



## **STAT226: Statistical Computing in R**

### **Sexual Dimorphism in Penguins**

Author: Samyak Shrestha

Date: 20th December, 2024

Professor Brian Jones

## I) Introduction

Sexual dimorphism in penguins, characterized by differences such as body size and beak dimensions, can influence various biological and ecological aspects such as survival strategies, breeding success, and adaptation to environmental pressures (Fairbairn DJ 1997; Pérez-Barbería, 2006). Particularly, the Palmer Archipelago of Antarctica presents a unique setting to explore these differences due to it being home to a diverse species of penguins. This project will involve predicting the sex of the penguin using a series of morphometric features such as flipper length, culmen length, culmen depth, and body mass. This difference in the degree and type of sexual dimorphism can provide valuable insights into how physical influence gender characteristics. Furthermore, this project will involve the use of three techniques we have learnt throughout the semester: Bootstrapping, Decision Trees, and Logistic Regression along with specific questions tailored to each.

The research was conducted and data collected were on *Pygoscelis* penguins nesting on several islands within the Palmer Archipelago west of the AP near Anvers Island over the austral summer seasons from 2007 to 2010 by Dr. Kristen Gorman at Palmer Station, which is part of the Antarctica Long Term Ecological Research network (Gorman et al., 2014; Palmer Archipelago Penguin Data, n.d.). The research conducted by Gorman and colleagues aimed to explore the relationship between sex-specific foraging behaviors and changes in the environment. During the breeding season, they collected penguins to conduct body measurements for right flipper length, culmen length, culmen depth, and body mass, and blood tests to determine the sex of the bird.

Table 1 below contains the variables of interest from these data:

Variable	Description
<b>Body mass (g)</b>	This quantitative variable measures the penguin's weight in grams.
<b>Culmen length (mm)</b>	This quantitative variable measures the length of the upper ridge of the penguin's beak.
<b>Culmen depth (mm)</b>	This quantitative variable measures the height of the penguin's bill.
<b>Flipper length (mm)</b>	This quantitative variable measures the length from the penguin's sternum to the tip of its right flipper.
<b>Sex</b>	This binary categorical variable identifies whether the penguin is male (1) or female (0).

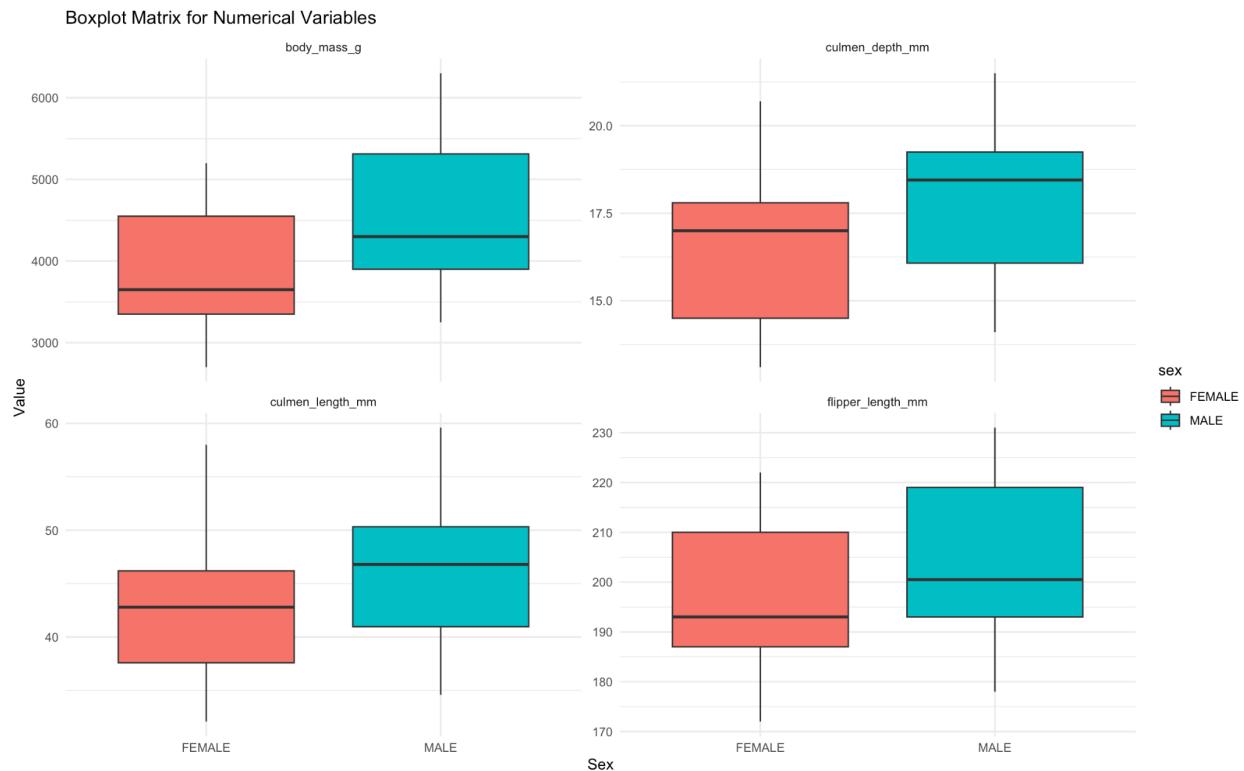
*Table 1. Variables of interest and their descriptions*

## II) Exploratory Data Analysis (EDA)

In this section, we explore the Palmer Archipelago penguin dataset to understand the key characteristics and relationships among the variables, particularly focusing on how they differ by sex (Male or Female). By visualizing and summarizing the data, we aim to identify patterns and potential predictors of penguin sex based on their physical measurements.

sex	mean_culmen_length	mean_culmen_depth	mean_body_mass	mean_flipper_length
	<dbl>	<dbl>	<dbl>	<dbl>
1 FEMALE	42.1	16.4	3862.	197.
2 MALE	45.9	17.9	4546.	205.

The summary table above reveals clear differences in physical measurements between male and female penguins. Males have higher mean values across all variables, including culmen length (45.9 mm vs. 42.1 mm), culmen depth (17.9 mm vs. 16.4 mm), body mass (4546 g vs. 3862 g), and flipper length (205 mm vs. 197 mm). These distinctions suggest that body mass, culmen length, and flipper length are the strongest indicators of sex, with culmen depth showing less separation.



