MATH 358 Notes : Honours Advanced Calculus

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These notes are based on lectures given by Professor John Toth at McGill University in Winter 2025. The subject of these lectures is **TODO**.
As a disclaimer, it is more than possible that I made some mistakes. Feel free to correct me or ask me anything about the content of this document at the following address : samy.lahloukamal@mcgill.ca

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1 Integration over Surfaces in $\mathbb{R}^3$

Definition (Surface Parametrization)**.** *Let $D \subset \mathbb{R}^2$ be a piecewise C^1 bounded domain and let $S = \phi(D)$ be the corresponding surface where $\phi : D \to S \subset \mathbb{R}^3$ is a C^1 mapping. We call D the parametrization domain.*

Definition (Regular Surfaces)**.** *Given a parametrized surface S with parameter ϕ and parametrization domain D, we say that S is regular if $\phi_u(u_0, v_0) \times \phi_v(u_0, v_0) \neq 0$ for all $(u_0, v_0) \in D$.*

Definition (Tangent Plane)**.** *Given a regular surface S with parameter ϕ and parametrization domain D, then for all $d \in D$, we can define the vector $n = \phi_u(d) \times \phi_v(d)$ which is normal to the surface at $\phi(d)$. From this, we define*

$$T_{\phi(d)}S = \{X \in \mathbb{R}^3 : (X - \phi(d)) \cdot n = 0\}$$

as the tangent plane to S at $\phi(d)$.

Definition (Area of a Surface)**.** *Given a regular surface S with parameter ϕ and parametrization domain D, we can define the area of S as*

$$\mathit{area}(S) = \iint_D \|\phi_u \times \phi_v\| dA.$$