MATHEMATICS

# Asymptotic bounds for $T(n) = T(n-2) + n$

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I am trying to figure out how to find the Asymptotic bounds for $T(n) = T(n-2) + n$ and I am pretty sure that I need to use the substitution method. I have what I believe is a proof using the Subtract and Conquer method, which was not a method that has been covered yet.

### Subtract and Conquer

$T(n) = T(n-2) + n$

Subtract and Conquer: $T(n) = aT(n-b) + f(n)$ when $n > 1$

$T(n) = [O(n^k),\ if\ a < 1 \mid O(n^{k+1}),\ if\ a = 1 \mid O(n^{ka^{n/b}}),\ if\ a > 1]$

$a = 1,\ b = 2,\ k = 1$

$T(n) = O(n^{1+1})$

$\equiv T(n) = \mathcal{O}(n^2)\ \blacksquare$

### I tried to formulate one by expanding the recurrence, but I just don't know where to go after doing so

$T(n) = T(n-2) + n$

$T(n-2) = [T(n-4) + (n-2)] + n$

$= T(n-4) + 2n - 2$

$T(n-4) = [T(n-6) + (n-4)] + 2n - 2$

$= T(n-6) + 3n - 6$

$T(n-6) = [T(n-8) + (n-6) + 3n - 6$

$= T(n-8) + 4n - 12$

Substitute k: $T(n-2k) + kn - c_1$

### And then with the substitution method:

$T(n) = T(n-2) + n$

Assume: $T(k) \le c \cdot k^2$ for $k < n$

$\le c(n-2)^2 + n$

$= cn^2 + (-4) + n$

$= cn^2 - (4-n)$

$(4-n) \ge 0 \; for \; n \ge 4$

$T(n) = O(n^2)$ ■

I'm not even sure if I'm in the ballpark here. If someone has a good article to read or some advice I would really appreciate it.

recursive-algorithms

asked Oct 9 '15 at 8:25

Jens Bodal

**103** 3

did you try GFs? It may get a bit messy though – Alex Oct 9 '15 at 11:28

Sorry what is "GFs"? – Jens Bodal Oct 9 '15 at 21:21

generating functions – Alex Oct 9 '15 at 21:44

## 2 Answers

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For even $n = 2k$,

$$T(2k) = T(2(k-1)) + 2k$$

or

$$T'(k) = T'(k-1) + 2k$$

which is a simple first order recurrence on $k$ (sum of integers), and

$$T(n) = T(2k) = 2\frac{k(k+1)}{2} + T_0 = \frac{n}{2}(\frac{n}{2}+1) + T_0.$$

For odd $n = 2k+1$,

$$T(2k+1) = T(2(k-1)+1) + 2k + 1$$

or

$$T'(k) = T'(k-1) + 2k + 1$$

$$T(n) = T(2k+1) = k(k+2) + T_1 = \frac{(n-1)(n+3)}{4} + T_1.$$

Hence whatever the initial values, $T(n) = \Theta(n^2)$ and $\lim_{n\to\infty} \dfrac{T(n)}{n^2} = \dfrac{1}{4}$.

edited Oct 9 '15 at 10:36

answered Oct 9 '15 at 10:27

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**157k** 13 94 256

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Maybe this help you:
$T(n) = \frac{1}{2}n.\,(n+1) - 3 + T(1) + T(2) - T(n-1)$ for $n = 3, 4, 5, \ldots$

answered Oct 9 '15 at 10:08

A.F.23
**608** 3 13

Sorry I don't think I understand enough to see the relation between this and my problem. Why $T(n) = \frac{1}{2}n$? – Jens Bodal Oct 9 '15 at 21:23

I just did some calculations to find relation between T(n) & T(n-1) like usual recursive functions hope you will be more happy with it instead of your original relation. It's easy to reach this. – A.F.23 Oct 9 '15 at 21:38