Experimental Design:
Maximizing Statapult Launch Distance

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I. 2^5 Factorial Design

Experimental Design

We begin our design by defining the factors of our experiment with chosen high (+) and low (-) settings for five different factors on our Statapult. For the simplification in data collection, we chose to keep our catapult *Tilt* constant, at level 2. The remaining factors – rubber band position on arm (*Arm*), ball position (*Position*), ball type (*Ball*), arm draw back height (*Height*), and rubber band attachment on post (*Post*) will be the factors in which we will change in our experiment. For the levels in our experiment, we chose to test a golf (+) and wiffle (-) ball as our levels for our *Ball* factor, while the rest of our factors correspond to settings on our Statapult: placement 3 (+) and placement 1 (-).

Following our factor assignments, our team began by setting up our equipment. We chose to have the Statapult launch our observations from the floor, with a measuring tape placed upon the floor beneath the release point of the Statapult arm. While conducting this experiment, our team was expected to find a way to account for the change in distance launched associated with rubberband fatigue. We were able to account for this source of variation by randomizing the order of our runs, thus reducing the chance of biased results. Before discussing our analysis, it is important for us to note that our 2^5 factorial design is a single replicate study due to the fact that our team was conducting our experiment during a short period of allotted time, or else replicates could have been made.

Exploratory Data Analysis

To begin our analysis, our team produced the following box plots of our data in order to establish a general idea of the center, shape, and spread of our data before we create our models.

Factor Name	Arm	Ball	Position	Height	Post
Appears in Model	A	B	C	D	E

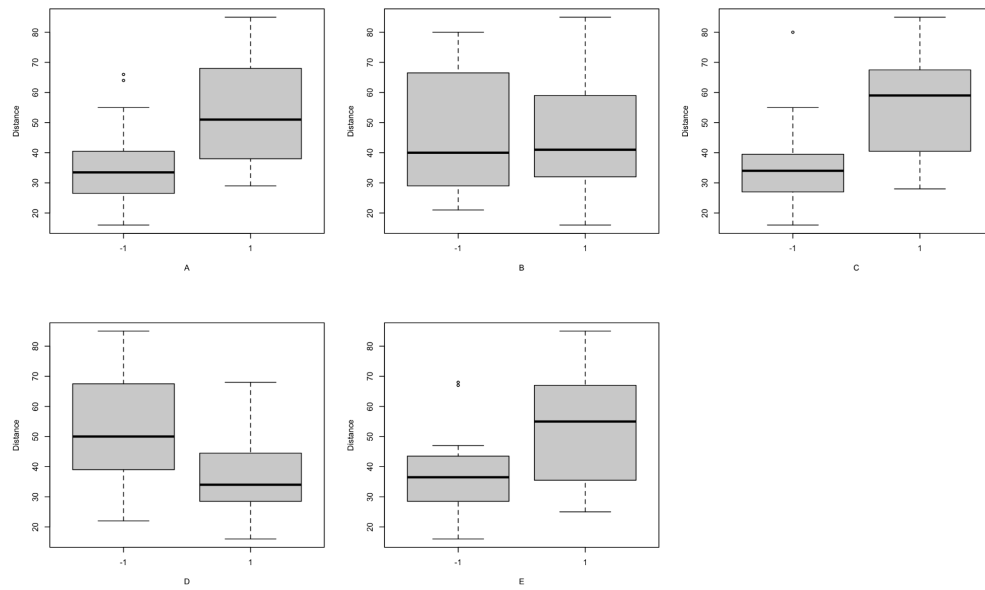**Table 1:** Key for Referencing Factors in Model

