Implied Volatility: A Cornerstone of Modern Financial Theory

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Abstract

Implied volatility stands as a cornerstone concept in options pricing and trading, playing a pivotal role in assessing market expectations of future volatility. This expository paper presents a comprehensive overview of implied volatility, spanning foundational works to recent studies. Beginning with Robert E. Whaley's introduction of the VIX index, key contributions such as the Black-Scholes model, volatility smile phenomenon, and stochastic volatility models are explored. The paper elucidates the intricacies of implied volatility modeling, forecasting methodologies, and implications for trading strategies. Further discussions encompass volatility dynamics, empirical evidence on return-volatility relationships, and practical applications in option pricing and risk management. Through a structured analysis, this paper aims to provide a thorough understanding of implied volatility, catering to researchers, practitioners, and enthusiasts in financial markets.

Introduction

Implied volatility represents a forward-looking and market-derived measure of the expected volatility of an underlying asset, crucially influencing the pricing of options. It is extracted from the market prices of options using models such as the Black-Scholes formula, reflecting the collective anticipation of market participants regarding future volatility. Unlike historical volatility, which is backward-looking and based on actual asset price changes, implied volatility projects forward, providing insights into market sentiments and potential future market dynamics.

This paper delves into the intricacies of implied volatility, tracing its conceptual and methodological evolution from its origins in the seminal Black-Scholes model to its current applications that include advanced stochastic models. It also covers the volatility smile phenomenon and its implications for the standard assumptions underpinning earlier models.

In addition to theoretical discussions, this paper explores the practical significance of implied volatility in financial markets. It discusses how traders and risk managers utilize implied volatility. The analysis extends to the impact of implied volatility on option pricing, trading strategies, risk management, and market behavior.

By offering a comprehensive review of both the theoretical frameworks and practical applications of implied volatility, this paper aims to enrich the understanding of this complex but critical concept in financial theory and practice. Through rigorous analysis, it seeks to provide valuable insights for both academic researchers and financial market practitioners.

Literature Review

Implied volatility, a pivotal concept in options pricing and trading, has garnered significant attention in financial research. This review explores seminal works and recent studies that illuminate various facets of implied volatility, including its modeling intricacies, forecasting methodologies, and implications for trading strategies.

Robert E. Whaley's groundbreaking paper "Derivatives on Market Volatility" ([a], pp. 71-84) stands as a cornerstone in the realm of implied volatility. This groundbreaking work introduced the VIX index, a pivotal measure of implied volatility in the S&P 500 options market. The VIX index provided a standardized framework for quantifying market volatility, revolutionizing risk management practices and derivatives trading strategies. Whaley argued that to compute an implied volatility, three types of information are required: an option valuation model, the values of the model's determinants (except for volatility), and an observed option price.

The study by Emanuel Derman and Iraj Kani, "Riding on a Smile" ([b], pp. 32-39), brought to light the phenomenon known as the volatility smile. This empirical regularity, characterized by the skewness observed in the implied volatility surface, challenged the assumptions of the Black-Scholes model, and propelled the exploration of more nuanced volatility models. Additionally, in the same paper, Derman and Kani introduced the concept of implied volatility surfaces, providing a framework to visualize the implied volatility of options across various strike prices and expiration dates, thereby further advancing the understanding and analysis of options pricing and risk management.

Fischer Black and Myron Scholes' paper " The Pricing of Options and Corporate Liabilities " ([c], pp. 637-654) laid the foundation for modern option-pricing theory with the introduction of the Black-Scholes model. While the model assumes constant volatility, its significance lies in providing a framework for subsequent research on implied volatility and volatility surfaces.

The research by Peter Carr and Dilip B. Madan, particularly their paper "Option Valuation Using the Fast Fourier Transform" ([d], pp. 61-73), marked a significant advancement in option-pricing models that accommodate stochastic volatility. The Carr-Madan Fourier-based option pricing method offered a robust approach for accurately pricing options under varying volatility regimes.

Bruno Dupire's pioneering work, "Pricing and Hedging with Smiles" ([e], pp. 18-20), introduced the local volatility model, which allowed for time-varying volatility. This innovative approach to modeling implied volatility surfaces shed light on the dynamics of option prices and spurred further innovations in volatility modeling.

Jim Gatheral's influential book "The Volatility Surface: A Practitioner's Guide" (2006) provided invaluable insights into implied volatility surfaces and their implications for option pricing and risk management. By elucidating the intricate relationship between option prices and implied volatility levels across different strike prices and maturities, Gatheral's work enriched our understanding of volatility dynamics in financial markets.

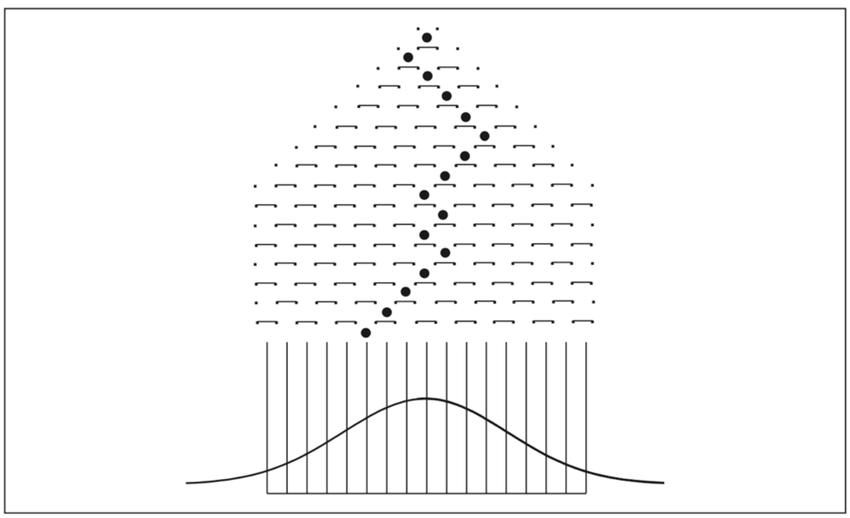
Steven L. Heston's research, particularly his paper "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options" ([g], pp. 327-343), introduced the Heston model, a widely used stochastic volatility model in options pricing. This model, with its ability to capture volatility evolution over time, offered a more nuanced depiction of option prices compared to the Black-Scholes model.

