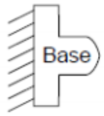
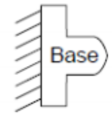
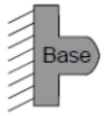
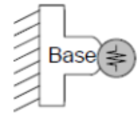
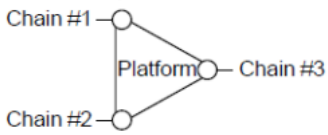
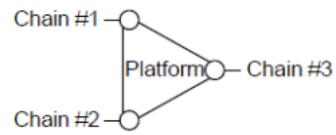
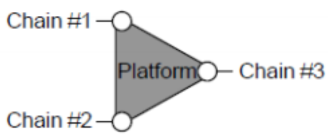
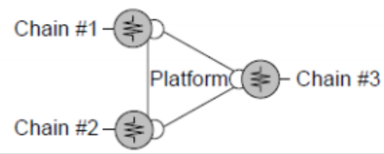
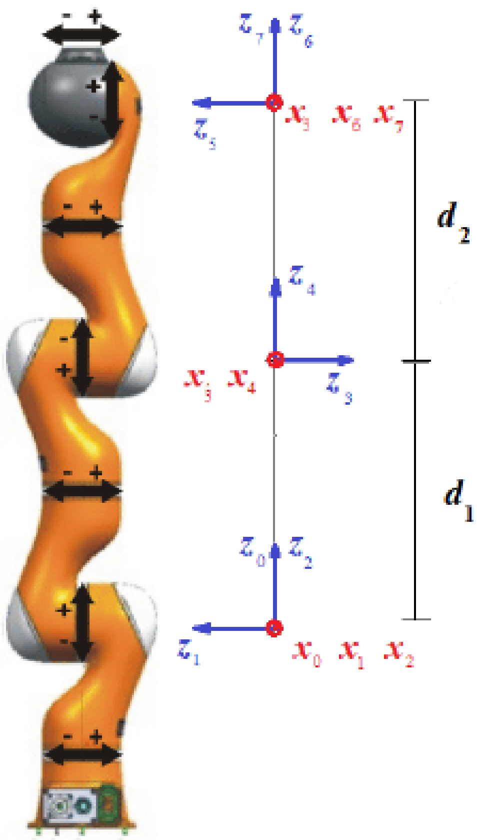
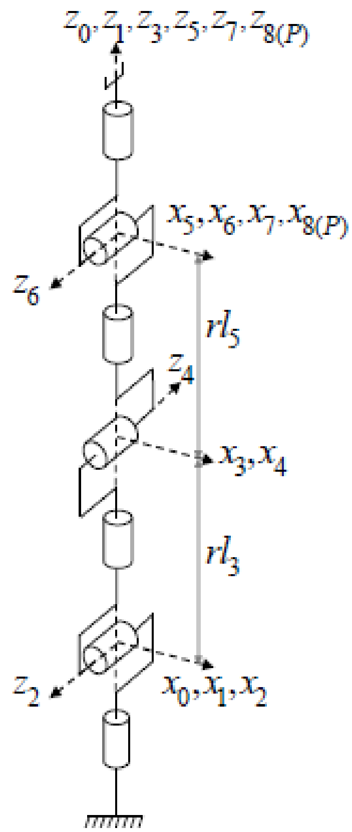
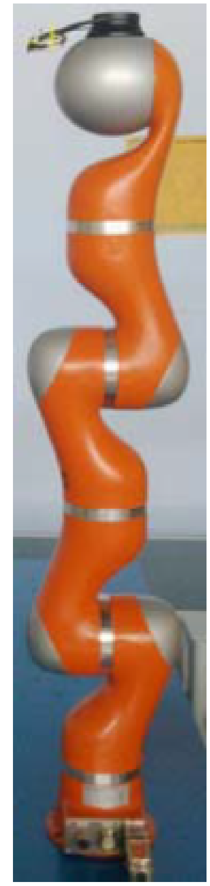
VJM model of KUKA_iiwa robot manipulator:

Manipulator components		VJM-model elements	
Component	Graphical presentation	Components	Graphical presentation
Actuated joint		Actuated joint + 6 d.o.f. virtual spring	
Passive joint (non-actuated)		Passive joint	
Rigid link		Rigid link	
Elastic link		Rigid link + 6 d.o.f. virtual spring	
Rigid base		Rigid base	
Elastic base		Rigid base + 6 d.o.f. virtual spring	
Rigid platform		Rigid platform	
Elastic platform		Rigid platform + set of 6 d.o.f. virtual spring (for each chain)	







$$
\mathbf{K} = \begin{bmatrix}
\frac{E\cdot S}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12\cdot E\cdot Iz}{L^3} & 0 & 0 & 0 & \frac{-6\cdot E\cdot Iz}{L^2} \\
0 & 0 & \frac{12\cdot E\cdot Iy}{L^3} & 0 & \frac{6\cdot E\cdot Iy}{L^2} & 0 \\
0 & 0 & 0 & \frac{G\cdot J}{L} & 0 & 0 \\
0 & 0 & \frac{6\cdot E\cdot Iy}{L^2} & 0 & \frac{4\cdot E\cdot Iy}{L} & 0 \\
0 & \frac{-6\cdot E\cdot Iz}{L^2} & 0 & 0 & 0 & \frac{4\cdot E\cdot Iz}{L}
\end{bmatrix}
$$

$$
\begin{bmatrix}
\mathbf{0} & \mathbf{J}_\theta & \mathbf{J}_q \\
\mathbf{J}_\theta^{\,T} & -\mathbf{K}_\theta & \mathbf{0} \\
\mathbf{J}_q^{\,T} & \mathbf{0} & \mathbf{0}
\end{bmatrix}
\cdot
\begin{bmatrix}
\mathbf{F} \\ \boldsymbol{\theta} \\ \Delta\mathbf{q}
\end{bmatrix}
=
\begin{bmatrix}
\Delta\mathbf{t} \\ \mathbf{0} \\ \mathbf{0}
\end{bmatrix}
$$

$$\boxed{\mathbf{K}_C = \mathbf{K}_C^0 - \mathbf{K}_C^0 \cdot \mathbf{J}_q \cdot \mathbf{K}_{Cq}}$$

where $\mathbf{K}_C^0 = \left(\mathbf{J}_\theta \cdot \mathbf{K}_\theta^{-1} \cdot \mathbf{J}_\theta^T\right)^{-1}$ *is classical Cartesian stiffness matrix*

The extended model of the KUKA manipulator:

$$\theta_7 \cdot R_{x,q_2} \cdot R_{x,\theta_8} \cdot T_{z,d_2} \cdot H_{3D,\theta_{14}} \cdot R_{z,q_3} \cdot R_{z,\theta_{15}} \cdot T_{z,d_3} \cdot H_{3D,\theta_{21}} \cdot R_{x,q_4} \cdot R_{x,\theta_{22}} \cdot T_{z,d_4} \cdot H_{3D,\theta_{28}} \cdot R_{z,q_5} \cdot R_{z,\theta_{29}} \cdot T_{z,d_5} \cdot H_{3D,\theta_{35}} \cdot R_{x,q_6} \cdot R_{x,\theta_{36}} \cdot T_{z,d_6}$$