*Figure 1. Boxplot matrix of key morphological traits stratified by sex*

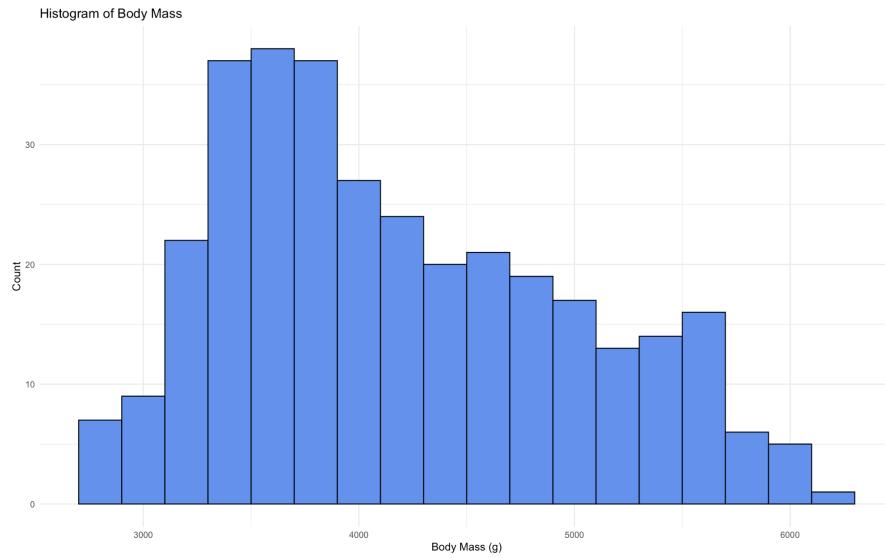
In Figure 1, we can observe clear differences between males and females across all variables. Males tend to have higher body mass, culmen length, and flipper length, as indicated by their

higher medians and wider distributions compared to females. Culmen depth, however, shows less pronounced separation, with some overlap in values between males and females. These distinctions suggest that body mass, culmen length, and flipper length may serve as strong predictors for determining the sex of penguins, while culmen depth might play a supplementary role.

### III) Bootstrapping

- 1) How does the bootstrap sampling distribution of the mean compare to the original dataset's distribution of body mass? What mathematical concepts are at play?

Body mass is one of the strongest predictors of penguin sex, as demonstrated in the EDA. However, the body mass measurements in the dataset are drawn from a specific sample of penguins. To better understand the variability and reliability of the observed mean body mass, we will construct a bootstrap sampling distribution and compare it to the original dataset.



*Figure 2. Histogram of Body Mass*

Figure 2 depicts the distribution of body mass values for the original dataset. This histogram reflects the actual variation in the penguins' body masses. The data appears slightly skewed to

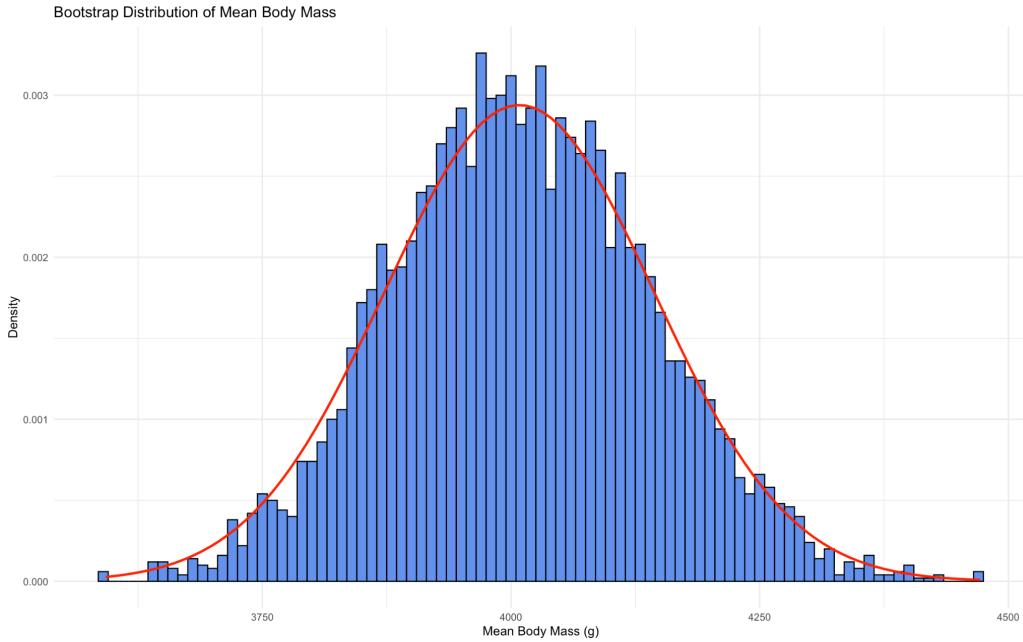
the right, with most penguins having body mass values concentrated around 4000 to 4500 grams, but with some heavier individuals contributing to a longer tail on the right. To better understand the sampling distribution of the mean, we employ bootstrap resampling, which allows us to approximate the sampling distribution of the mean by repeated sampling with replacement from the observed data.

I used the following R-code to calculate the bootstrap sampling distribution of the mean body mass:

```
num_samp <- 5000

sigma_original <- sd(penguins_samp$body_mass_g) #standard
deviation of the original sample

resamp_summary <- 1:num_samp %>%
  map_dfr(~ penguins_samp %>%
    slice_sample(n = sampszie, replace = TRUE) %>%
    summarize(
      mean_mass = mean(body_mass_g),
      sd_mass = sd(body_mass_g),
      scale_var = (sampszie - 1) * var(body_mass_g) /
        sigma_original^2
    )
  )
```



*Figure 3. Bootstrap distribution of mean body mass*

In Figure 3, each bar corresponds to the mean body mass of one of many bootstrap samples, and the red curve represents a fitted normal distribution. This distribution is much narrower than the original because it reflects the variability of the sample means rather than the variability of individual body masses. There are two mathematical concepts at work here:

- i) The **Weak Law of Large Numbers** states that as the sample size ( $n$ ) increases, the sample mean ( $\bar{X}$ ) converges in probability to the population mean ( $\mu$ ):

$$P(|\bar{X} - \mu| > \epsilon) \rightarrow 0 \quad \text{as } n \rightarrow \infty$$

In Figure 3, the mean of the bootstrap sampling distribution closely approximates the true population mean of body mass because the repeated sampling process simulates large sample sizes. Each bootstrap sample mean clusters around the true mean ( $\mu$ ), with less variability as  $n$  increases. This behaviour demonstrates the WLLN, where the mean of the sampling distribution becomes a consistent estimator of the population mean.

ii) The **Central Limit Theorem** states that regardless of the population's original distribution, the sampling distribution of the sample mean ( $X_{\bar{}}^{} \bar{}$ ) approaches a normal distribution as  $n$  increases:

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

In Figure 3, the bootstrap sampling distribution follows a bell-shaped curve and is approximately normal, even though the original dataset is slightly skewed. This normality arises because each bootstrap sample aggregates many observations, averaging out the population's skewness.