Figure 1: Box Plots of Tested Factors at (+) and (-) Levels

In addition to box plots, our team produced interaction plots between each of our factors in order for us to identify potential interactions which may appear to be significant to our later analysis.
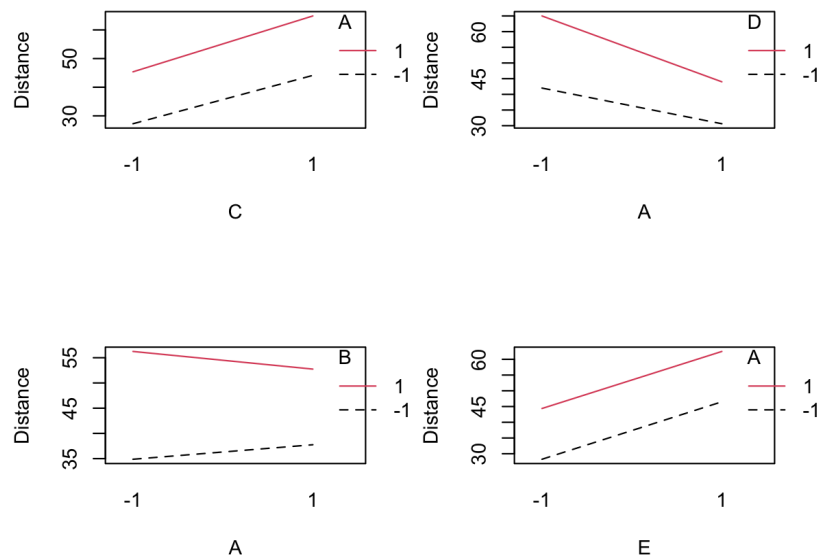

Figure 2: Series of Produced Interaction Plots

Due to the fact that each plot appears to display parallel behavior, we can conclude that interaction between factors is most likely not going to be important in our analysis. After conducting our initial EDA, our team began with the building of our model. We begin by

defining a model which contains all main effects and interactions. Before our team conducted model analysis, we first had to ensure that the conditions of our model were satisfied.
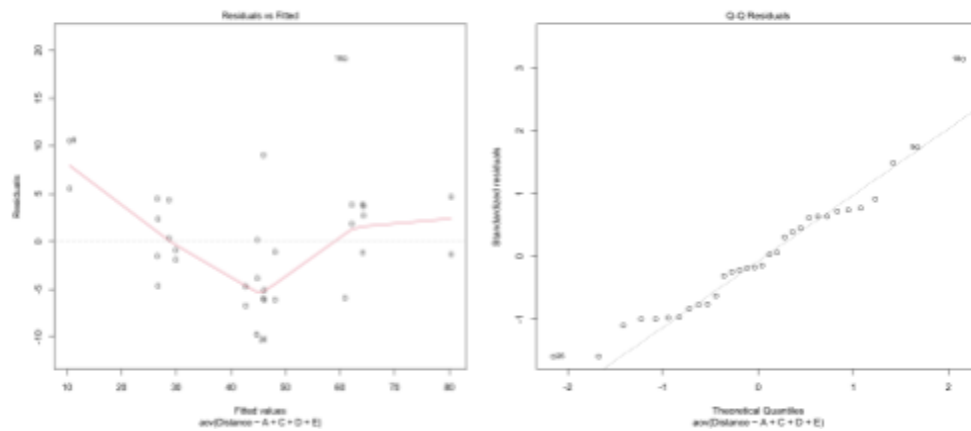

Figure 3: Plots to Assess Conditions – Residuals vs Fits (L) and QQ Plot (R)