Volatility

Before delving into implied volatility, it's essential to grasp the concept of volatility([h], pp. 69). In a sense, volatility is a measure of the speed of the market. Markets that change at a slow pace are considered to have low volatility, whereas markets that change rapidly are seen as having high volatility. Naturally, some markets are more prone to fluctuations (volatility) than others.

Figure 1 below illustrates a pinball maze where a dropped ball navigates through a series of nails. With each encounter, the ball has a 50 percent chance of moving left or right. As it progresses downward, encountering more nails, the ball eventually reaches the bottom and settles into one of the troughs. Most balls land in the central area of the maze, with fewer balls landing in troughs situated further from the center. With an increased number of balls dropped into the maze, a bell-shaped or normal distribution pattern emerges.

Figure1. Random Walk and Normal Distribution [h]



Louis Bachelier [i] in 1900, first assumed that underlying contract prices follow a normal distribution. However, this assumption has logical flaws, leading to modifications over time to align with real-world conditions.

Volatility is mathematically defined as one standard deviation in percentage terms over a one-year period. Thus, volatility is proportional to the square root of time. For convenience, many traders assume that there are 256 trading days in a year since the square root of 256: 16.

$$volatility_t = volatility_{annual} \times \sqrt{t} = \frac{volatility_{annual} \times \sqrt{t}}{16}$$

Similar to how interest can be compounded at various intervals, volatility can also be compounded at different intervals (as exemplified by the Black-Scholes model, which operates in continuous time). When the percentage price changes follow a normal distribution, the continuous compounding of these changes yields a lognormal distribution of prices at expiration.

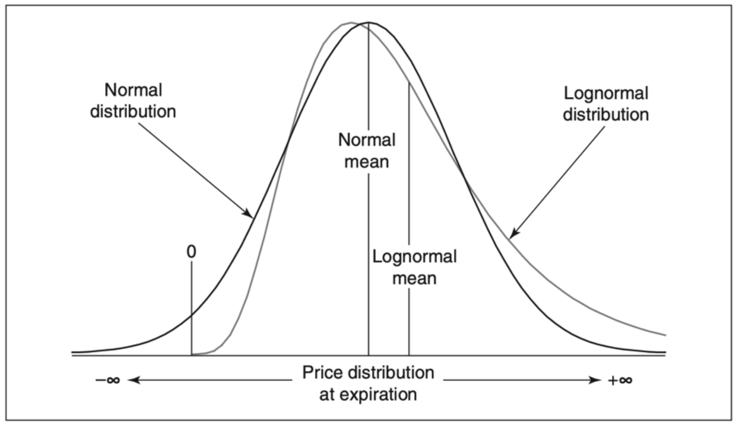
Table 1. Return of ± 10% on $1,000

Rate of Payment	**Value after One Year**	**Yield (Annualized)**
10%	$1,100	10%
-10%	$900	-10%
10% compounded continuously	1,105.17	11.05%
-10% compounded continuously	904.83	-9.52%

Table 1 above shows how the continuously compounded rates, despite having the same nominal percentage change, result in slightly different yields compared to their simple annual counterparts because of compounding over time. This phenomenon underscores the transformative effect of continuous compounding on the assumed normal distribution of price changes, reflecting the compounded impact of continuous fluctuations over time. These changes result in the lognormal distribution, in which the entire distribution is skewed towards the upside. This skew arises because upside price changes generally exhibit greater magnitude, in absolute terms, than downside price changes.

$$Y = e^X, where\ X \sim N(\mu, \sigma^2)$$

Figure 2. Lognormal Distribution [h]

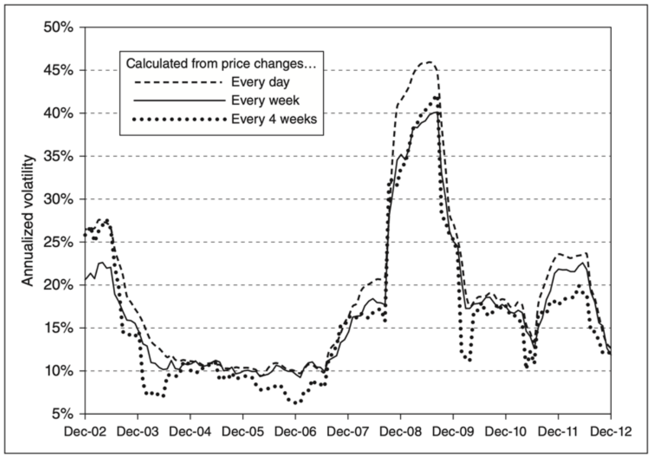


$$f(x) = \frac{1}{\sigma x \sqrt{2\pi}} e^{\frac{-(\ln x - \mu)^2}{2\sigma^2}}, x \geq 0$$

We can categorize volatility into two types: historical volatility, linked to an underlying asset, and implied volatility, linked to the derivative instrument. Realized volatility represents the annualized standard deviation of percentage price fluctuations of an underlying asset over a specified timeframe.

In Figure 3 below and in general, there is no conclusive evidence indicating that opting for one interval over another consistently leads to higher or lower volatility.

Figure 3. S&P 500 Index 250-day historical volatility [h]



Volatility Measurement

By definition, volatility is the square root or variance, that is, the standard deviation defined as:

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

The impact of a stock becoming ex-dividend can create the appearance of volatility, even when none exists. Failure to account for this adjustment could result in our volatility estimation being off by several percentage points. For instance, let's consider a scenario where there's a 5% price drop attributable to the stock going ex-dividend. On an annualized basis (i.e., 0.05 multiplied by 16) this appears as a substantial 80% move, which is evidently significant.

Dividends

There are various methods for making this adjustment ([q], pp. 17). One approach involves deducting the dividend from the price prior to the ex-dividend date. While this preserves the absolute values of the day-to-day changes before the ex-dividend date, repeated application of this method across a series of dividends may result in seemingly negative stock prices.