Together, the WLLN and CLT explain why the bootstrap sampling distribution of the mean is centered around the population mean ( $\mu$ ) and has a normal shape. The WLLN ensures that the sample mean converges to the true mean, while the CLT explains why this distribution becomes normal with reduced variance as  $n$  increases.

### Comparison of favstats

	min	Q1	median	Q3	max	mean	sd	n	missing
	2700	3550	4050	4775	6300	4207.057	805.2158	333	0

Mean distribution of the original population (up)

	min	Q1	median	Q3	max	mean	sd	n	missing
	3592.5	3921.667	4007.083	4098.333	4471.667	4010.555	129.9637	5000	0

Bootstrap mean distribution(up)

The original body mass distribution has a mean of 4207.057 g with a standard deviation of 805.2158 g, reflecting the variation in individual penguins' body masses. In contrast, the bootstrap sampling distribution of the mean has a mean of 4010.555 g and a much smaller standard deviation of 129.9637 g, as calculated by resampling the data. This narrower spread reflects the reduced variability in the sample mean compared to individual data points, consistent with the CLT. Additionally, while the original data has a broader range, the bootstrap distribution

of means is more concentrated, highlighting its role in estimating the true population mean with greater precision.

- 2) Use the bootstrap to generate and interpret a 95% confidence interval for the mean body mass of penguins.

I used the following code to calculate the bootstrap confidence interval:

```
lower1 <- quantile(resamp_summary$mean_mass, 0.025)  
upper1 <- quantile(resamp_summary$mean_mass, 0.975)  
c(lower1, upper1)
```

2.5%	97.5%
3756.667	4267.521

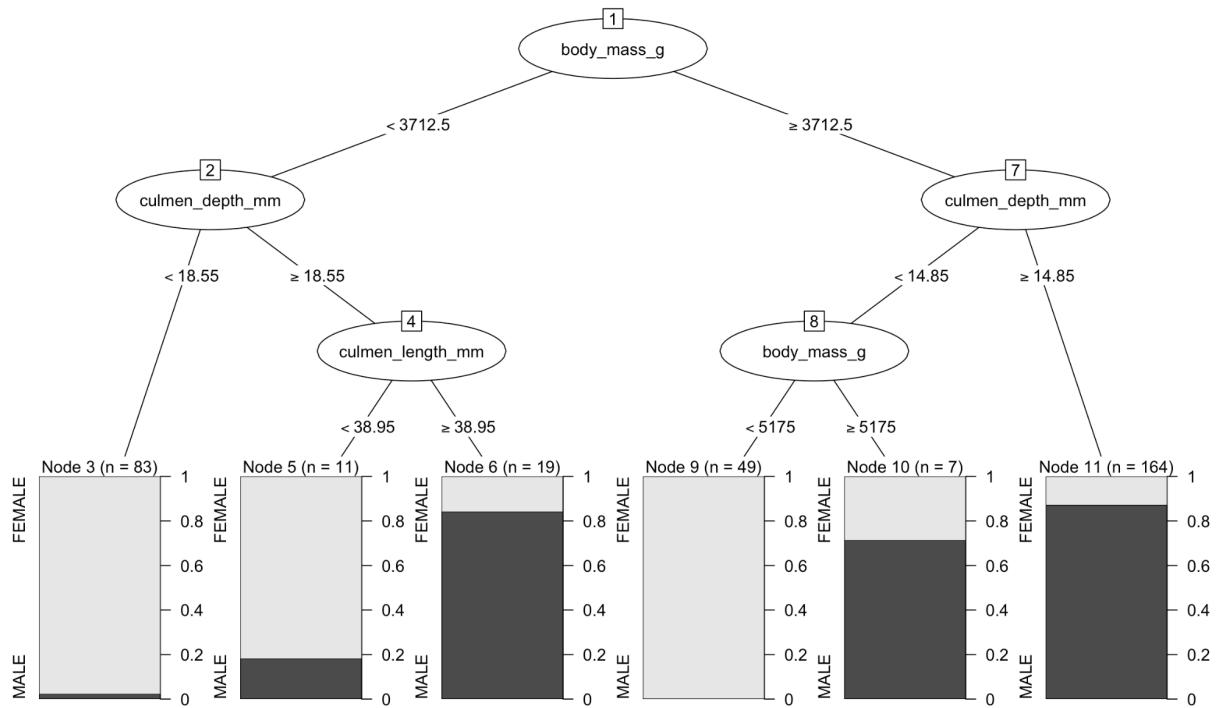
Based on the bootstrapped samples, this interval indicates that we are 95% confident that the true population mean of penguin body mass falls between 3756.667 grams and 4267.521 grams.

#### IV) Decision Trees

- 3) What are the most significant physical characteristics (e.g., body mass, culmen length, or culmen depth) for predicting the sex of penguins?

I used the following code to create my decision tree as seen in Figure 4:

```
mod_dtrees <- decision_tree(mode = "classification") %>%  
  set_engine("rpart") %>%  
  fit(sex ~ culmen_length_mm + culmen_depth_mm + body_mass_g +  
       flipper_length_mm, data = data_tree)  
  plot(as.party(mod_dtrees$fit))  
  title("Decision Tree for Penguin Sex Prediction")
```



*Figure 4. Decision Tree for Penguin Sex Prediction*

Based on Figure 4, the decision tree predicts the sex of penguins (Male or Female) based on features like body mass, culmen depth, and culmen length. The most important predictor is body mass, with penguins weighing less than 3712.5 grams generally classified as Female, and those weighing more classified as Male. Culmen depth is the second key feature, where higher values ( $\geq 18.55$  mm) are indicative of Male penguins. Culmen length further refines the classification, with longer culmen lengths ( $\geq 38.95$  mm) more associated with Male penguins. Overall, the tree shows that penguins with lower body mass and smaller culmen dimensions are likely Female, while those with higher body mass and larger culmen dimensions are likely Male.

