Report Instructions - SF2980 Risk Management

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Instructions

Objectives

The projects are intended as open ended exercises suitable for deeper investigation of some selected topics. Students will

- practice by applying the theory and methods on real examples
- develop their own statistical software functions in programs such as "R" or "MatLab" to solve the problems
- make qualitative and quantitative statements and conclusions about the risk management problems studied in the projects
- present the problems, relevant theory, results and conclusions in written reports
- present the problems, relevant theory, results and conclusions in oral presentations and organized discussions.

Format

- Students will work on the projects in groups of at most four people.
- The projects will be presented and evaluated in written form by handing in two written reports.
- The projects will be presented orally at two presentation seminars.

Grading

On each report you will be given a score in the range 0-25; the total score for the two reports is at most 50. A combined score of at least **25 pts is needed to pass** the assignments part of the course. Grades will be based on the following criteria.

- **Objectives.** A clear description of the project and its objectives.
- **Mathematical background.** A clear and concise presentation of the most relevant mathematical background.
- **Results.** A clear and concise presentation of your solution and results. You may also add your own explorations/extensions that you find relevant.
- **Summary.** A brief complete summary of your results.

Report Template

To get started with the report there is a report template in LaTeX available for download on the Canvas page. Please use the sections outlined in the report template. If you are not able to use LaTeX to typeset your report you may use other software (e.g. Word, Pages, etc.)

KTH FinanceLab

Some parts of the projects will be done with the help of KTH FinanceLab. We will mainly use FinanceLab to incorporate real data into the assignments, but all coding can be done in "R" or "Matlab". You can log in to KTH FinanceLab at

`http://www.math.kth.se/matstat/finance/financelab/index.html`.

If you experience problems using FinanceLab you may get support by Adam Lindhe `adlindhe@kth.se`.

Report I

In the first report (Report I), due on **November 22 at 23:59**, you will present your analysis and results on the **mandatory project seminar on Nov 28**. The report must be be **typeset on a computer** (no handwriting) and handed in by uploading it on Canvas in PDF format. You do not have to supply your source code.

Pension savings

Assignments (a) and (b) are almost identical to Project 8 p. 229 in the book, assignments (c, d, e) are new.

Consider a yearly investment of $1,000$ dollars in long positions in a portfolio of stocks and a risk-free one-year zero coupon bond over a 30-year period. The yearly returns on the portfolio of stocks year k is modeled as $R_k = e^{\mu+\sigma Z_k}$, where Z_k is standard Normally distributed. The yearly returns are assumed to be independent. The yearly return on the risk-free bond is assumed to be $e^{0.01}$. Each year, after the investment of $1,000$ dollars has been added, the entire portfolio is rebalanced such that the fraction of the total portfolio value invested in the stocks at the beginning of year k is $p(1-c(k-1)/30)$, where $p, c \in [0, 1]$.

(a) Determine a function f such that the value of the pension savings in 30 years can be expressed as $V_{30} = f(\mu, \sigma, p, c, Z_1, \dots, Z_{30})$.

Simulate a sample of suitable size n from the distribution of $(Z_1, \dots, Z_{30})$ and use this sample to determine the empirical distribution F_n of V_{30} for a range of values of the parameters μ, σ, p, c.

(b) Set $\mu = 0.03$ and $\sigma = 0.2$ and investigate the effects on the empirical distribution $F_n(p, c)$ of V_{30} of varying p and c. Suggest a suitable criterion for selecting the optimal empirical distribution $F_n(p, c)$ and determine the optimizer (p, c). For the optimal portfolio you must report the mean $E[V_{30}]$ and the 1%-quantile: $F^{-1}_{V_{30}}(0.01)$.

(c) Sweden's pension system has transited from traditional life insurance to a system based on fund insurance. Roughly speaking fund insurance works as in (a) and (b) where the risky asset can be viewed as a portfolio selected by the customer. In traditional life insurance the individual participates in a collective where there is a guaranteed return as well as a potential for higher returns for the collective if the investments in the risky asset (portfolio) performs well. One version of traditional life insurance works as follows (there are some variations on this theme).

There is a guaranteed annual return $\bar{r}$ on all investments. The guaranteed return is typically slightly lower than the risk-free rate. The fund manager of the traditional life insurance policy then invests the necessary amount in the risk-free asset to cover the guarantee and the remaining surplus in the risky asset to generate a higher return. For the investor with a yearly investment of 1,000 dollars, the guaranteed amount after k years is G_k where

$$\begin{aligned} G_0 &= 0, \\ G_k &= 1000 \sum_{j=1}^{k} e^{\bar{r}j}, \quad k \geq 1. \end{aligned}$$

Let $V_0 = 0$ and V_k, $k \geq 1$, be the value of the portfolio after k years. At the beginning of year $k+1$ the amount $G_{k+1}e^{-r}$ is invested in the bank account and $V_k + 1000 - G_{k+1}e^{-r}$ is invested in the risky asset.

Let $\bar{r} = 0.005$ and plot a histogram of V_{30} using the traditional life insurance strategy and compare it to your optimal solution in (b). You must also report the mean $E[V_{30}]$ and the 1%-quantile: $F^{-1}_{V_{30}}(0.01)$.

(d) It may be difficult to generate large profits using the strategy in (c) and a way to obtain larger profits is to have a leverage in the exposure to stocks. In this case it is assumed that the market will not experience a crash of more than, say, 50% over one year.

At the beginning of year $k+1$, the surplus over the guarantee is invested in the stock with a leverage of 2 (the 2 is 1/0.5). That is, the amount $2(V_k + 1000 - G_{k+1}e^{-r})$ is invested in the stock and the remaining amount, $V_k + 1000 - 2(V_k + 1000 - G_{k+1}e^{-r})$ in the bond. This type of strategy is often referred to as a constant proportion portfolio insurance (CPPI).

Note that if the stock price falls by more than 50% during year $k+1$ the value V_{k+1} is less than the guarantee G_{k+1}. In practice one would monitor the stock price continuously and rebalance to make sure that this "never" happens. For this project you may assume that if you fall behind the guarantee, that is $V_{k+1} < G_{k+1}$, then you invest the future yearly investments in the bond until $V_{k+m} \geq G_{k+m}$ and then proceed according to the strategy, or until the end (whatever comes first).

Compare the histogram of V_{30} to the results in (b) and (c). You must also report the mean $E[V_{30}]$ and the 1%-quantile: $F^{-1}_{V_{30}}(0.01)$.

(e) In assignments (b - d) you have used a log-normal distribution for the annual returns of the portfolio of stocks. In this exercise you should replace the log-normal distribution by an empirical distribution based on monthly historical data and use the historical simulation approach. You may assume that monthly returns are independent, or use historical simulation based on, say, annual returns with overlapping segments of monthly returns. You must clearly specify your choice of historical simulation approach. You may select a portfolio consisting of one or several funds from the Swedish Pension System (PPM). You can find historical data from the PPM system at:

- `https://www.pensionsmyndigheten.se/nyheter-och-press/nyheter-fondtorg/statistik-om-pre`

You may also select your own portfolio of stocks or other funds. You may find historical data at

- `http://www.nasdaqomxnordic.com/shares`
- `http://www.nasdaqomxnordic.com/Funds`

You can also find historical data on a large number of financial products at KTH Finance Lab:

- `http://www.math.kth.se/matstat/finance/financelab/index.html`

Repeat the analysis in (b, c, d) using your historical simulation approach and compare the results.