A better method is to multiply by an adjustment factor that leaves the percentage changes unaffected.

$$1 - \frac{dividend}{price}$$

Prices preceding the ex-dividend date undergo multiplication by this factor. This process is termed *backward adjustment*. Alternatively, one can opt for *forward adjustment* of prices, where the current price differs from the adjusted price.

In short, distinguishing between mean returns (drift) and variance poses a significant challenge, a central issue in many discussions about trading methods and outcomes. Estimates of mean return tend to be highly uncertain, particularly with small sample sizes. Therefore, we typically set the mean return to zero, enhancing measurement accuracy by eliminating a source of noise. This yields:

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i)^2$$

To estimate the variance of sample returns, we make the adjustment to have an unbiased estimator:

$$s^2 = \frac{N}{N+1}\sigma^2$$

Implied Volatility

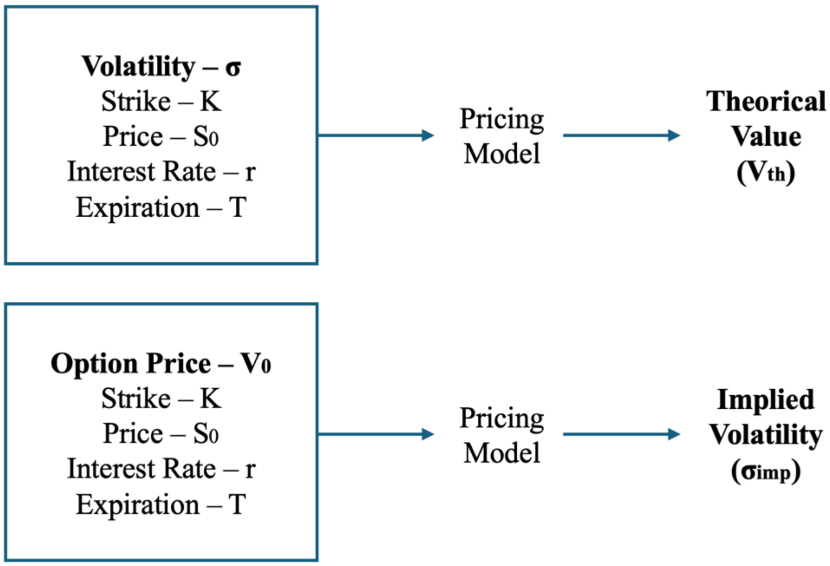
Essentially, implied volatility reflects the market's agreed-upon prediction of the underlying asset's volatility during the option's duration.

In options pricing, the theoretical value of an option is determined by various factors such as the current stock price, the option's strike price, time to expiration, interest rates, and volatility. Suppose that the calculated theoretical value differs from the actual market price, i.e., assuming value for the pricing suggests a discrepancy. There are many forces in the market that cannot be easily identified or quantified. However, one possible approach to addressing the question is to assume uniformity among traders, presuming that all market participants are utilizing the same theoretical pricing model for the option being traded.

It's improbable that either the time to expiration or the exercise price is the cause, as these factors are inherent in the option contract. Additionally, interest rates are usually the least important of the inputs into a theoretical pricing model. This implies that the cause for the discrepancy is the volatility. Essentially, it appears the market is operating on a different volatility assumption than ours.

To obtain the volatility used by the market, we need to determine the level of volatility required to generate a theoretical value equal to the option's market price when keeping all other variables constant, including time to expiration, exercise price, underlying price, and interest rates.

Figure 4. Finding Implied Volatility



In essence, in Figure 4 above, we are employing the theoretical pricing model in reverse to find the unknown volatility. Practically, this is more complex because conventional theoretical pricing models aren't designed for such backward calculations. Nevertheless, there are several relatively straightforward algorithms capable of efficiently determining implied volatility when all other parameters are known.

Implied volatility is not only a function of the variables within the theoretical pricing model but also of specific model employed. In certain cases, various models can produce notably diverse implied volatilities for the same options. Challenges may arise when inputs are outdated; relying on old option prices in the face of changing market conditions, especially with options that are too far in or out of the money, can result in misleading or incorrect estimates of implied volatility.

Market implied volatility is always changing because option prices and other market conditions are always changing. It's like the market is continuously gathering information from all traders to decide on a common volatility level for each contract expiration. However, traders don't discuss and vote on the volatility; it's determined by their actions in the market. When bids and offers are exchanged, the trading price of an option reflects a balance between supply and demand, which is captured as implied volatility.

Estimating Future Volatility

Time Series Analysis

To forecast volatility when limited historical data is available, a simple approach used is to calculate the average of the available historical volatility measures. In this scenario, with historical volatilities for different time periods (6-week, 12-week, 26-week, and 52-week).This involves assigning different weights to each historical volatility measure based on their perceived importance or relevance.

$$future\ volatility = \sum_{i=0}^{n} w_i \sigma_i$$

Imagine we're examining options with extended expiration dates. In such cases, the tendency of volatility to revert to its mean over prolonged periods tends to diminish the significance of short-term volatility fluctuations. In fact, when dealing with options expiring over extensive time frames, the most appropriate volatility forecast often aligns with the long-term average volatility of the asset. Consequently, the importance we attribute to various volatility data points will hinge on the remaining time to expiration for the options under consideration.

The analysis of data series to forecast future values is commonly known as time-series analysis. Applying time-series models to forecast volatility requires a series of independent data points. However, in our examples, the volatilities used for prediction are not truly independent because they overlap. For instance, the 52-week volatility overlaps with the 26-, 12-, and 6-week volatilities, creating dependence between data points. Instead of using historical volatilities as data points, we could consider using underlying returns, which form a genuine time series and may be suitable for applying time-series models.

The exponentially weighted moving average (EWMA) model is frequently employed as a time-series model to predict forthcoming volatility.

$$\sigma^2 = \sum_{i=1}^{n} \alpha_i \sigma_i$$

In this model, greater weight is given to more recent return. We also use the constraint that all weights ad to one and recent returns have more wight.

$$\sum_{i=1}^{n} \alpha_i = 1 \; and \; \alpha_n > \alpha_{n-1}$$

Other widely used time-series models for volatility forecasting stem from Robert Engle's ARCH model, developed in 1982. These techniques evolved into the generalized autoregressive conditional heteroskedasticity (GARCH) family of models. GARCH models consist of three components: a volatility estimate (like EWMA), a correlation component reflecting successive returns, and a mean-reversion component.