- 4) How well does the decision tree classify the sex of penguins, and what is the accuracy of its predictions?

I used the following code to calculate the accuracy and confusion matrix for the decision tree:

```
pred_tree <- data_tree %>%
  bind_cols(
    predict(mod_dtrees, new_data = data_tree, type = "class")
  ) %>%
  rename(sex_tree = .pred_class)
accuracy(pred_tree, truth = sex, estimate = sex_tree)
confusion_tree <- pred_tree %>%
  conf_mat(truth = sex, estimate = sex_tree)
print(confusion_tree)
```

.metric	.estimator	.estimate	Truth
<chr>	<chr>	<dbl>	Prediction
accuracy	binary	0.910	FEMALE
			MALE

The decision tree achieved an accuracy of approximately 0.910 (91%), indicating strong predictive performance. The confusion matrix shows that the model achieves 91% accuracy, correctly classifying 164 males and 139 females, with 30 misclassifications (4 false positives and 26 false negatives).

## V) Logistic Regression

Decision trees inherently prioritize variables with the most predictive power, and since flipper length is absent, it likely does not provide substantial predictive information for classifying sex of the penguins. Thus, I choose to drop it from my logistic Regression model. My final model is:

$$\text{logit}\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \beta_1 \text{BodyMass} + \beta_2 \text{CulmenDepth} + \beta_3 \text{CulmenLength}$$

- 5) What is the relationship between culmen depth, culmen length, body mass, and the probability of a penguin being male?

I used the following R code:

```
data_split <- initial_split(penguins, prop = 0.8, strata = sex)
train_data <- training(data_split)
test_data <- testing(data_split)
mod_full <- logistic_reg(mode = "classification") %>%
  set_engine("glm") %>%
  fit(sex ~ culmen_length_mm + culmen_depth_mm + body_mass_g,
  data = train_data)
```

```

Call:
stats::glm(formula = sex ~ culmen_length_mm + culmen_depth_mm +
  body_mass_g, family = stats::binomial, data = data)

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -5.968e+01 7.807e+00 -7.645 2.10e-14 ***
culmen_length_mm 8.211e-02 4.763e-02  1.724  0.0847 .
culmen_depth_mm 2.027e+00 2.736e-01  7.406 1.30e-13 ***
body_mass_g    5.085e-03 7.223e-04  7.040 1.92e-12 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 368.74 on 265 degrees of freedom
Residual deviance: 128.20 on 262 degrees of freedom
AIC: 136.2

Number of Fisher Scoring iterations: 7

```

**Figure 6.** Summary output of the Logistic Model

From Figure 6, culmen depth (coefficient =  $\approx 2.027$ ,  $p < 0.001$ ) and body mass (coefficient =  $0.005$ ,  $p < 0.001$ ) are highly significant predictors of being male, with increases in these variables strongly associated with higher odds of being male. The coefficient for culmen depth is  $\approx 2.027$ , which means that for each millimeter increase in culmen depth, the log-odds of being male increase by  $\approx 2.027$ . Similarly, the coefficient for body mass is  $\approx 0.005$ , which means that for each one gram increase in body mass, the log-odds of being male increase by  $\approx 0.005$ . Culmen length has a smaller, marginally significant effect (coefficient =  $\approx 0.082$ ,  $p = 0.085$ ). Again, the coefficient for culmen length is  $\approx 0.082$ , which means that for each one millimeter increase in culmen length, the log-odds of being male increase by  $\approx 0.082$ . The model significantly improves over the null model, as indicated by the large reduction in deviance (368.74 to 128.20) and an AIC of 136.2.

- 6) How accurate is the logistic regression model in predicting the sex of penguins, and how does its performance compare to the decision tree?

I used the following R code:

```
full_accuracy <- accuracy(pred_full, truth = sex, estimate =  
sex_full)  
  
confusion_full <- pred_full %>%  
  
conf_mat(truth = sex, estimate = sex_full)  
  
print(confusion_full)
```

.metric	.estimator	.estimate	Truth	
			Prediction	FEMALE MALE
1	accuracy	binary	0.925	FEMALE 32 4 MALE 1 30

The logistic regression model achieved an accuracy of approximately 0.925 (92.5%), indicating strong predictive performance on the test dataset. The confusion matrix shows that the model correctly classified 32 females and 30 males, while misclassifying only 5 cases (1 false positive and 4 false negatives).

Model Accuracy
Decision Tree 0.9099099
Logistic Regression 0.9253731

Overall, both the decision tree and logistic regression models performed well on predicting the sex of the penguin as seen in their accuracy values of over 90%.

## VI) Conclusion

In this project, we analyzed the Palmer Archipelago penguin dataset to predict the sex of penguins using physical measurements like body mass, culmen length, flipper length, and

culmen depth. Through exploratory data analysis, we observed distinct differences in these features between males and females. Using decision trees and logistic regression, we achieved strong predictive performance, with accuracies of  $\approx 91\%$  and  $\approx 92.5\%$ . The logistic model indicates that culmen depth is a particularly strong predictor of sex, more so than body mass. This aligns with biological understanding that sexual dimorphism in some bird species can be prominently seen in features like beak depth (Owens, 1988). The exclusion of flipper length from the final model suggests that, although potentially informative, this variable does not provide additional predictive power beyond what body mass and culmen depth already offered in this specific dataset and model setup. Additionally, bootstrap resampling allowed us to estimate the sampling distribution of the mean body mass and construct confidence intervals.



## VII) Citations

Gorman, K. B., Williams, T. D., & Fraser, W. R. (2014). Ecological Sexual Dimorphism and Environmental Variability within a Community of Antarctic Penguins (Genus *Pygoscelis*). *PLOS ONE*, 9(3), e90081.

