

Asymptotic bounds for T(n) = T(n-2) + n

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I am trying to figure out how to find the Asymptotic bounds for T(n) = T(n-2) + n and I am pretty sure that I need to use the substitution method. I have what I believe is a proof using the <u>Subtract and Conquer</u> method, which was not a method that has been covered yet.

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Subtract and Conquer

$$\star$$

$$T(n) = T(n-2) + n$$

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Subtract and Conquer: T(n) = aT(n-b) + f(n) when n > 1

$$T(n) = [O(n^k), \ if \ a < 1 \ | \ O(n^{k+1}), \ if \ a = 1 \ | \ O(n^{ka^{n/b}}), \ if \ a > 1]$$

$$a=1,\;b=2,\;k=1$$

$$T(n) = O(n^{1+1})$$

$$\equiv T(n) = \mathcal{O}(n^2)$$
 $lacksquare$

I tried to formulate one by expanding the recurrence, but I just don't know where to go after doing so

$$T(n) = T(n-2) + n$$

$$T(n-2) = [T(n-4) + (n-2)] + n$$

$$=T(n-4)+2n-2$$

$$T(n-4) = [T(n-6) + (n-4)] + 2n - 2$$

$$=T(n-6)+3n-6$$

$$T(n-6) = [T(n-8) + (n-6) + 3n - 6]$$

$$=T(n-8)+4n-12$$

Substitute k: $T(n-2k) + kn - c_1$

And then with the substitution method:

$$T(n) = T(n-2) + n$$

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Assume:
$$T(k) \le c \cdot k^2$$
 for $k < n$

$$\leq c(n-2)^2 + n$$

$$=cn^2 + (-4) + n$$

$$= cn^2 - (4-n)$$

$$(4-n) \geq 0$$
 for $n \geq 4$

$$T(n) = O(n^2)$$

I'm not even sure if I'm in the ballpark here. If someone has a good article to read or some advice I would really appreciate it.

recursive-algorithms



did you try GFs? It may get a bit messy though - Alex Oct 9 '15 at 11:28

Sorry what is "GFs"? - Jens Bodal Oct 9 '15 at 21:21

generating functions - Alex Oct 9 '15 at 21:44

2 Answers



For even n=2k,



$$T(2k) = T(2(k-1)) + 2k$$



or



$$T'(k) = T'(k-1) + 2k$$

which is a simple first order recurrence on k (sum of integers), and

4)

$$T(n) = T(2k) = 2rac{k(k+1)}{2} + T_0 = rac{n}{2}(rac{n}{2}+1) + T_0.$$

For odd n = 2k + 1,

$$T(2k+1) = T(2(k-1)+1) + 2k + 1$$

or

$$T^{1}(I_{2}) = T^{1}(I_{2} - 1) + 2I_{2} + 1$$

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$$T(n) = T(2k+1) = k(k+2) + T_1 = rac{(n-1)(n+3)}{4} + T_1.$$

Hence whatever the initial values,
$$T(n)=\Theta(n^2)$$
 and $\lim_{n\to\infty} \frac{T(n)}{n^2}=\frac{1}{4}.$

edited Oct 9 '15 at 10:36

answered Oct 9 '15 at 10:27



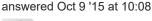


Maybe this help you:
$$T(n)=rac{1}{2}n.\ (n+1)-3+T(1)+T(2)-T(n-1) ext{ for } n=3,4,5,\dots$$











Sorry I don't think I understand enough to see the relation between this and my problem. Why $T(n) = \frac{1}{2}n$? Jens Bodal Oct 9 '15 at 21:23 🎤

I just did some calculations to find relation between T(n) & T(n-1) like usual recursive functions hope you will be more happy with it instead of your original relation. It's easy to reach this. - A.F.23 Oct 9 '15 at 21:38