Volatility Term Structure

The term structure of volatility refers to the pattern of how implied volatility differs for options with the same underlying asset but different expiration dates. It effectively represents the market's expectations of future volatility over various time horizons. By examining the term structure, investors and analysts can gauge how volatility is anticipated to evolve over time.

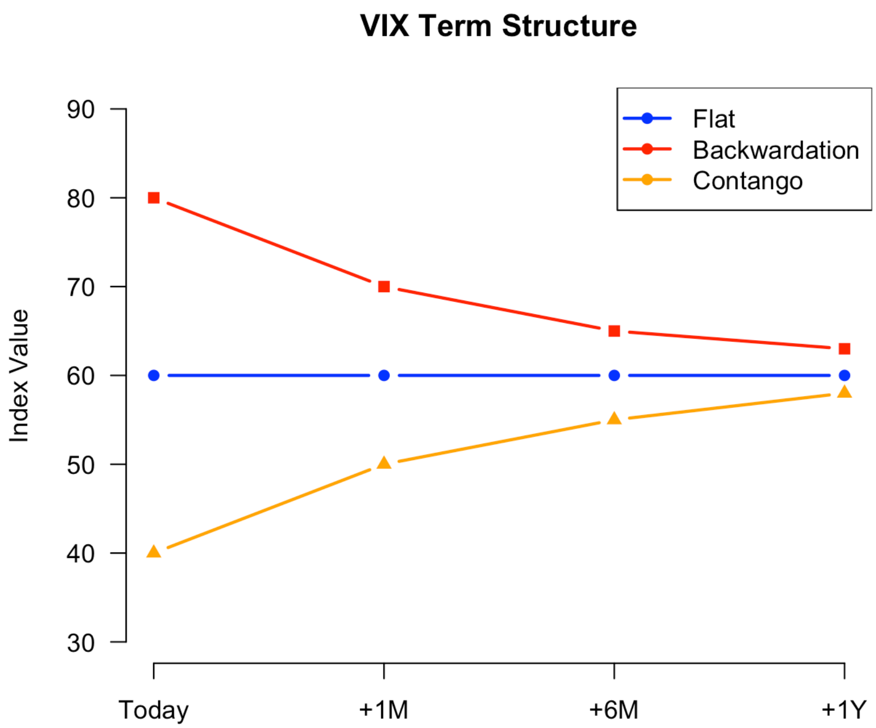
Economic indicators and macroeconomic events, such as key economic reports, monetary policy decisions, and geopolitical events, play a significant role in shaping market expectations of future volatility. These factors can lead to substantial shifts in the term structure as market participants adjust their forecasts based on new information. Additionally, market sentiment, driven by the anticipation of specific events like elections, regulatory changes, or corporate earnings reports, also profoundly influences the term structure. Whether the general market sentiment is bullish or bearish, it plays a crucial role in determining how investors and analysts perceive future risks and opportunities, thereby molding expectations of volatility across different time horizons.

Understanding the Volatility Term Structure trough VIX Futures

The Chicago Board Options Exchange (CBOE) initiated trading of VIX futures contracts in 2004, with these contracts settling to the VIX value at the opening on their expiration Wednesday, each point of volatility priced at $1,000. VIX futures are distinctive in comparison to traditional futures markets, primarily due to two reasons: the term structure of VIX futures that affects pricing as market conditions vary, and the challenge in replicating an underlying VIX position. Unlike stock index futures where traders can replicate positions by trading the constituent stocks, or physical commodity futures where one can buy the commodity, replicating a VIX position typically involves options, which is not always practical.

Three primary patterns are typically observed in the VIX term structure, with Contango being the most common. Contango, or a Normal pattern, occurs when longer-term options display higher implied volatilities compared to shorter-term options, suggesting an increasing uncertainty or risk as the future unfolds. This is often seen in VIX futures, where long-term maturities trade at higher prices than short-term ones, such as during August 2012. In contrast, the Inverted or Backwardation pattern, where shorter-term options have higher implied volatilities than longer-term counterparts, is less common and typically emerges during periods of acute market stress or turmoil, like during the financial crisis in the latter half of 2008 when VIX futures transitioned from contango to backwardation. The Flat pattern, where implied volatilities are similar across various expiration dates, indicates a stable and consensus-driven market view on future volatility.

Figure 5. Contango, Backwardation, and Market Equilibrium Trends



Strategic Considerations for Trading VIX Futures

In VIX futures trading, strategies are closely tied to market dynamics, with traders buying futures when they expect a rise and selling when they anticipate a fall. This trading behavior aligns with the VIX futures' reflection of the market's expectations for SPX implied volatility at the contract's maturity. Traders often leverage the term structure, such as through calendar spreads, to capitalize on variances between short-term and long-term volatility expectations. Shifts in this structure can indicate changes in market conditions or sentiment, playing a critical role in decisions about portfolio rebalancing and risk management.

When developing a basic futures trading strategy, traders should keep several key points in mind: In a contango situation where the VIX term structure is upward sloping, VIX futures prices tend to decrease over time if the index value remains unchanged. Changes in the prices of VIX futures contracts typically lag behind rapid shifts in the index value. At the time of expiration, futures prices must align with the index prices. Directly replicating the index is often not feasible for most traders; thus, the futures prices should be considered on their own merits rather than strictly in relation to the index price.

Trading VIX futures might seem complex due to these unique aspects, but it's not necessarily more complicated than other futures markets; it's just different. Understanding these differences is crucial. A trader might find buying VIX futures profitable if they anticipate an increase in the index value, particularly near the expiration or if a significant rise is expected that could flip the term structure from contango to backwardation. Conversely, selling VIX futures could be profitable if a decrease in the index value is expected near expiration or a significant drop is anticipated that could reverse the term structure from backwardation to contango. However, traders should manage their expectations regarding how quickly these changes in the futures prices might occur.