<https://journals.plos.org/plosone/article/file?id=10.1371/journal.pone.0090081&type=printable>

Pérez-Barbería, J. (2006). Sexual Segregation in Vertebrates: Ecology of the Two Sexes. Based on a workshop held in Cambridge, September 2002. Edited by K E Ruckstuhl and P Neuhaus. *The Quarterly Review of Biology*, 81(4), 424–425.

<https://www.journals.uchicago.edu/doi/10.1086/511617>

Fairbairn DJ (1997). Allometry for sexual size dimorphism: pattern and process in the coevolution of body size in males and females. *Annu Rev Ecol Syst* 28: 659–687.

<https://www.annualreviews.org/content/journals/10.1146/annurev.ecolsys.28.1.659>

*Palmer Archipelago (Antarctica) penguin data*. (n.d.). from

<https://www.kaggle.com/datasets/parulpandey/palmer-archipelago-antarctica-penguin-data>

Owens, I. P. F., & Hartley, I. R. (1998, March 7). *Sexual dimorphism in birds: Why are there so many different forms of dimorphism?*. Proceedings of the Royal Society B: Biological Sciences. <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC1688905/>

All images were sourced from the open source website “Unsplash” (<https://unsplash.com/>)

# Experimental Design: Maximizing Statapult Launch Distance

Madison Buckwalter

Kate Ford

Samyak Shrestha

Melissa Vu

Professor Leatherman — STAT 306

April 21, 2025

## I. $2^5$ Factorial Design

### Experimental Design

We begin our design by defining the factors of our experiment with chosen high (+) and low (-) settings for five different factors on our Statapult. For the simplification in data collection, we chose to keep our catapult *Tilt* constant, at level 2. The remaining factors – rubber band position on arm (*Arm*), ball position (*Position*), ball type (*Ball*), arm draw back height (*Height*), and rubber band attachment on post (*Post*) will be the factors in which we will change in our experiment. For the levels in our experiment, we chose to test a golf (+) and wiffle (-) ball as our levels for our *Ball* factor, while the rest of our factors correspond to settings on our Statapult: placement 3 (+) and placement 1 (-).

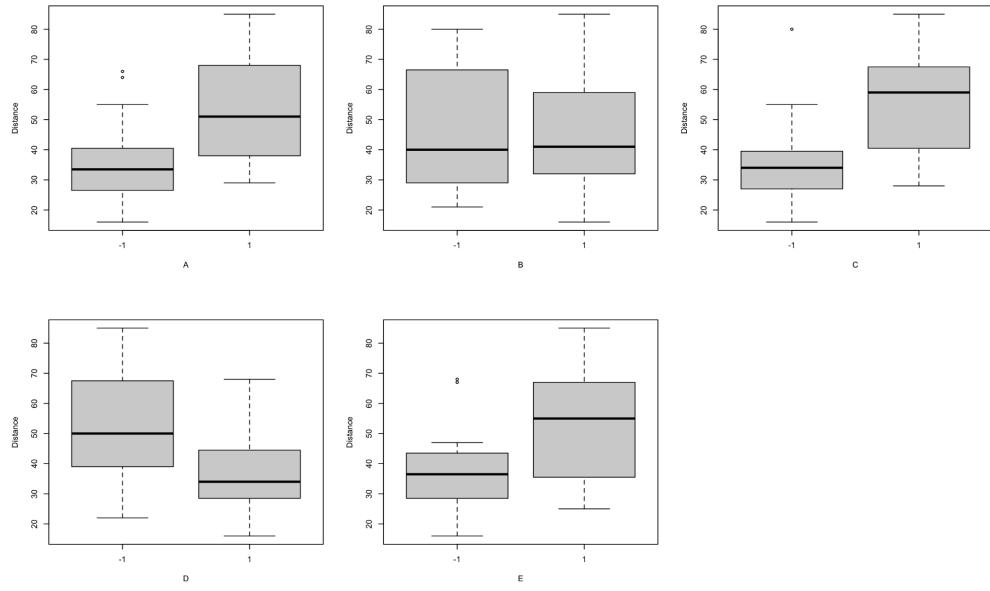
Following our factor assignments, our team began by setting up our equipment. We chose to have the Statapult launch our observations from the floor, with a measuring tape placed upon the floor beneath the release point of the Statapult arm. While conducting this experiment, our team was expected to find a way to account for the change in distance launched associated with rubberband fatigue. We were able to account for this source of variation by randomizing the order of our runs, thus reducing the chance of biased results. Before discussing our analysis, it is important for us to note that our  $2^5$  factorial design is a single replicate study due to the fact that our team was conducting our experiment during a short period of allotted time, or else replicates could have been made.

### Exploratory Data Analysis

To begin our analysis, our team produced the following box plots of our data in order to establish a general idea of the center, shape, and spread of our data before we create our models.

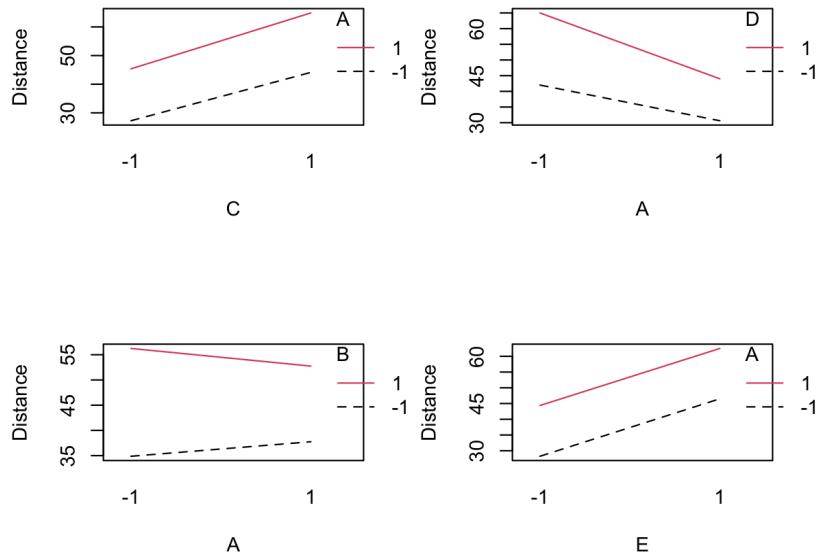
Factor Name	<i>Arm</i>	<i>Ball</i>	<i>Position</i>	<i>Height</i>	<i>Post</i>
Appears in Model	A	B	C	D	E