Above, we provided the two plots in which our team used to assess if our model met the conditions of mean zero, equal variance, and normality. Regarding the plot to the left, our team was concerned about the clustering pattern our data displays, but this is to be expected from the construction of our data having clusters of observations at 25, 45, and 60 inches. In addition, despite the dip in our residuals, our team concluded that our data displays mean zero in addition to equal variance. On the other hand, the check for normality using our QQ plot was much more simple, as there was no question that our residuals follow the QQ line and do not deviate too significantly. Hence, we assured that our model met the proper conditions for analysis. Next, in order to identify which factors have significant effects on our response variable, we run the half normality plot as below.
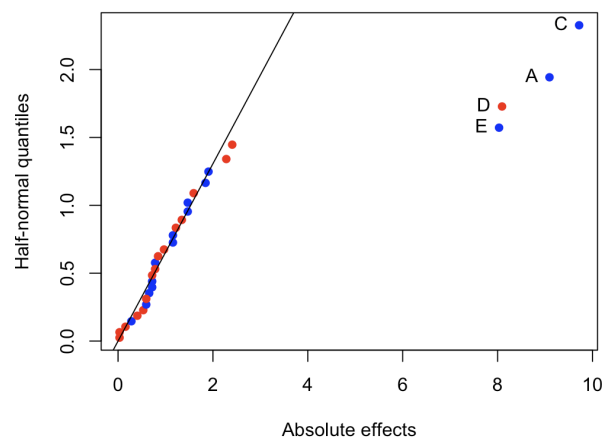

Figure 4: Half Normal Probability Plot

From the plot output above, we can see that points A, C, D, and E notably deviate far from the quantile line, indicating that these factors have large effects and are statistically significant in determining the response variable – Distance (in). Thus, we will only consider factors A,C, D, and E in our design throughout the rest of our analysis.

$$Y = \beta_0 + \beta_A A + \beta_C C + \beta_D D + \beta_E E + \epsilon$$

Equation 1: 2^5 Design Model

Design Analysis

Now that we have assessed conditions and identified the terms in our model which our team will include for our design analysis, we may begin testing our model using our ANOVA procedure. In total we will conduct two hypothesis tests, one in which we will be testing our model against the null model and the other we will test the significance of the main effects in our model – since we do not have any interaction terms, these are the highest order terms.

Our team begins the test of our chosen model versus the null model. Consider the null hypothesis that our chosen model is indistinguishable from the null model in predicting *Distance*, with the alternative being that our chosen model has at least one term in our model which has a non-zero effect on *Distance*. With $F_4 = 56.103$ and associated p-value = 1.081E-12 <<< 0.05, we have sufficient evidence to reject the null hypothesis. Thus, we can conclude that our chosen model contains at least one non-zero term which predicts *Distance* better than the null model. Since we can only conclude that there is at least one term in our model which has an effect on *Distance*, we must perform a full ANOVA on our model to determine which predictor/predictors in our model are responsible for this. Consider the null hypothesis to be that a chosen predictor in our model has zero effect on predicting *Distance*, where the alternative is that our predictor has a non-zero effect on *Distance* and belongs in our model. Below, is a table of all of the reported F and p-values for each term in our model.

Predictor	A *Arm*	C *Position*	D *Height*	E *Post*
$F_{1,\,27}$	60.418	69.008	47.861	47.124
P-value	2.33E-8	6.46E-9	1.96E-7	2.24E-7

Table 2: Reported F and p-values for Main Effect Hypothesis Testing

With all terms above having a proper F and associated p-value much less than our significant level ($\alpha = 0.05$), we have sufficient evidence to conclude that all terms in our model have a non-zero effect in predicting *Distance* for our data set. Hence, we can conclude that our chosen model is proper for our team to use as a means of analysis of our 2^5 Factorial Design.

Now that we have confirmed the predictors in our model to be significant, we can identify a recommendation of a setting which will maximize *Distance*. Using Figure (1), we can identify the levels of each factor which are associated with a greater mean *Distance* response to build our final recommendation. Based on the analysis of our data, we can make a final recommendation that in order to maximize distance launched from the Statapult – regardless of ball type – let *Arm* be at placement 3 (+), *Position* to be at placement 3 (+), *Height* be at placement 1 (-), and *Post* to be at placement 3 (+).