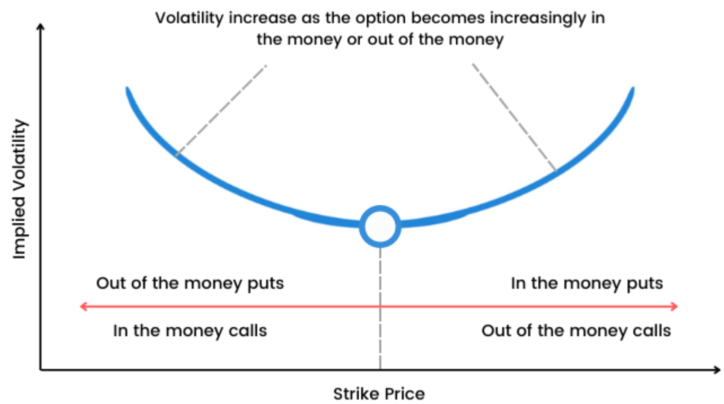
Volatility Smile

The volatility smile is a pattern in which implied volatility is graphed against strike prices for options with the same maturity. Typically, the graph shows higher volatilities for deep in-the-money and out-of-the-money options, creating a U-shaped curve known as the "smile." The volatility smile implies that the distribution of returns for the underlying asset is leptokurtic – indicating fat tails and greater kurtosis than that of a normal distribution.

The volatility smile's existence often indicates that the market is straying from the log-normal distribution of returns, a fundamental assumption of the Black-Scholes model. Market activities and sentiment, driven by events like mergers or economic turmoil, increase the allure of options with strike prices that are either much higher or lower than the current market price, thus deepening the smile. These patterns are further shaped by the collective psychology of the market participants, where fear can amplify demand for downside protection and greed can do the same for upside potential.

In the realm of financial markets, the volatility smile varies notably across different asset classes. Equity markets commonly exhibit a marked smile, particularly in times of turbulence, as investors price in significant tail risks. In currency markets, the smile's shape can fluctuate depending on the stability of the currency in question and the prevailing economic conditions, reflecting the uncertainties inherent in forex trading. Meanwhile, commodity markets demonstrate their own unique smile dynamics that are often influenced by sudden supply shocks or geopolitical tensions, which can drastically alter the demand-supply equation and, consequently, price volatilities.

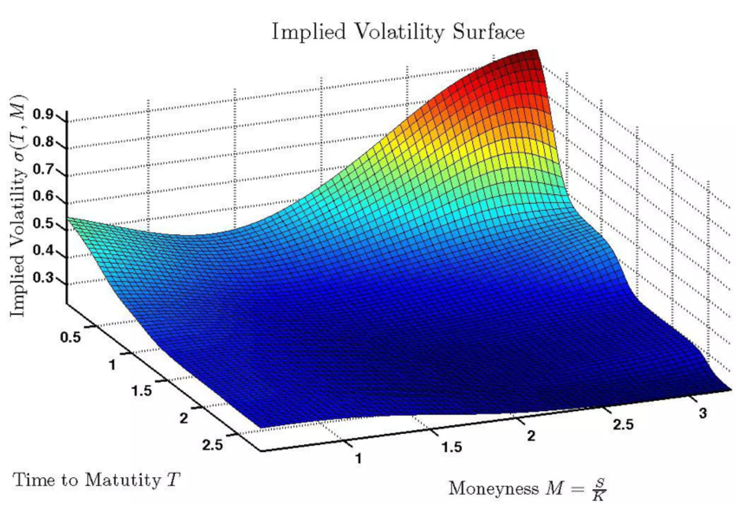
Figure 6. Implied Volatility vs. Strike Price [t]



Implied Volatility Surfaces

An implied volatility surface is a three-dimensional graph displaying the implied volatility of options across different strikes and maturities. It provides a visual representation of how implied volatility changes for different option contracts on the same underlying asset, considering both the strike price and time to expiration.

Figure 7. Implied Volatility Surface [r]



The Black-Scholes Model: The Cornerstone of Modern Options-pricing Theory

The Black-Scholes model, attributed to economists Fischer Black and Myron Scholes ([c], 637-654), revolutionized the field of financial economics by providing a systematic method for valuing European-style options. Recognized with a Nobel Prize, the model presumes that the underlying asset price follows a lognormal distribution and factors in the risk-free rate, strike price, and time to expiration to determine an option's price. Despite its groundbreaking insights, the model simplifies some real market behaviors by assuming a constant volatility and risk-free rate, which overlooks complex market dynamics such as the volatility smile. Nonetheless, its foundational principles continue to underpin contemporary financial theory and practice.

The Black-Scholes model operates under several critical assumptions. It posits that the stock price behaves according to a lognormal distribution, implying that prices can only take positive values and tend to have a compound growth rate. Market efficiency is another cornerstone of the model, where prices reflect all available information, and as such, arbitrage opportunities are non-existent. A key simplification in the model is the presumption that both the volatility of the underlying asset and the risk-free interest rate remain constant over the option's lifespan. Additionally, the model assumes that the underlying asset does not pay any dividends during the life of the option.

The model also assumes that the stock price follows a geometric Brownian motion with constant drift and volatility. This is mathematically represented as:

$$dS_T = \mu dS_t + \sigma S_t dW_t$$

Where S_t is the asset price at time t, μ is the expected return, σ is the volatility, and $dW_t \sim \mathcal{N}(0, dt)$, represents the increment of a Wiener process.

Modeling Stock Price Evolution with the Black-Scholes Model: A Risk-Neutral Perspective

The realized return between times t and T is expressed by $R(t, T)$, which is the natural logarithm of the price ratio $\left(\frac{S_T}{S_t}\right)$. Consequently, the future stock price (S_T) is equal to the initial price (S_t) multiplied by the exponential of $R(t, T)$. Under the assumption that stock price behaves according to a lognormal distribution, we get that the return behaves according to the normal distribution, i.e., $R(0, T) \sim \mathcal{N}(m, \nu^2)$, where $\nu^2 = \sigma^2 T$.