**Table 1:** Key for Referencing Factors in Model



**Figure 1:** Box Plots of Tested Factors at (+) and (-) Levels

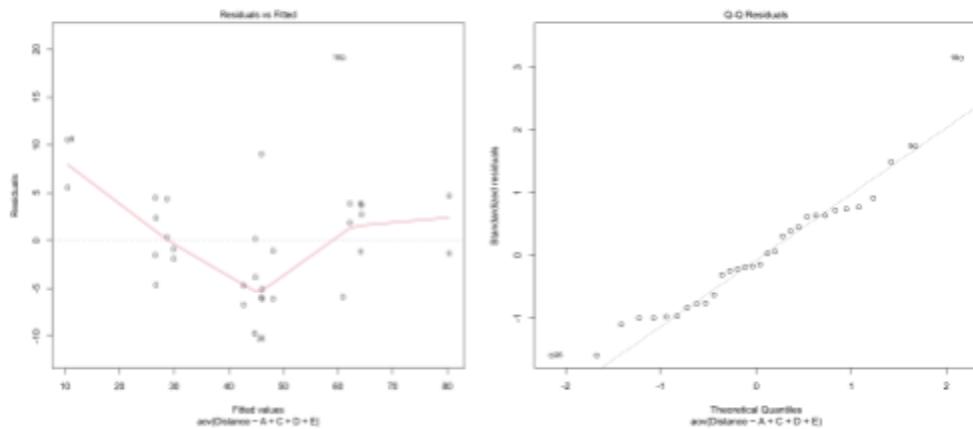
In addition to box plots, our team produced interaction plots between each of our factors in order for us to identify potential interactions which may appear to be significant to our later analysis.



**Figure 2:** Series of Produced Interaction Plots

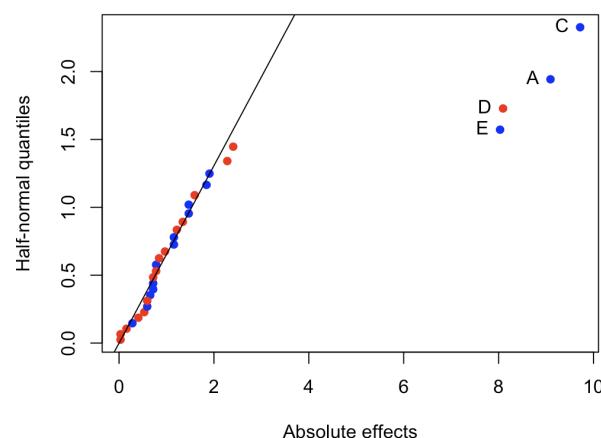
Due to the fact that each plot appears to display parallel behavior, we can conclude that interaction between factors is most likely not going to be important in our analysis. After conducting our initial EDA, our team began with the building of our model. We begin by

defining a model which contains all main effects and interactions. Before our team conducted model analysis, we first had to ensure that the conditions of our model were satisfied.



**Figure 3:** Plots to Assess Conditions – Residuals vs Fits (L) and QQ Plot (R)

Above, we provided the two plots in which our team used to assess if our model met the conditions of mean zero, equal variance, and normality. Regarding the plot to the left, our team was concerned about the clustering pattern our data displays, but this is to be expected from the construction of our data having clusters of observations at 25, 45, and 60 inches. In addition, despite the dip in our residuals, our team concluded that our data displays mean zero in addition to equal variance. On the other hand, the check for normality using our QQ plot was much more simple, as there was no question that our residuals follow the QQ line and do not deviate too significantly. Hence, we assured that our model met the proper conditions for analysis. Next, in order to identify which factors have significant effects on our response variable, we run the half normality plot as below.



**Figure 4:** Half Normal Probability Plot

From the plot output above, we can see that points A, C, D, and E notably deviate far from the quantile line, indicating that these factors have large effects and are statistically significant in determining the response variable – Distance (in). Thus, we will only consider factors A, C, D, and E in our design throughout the rest of our analysis.

$$Y = \beta_0 + \beta_A A + \beta_C C + \beta_D D + \beta_E E + \epsilon$$

**Equation 1:**  $2^5$  Design Model

## Design Analysis

Now that we have assessed conditions and identified the terms in our model which our team will include for our design analysis, we may begin testing our model using our ANOVA procedure. In total we will conduct two hypothesis tests, one in which we will be testing our model against the null model and the other we will test the significance of the main effects in our model – since we do not have any interaction terms, these are the highest order terms.

Our team begins the test of our chosen model versus the null model. Consider the null hypothesis that our chosen model is indistinguishable from the null model in predicting *Distance*, with the alternative being that our chosen model has at least one term in our model which has a non-zero effect on *Distance*. With  $F_4 = 56.103$  and associated  $p\text{-value} = 1.081\text{E-}12 << 0.05$ , we have sufficient evidence to reject the null hypothesis. Thus, we can conclude that our chosen model contains at least one non-zero term which predicts *Distance* better than the null model. Since we can only conclude that there is at least one term in our model which has an effect on *Distance*, we must perform a full ANOVA on our model to determine which predictor/predictors in our model are responsible for this. Consider the null hypothesis to be that a chosen predictor in our model has zero effect on predicting *Distance*, where the alternative is that our predictor has a non-zero effect on *Distance* and belongs in our model. Below, is a table of all of the reported F and p-values for each term in our model.

Predictor	A <i>Arm</i>	C <i>Position</i>	D <i>Height</i>	E <i>Post</i>
$F_{1,27}$	60.418	69.008	47.861	47.124
P-value	2.33E-8	6.46E-9	1.96E-7	2.24E-7

**Table 2:** Reported F and p-values for Main Effect Hypothesis Testing

With all terms above having a proper F and associated p-value much less than our significant level ( $\alpha = 0.05$ ), we have sufficient evidence to conclude that all terms in our model have a non-zero effect in predicting *Distance* for our data set. Hence, we can conclude that our chosen model is proper for our team to use as a means of analysis of our  $2^5$  Factorial Design.