Predictor	A *Arm*	B *Ball*	C *Position*	D *Height*	E *Post*
Level	(+)	Not significant	(+)	(-)	(+)

Table 3: Final Recommendation for Factor Levels from 2^5 Design Analysis

II. 2_V^{5-1} Half Fraction Factorial Design

Experimental Design

To reduce our data in order to simulate a half fraction factorial design, as if we had only been able to conduct 16 runs, we blocked our data according to the defining relation $I \equiv ABCDE$. We alias I with the highest order interaction term, which is the five-way interaction between each factor we changed in the statapult: arm, position, ball, height, and post. This resulted in two blocks with 16 effects in each, one of which became the entirety of our data for this portion of the analysis. The defining contrast $L = \alpha_{arm}x_{arm} + \alpha_{position}x_{position} + \alpha_{ball}x_{ball} + \alpha_{height}x_{height} + \alpha_{post}x_{post}$ can equal 0 mod 2 or 1 mod 2. Block 1, the principal block, includes the treatment combinations for which $L = 1\ mod\ 0$, while Block 2 includes those for which $L = 0\ mod\ 1$. We chose to analyse the treatment combinations included in Block 2, leaving us with 16 runs on which to perform a

similar analysis as above. As the largest treatment effect in Block 2 is abcde, a word of length = 5, the resolution of this design is V, so the final notation of this design is 2_V^{5-1}.

Exploratory Data Analysis

We anticipate that the analysis of the reduced data will resemble the full 32 run factorial experiment described above because of the rigorous blocking procedure. In an initial exploratory analysis, we find similar results in the box plots comparing each factor at high and low levels.
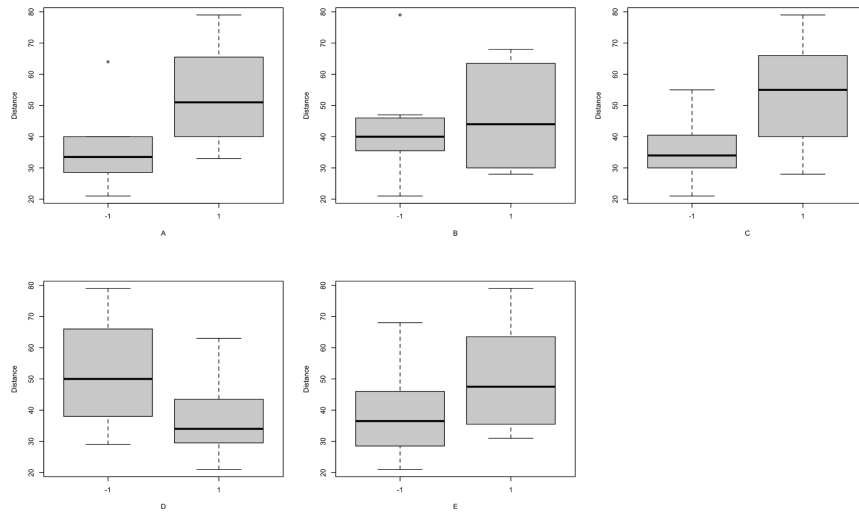

Figure 5: Box Plot exploration of the data analyzed in the half fraction design.

Similarly, we find no evidence of interaction in the subsetted data as seen in the plots below.
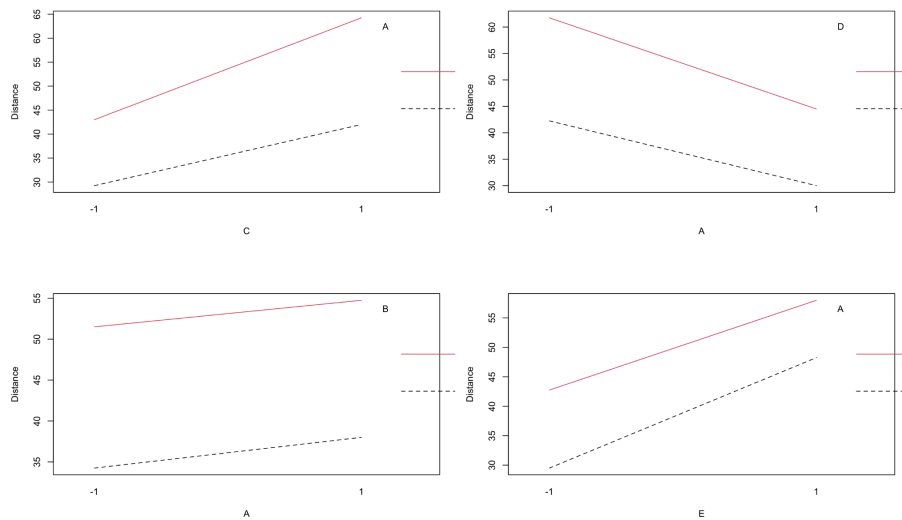