with: $H_{3D} = T_x \cdot T_y \cdot T_z \cdot R_x \cdot R_y \cdot R_z$

```
clear all, clc
Tbase = eye(4);
Ttool = eye(4);


%% links stiffness modeling
% Actuator stiff 1
k0 = 1e6;

%LINK1
%material and shape parameters
E = 70 *10e9; %Young's modulus
G = 25.5*10e9; %shear modulus
robot.d(1) = 10*10e-3;
robot.L(1) = 0.21;

%for cylinder
S = pi*robot.d(1)^2/4;
Iy = pi*robot.d(1)^4/64;
Iz = pi*robot.d(1)^4/64;
J = Iy + Iz;


k1 = [E*S/robot.L(1) 0                  0                  0          0                  0;
0           12*E*Iz/robot.L(1)^3  0                  0          0                  6*E*Iy/robot
0           0                  12*E*Iy/robot.L(1)^3 0          -6*E*Iy/robot.L(1)^2 0;
0           0                  0                  G*J/robot.L(1) 0                  0;
0           0                  -6*E*Iy/robot.L(1)^2 0          4*E*Iy/robot.L(1)     0;
0           6*E*Iy/robot.L(1)^2   0                  0          0                  4*E*Iz/robot


% Actuator stiff 2
k2 = 1e6;

%LINK2
%material and shape parameters
robot.d(2) = 10*10e-3;
robot.L(2) = 0.21;
S = pi*robot.d(2)^2/4;

k3 = [E*S/robot.L(2) 0                  0                  0          0                  0;
0           12*E*Iz/robot.L(2)^3  0                  0          0                  6*E*Iy/robot
0           0                  12*E*Iy/robot.L(2)^3 0          -6*E*Iy/robot.L(2)^2 0;
0           0                  0                  G*J/robot.L(2) 0                  0;
0           0                  -6*E*Iy/robot.L(2)^2 0          4*E*Iy/robot.L(2)     0;
0           6*E*Iy/robot.L(2)^2   0                  0          0                  4*E*Iz/robot

% Actuator stiff 3
k4 = 1e6;

%LINK3
%material and shape parameters
robot.d(3) = 10*10e-3;
robot.L(3) = 0.21;
S = pi*robot.d(3)^2/4;
```

```
k5 = [E*S/robot.L(3) 0                  0                  0           0                  0;
0           12*E*Iz/robot.L(3)^3  0                  0           0                  6*E*Iy/robot
0           0                  12*E*Iy/robot.L(3)^3 0           -6*E*Iy/robot.L(3)^2 0;
0           0                  0                  G*J/robot.L(3) 0                  0;
0           0                  -6*E*Iy/robot.L(3)^2 0           4*E*Iy/robot.L(3)      0;
0           6*E*Iy/robot.L(3)^2   0                  0           0                  4*E*Iz/robot

% Actuator stiff 4
k6 = 1e6;

%LINK4
%material and shape parameters
robot.d(4) = 10*10e-3;
robot.L(4) = 0.2;
S = pi*robot.d(4)^2/4;

k7 = [E*S/robot.L(4) 0                  0                  0           0                  0;
0           12*E*Iz/robot.L(4)^3  0                  0           0                  6*E*Iy/robot
0           0                  12*E*Iy/robot.L(4)^3 0           -6*E*Iy/robot.L(4)^2 0;
0           0                  0                  G*J/robot.L(4) 0                  0;
0           0                  -6*E*Iy/robot.L(4)^2 0           4*E*Iy/robot.L(4)      0;
0           6*E*Iy/robot.L(4)^2   0                  0           0                  4*E*Iz/robot

% Actuator stiff 5
k8 = 1e6;

%LINK5
%material and shape parameters
robot.d(5) = 10*10e-3;
robot.L(5) = 0.2;
S = pi*robot.d(5)^2/4;

k9 = [E*S/robot.L(5) 0                  0                  0           0                  0;
0           12*E*Iz/robot.L(5)^3  0                  0           0                  6*E*Iy/robot
0           0                  12*E*Iy/robot.L(5)^3 0           -6*E*Iy/robot.L(5)^2 0;
0           0                  0                  G*J/robot.L(5) 0                  0;
0           0                  -6*E*Iy/robot.L(5)^2 0           4*E*Iy/robot.L(5)      0;
0           6*E*Iy/robot.L(5)^2   0                  0           0                  4*E*Iz/robot

% Actuator stiff 6
k10 = 1e6;

%LINK6
%material and shape parameters
robot.d(6) = 10*10e-3;
robot.L(6) = 0.126;
S = pi*robot.d(6)^2/4;

k11 = [E*S/robot.L(6) 0                  0                  0           0                  0;
0           12*E*Iz/robot.L(6)^3  0                  0           0                  6*E*Iy/robot
0           0                  12*E*Iy/robot.L(6)^3 0           -6*E*Iy/robot.L(6)^2 0;
0           0                  0                  G*J/robot.L(6) 0                  0;
0           0                  -6*E*Iy/robot.L(6)^2 0           4*E*Iy/robot.L(6)      0;
0           6*E*Iy/robot.L(6)^2   0                  0           0                  4*E*Iz/robot

% Actuator stiff 7
k12 = 1e6;

%LINK7 (tool)
%material and shape parameters
```

```
robot.d(7) = 10*10e-3;
robot.L(7) = 0.05;
S = pi*robot.d(7)^2/4;

k13 = [E*S/robot.L(7) 0                  0                 0          0                 0;
0           12*E*Iz/robot.L(7)^3  0                 0           0                 6*E*Iy/robot
0           0                  12*E*Iy/robot.L(7)^3 0           -6*E*Iy/robot.L(7)^2 0;
0           0                  0                 G*J/robot.L(7) 0                 0;
0           0                  -6*E*Iy/robot.L(7)^2 0           4*E*Iy/robot.L(7)      0;
0           6*E*Iy/robot.L(7)^2   0                 0           0                 4*E*Iz/robot

% K theta matrixshow
Kt = [k0 zeros(1,42) zeros(1,6)
    zeros(6,1) k1 zeros(6,36) zeros(6,6)
    zeros(1,1) zeros(1,6) k2 zeros(1,35) zeros(1,6)
    zeros(6,1) zeros(6,6) zeros(6,1) k3 zeros(6,29) zeros(6,6)
    zeros(1,1) zeros(1,6) zeros(1,1) zeros(1,6) k4 zeros(1,28) zeros(1,6)
    zeros(6,1) zeros(6,6) zeros(6,1) zeros(6,6) zeros(6,1) k5 zeros(6,22) zeros(6,6)
    zeros(1,1) zeros(1,6) zeros(1,1) zeros(1,6) zeros(1,1) zeros(1,6) k6 zeros(1,21) zeros(1,6
    zeros(6,1) zeros(6,6) zeros(6,1) zeros(6,6) zeros(6,1) zeros(6,6) zeros(6,1) k7 zeros(6,15
    zeros(1,1) zeros(1,6) zeros(1,1) zeros(1,6) zeros(1,1) zeros(1,6) zeros(1,1) zeros(1,6)  k
    zeros(6,1) zeros(6,6) zeros(6,1) zeros(6,6) zeros(6,1) zeros(6,6) zeros(6,1) zeros(6,6) ze
    zeros(1,1) zeros(1,6) zeros(1,1) zeros(1,6) zeros(1,1) zeros(1,6) zeros(1,1) zeros(1,6) ze
    zeros(6,1) zeros(6,6) zeros(6,1) zeros(6,6) zeros(6,1) zeros(6,6) zeros(6,1) zeros(6,6) ze
    zeros(1,1) zeros(1,6) zeros(1,1) zeros(1,6) zeros(1,1) zeros(1,6) zeros(1,1) zeros(1,6) ze
    zeros(6,1) zeros(6,6) zeros(6,1) zeros(6,6) zeros(6,1) zeros(6,6) zeros(6,1) zeros(6,6) ze

Kt
```