Now that we have confirmed the predictors in our model to be significant, we can identify a recommendation of a setting which will maximize *Distance*. Using Figure (1), we can identify the levels of each factor which are associated with a greater mean *Distance* response to build our final recommendation. Based on the analysis of our data, we can make a final recommendation that in order to maximize distance launched from the Statapult – regardless of ball type – let *Arm* be at placement 3 (+), *Position* to be at placement 3 (+), *Height* be at placement 1 (-), and *Post* to be at placement 3 (+).

Predictor	A <i>Arm</i>	B <i>Ball</i>	C <i>Position</i>	D <i>Height</i>	E <i>Post</i>
Level	(+)	Not significant	(+)	(-)	(+)

**Table 3:** Final Recommendation for Factor Levels from  $2^5$  Design Analysis

## II. $2^{5-1}$ Half Fraction Factorial Design

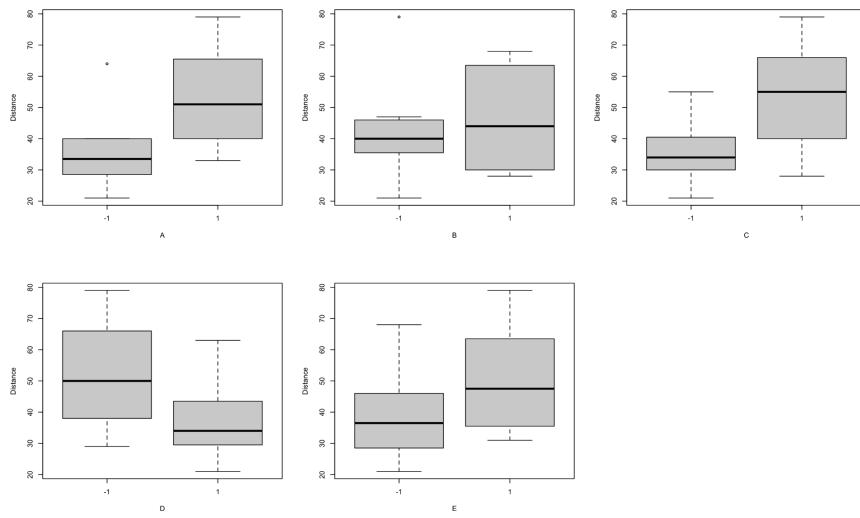
### Experimental Design

To reduce our data in order to simulate a half fraction factorial design, as if we had only been able to conduct 16 runs, we blocked our data according to the defining relation  $I \equiv ABCDE$ . We alias  $I$  with the highest order interaction term, which is the five-way interaction between each factor we changed in the statapult: arm, position, ball, height, and post. This resulted in two blocks with 16 effects in each, one of which became the entirety of our data for this portion of the analysis. The defining contrast  $L = \alpha_{arm}x_{arm} + \alpha_{position}x_{position} + \alpha_{ball}x_{ball} + \alpha_{height}x_{height} + \alpha_{post}x_{post}$  can equal 0 mod 2 or 1 mod 2. Block 1, the principal block, includes the treatment combinations for which  $L = 1 \text{ mod } 0$ , while Block 2 includes those for which  $L = 0 \text{ mod } 1$ . We chose to analyse the treatment combinations included in Block 2, leaving us with 16 runs on which to perform a

similar analysis as above. As the largest treatment effect in Block 2 is abcde, a word of length = 5, the resolution of this design is V, so the final notation of this design is  $2_V^{5-1}$ .

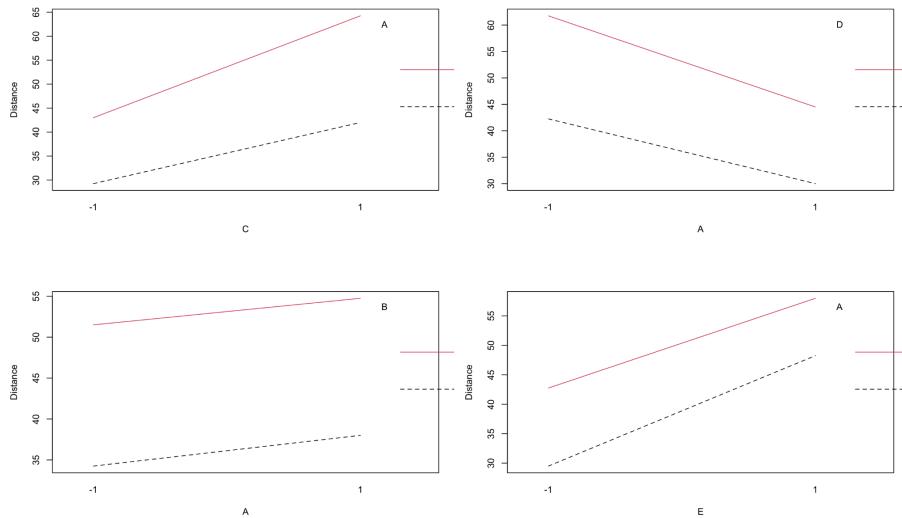
## Exploratory Data Analysis

We anticipate that the analysis of the reduced data will resemble the full 32 run factorial experiment described above because of the rigorous blocking procedure. In an initial exploratory analysis, we find similar results in the box plots comparing each factor at high and low levels.



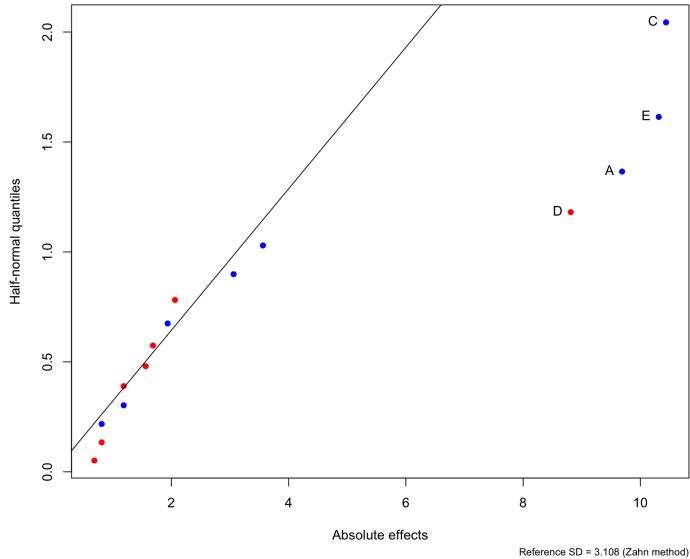
**Figure 5:** Box Plot exploration of the data analyzed in the half fraction design.

