- [x] d. f has local minimum at $x = 3\sqrt{3}$. ✗
- [ ] e. $x = \pm 3$ are horizontal asymptotes.

Your answer is partially correct.

You have correctly selected 1.

The correct answers are: f has local maximum at $x = 3\sqrt{3}$

, f has local minimum at $x = -3\sqrt{3}$