Figure 6: Interaction plots for factors in the half fraction factorial design.

We define an initial model with every possible term, which includes main effects and two-way interactions given the limitations of having only 16 runs. With the half normality plot below , we identify which factors have significant effects on our response variable in the half fraction factorial design.
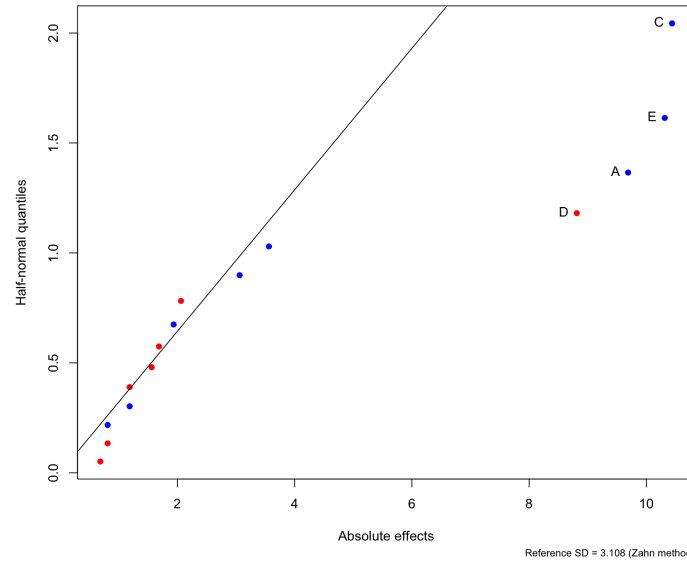

Figure 7: Half Normal Probability Plot for Half Fraction Factorial Design

From the plot output above, we can see that points A, C, D, and E notably deviate far from the quantile line, indicating that these factors have large effects and are statistically significant in determining the response variable – Distance (in). Thus, we build a model to analyze distance traveled in the half faction design:

$$Y = \beta_0 + \beta_A A + \beta_C C + \beta_D D + \beta_E E + \epsilon$$

Equation 2: 2_V^{5-1} Design Model

We also assess the conditions of this model with the plots below, which indicate some concern about the zero mean and equal variance conditions. The QQ plot confirms the normality condition has been met, and we will proceed with analysis despite light concerns for other conditions being met.
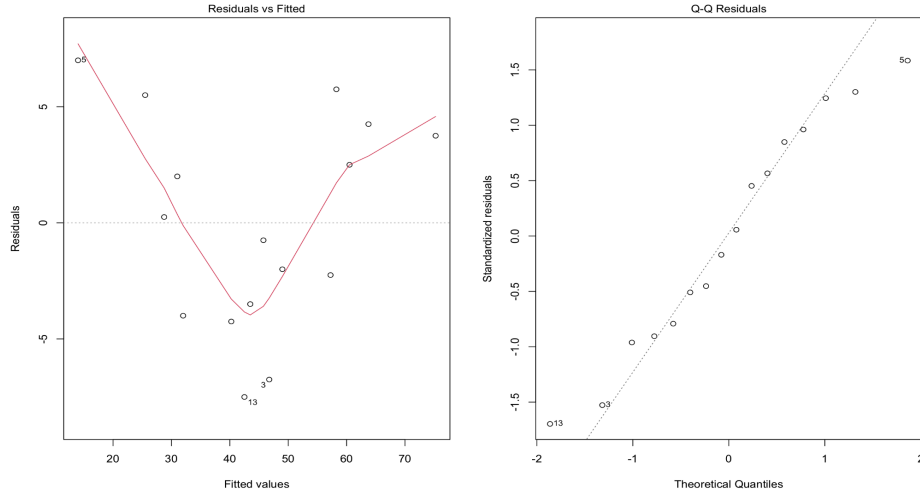
Figure 8: Condition-checking figures for the half fraction factorial design.