Similarly, we find no evidence of interaction in the subsetted data as seen in the plots below.



**Figure 6:** Interaction plots for factors in the half fraction factorial design.

We define an initial model with every possible term, which includes main effects and two-way interactions given the limitations of having only 16 runs. With the half normality plot below , we identify which factors have significant effects on our response variable in the half fraction factorial design.



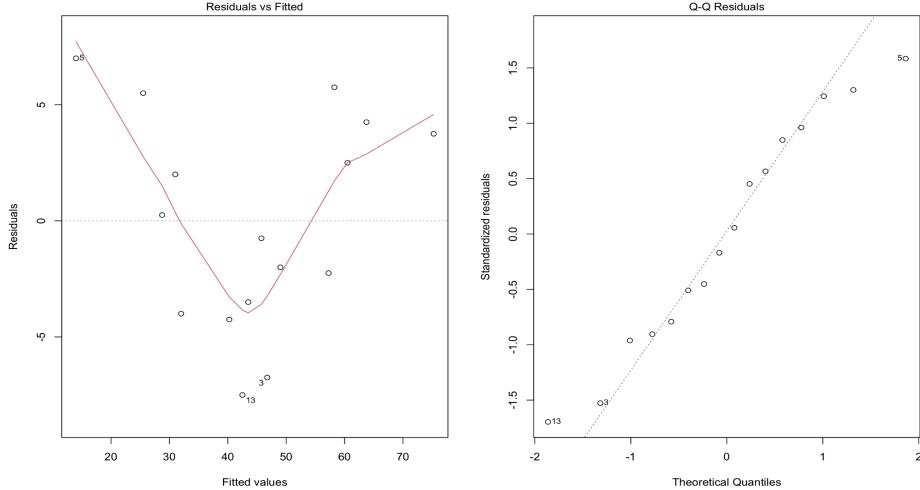
**Figure 7:** Half Normal Probability Plot for Half Fraction Factorial Design

From the plot output above, we can see that points A, C, D, and E notably deviate far from the quantile line, indicating that these factors have large effects and are statistically significant in determining the response variable – Distance (in). Thus, we build a model to analyze distance traveled in the half fraction design:

$$Y = \beta_0 + \beta_A A + \beta_C C + \beta_D D + \beta_E E + \epsilon$$

**Equation 2:**  $2^{5-1}$  Design Model

We also assess the conditions of this model with the plots below, which indicate some concern about the zero mean and equal variance conditions. The QQ plot confirms the normality condition has been met, and we will proceed with analysis despite light concerns for other conditions being met.



**Figure 8:** Condition-checking figures for the half fraction factorial design.

## Design Analysis

To conduct our two hypothesis tests for this reduced data, we build a null model and run a general model utility test. With an F statistic of 33.891 and an associated p value of 3.971e-06, we have sufficient evidence to reject the null hypothesis. Thus, we know that at least one factor in our model has a nonzero effect on Distance. We run a full ANOVA to examine the main effects of the factors in our model, finding each individually to have a nonzero effect on Distance, as demonstrated by Table 4:

Predictor	A <i>Arm</i>	C <i>Position</i>	D <i>Height</i>	E <i>Post</i>
$F_{1,27}$	40.691	45.619	30.633	18.621
P-value	5.231e-05	3.140e-05	0.0001768	0.0012245

**Table 4:** Reported F and p-values for Main Effect Hypothesis Testing in the Half Fraction Factorial

Finally, based on these factors being significant and using the visual comparisons of factor levels in Figure 5, we recommend the following levels of each factor in order to maximize distance traveled:

Predictor	A <i>Arm</i>	B <i>Ball</i>	C <i>Position</i>	D <i>Height</i>	E <i>Post</i>
Level	(+)	Not significant	(+)	(-)	(+)

**Table 5:** Final Recommendation for Factor Levels from  $2^5$  Design Analysis

### III. Comparison

Both factorial ( $2^5$ ) and half fractional factorial ( $2^{5-1}$ ) designs provide consistent results regarding identifying the key factors determining the Statapult's launch distance. In both of our designs, factors A, C, D, and E are found to have statistically significant effects on the response variable.

Moreover, the final recommendation for factor settings to maximize launch distance are also identical in both designs: high level for Arm (A), high level for Position (C), low level for Height (D), and high level for Post (E). This result indicates that despite the reduced number of runs in the half fractional factorial design, we were able to obtain the similar insights outcome with half the experimental effort.

Considering the trade-off between these two designs, the full factorial ( $2^5$ ) design enables the estimation of all main and interaction effects by having a higher resolution. However, this type of design requires a higher number of runs, making the experiment more resource-intensive and time-consuming. In contrast, the fractional factorial ( $2^{5-1}$ ) design reduces experimental burden with less runs. While this introduces the possibility of confounding higher-order interactions, the half-fractional model successfully identified the same significant main effects (factors A, C, D, and E) and yielded identical recommendations for optimal factor levels, which demonstrates its efficacy in this context.

### III. Discussion & Conclusion

Ultimately, our results show that to achieve the farthest launch distance using the Statapult, one should set the Arm, Position, and Post factors to their high settings, while setting Height to its low setting. Ball type had no significant effect, allowing greater flexibility in material use without affecting performance. Even though both designs identified four similar significant factors - Arm (A), Position (C), Height (D), and Post (E) - in maximizing the Statapult's launch Distance (in), we can consider which design to use based on our preferences for future experiments.

From a practical standpoint, the full factorial design offered a complete view of all possible interactions and ensured no aliasing of main effects. This level of comprehensiveness is useful in experimental settings where interactions may be present or suspected. However, our

data showed little to no interaction among factors, supported by largely parallel interaction plots in EDA, suggesting that a simpler design such as the half fractional factorial one would have been sufficient.

The half-fractional design effectively identified the same significant main effects and optimal settings while requiring only half the number of experimental runs. Using this type of design is desirable in scenarios where there are time constraints and limited materials. In our Statapult experiment, the same key factors were identified in both designs. Therefore, the half-fractional design is not only more efficient but just as effective, which is ideal for your goal of maximizing Statapult launch distance with minimal effort.

## References

Montgomery, Douglas C. (2019). Design and Analysis of Experiments, Tenth Edition. New York Wiley.