Kt = *49x49 double*

```
   1.0e+10 * ···
    0.0001         0         0         0         0         0         0         0
         0    2.6180         0         0         0         0         0         0
         0         0    0.4452         0         0         0    0.0467         0
         0         0         0    0.4452         0   -0.0467         0         0
         0         0         0         0    0.0012         0         0         0
         0         0         0   -0.0467         0    0.0065         0         0
         0         0    0.0467         0         0         0    0.0065         0
         0         0         0         0         0         0         0    0.0001
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
     ⋮
```

```
%% lengths  of links used in Forward kinematics, the thetas deviations and the configuration a
d = [robot.L(1), robot.L(2), robot.L(3), robot.L(4), robot.L(5), robot.L(6), robot.L(7)];
t= zeros(1, 49);
q= [ 0 0 0 0 0 0 0]
```

q = *1x7 double*

```
     0     0     0     0     0     0     0
```

```
% Jacobian and final result for Kc
Jth = Jt(Tbase,Ttool,q,t,d);
Kc =inv(Jth*inv(Kt)*Jth')
```

Kc = *6x6 double*

```
   1.0e+08 *
    0.2702         0   -0.3107         0   -0.1074         0
         0    0.0253         0    0.0145         0    0.0002
   -0.3107         0    8.4001         0   -0.6457         0
         0    0.0145         0    0.0112         0    0.0001
   -0.1074         0   -0.6457         0    0.1447         0
         0    0.0002         0    0.0001         0    0.0023
```

```
F  =[1000; 1000; 0; 0; 0; 0];
% showing the deflection map
showDeflection(F, 'VJM')
```