Project II - SF2975 Financial Derivatives

Henrik Hult, Fredrik Viklund, Alexander Aurell, Viktor Nilsson

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Project II: Calibration of the Hull-White Model

Preliminaries

The objective is to calibrate a one-factor Hull-White model to market data. This data consists of a zero coupon yield curve that you will obtain by running the provided Python notebook which produces a yield curve calibrated against observed par coupon rates. Insert a recent set of the daily updated U.S. treasury par rates from their website, or another bond of your choice, into the notebook and run it.

Hull-White Model

Recall that the one-factor *Hull-White model* for the continuously compounded yearly short rate is given as follows:

$$dr(t) = (\Theta(t) - ar(t))dt + \sigma dW(t), \quad r(0) = r_0. \tag{1}$$

The time t refers to years, with the longest maturity being $T^* < \infty$. Let $a = 10\%$ and $\sigma = 1\%$, say. $\Theta(t)$ is a deterministic function.

Assignments

1. The first objective is to compute the Hull-White term structure $p(t, T^*)$ numerically, from the zero coupon yield curve implied by market data. This can be done in the following steps.

   (a) Export a continuously compounded zero coupon yield curve $y^*(0, T)$, bootstrapped from market data of your choice, using the Python notebook. Transform the yield curve to a forward rate curve $f^*(0, T)$, $0 \leq T \leq T^*$. Plot $y^*(0, T)$, $f^*(0, T)$ and $p^*(0, T)$.

   (b) Express $\Theta(t)$ in terms of the forward rate curve and plot it.

(c) Simulate the short rate $r(t)$ and plot it. This means sampling one path of $r(t), 0 \leq t \leq T^*$, using the $\mathbb{Q}$-dynamics in Equation (1). Add $\Theta(t)/a$ to the same plot. How do these relate?

(d) Assume having N such simulations $\{r_i(t)\}_{i=1}^N$. Briefly describe a Monte-Carlo approach to computing $p(0, T^*)$. Optional: Implement it and check that it coincides with the data $p^*(0, T^*)$ for a large N.

(e) Use the steps (a)-(c) to calculate the Hull-White term structure $p(t, T^*)$, i.e., conditioned on your simulated short rate $r(t)$, and plot it.

Details can be found in Chapter 24 of Björk. Illustrate and comment on your results!

2. Derive formulas for pricing caplets and swaptions in the Hull-White model.

   (a) The formulas can be expressed using the formula for zero-bond put options as described on page 5 of the paper "Calibration Methods of Hull-White Model" that can be found on the course Canvas page. Hint: In the case of swaptions, use Jamshidian's trick (see the article and/or Wikipedia).

   (b) Derive the pricing formula $ZBP(T_F, T_P, K)$ for a zero bond put option with fixing time T_F, paying time T_P and strike K. In Chapter 26 of Björk, this is derived for the call case. Adapt the arguments of Chapter 26.5 to get the formula for put options, corresponding to Proposition 26.13.

3. Use the formulas you derived together with your calibrated model to numerically compute prices of either caplets or swaptions (or both), chosen as you wish. Do not use Monte-Carlo for this.