Solutions to Fourier Analysis - Stein & Shakarchi

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Preface

The goal of this document is to share my personal solutions to the exercises in the book Fourier Analysis by Stein & Shakarchi during my reading.
As a disclaimer, the solutions are not unique and there will probably be better or more optimized solutions than mine. Feel free to correct me or ask me anything about the content of this document at the following address : samy.lahloukamal@mcgill.ca

Contents

Chapter 1

The Genesis of Fourier Analysis

Chapter 2

Basic Properties of Fourier Series

Chapter 3

Convergence of Fourier Series

Chapter 4

Some Applications of Fourier Series

Chapter 5

The Fourier Transform on $\mathbb{R}$

Chapter 6

The Fourier Transform on $\mathbb{R}^d$

Chapter 7

Finite Fourier Analysis

Exercise 1
Let f be a function on the circle. For each $N \geq 1$ the discrete Fourier coefficients of f are defined by

$$a_N(n) = \frac{1}{N} \sum_{k=1}^{N} f(e^{2\pi ik/N}) e^{-2\pi ikn/N}, \qquad \text{for } n \in \mathbb{Z}.$$

We also let

$$a(n) = \int_0^1 f(e^{2\pi ix}) e^{2\pi inx} dx$$

denote the ordinary Fourier coefficients of f.

(a) Show that $a_N(n) = a_N(n + N)$.

(b) Prove that if f is continuous, then $a_N(n) \to a(n)$ as $N \to \infty$.

Solution

(a) **TODO**

(b) **TODO**

Chapter 8

Dirichlet's Theorem