Algebraic Geometry : Homework 3

Samy Lahlou

Exercise 4: Describe all projective varieties in $\mathbb{P}^1(k)$ and $\mathbb{P}^2(k)$.

Solution : We know that the varieties in $\mathbb{A}^1(k)$ are the singletons, the empty space or the whole space. It follows that the projective varieties in $\mathbb{P}^1(k)$ are the empty set, the points (including the point at infinity H_∞) and $\mathbb{P}^1(k)$.

In $\mathbb{A}^2(k)$, we know that the varieties are the emoty set, the singletons, the irreducible plane curves $V(F)$ where F is irreducible, and the space $\mathbb{A}^2(k)$. It follows that the projective varieties in $\mathbb{P}^2(k)$ are the empty set, points (including point on H_∞), $V(f)$ where f is an homogeneous irreducible polynomial, H_∞ and $\mathbb{P}^2(k)$ using Exercise 3.