New Proof of the Divergence of the Harmonic Series

Samy Lahlou

January 5, 2025

- talk about the doccument with the different proofs of the divergence (citation at the end)
- state the Alternating Series test (prove it ?)
- prove that the Alternating Harmonic Series converges
- lemma for grouping terms
- proof by contradiction of the divergence of the harmonic series
- ask to be contacted if a similar proof already exists

Unrigorous proof:
Suppose that he Harmonic Series Converges, define

$$H = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots$$

Since we know that the series

$$L = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$$

converges to a nonzero real number, then

$$\begin{aligned} H - L &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots \\ &\quad - \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots\right) \\ &= 0 + 2 \cdot \frac{1}{2} + 0 + 2 \cdot \frac{1}{4} + 0 + 2 \cdot \frac{1}{6} + \dots \\ &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots \\ &= H \end{aligned}$$

which implies

$$L = 0$$

A contradiction, so the Harmonic Series Diverges.