The risk-neutral expected value of S_T is denoted as $\mathbb{E}^*[S_T] = S_0 e^{m + \frac{\sigma^2 T}{2}}$, and since it assumes markets are efficient, under risk neutral probability we also get that $\mathbb{E}^*[S_T] = S_0 e^{rT}$. This leads us to denote the mean for the returns as $(r - \frac{\sigma^2}{2})T$. Thus, we get that under the risk neutral probability $R(0, T) \sim \mathcal{N}((r - \frac{\sigma^2}{2})T, \ \sigma^2 T)$. Returns can be expressed as a linear combination of the of the standard normal, yielding:

$$S_T = S_0 e^{\left(r - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}Z}$$

This formulation connects the lognormal behavior of stock prices with the normal distribution of returns, underlying the Black-Scholes model's use of continuous compounding and risk-neutral valuation in option-pricing.

Heston Model: A Stochastic Volatility Framework for Options-pricing

The Heston model stands out due to its mathematical rigor and practical applicability in financial markets. Proposed by Steven L. Heston in 1993 ([g], 327,343), the model provides a closed-form solution for pricing options by incorporating stochastic volatility. This model is particularly noted for its ability to describe the volatility smile.

The Heston model assumes that the variance of the asset returns follows a stochastic process, which is distinct from the Black-Scholes model that assumes constant volatility. The key feature of the Heston model is the specification of two stochastic differential equations (SDEs):

$$dS_T = \mu dS_t + \sqrt{\nu_t} S_t dW_t^{\,S}$$

Where S_t is the asset price at time t, μ is the expected return, ν_t is the instantaneous variance, and $dW_t^{\,S} \sim \mathcal{N}(0, dt)$, represents the increment of a Wiener process related to the asset price.

$$d\nu_t = \kappa(\theta - \nu_t)d_t + \sigma\sqrt{\nu_t} S_t dW_t^{\,\nu}$$

Where κ represents the mean reversion rate, θ the long-term variance, σ is the volatility of the volatility, ν_t is the instantaneous variance, and $dW_t^{\,\nu} \sim \mathcal{N}(0, dt)$, represents the increment of a Wiener process related to the variance.

The correlation between the two Wiener processes, dW_t^{S} and dW_t^{v}, is denoted by ρ, introducing the leverage effect, which captures the negative correlation between asset returns and changes in volatility.

The Heston model is advantageous because it provides a flexible framework to capture the dynamic behavior of volatility, which is more aligned with observed market phenomena than static models. The model's ability to generate realistic volatility smiles and surfaces makes it invaluable for pricing exotic options and structured products where standard models might fail. In practical terms, the Heston model has been widely applied in risk management, allowing traders to hedge more effectively against the risks inherent in volatile markets. Furthermore, it serves as a foundational tool in the calibration of market data, helping to extract market expectations of future volatility from option prices.

Calibrating the Heston model involves fitting the model parameters (κ, θ, σ, ρ) to market data, typically observed option prices. This process, however, can be computationally intensive and sensitive to the initial guesses for the parameters, highlighting a significant challenge in its practical implementation. Moreover, while the Heston model is powerful, it does assume that jumps in asset prices do not occur, which can be a limitation during market turmoil or for assets that exhibit jump behaviors.

Implied Volatility Indices

The inaugural volatility index was the initial CBOE Volatility Index (VIX) unveiled by the Chicago Board Options Exchange (CBOE) in 1993, built upon the foundational research of Whaley ([a]). The VIX Index swiftly emerged as the standard for measuring volatility in the U.S. stock market, prompting other exchanges to emulate the successful model of the CBOE and introduce their own implied volatility indices. In 2021, there were 68 such indices spanning various asset classes beyond equities ([n], pp. 303). Volatility indices play a crucial role in enhancing transparency in derivatives markets, as volatility serves as a fundamental component of derivative pricing.

There are typically two approaches to computing a volatility index: one utilizes Black-Scholes at-the-money implied volatilities, while the other employs a model-free calculation based on variance swaps. Despite their differences, the estimates from these methods are closely correlated, as they both offer market-derived forecasts of the future realized volatility of the underlying asset.

It is worth noting that while standard Black & Scholes implied volatility predicts future volatility around the current level (at-the-money strike price), variance swaps provide an estimate of variance that is independent of the market level. The variance calculation needs perfectly liquid options for every exercise price for accuracy. Consequently, less liquid out-of-the-money options are excluded to ensure reliability.

The computation of the original VIX (VXO) relies on the Black-Scholes implied volatilities of at-the-money S&P100 options, as outlined by Wiley in 1993. On September 22, 2003, CBOE introduced a new VIX, based on future variance fair value by Demeterfi et al. [p]. Calculated directly from market observables, it now uses S&P500 options instead of S&P100. Additionally, despite being variance-based, VIX is still quoted as the square root of variance for comparability with option implied volatility.

Table 2. Overview of relevant Implied Volatility Indices

Name (Symbol)	Underlying Asset	Provider	Methodology	Time Range	Available Data since
Stock Indexes Volatility Indexes					
CBOE Volatility Index (VIX)	S&P 500 Index	CBOE	Model-free	30-day	Jan-90
CBOE S&P 100 Volatility Index (VXO)	S&P 100 Index	CBOE	Black Scholes	30-day	Jan-86
CBOE Short-Term Volatility Index (VXST)	S&P 500 Index	CBOE	Model-free	9-day	Jan-11
CBOE One-Year Volatility Index (VIX1Y)	S&P 500 Index	CBOE	Model-free	1 year	Jan-07
CBOE NASDAQ Volatility Index (VXN)	NASDAQ-100 Index	CBOE	Model-free	30-day	Jan-95
Commodity Volatility Indexes					
CBOE/COMEX Gold Volatility Index (GVX)	COMEX Gold futures (GC)	CBOE	Model-free	30-day	Sep-10
CBOE/NYMEX WTI Volatility Index (OIV)	WTI Crude Oil Futures (CL)	CBOE	Model-free	30-day	Sep-10
Currency Volatility Indexes					
CBOE/CME FX Euro Volatility Index (EUVIX)	USD/Euro futures (6E)	CBOE	Model-free	30-day	Jan-07
CBOE/CME FX British Pound Volatility Index (BPVIX)	USD/GBP futures (6B)	CBOE	Model-free	30-day	Jan-07
Interest Rates Volatility Indexes					
CBOE/CBOT 10-year U.S. Treasury Note Volatility Index (TYVIX)	10-year Treasury Notes futures (TY)	CBOE	Model-free	30-day	Jan-03
CBOE Interest Rate Swap Volatility Index (SRVX)	10-year USD interest rate swaps	CBOE	Model-free	one-year	Jun-12
Stocks and Equity ETFs Volatility Indexes					
CBOE Emerging Markets ETF Volatility Index (VXEEM)	iShares MSCI Emerging Markets Index (EEM)	CBOE	Model-free	30-day	Mar-11
CBOE Energy Sector ETF Volatility Index (VXXLE)	Energy Select Sector SPDR (XLE)	CBOE	Model-free	30-day	Mar-11
Volatility of Volatility Indexes					
CBOE VIX of VIX Index (VVIX)	VIX futures	CBOE	Model-free	30-day	Jun-06