Design Analysis

To conduct our two hypothesis tests for this reduced data, we build a null model and run a general model utility test. With an F statistic of 33.891 and an associated p value of 3.971e-06, we have sufficient evidence to reject the null hypothesis. Thus, we know that at least one factor in our model has a nonzero effect on Distance. We run a full ANOVA to examine the main effects of the factors in our model, finding each individually to have a nonzero effect on Distance, as demonstrated by Table 4:

Predictor	A *Arm*	C *Position*	D *Height*	E *Post*
$F_{1,\,27}$	40.691	45.619	30.633	18.621
P-value	5.231e-05	3.140e-05	0.0001768	0.0012245

Table 4: Reported F and p-values for Main Effect Hypothesis Testing in the Half Fraction Factorial

Finally, based on these factors being significant and using the visual comparisons of factor levels in Figure 5, we recommend the following levels of each factor in order to maximize distance traveled:

Predictor	A *Arm*	B *Ball*	C *Position*	D *Height*	E *Post*
Level	(+)	Not significant	(+)	(-)	(+)

Table 5: Final Recommendation for Factor Levels from 2_V^{5-1} Design Analysis

III. Comparison

Both factorial (2^5) and half fractional factorial (2^{5-1}) designs provide consistent results regarding identifying the key factors determining the Statapult's launch distance. In both of our designs, factors A, C, D, and E are found to have statistically significant effects on the response variable.

Moreover, the final recommendation for factor settings to maximize launch distance are also identical in both designs: high level for Arm (A), high level for Position (C), low level for Height (D), and high level for Post (E). This result indicates that despite the reduced number of runs in the half fractional factorial design, we were able to obtain the similar insights outcome with half the experimental effort.

Considering the trade-off between these two designs, the full factorial (2^5) design enables the estimation of all main and interaction effects by having a higher resolution. However, this type of design requires a higher number of runs, making the experiment more resource-intensive and time-consuming. In contrast, the fractional factorial (2^{5-1}) design reduces experimental burden with less runs. While this introduces the possibility of confounding higher-order interactions, the half-fractional model successfully identified the same significant main effects (factors A, C, D, and E) and yielded identical recommendations for optimal factor levels, which demonstrates its efficacy in this context.

IIII. Discussion & Conclusion

Ultimately, our results show that to achieve the farthest launch distance using the Statapult, one should set the Arm, Position, and Post factors to their high settings, while setting Height to its low setting. Ball type had no significant effect, allowing greater flexibility in material use without affecting performance. Even though both designs identified four similar significant factors - Arm (A), Position (C), Height (D), and Post (E) - in maximizing the Statapult's launch Distance (in), we can consider which design to use based on our preferences for future experiments.

From a practical standpoint, the full factorial design offered a complete view of all possible interactions and ensured no aliasing of main effects. This level of comprehensiveness is useful in experimental settings where interactions may be present or suspected. However, our

data showed little to no interaction among factors, supported by largely parallel interaction plots in EDA, suggesting that a simpler design such as the half fractional factorial one would have been sufficient.

The half-fractional design effectively identified the same significant main effects and optimal settings while requiring only half the number of experimental runs. Using this type of design is desirable in scenarios where there are time constraints and limited materials. In our Statapult experiment, the same key factors were identified in both designs. Therefore, the half-fractional design is not only more efficient but just as effective, which is ideal for your goal of maximizing Statapult launch distance with minimal effort.

References

Montgomery, Douglas C. (2019). Design and Analysis of Experiments, Tenth Edition. New York
Wiley.