Implied Volatility Dynamics: Insights into Market Behavior

A fundamental principle of finance is the positive expected return-risk trade-off. However, the contemporaneous empirical return-volatility relation is negative. Contrary to a fundamental principle of finance, which suggests a positive expected return-risk trade-off, empirical evidence reveals a negative contemporaneous relationship between returns and volatility. Additionally, extensive empirical literature, such as Campbell's and Hentschel's "No news is good news" ([k], pp. 281–318), documents this relationship as asymmetric, meaning that a negative return (positive volatility) shock has a more significant impact relative to a positive return (negative volatility) shock.

Three main theories attempt to explain the asymmetric negative return-volatility relation. The leverage hypothesis, proposed by Black ([j], 177–181), suggests that declining stock prices lead to a higher debt-to-equity ratio, making the firm riskier and causing equity volatility to increase. The volatility feedback hypothesis, developed by Poterba and Summers ([l], pp. 27–59), and Campbell and Hentschel ([k], pp.281–318), proposes that positive shocks to variance imply higher future expected variance and, consequently, higher expected future returns, but only if accompanied by a drop in stock prices today. This leads to negative returns due to a volatility feedback effect.

A newer approach utilizes behavioral reasons instead of fundamental factors. Low ([m], pp. 527–546) first provides a behavioral explanation based on loss aversion, suggesting that the risk-return relation is both asymmetric and non-linear, resembling a downward-sloping S-curve. This is related to the loss aversion principle. Subsequent studies have explored the negative asymmetric impact of stock market returns on volatility using behavioral explanations, associating substantial downward movements in asset prices with fear of risk and consecutive price run-ups with exuberance.

Practical Applications of Implied Volatility

Option Pricing under Black-Scholes Model

Under the lognormal distribution assumption in the Black-Scholes model, a 110-strike call option is theoretically always more valuable than a 90-strike put when the forward price of the underlying asset is 100. This stems from the properties of the lognormal distribution, which inherently assumes prices cannot fall below zero, skewing probabilities toward the right. In this model, where asset prices are modeled to follow a geometric Brownian motion, the distribution of future prices at a given expiration is lognormal. This skews the probability density of asset prices towards higher values, increasing the likelihood of the call option finishing in-the-money due to its potential for unlimited upside beyond the strike price. Conversely, the put option, benefiting from declines in the asset price, is less likely to end in-the-money due to the tapered probability of extremely low prices in the lognormal distribution. This theoretical assessment holds true under perfect market conditions that fully align with the Black-Scholes model's assumptions, which is seldom the case in real-world scenarios where factors such as volatility smile and market inefficiencies influence pricing.

Important principles to note:

1. A change in volatility will impact an at-the-money option more significantly than an equivalent in-the-money or out-of-the-money option in total points.

2. In terms of percentage, a change in volatility will have a greater impact on an out-of-the-money option compared to an equivalent in-the-money or at-the-money option.

3. Volatility changes will impact long-term options more than equivalent short-term ones.

Under the Black-Scholes model, the sensitivity of option prices to changes in volatility is captured by Vega, a Greek letter representing the rate of change of an option's price per one percent change in implied volatility. This sensitivity varies based on the option's moneyness and time to expiration. As noted, an at-the-money (ATM) option experiences more significant changes in dollar terms compared to in-the-money (ITM) or out-of-the-money (OTM) options due to its higher Vega at this moneyness level. However, the percentage impact of volatility is greater for OTM options because their premiums are lower; any increase in volatility disproportionately enhances their relative value, reflecting higher potential for profit in dramatic market shifts.

Additionally, longer-term options are more sensitive to changes in volatility because they have more time until expiration, allowing more opportunity for the underlying asset to experience significant price shifts. These nuanced effects of volatility underscore its critical role in option-pricing and risk management strategies, emphasizing why market practitioners must diligently monitor volatility indicators to align their trading and hedging strategies effectively with prevailing market conditions.

Value at Risk (VaR)

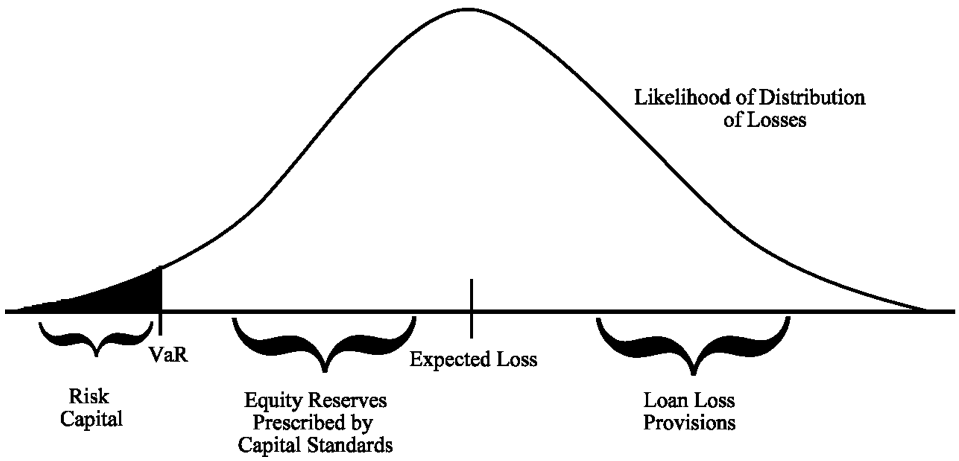
Value-at-Risk (VaR) is a widely used risk management tool that quantifies the potential loss in value of a risky asset or portfolio over a defined period for a given confidence interval. Essentially, it provides a statistical measure of the maximum potential loss that an investment might suffer with a certain degree of confidence over a set time horizon. For example, a one-day 95% VaR of $1 million means that there is only a 5% chance that the portfolio will lose more than $1 million in a single day. Furthermore, an implied volatility index improves risk management procedures; for instance, it can serve as an input parameter for Value-at-Risk (VaR) computations.

Integrating implied volatility into the VaR model, enhances the accuracy of risk estimates. By incorporating this dynamic measure, the VaR calculation becomes more responsive to current market conditions, offering a more precise assessment of potential financial exposure.

Given a probability level $\alpha \in [0,1]$,and a reference financial instrument r, the value-at-risk (VaR) at level α for the final net worth X which follows distribution $\mathbb{P}$, is defined as the negative value of the α-quantile ${q_\alpha}^+$ of the ratio X/r ([s], pp.13), that is:

$$VaR_\alpha(X) = -\inf\{x \mid \mathbb{P}[X \leq x \bullet r] > \alpha\} = {F_Y}^{-1}(1-\alpha)$$

Figure 8. Value at Risk (VaR) [u]



Implication on Trading Strategies

The Volatility Index (VIX) is a crucial metric for traders and investors, providing a real-time market estimate of expected volatility derived from S&P 500 index options. Known colloquially as the "fear gauge," the VIX reflects investor sentiment and market expectations for volatility over the coming 30 days. When the VIX is high, it signals increased market volatility and uncertainty, which often corresponds with market downturns or financial instability. Conversely, a low VIX suggests confidence and stability in the market. Traders use these signals to adjust their strategies; for instance, a high VIX might lead to strategies that protect against downside risk, such as buying protective put options or moving into safer, less volatile assets.

In terms of its impact on implied volatility, the VIX directly influences options pricing and trading. Implied volatility represents the market's forecast of a likely movement in a security's price and is a critical component in the pricing models of options. Since the VIX measures the market’s expectation of volatility, it naturally affects the implied volatility used in these models. When the VIX increases, implied volatility tends to rise, leading to more expensive options premiums. This relationship allows traders to speculate on future volatility or hedge against potential price movements in the underlying assets. Thus, understanding the implications of VIX readings can empower investors to make more informed decisions, adapting their trading strategies to either capitalize on or protect against expected changes in market volatility.

References

[a] Whaley, R. E. (1993). Derivatives on Market Volatility. Journal of Derivatives, 1(1).

[b] Derman, E., & Kani, I. (1994). Riding on a Smile. Risk, 7(2).

[c] Black, F., & Scholes, M. (1973). "The Pricing of Options and Corporate Liabilities." Journal of Political Economy, 81(3), 637-654.

[d] Carr, P., & Madan, D. B. (1999). "Option Valuation Using the Fast Fourier Transform." Journal of Computational Finance, 2(4).

[e] Dupire, B. (1993). "Modeling Volatility Smile." Risk Magazine, 7(1), 71-77.

[f] Gatheral, J. (1997). Implied Volatility Surfaces: Theory and Application. Quantitative Finance, 1(1).

[g] Heston, S. L. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. Review of Financial Studies, 6(2).

[h] Natenberg, S. (1994). Option Volatility and Pricing: Advanced Trading Strategies and Techniques. McGraw-Hill.

[i] Bachelier, Louis. Louis Bachelier's Theory of Speculation. Translated by Mark Davis and Alison Etheridge, Princeton University Press, 2006.

[j] Black, F. (1976). Studies of stock market volatility changes. Proceedings of the American Statistical Association, Business and Economic Statistics Section.

[k] Campbell, J. Y., & Hentschel, L. (1992). No news is good news: An asymmetric model of changing volatility in stock returns. Journal of Financial Economics, 31(3).

[l] Poterba, J. M., & Summers, L. H. (1988). Mean reversion in stock prices: Evidence and implications. Journal of Financial Economics, 22(1).

[m] Low, C. (2004). The fear and exuberance from implied volatility of S&P 100 index options. Journal of Business, 77(3).

[n] Fassas, Athanasios P., and Costas Siriopoulos. “Implied Volatility Indices – A Review.” The Quarterly review of economics and finance 79 (2021): 303–329. Web.

[o] Quantitative Market Intelligence. (2021) “Multi-Factor Identification of Management and Market Signals (MIM2)”

[p] Demeterfi, K., Derman, E., Kamal, M., & Zou, J. (1999). A guide to volatility and variance swaps. The Journal of Derivatives, 6(4), 9–32.

[q] Sinclair, E. (Volatility Trading). Wiley Trading, John Wiley & Sons, 2008.

[r] Investopedia. (n.d.). Volatility Surface Explained. Retrieved April 28, 2024, from https://www.investopedia.com/articles/stock-analysis/081916/volatility-surface-explained.asp

[s] Artzner, P., Delbaen, F., Eber, J.-M., & Heath, D. (1998). Coherent measures of risk. Université Louis Pasteur, Strasbourg; Eidgenössische Technische Hochschule, Zürich; Société Générale, Paris; Carnegie Mellon University, Pittsburgh, Pennsylvania. July 22, 1998.

[t] Elearnmarkets. 'Volatility Smile.' Elearnmarkets.com. Accessed May 3, 2024. https://www.elearnmarkets.com/school/units/option-greeks-1/volatility-smile.

[u] Gustafson, C. (2004). Limitations of Value-at-Risk (VaR) for Budget Analysis. Agribusiness & Applied Economics Report.